

The Theory of GRB Afterglows

Max Planck Institute for Extraterrestrial Physics

WS03/04 Student Seminar 16.1.04

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Overview

- The Basic Picture
- The Dynamics of Relativistic Shocks
- The Emission Model for a Blast Wave
- Spectral Evolution and Light Curves
- Jet Structure
- Fe lines in GRB Afterglows

The Basic Picture

- What can we learn?
 - Where does the energy come from?
=> shock dynamics
 - Where do the fast e^- come from?
=> Fermi Process
 - Where do the Photons come from?
=> synchrotron emission
 - What do we see? Where did we go wrong?
=> Application of Basic Model

The Standard Blastwave Afterglow Model

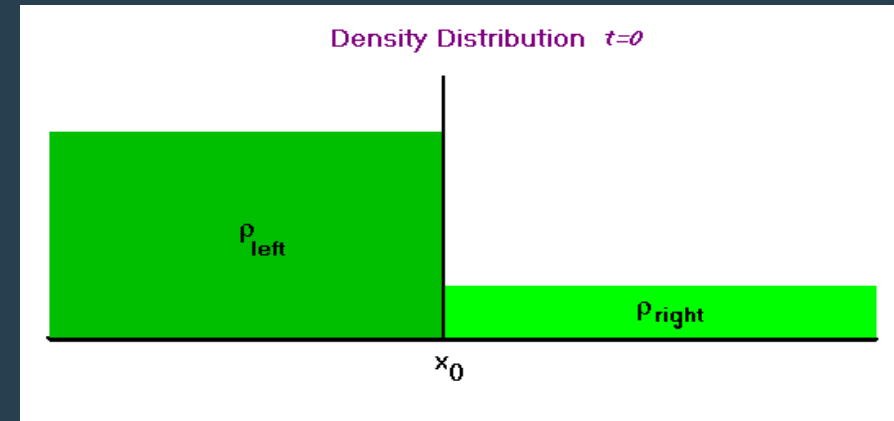
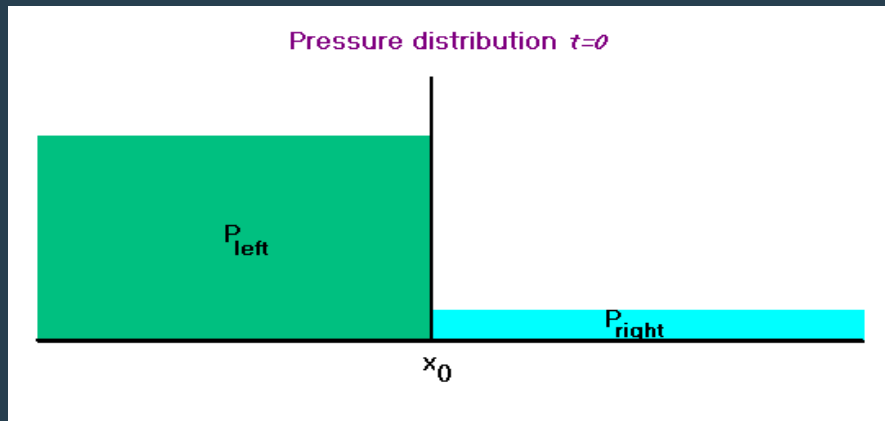
Assumptions:

- Spherical outflow
- Homogeneous surrounding medium
- Highly relativistic
- Adiabatic expansion
- Impulsive energy input
- Only forward shock

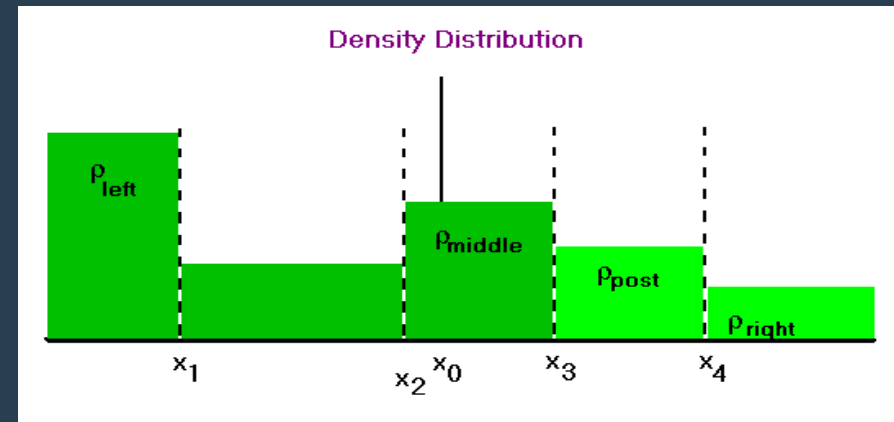
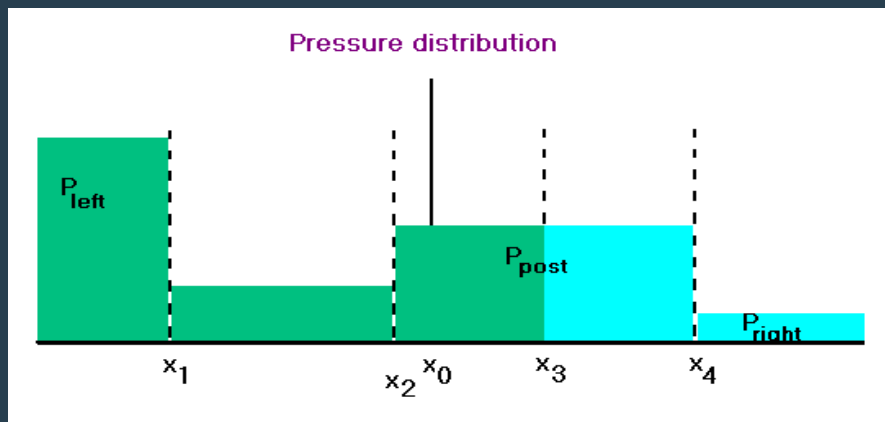
Dynamics of Shock Waves

