# Core Collapse Supernova Mechanism, Nuclear Statistical Equilibrium (NSE) and Radioactive Abundances

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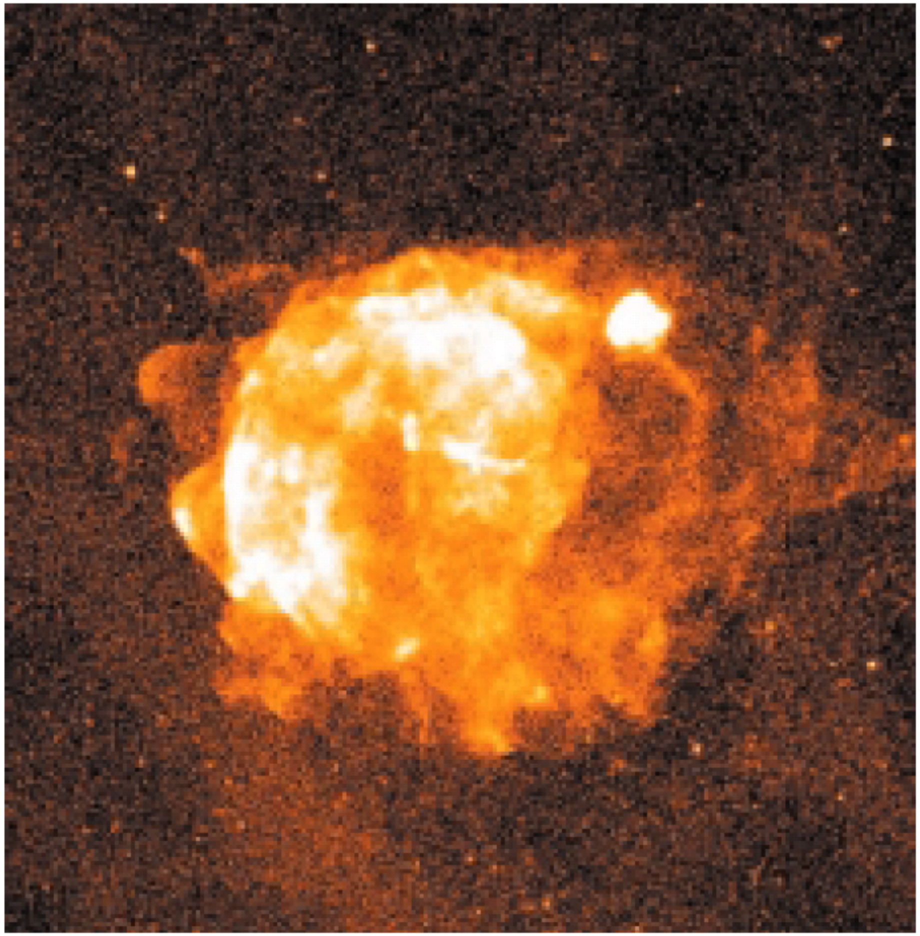
# Content

# Introduction

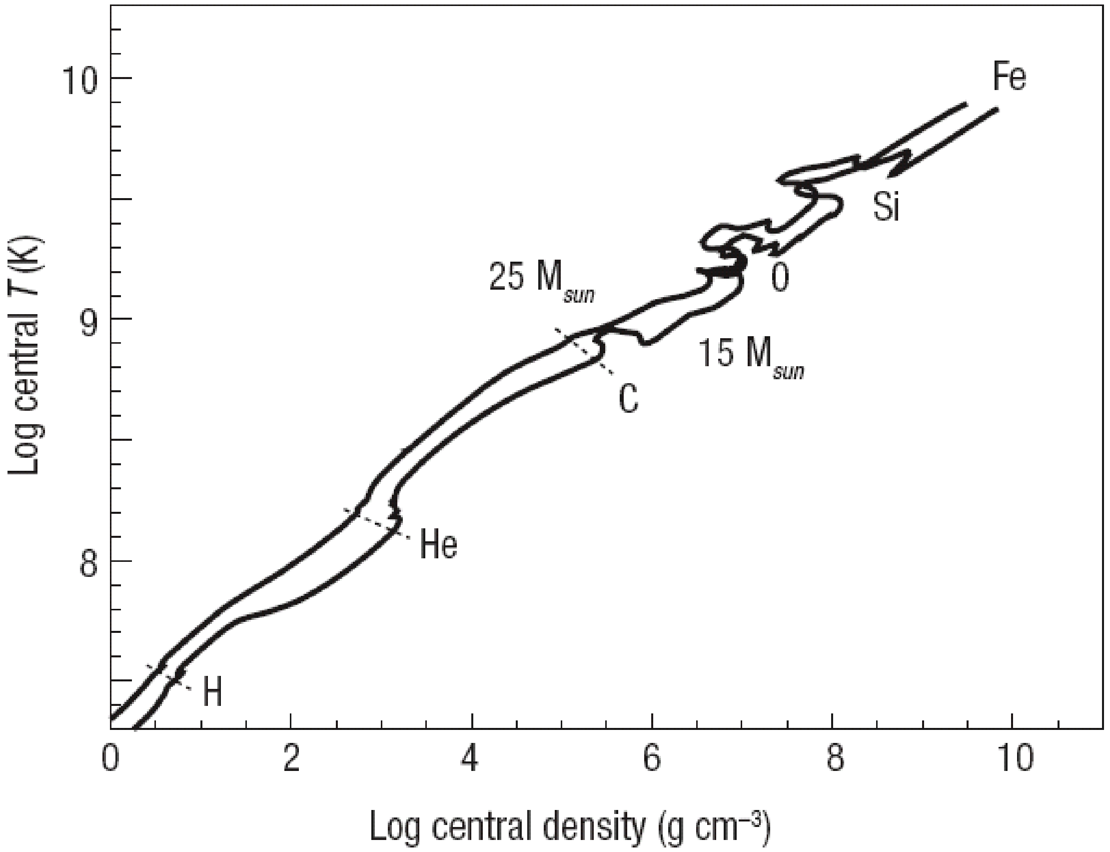
- Two types of supernovae:
  - Thermonuclear explosions of accreting white dwarfs (Type Ia)
  - Core collapse supernovae (Type II, Ib, Ic)
- Classification:
  - Type I is defined by a lack of hydrogen lines in its spectrum
  - Type II has those lines



# Core collapse supernova

- Starting point: stars heavier than 8 solar masses
- Power: $10^{53}$ erg $s^{-1}$ = $10^{46}$ J $s^{-1}$ ; released by neutrinos
- End product:
  - Neutron star
    - Radius: ca. 10 km
    - Density: nuclear matter
    - 90% Neutrons, 10% Protons
  - Black hole (if the progenitor star had a mass roughly heavier than 25 solar masses)

# Evolution of a massive star



- Each time a fuel runs out, the star contracts and heats up, until the next burning stage is reached
- Overall, the life of a massive star is a continued contraction

# Evolution of a massive star

- The evolution accelerates greatly after helium burning
- Reason: energy loss due to neutrino cooling
- Neutrinos are produced for example by pair annihilation:

$$e^- + e^+ \rightarrow \nu + \overline{\nu}$$

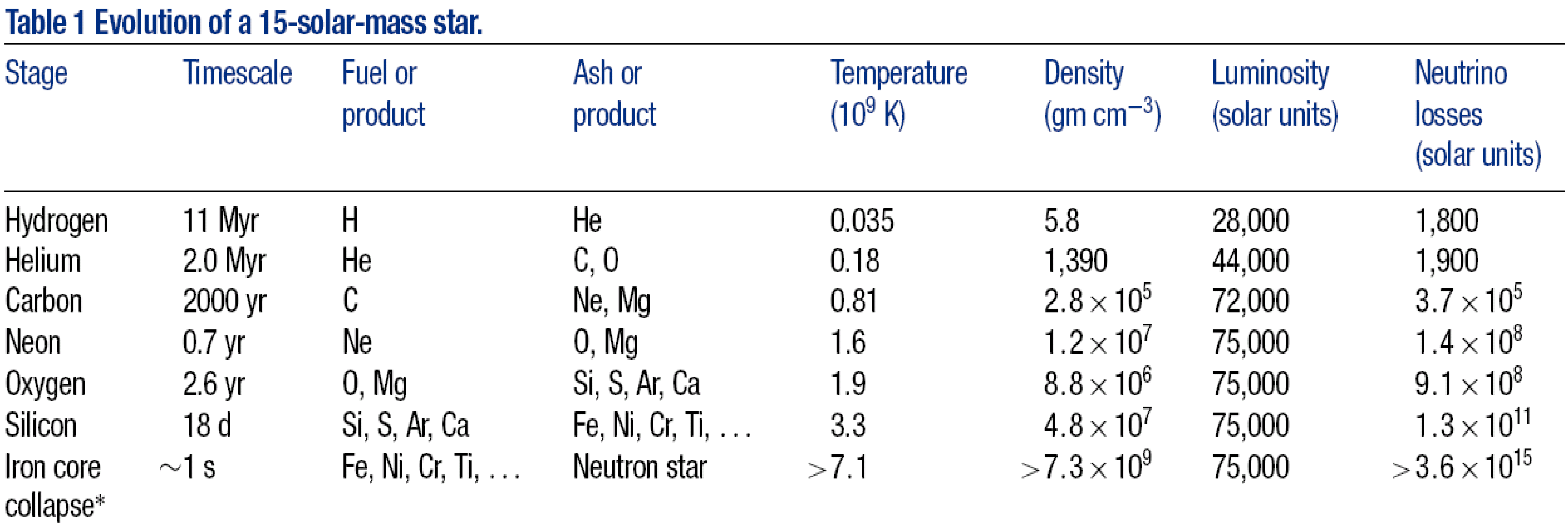
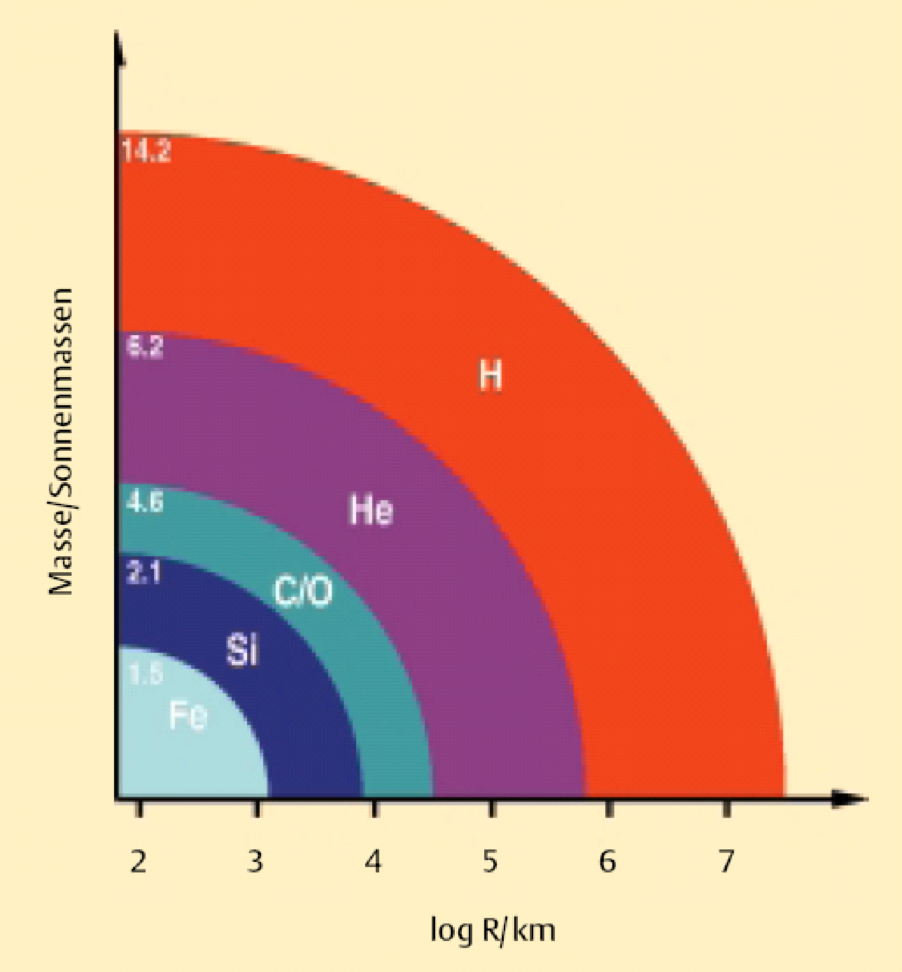


**Table 1 Evolution of a 15-solar-mass star.**

| Stage | Timescale | Fuel or product | Ash or product | Temperature ($10^9$ K) | Density (gm $\mathrm{cm}^{-3}$) | Luminosity (solar units) | Neutrino losses (solar units) |
|---|---|---|---|---|---|---|---|
| Hydrogen | 11 Myr | H | He | 0.035 | 5.8 | 28,000 | 1,800 |
| Helium | 2.0 Myr | He | C, O | 0.18 | 1,390 | 44,000 | 1,900 |
| Carbon | 2000 yr | C | Ne, Mg | 0.81 | $2.8 \times 10^5$ | 72,000 | $3.7 \times 10^5$ |
| Neon | 0.7 yr | Ne | O, Mg | 1.6 | $1.2 \times 10^7$ | 75,000 | $1.4 \times 10^8$ |
| Oxygen | 2.6 yr | O, Mg | Si, S, Ar, Ca | 1.9 | $8.8 \times 10^6$ | 75,000 | $9.1 \times 10^8$ |
| Silicon | 18 d | Si, S, Ar, Ca | Fe, Ni, Cr, Ti, . . . | 3.3 | $4.8 \times 10^7$ | 75,000 | $1.3 \times 10^{11}$ |
| Iron core collapse* | $\sim$1 s | Fe, Ni, Cr, Ti, . . . | Neutron star | $>$7.1 | $>7.3 \times 10^9$ | 75,000 | $>3.6 \times 10^{15}$ |