The Riemann (Sod) Shock Tube

Pre-shock



Post-shock



Dynamics of Relativistic Shock Waves

Same Principle: The Blandford McKee Solution

Continuity of

- Energy

$$w \gamma^2 \beta$$

- Momentum

$$w \gamma^2 \beta^2 + p$$

- Particle Flux

$$n \gamma \beta$$

Results in jump conditions

$$\frac{e_2}{n_2} = \gamma_2 \frac{w_1}{n_1}$$

$$\frac{n_2}{n_1} = \frac{g_2 \gamma_2 + 1}{\gamma_2 - 1}$$

$$\Gamma^2 = \frac{(\gamma_2 + 1)[g_2(\gamma_2 - 1) + 1]^2}{g_2(2 - g_2)(\gamma_2 - 1) + 2}$$

Dynamics of Relativistic Shock Waves

With some assumptions

- Ultrarelativistic shock
- Equation of state
- "Adiabatic" Evolution

$$\Gamma \gg 1$$

$$p = \frac{1}{3} e$$

And similarity Variables

$$\chi = [1 + 2(m+1)\Gamma^2](1 - r/t)$$

$$\Gamma^2 \propto t^{-m}, m > -1$$

We get self similar jump conditions

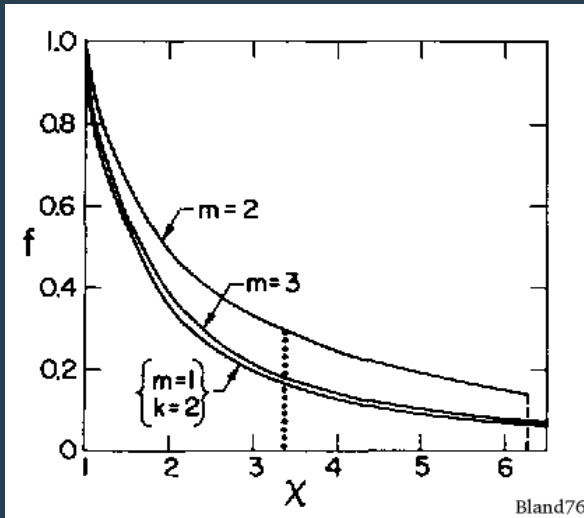
$$p = \frac{2}{3} w_1 \Gamma^2 f(\chi)$$

$$y^2 = \frac{1}{2} \Gamma^2 g(\chi)$$

$$n' = 2n_1 \Gamma^2 h(\chi)$$

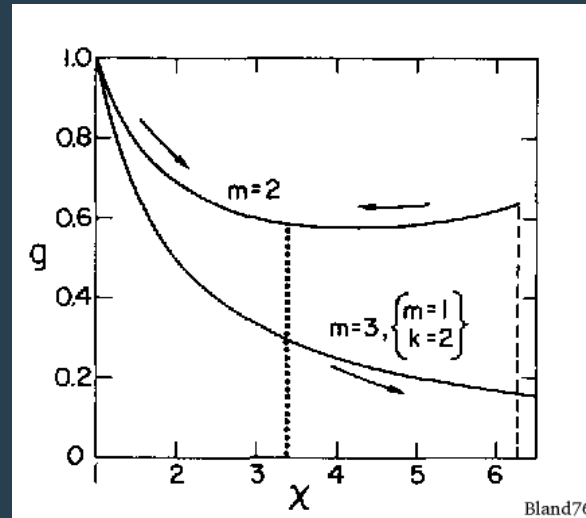
$$\chi \geq 1 \text{ and } f(1) = g(1) = h(1) = 1$$

The Blandford McKee Similarity Solution



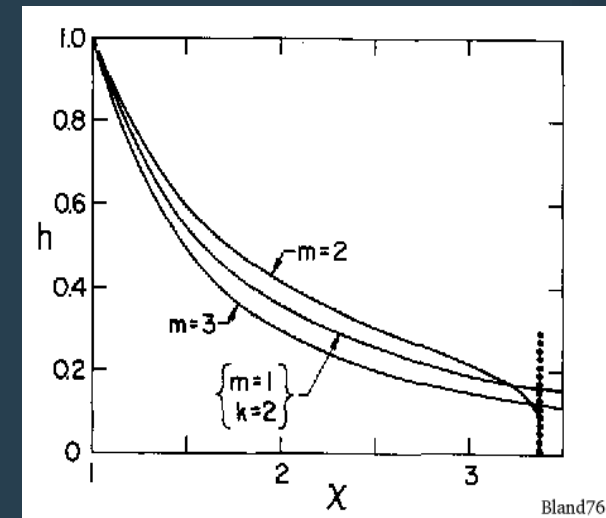
Normalized pressure

$$p = \frac{2}{3} w_1 \Gamma^2 f(\chi)$$



Velocity variation

$$y^2 = \frac{1}{2} \Gamma^2 g(\chi)$$



Normalized density

$$n' = 2n_1 \Gamma^2 h(\chi)$$

Acceleration Mechanism

Properties of the accelerator:

- Homogeneous Gas Surroundings
- Gas possibly relativistic itself
- "some" magnetic fields