# The star before the collapse

- Onion shell structure (see right)
- The inner core consists of iron-group elements (Fe, Ni, Cr, Ti, …)
- No further nuclear fusion is possible in the iron core
- Electrons supply most of the pressure, that stabilizes the star
- Photodisintegration of iron nuclei destabilizes the iron core; e.g.:
  $$\gamma + {}^{56}_{26}\mathrm{Fe} \rightleftharpoons 13\alpha + 4n$$
- Because of the high temperature and pressure, these reactions are in equilibrium (NSE)



# Nuclear Statistical Equilibrium (NSE)

- For a realistic calculation, one must treat the entire set of nuclei
- Typical reactions:
  - $(Z,A)+\gamma \rightleftharpoons (Z-1,A-1)+p$
  - $(Z,A)+\gamma \rightleftharpoons (Z,A-1)+n$
  - $(\gamma,\alpha),(n,\alpha),(p,\alpha),(n,p)$
- Chain of reactions, which are all in equilibrium:

$$(Z,A)+\gamma \rightleftharpoons \ \dots \ \rightleftharpoons Z\,p+(A-Z)\,n$$

- From thermodynamics we know:

$$\mathrm{d}U = T\mathrm{d}S - p\,\mathrm{d}V + \sum_i \mu_i \mathrm{d}N_i$$

- This gives in equilibrium: $\sum_i \mu_i \mathrm{d}N_i \overset{!}{=} 0$

and hence: $\boxed{\mu(Z,A) = Z\mu_p + (A-Z)\mu_N .}$

# Nuclear Statistical Equilibrium (NSE)

$$\mu(Z, A) = Z\mu_p + (A - Z)\mu_N. \qquad (1)$$

- The nuclei obey Boltzmann statistics for each species i:

$$n_i = g_i \left(\frac{m_i kT}{2\pi\hbar^2}\right)^{3/2} \exp\left(\frac{\mu_i - m_i c^2}{kT}\right)$$

where $g_i$ is the nuclear partition function: $g_i = \sum_r (2I_r + 1) \cdot e^{-E_r/kT}$

$$\Rightarrow \quad \mu_i = m_i c^2 + kT \cdot \ln\left[\frac{n_i}{g_i}\left(\frac{2\pi\hbar^2}{m_i kT}\right)^{3/2}\right] \qquad (2)$$

- By inserting (2) into (1) for each species, one obtains the Saha equation:

$$\boxed{n(Z, A) = \frac{g(Z, A) A^{3/2}}{2^A} n_p^Z n_n^{A-Z} \left(\frac{2\pi\hbar^2}{m_u kT}\right)^{3/2(A-1)} \exp\left(\frac{E_B(Z, A)}{kT}\right)}$$

with the nuclear binding energy: $E_B(Z, A) = c^2 \left[Z m_p + (A - Z) m_n - M(Z, A)\right]$

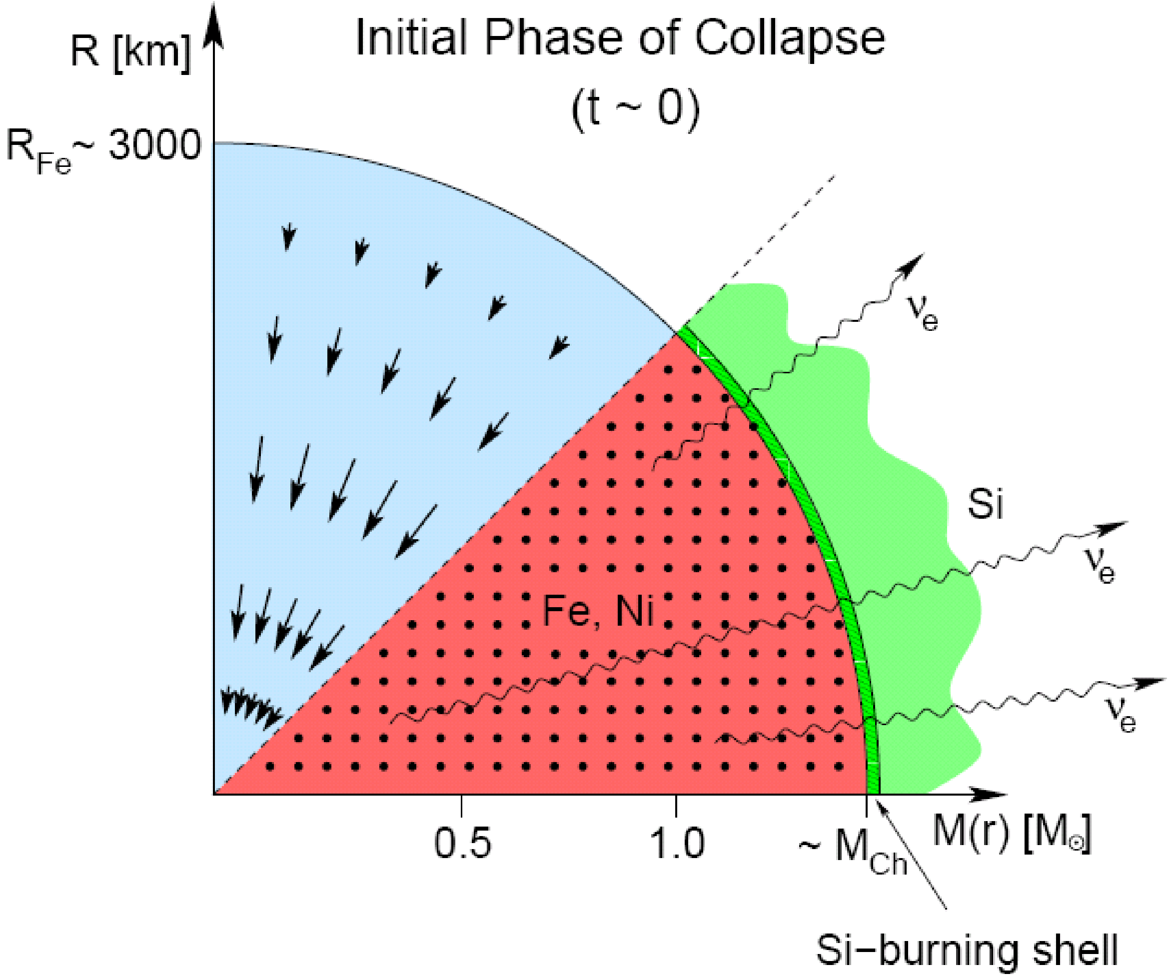
# NSE: Solving the Saha equation

$$\boxed{n(Z,A) = \frac{g(Z,A)A^{3/2}}{2^A} n_p^Z n_n^{A-Z} \left(\frac{2\pi\hbar^2}{m_u kT}\right)^{3/2(A-1)} \exp\left(\frac{E_B(Z,A)}{kT}\right)}$$

- In order to solve the Saha equation, one must specify $n_p$ and $n_n$ via:
  - baryon conservation: $\sum_i n_i A_i = n = \frac{\rho}{m_u}$
  - charge conservation: $\sum_i n_i Z_i = nY_e$ with: $Y_e \equiv \frac{n_e}{n}$
- In NSE, the composition of the matter is determined by $\rho$, *T* and *Y<sub></sub>*

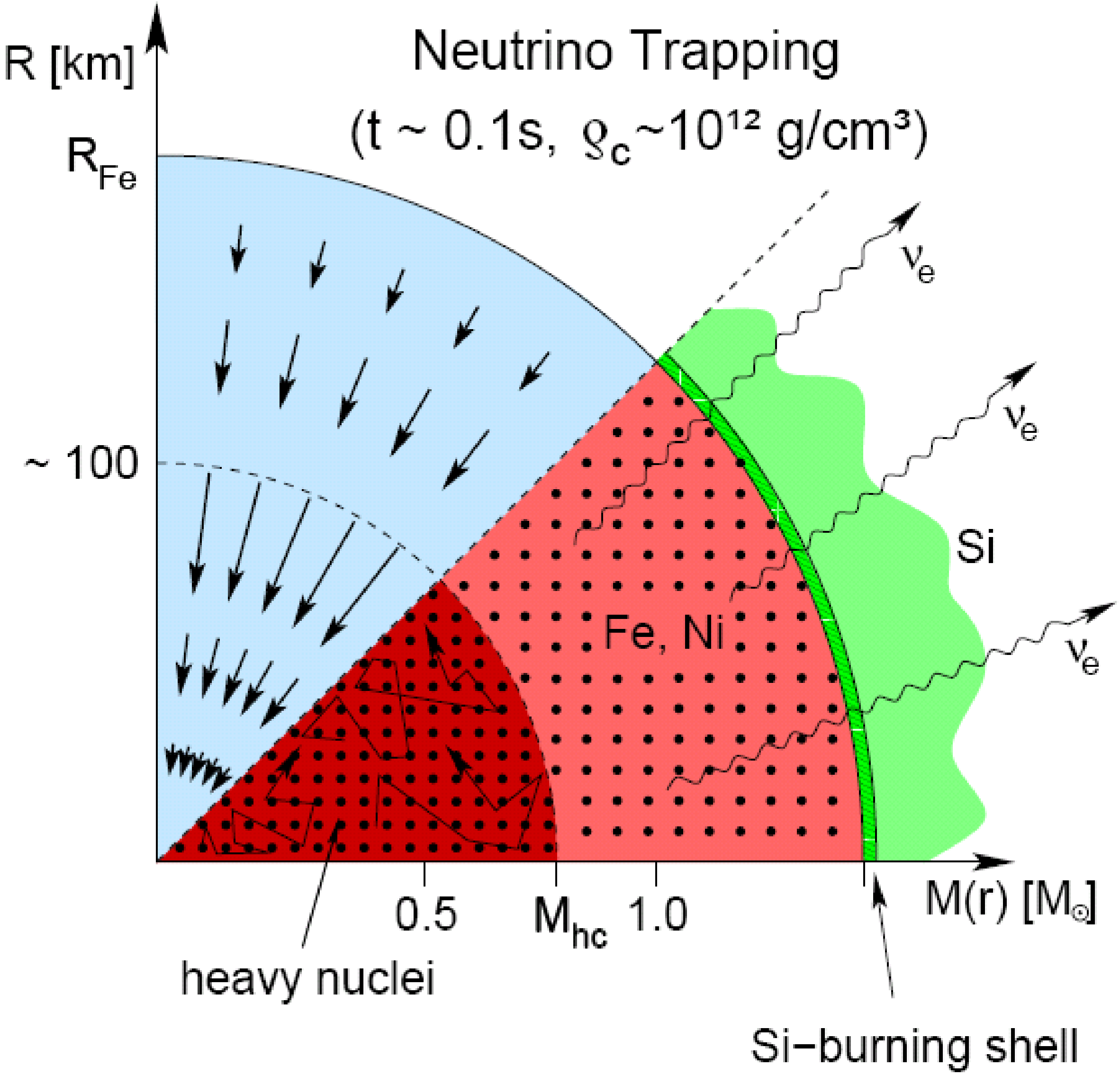
# The beginning of the collapse

- When the iron core reaches a mass of about 1.5 solar masses, it becomes unstable:
- Photodisintegration of iron nuclei in NSE uses up energy
- Neutrino cooling
- Electron capture by protons:
  - Pressure gets reduced even more
  - Neutronization of the matter
  - $e^- + p \rightarrow \nu_e + n$
- Self-enhancing process



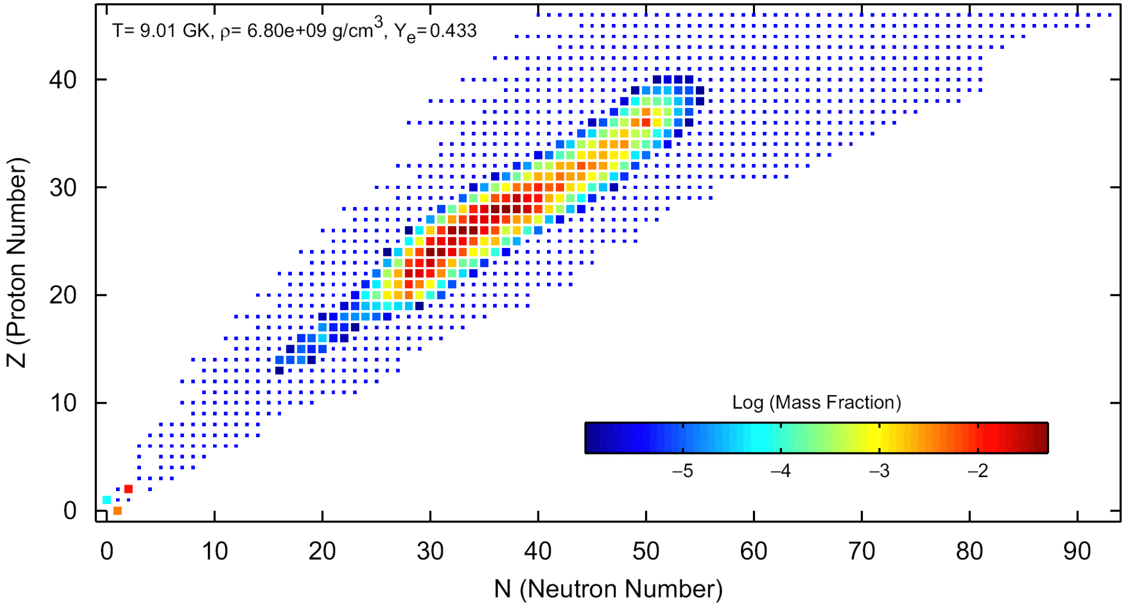
# Core collapse mechanism Pic. 2

- Neutrino trapping: diffusion time > collapse time
- NSE shifts to heavier and more neutron-rich nuclei