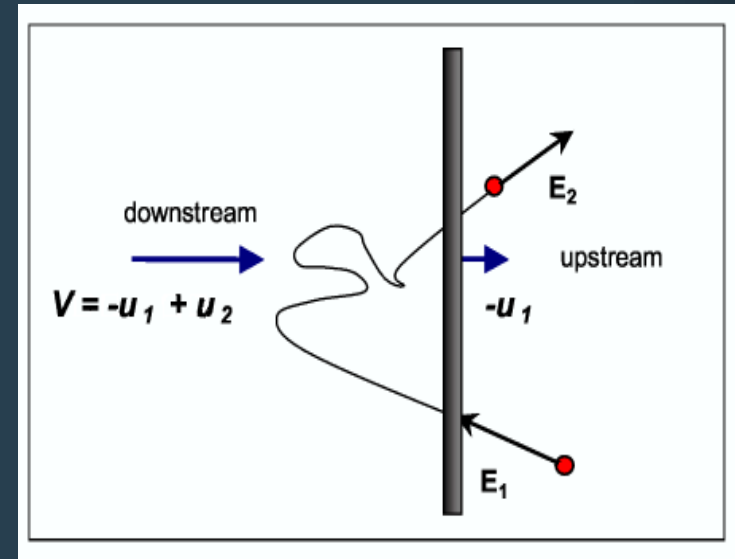
and, most importantly

- Highly relativistic shock wave

=> Acceleration by 1st order Fermi Process

The Fermi Process

- Random walk of particles
- Particles can repeatedly cross the shock wave
- Magnetic turbulence scatter particles downstream
- Particles gain Energy as $dE \propto \beta E$
- Particles have limited probability of staying in accelerator $dN \propto p N$



Shock scattering particle

The Fermi Process

After k scatterings in Accelerator:

- Energy

$$E = E_0 \beta^k$$

- Particles left

$$N = N_0 p^k$$

eliminate k :

$$\frac{\ln(N/N_0)}{\ln(E/E_0)} = \frac{\ln p}{\ln \beta}$$

$$\frac{N}{N_0} = \left(\frac{E}{E_0}\right)^{\ln(p-\beta)}$$

$$\rightarrow N(\gamma_e) \propto \gamma_e^{-\alpha}$$

\Rightarrow Power-law distribution

The Emission Mechanism

Synchrotron Radiation

- Larmor Frequency
- Relativistic: Cyclotron
- e^- relativistic \Rightarrow dipole characteristic is beamed
- Pulselength in e^- frame
- Doppler shifted
To lab-frame

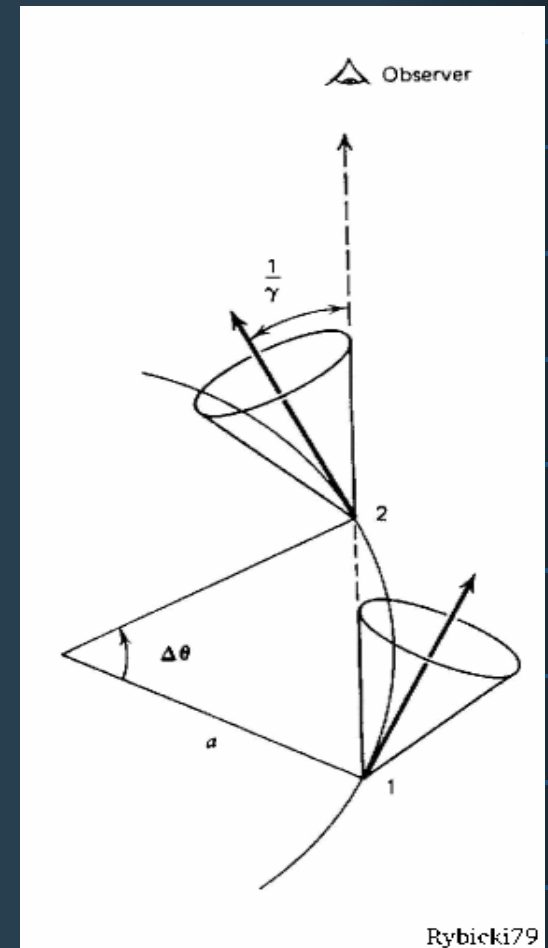
$$\omega_L = \frac{e B}{m_e c}$$

$$\omega_B = \frac{\omega_L}{\gamma_e}$$

$$\Delta\theta \approx \gamma_e^{-1}$$

$$\Delta t' = \frac{\Delta\theta}{\omega_L} = \frac{1}{\gamma_e \omega_L}$$

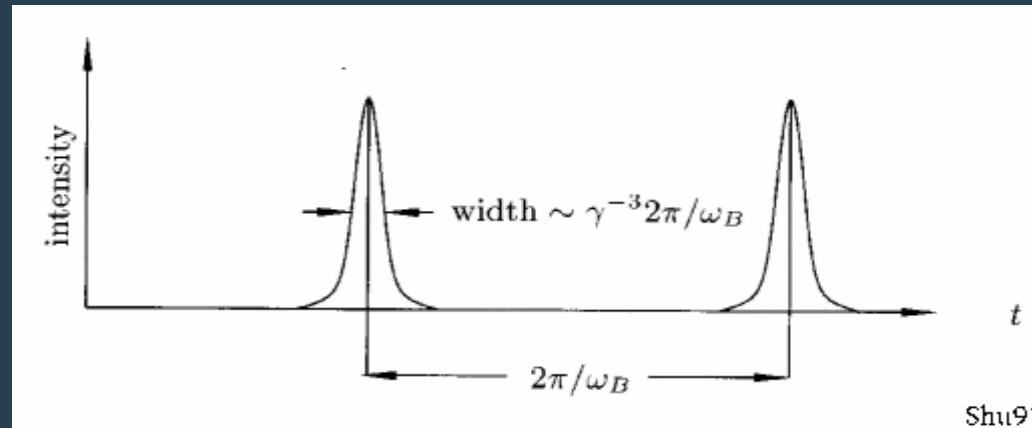
$$\Delta t = \frac{1}{\gamma^3 \omega_B}$$



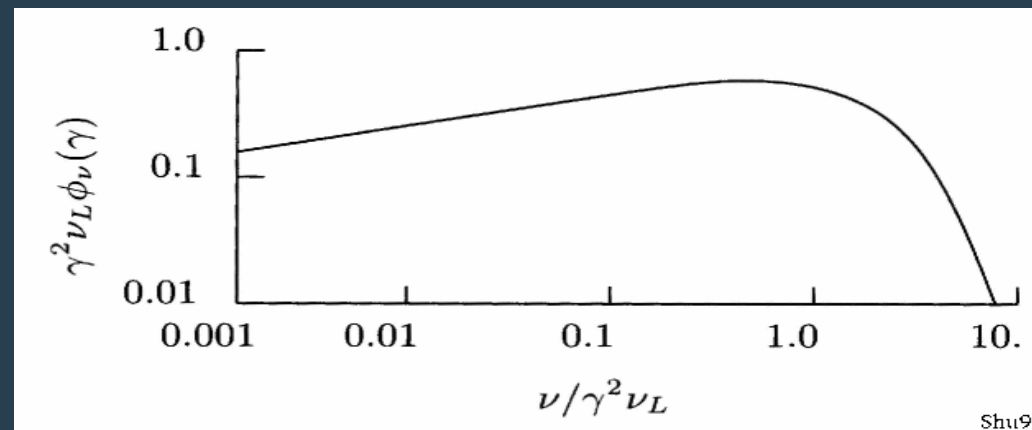
Beaming in synchrotron Emission

The Emission Mechanism

Derive the spectrum by Fourier-Transformation of E-Field:



Time dependence of E-Field



Fourier spectrum

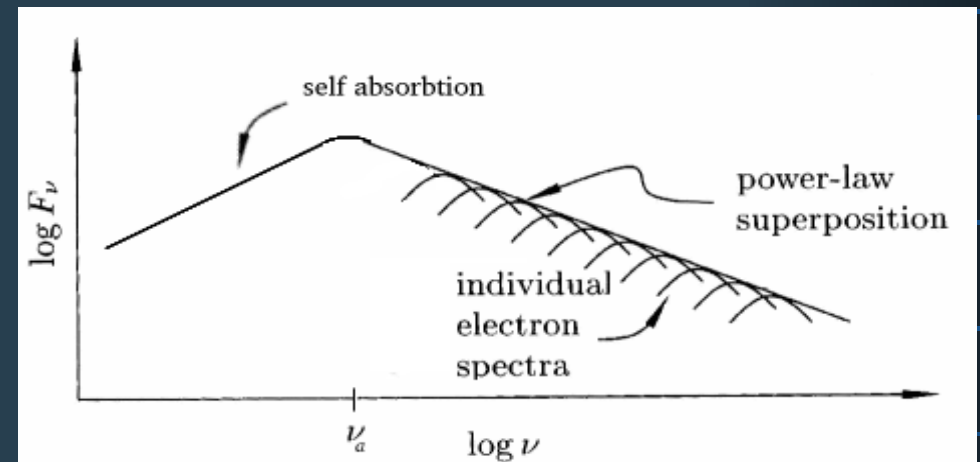
The Emission Mechanism

Spectrum of e^- powerlaw distribution

- Strong peak of single e^- at ν_L
- Power law e^- distribution
- Results in power law spectrum
- Below ν_a :

$$T_b < T_e \quad \text{where} \quad T_b \propto n \nu y^{-(2+\alpha)}$$
$$3kT_e = \gamma m_e c^2$$

- e^- Gas becomes optically thick



Basic synchrotron spectrum

Spectral Evolution and Light Curves

- More detailed Analysis necessary

(eg Sari, Piran & Narayan '98)