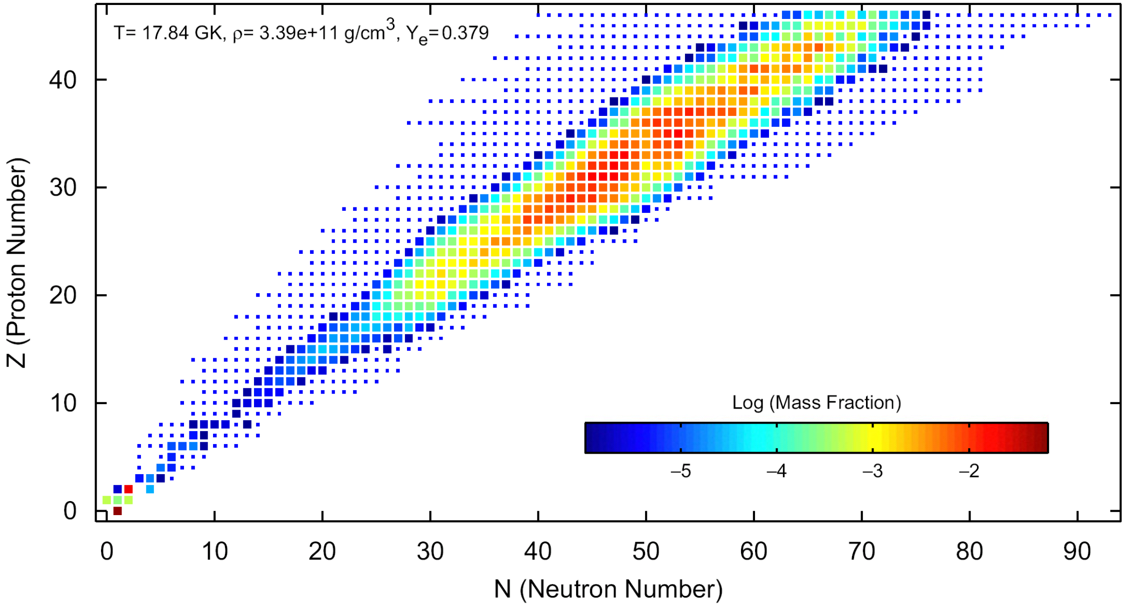


# NSE abundance distributions

- Before the collapse:

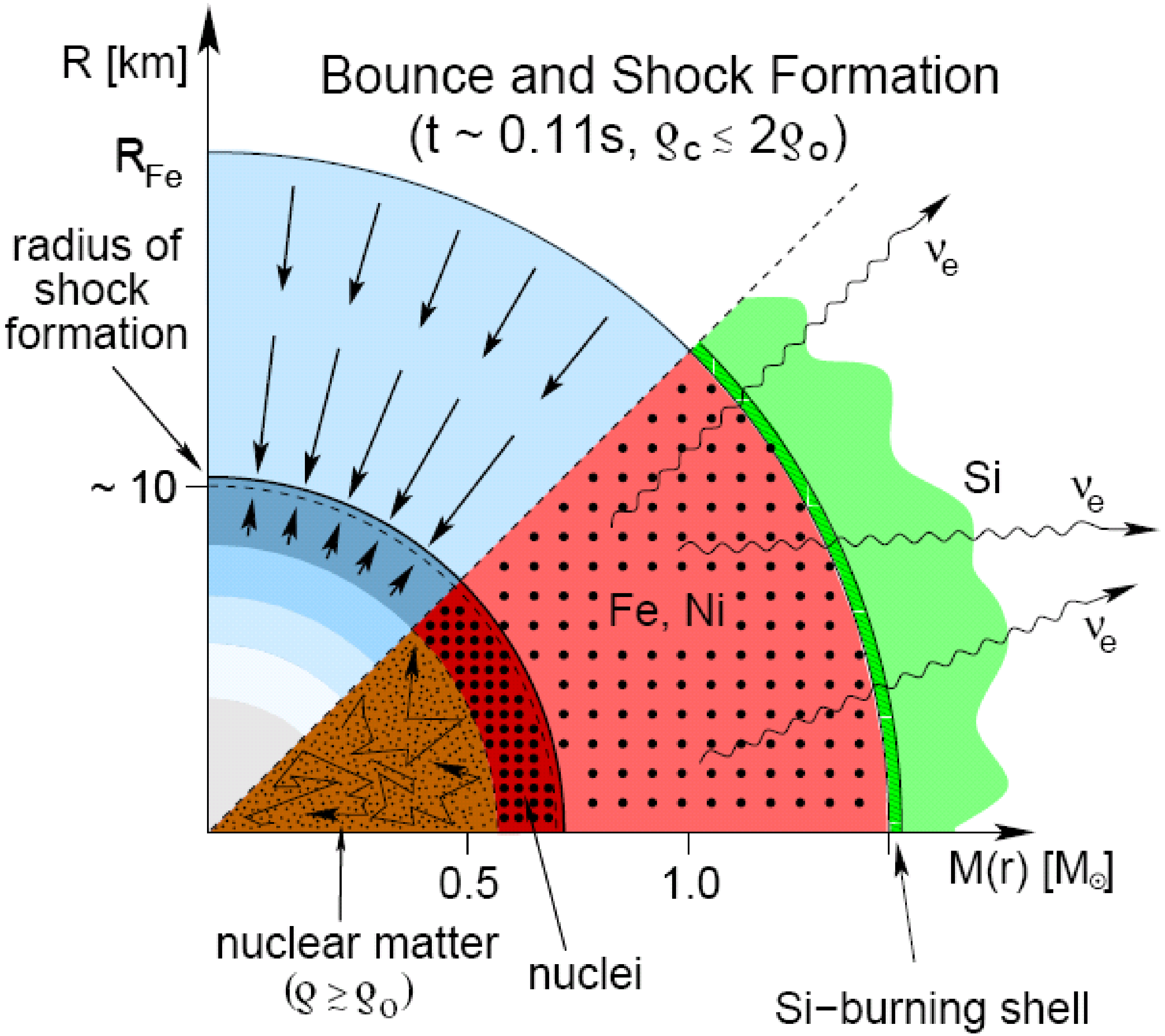


- During neutrino trapping:



# Core collapse mechanism Pic. 3

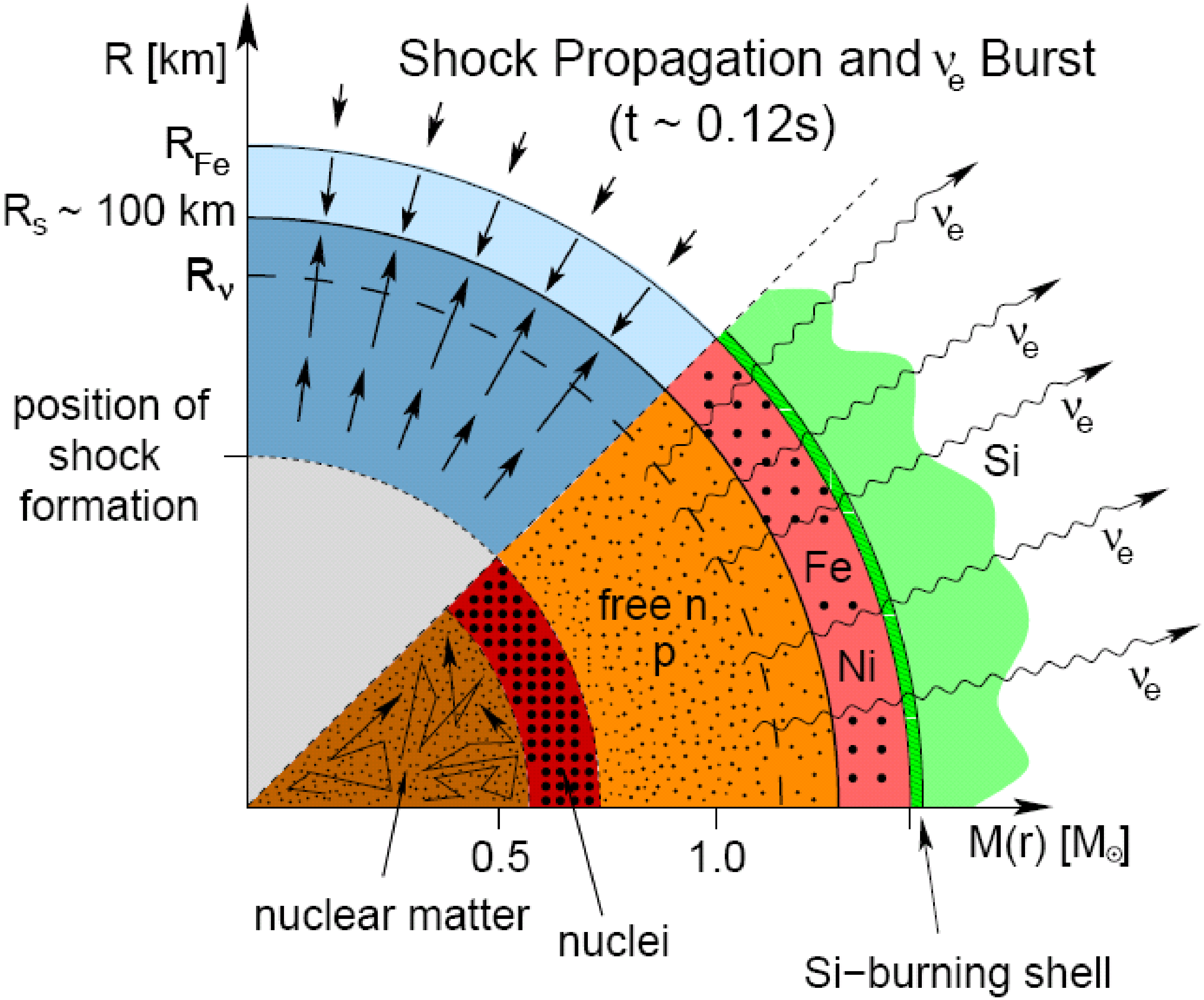
- Nuclear density: $\rho_0 \approx 10^{14}\,\mathrm{g/cm^3}$
- Matter cannot get compressed further
- → Bounce and Shock Formation



# Core collapse mechanism Pic. 4

- Shock dissociates iron nuclei into free protons and neutrons
- This uses up ca. 8-9 MeV per Nucleon
- Neutrino burst at shock breakout:

  $e^- + p \rightarrow \nu_e + n$
- Eventually, the shock wave stalls
- No explosion due to the prompt shock!



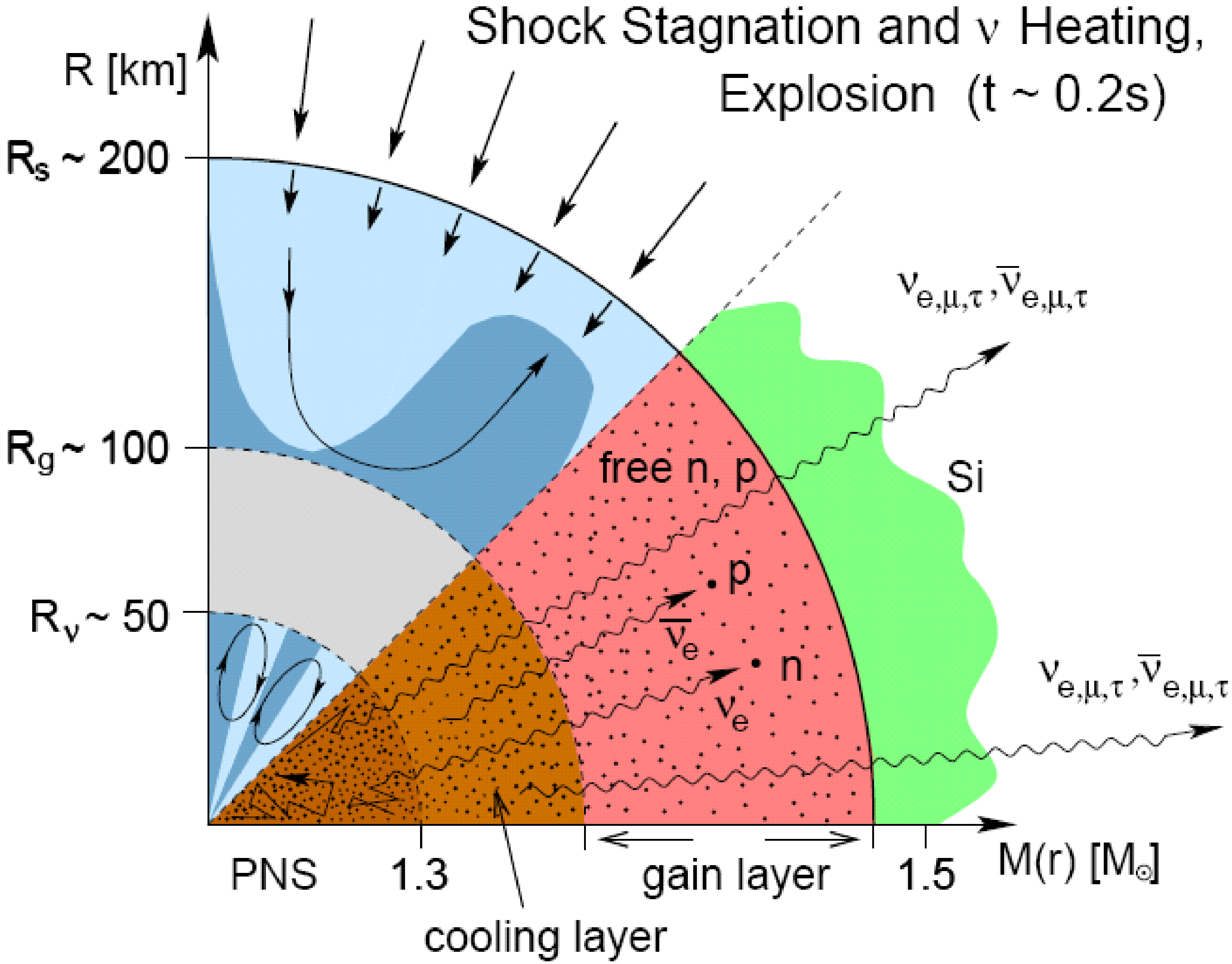
# How can the shock get revived?