- Modified e- distribution with minimum Lorentz Faktor γ_m

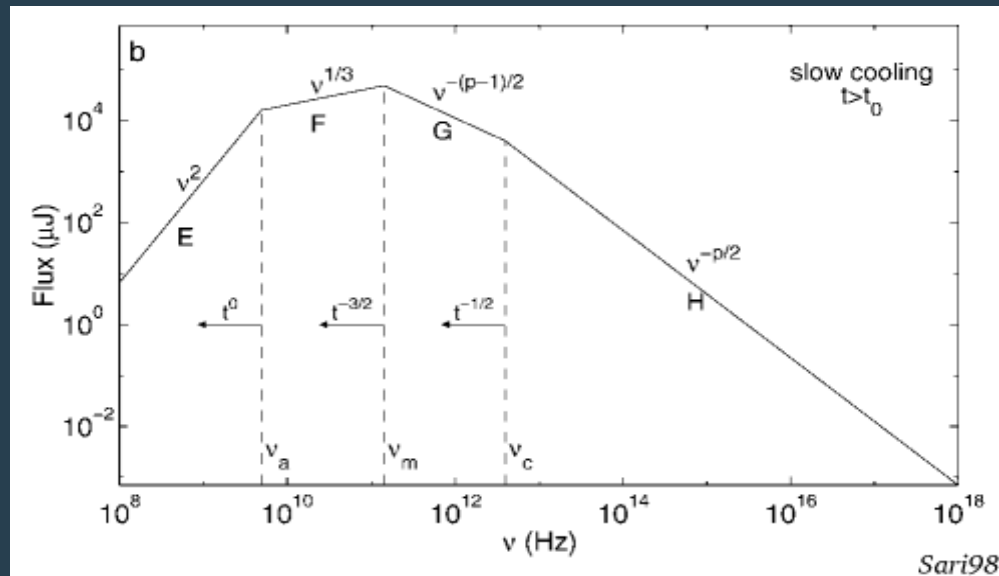
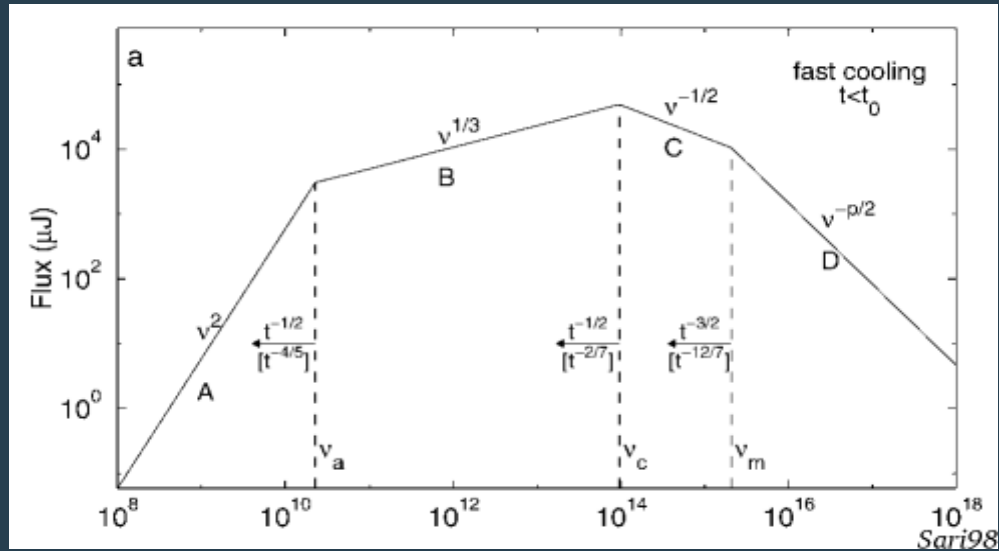
$$N(\gamma_e)d\gamma_e \propto \gamma_e^{-p}d\gamma_e, \gamma_e \geq \gamma_m$$

- Include radiation losses above critical Frequency ν_c

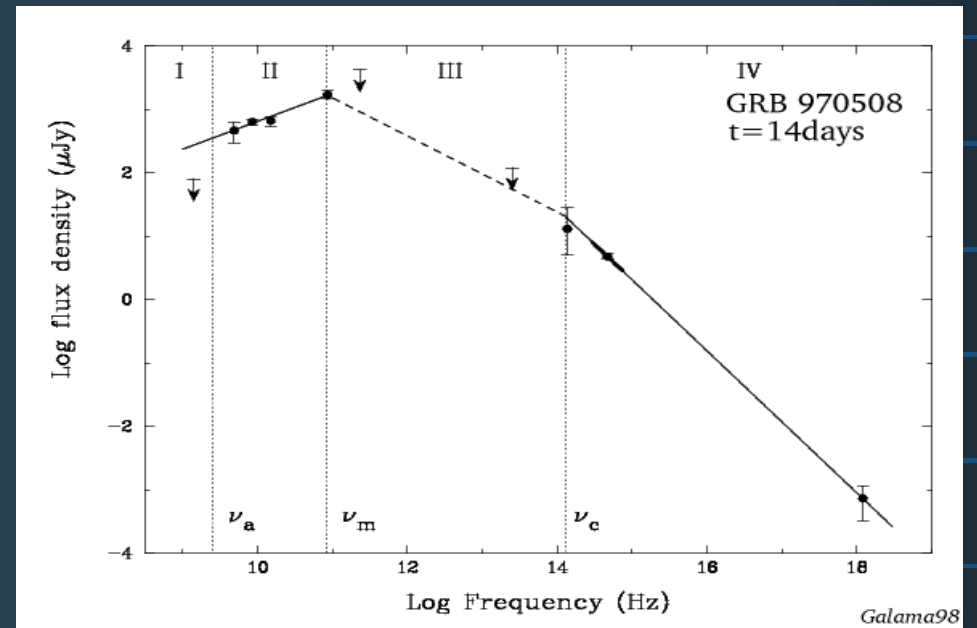
$$E_e(\gamma_c) = P(\gamma_c)t$$

- Allow adiabatic or radiative evolution

Spectral Evolution and Light Curves

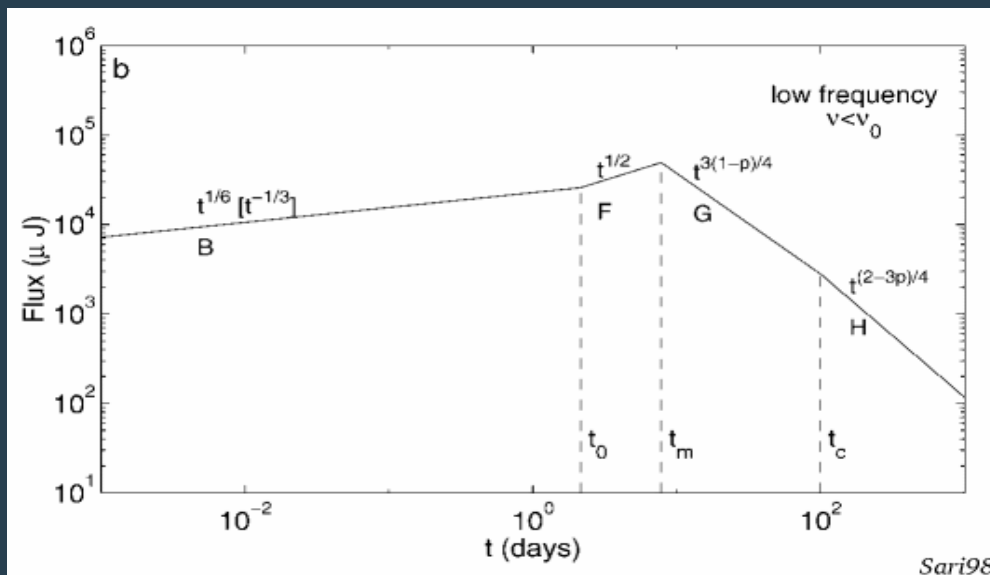
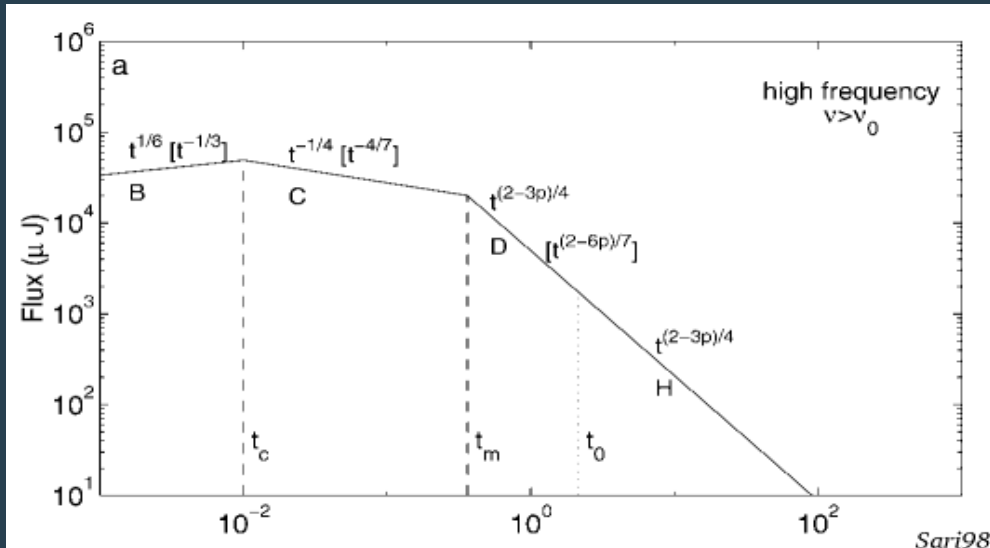


Model spectrum

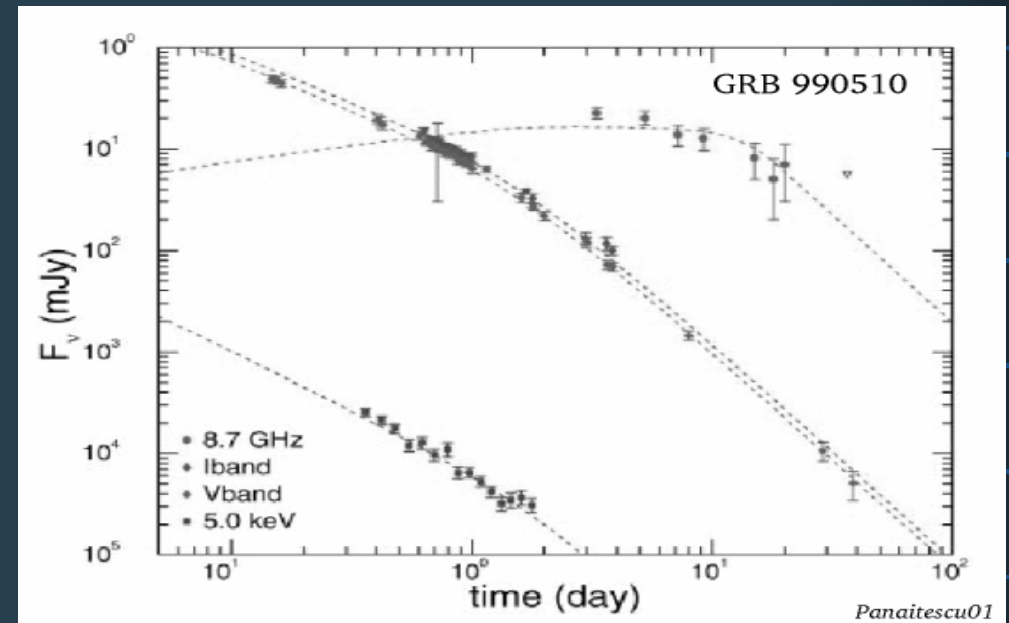


Actual data

Spectral Evolution and Light Curves



Model light curve

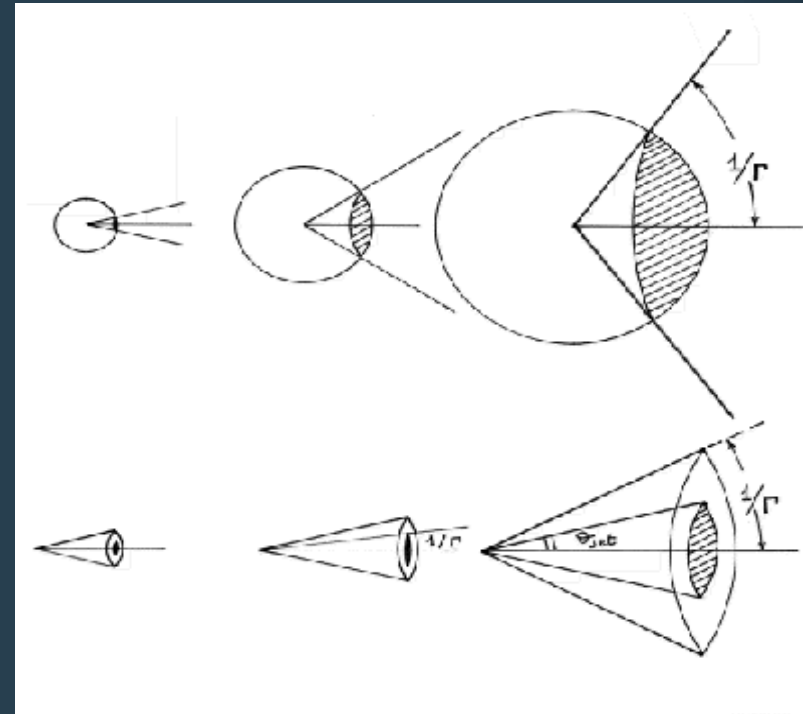


Model doesn't fit data too well...