- Possible solution: “delayed neutrino-heating mechanism”
- Where do the neutrinos come from?
  - $\nu_e$ can diffuse out of the proto-neutron star
  - Neutrino cooling:
    - Inverse β-decay: $e^- + p \longrightarrow n + \nu_e$

      $e^+ + n \longrightarrow p + \overline{\nu_e}$
    - Pair annihilation: $e^- + e^+ \longrightarrow \nu + \overline{\nu}$
    - Bremsstrahlung: $N + N \longrightarrow N + N + \nu + \overline{\nu}\ ; \qquad N = p, n$
- Neutrinos carry most of the energy released during collapse, 10% of that energy would be enough to reanimate the shock
- Neutrino heating:
  - $v_e + n \longrightarrow e^- + p,$
  - $\bar{v}_e + p \longrightarrow e^+ + n$

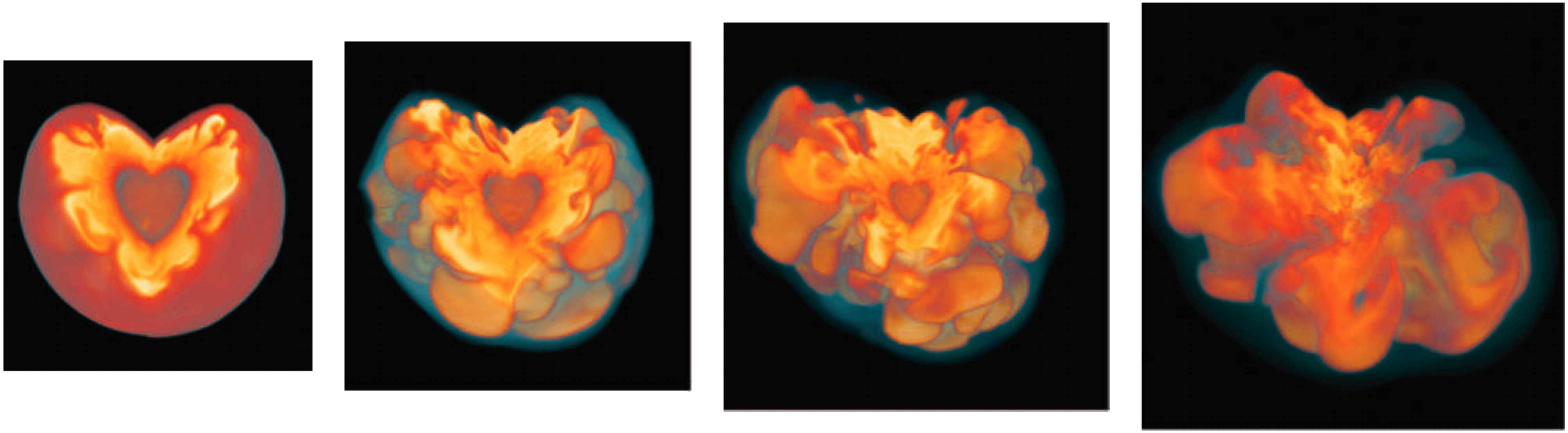
# Core collapse mechanism Pic. 5

- Competition between neutrino heating and cooling
- Neutrino heating causes convective overturn and increases pressure behind the shock → "hot bubble"
- Persistent neutrino heating drives the shock outwards again → Explosion!



# Problems with the “delayed neutrino-heating mechanism”

- In spherically symmetric simulations, only stars with 8-10 solar masses and a ONeMg core explode
- However, multi-dimensional models including convection could solve this puzzle



# Nucleosynthesis

Abundances come from the following processes:

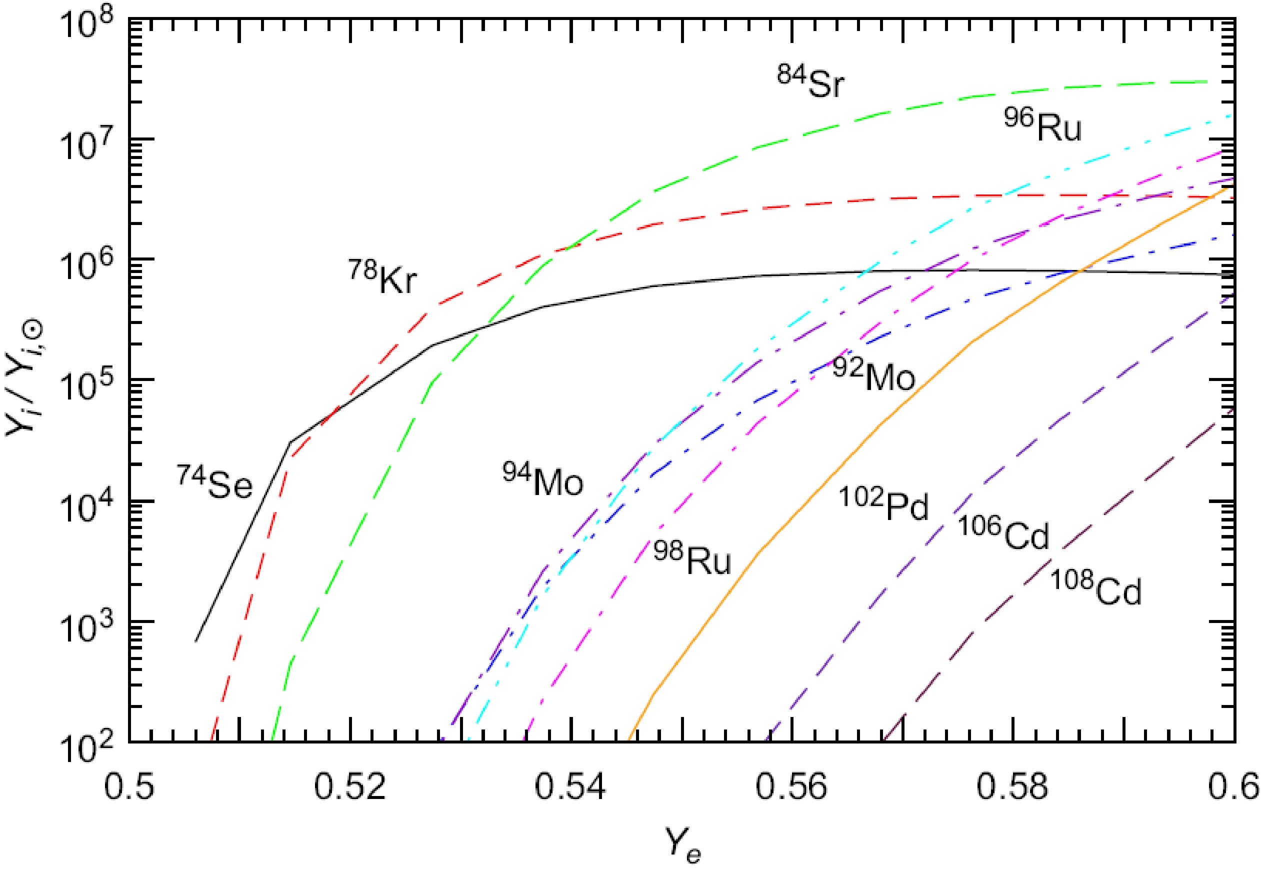
- During the lifetime of the star:
  - Nuclear fusion
    - Almost all nuclei of the iron group get dissociated during core collapse
- While or after the supernova explosion:
  - Explosive burning
  - p-process
  - possibly r-process (?)