Jets and Limb-Brightening

Further generalization of model:

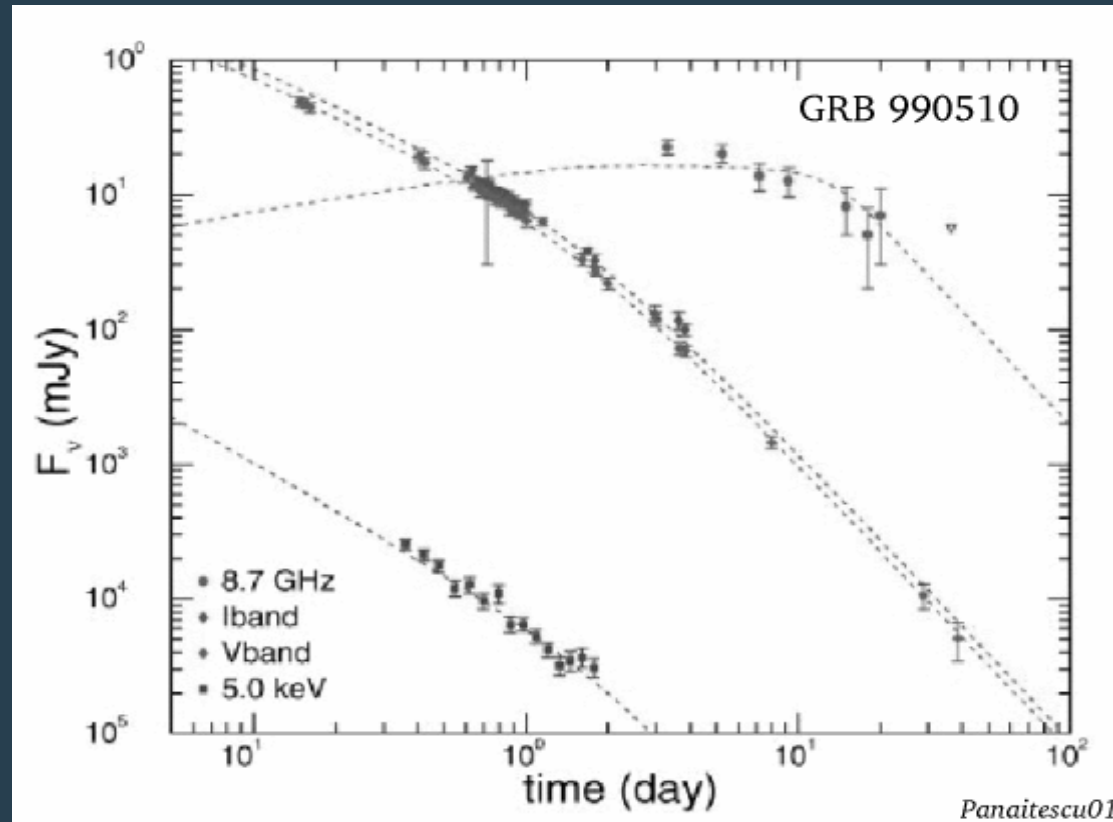
- Jet geometry
- Limb brightening
- Includes spherical geometry as long as line of sight is inside jet angle (casually disconnected)



Spherical and jet geometry with relativistic beaming

Jets and Limb-Brightening

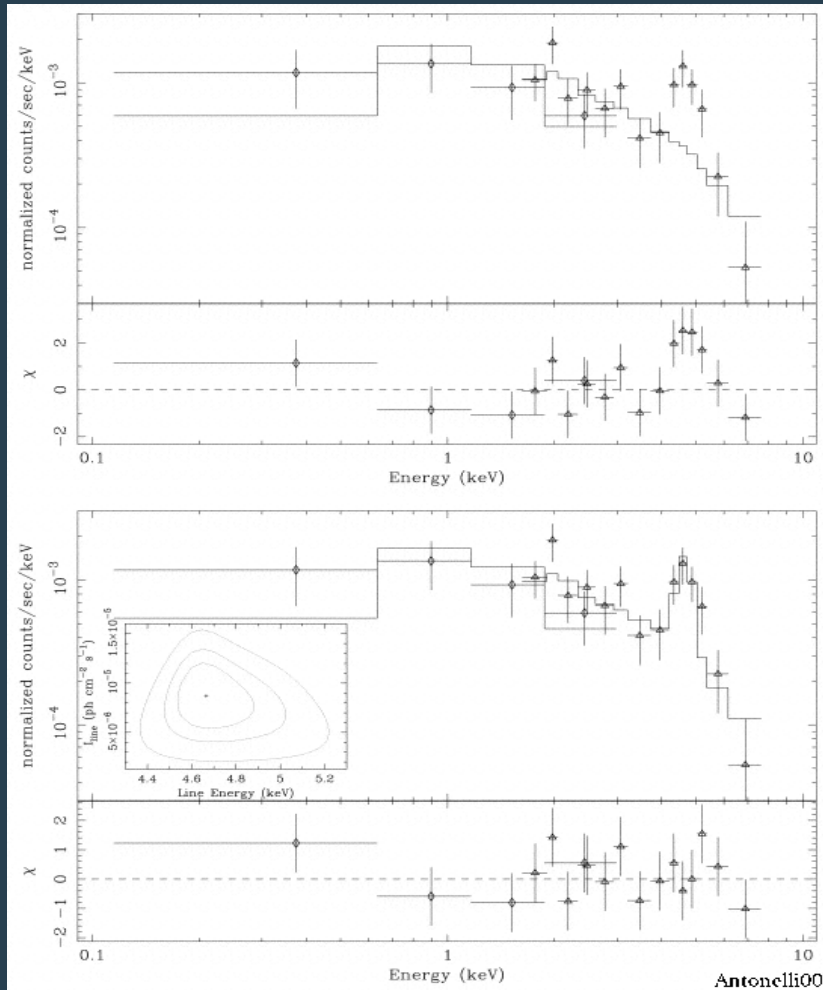
Strong breaks in Lightcurve attributed to Limb Brightening



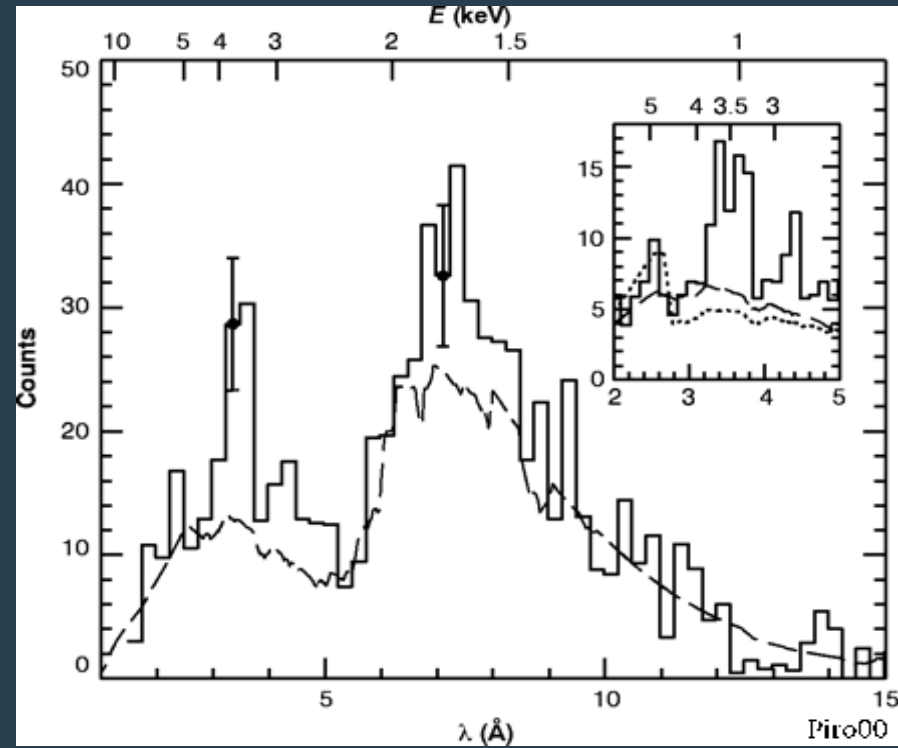
Jet model fits data

Fe K-Lines

Observation: Fe-K lines in X-Ray



BeppoSAX data of Fe-K line in GRB000214 Afterglow

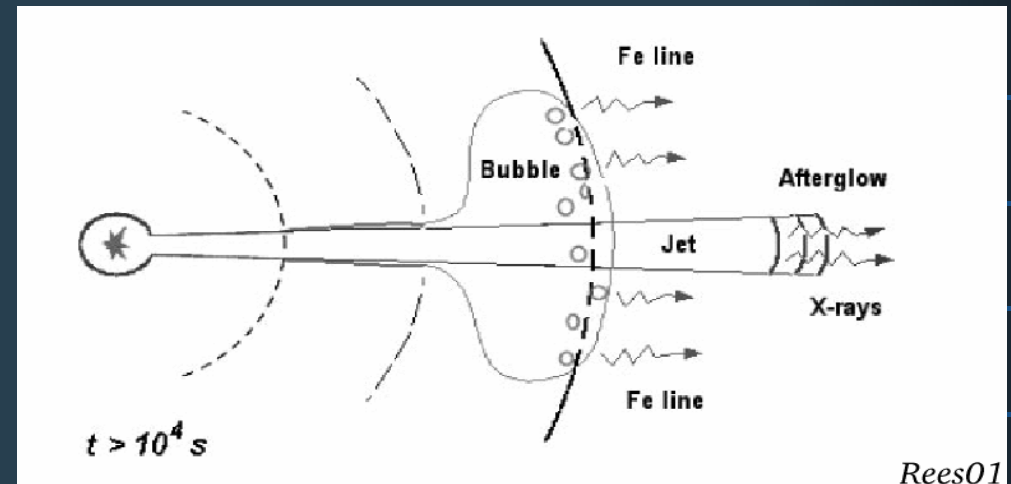


Chandra data of Fe-K line in GRB 991216

Fe K-Lines

The "nearby model"

- Jet ploughs through progenitor
- Waste heat pumped into lower envelope
- Bubble of heat rises up after ~ 1 day
- Needs only $10^{-5} M_{\odot}$ of Fe



The bubble of waste heat, breaking through the envelope

Conclusion

- Quite basic model lets you understand
 - Relativistic shock dynamics
 - Acceleration processes
 - Radiation processes
- Basic model can be extended to include e.g.
 - Jet Geometry
 - Limb brightening
 - Iron lines
 - ...

Literature Cited

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