# Nucleosynthesis: explosive burning

- Explosive oxygen and silicon burning
  - Between 3-4 billion K (oxygen) and 4-5 billion K (silicon)
  - Products are similar to the normal burning
  - “alpha-rich freeze out”:
    - oxygen burning produces many alpha particles
    - At low densities and at high expansion times, the alpha particles cannot reassemble
    - Radioactive abundances: $^{44}$Ti, $^{56,57}$Ni, etc.
- Explosive neon and carbon burning
  - Between 2-3 billion K
  - Primary products are similar to the normal burning
  - Neutron-rich isotopes, e.g.: $^{60}$Fe (radioactive)

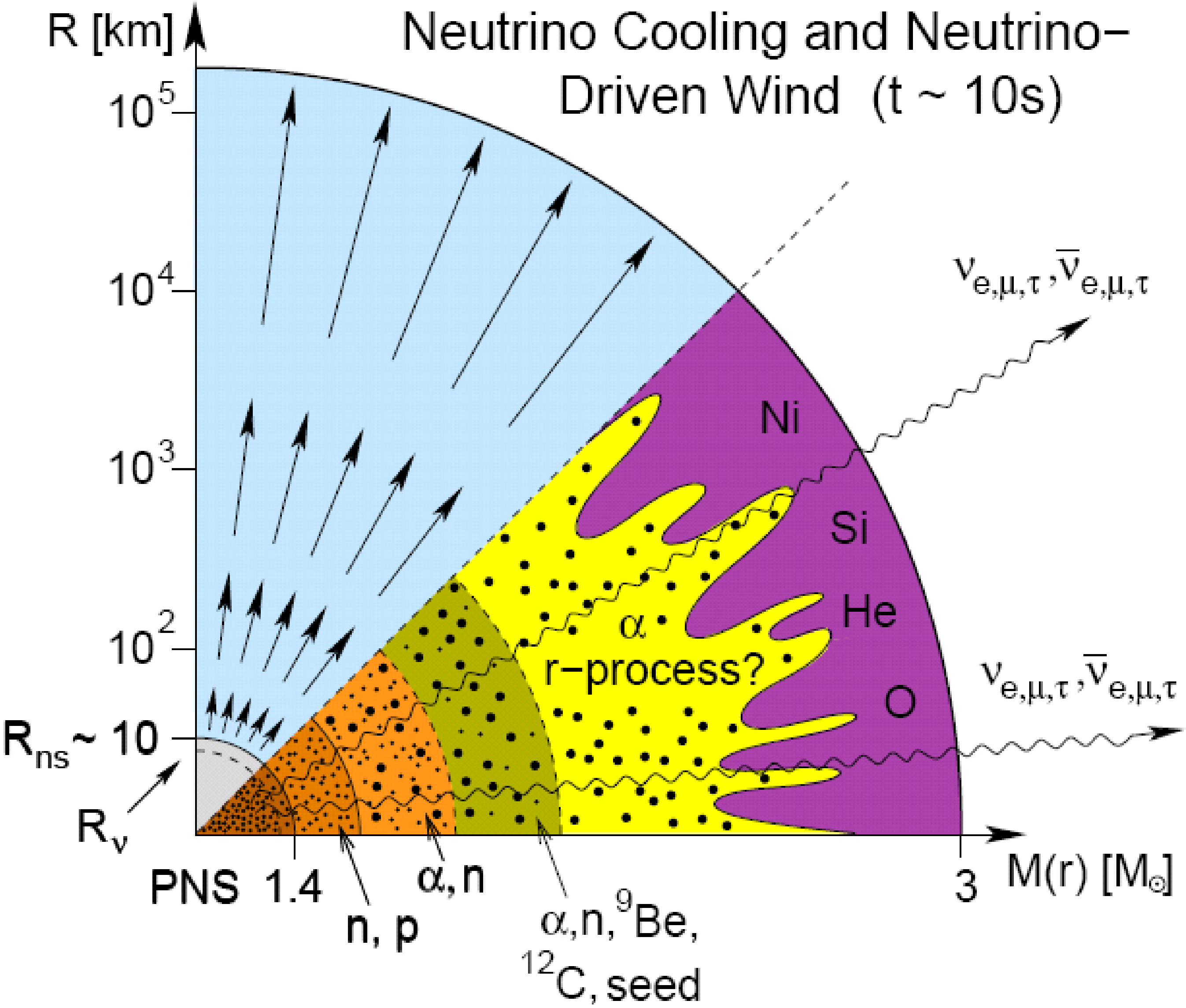
# Nucleosynthesis: p-process

- Early ejecta are rather proton-rich ( $Y_e \approx 0.57$ )
- → p-process
  - Rapid proton capture (rp-process)
  - $\overline{\nu_e} + p \rightarrow n + e^+$ (νp-process)



# Nucleosynthesis: r-process?

- r-process = rapid neutron capture
- r-process could produce heavy neutron-rich nuclei with A ≈ 80 - 240
- Later ejacta may be neutron-rich ($Y_e < 0.5$)
- α-particles do not capture neutrons, because $^5$He is unstable
- But: wind may be 2-4 times too dense to produce the very heavy nuclei...



# Summary

- Prompt shock caused by core collapse is not able to trigger a explosion.
- Possible solution: Delayed neutrino heating
- Core collapse supernovae are possible sites for the r-process.
- At high temperatures and high densities, nuclear reactions are in equilibrium similar to the chemical equilibrium. This is called NSE.

# The End

Thank you
for your attention!

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