

Modelling Thermonuclear Supernovae

- General to Supernovae
 - Numerical Methods
 - Model & Simulation

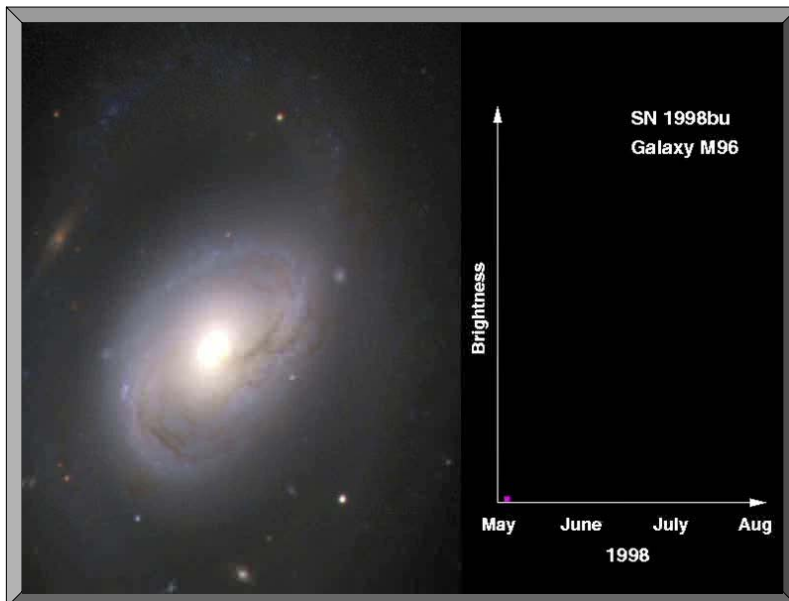
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Classification of Supernovae

- I: no hydrogen lines
 - Ia: strong silicon lines
 - Ib/c: weak/strong helium lines
- II: with hydrogen lines

Why are type Ia supernovae so important?

- homogeneous spectral properties and light curves
 - convenient for distance indicators
- ⇒ good knowledge about the explosion necessary
- ⇒ determining reasons for differences in luminosity



Constraints of the model

- absence of hydrogen and the presence of a silicon absorption line
- homogeneous light curves
- characteristics of the light curve

⇒ White Dwarf

Models

- may be more types of progenitor
- white dwarf in a binary system (single-degenerate)
 - Chandrasekhar mass
 - Sub-Chandrasekhar mass
- merging of two white dwarfs (double-degenerate)

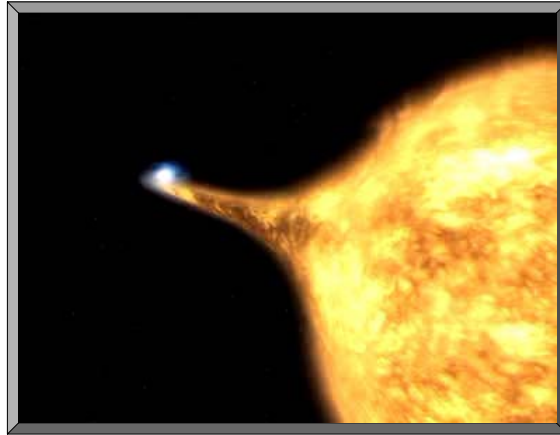
Advantages of single generate scenario

- a similar amount of energy
(explosion at a fix mass)
- similar development of the explosion

Principle of the explosion

- White Dwarf (C+O) accreting mass from his companion star
- Chandrasekhar-mass: fusion begins
- heat conduction: flame burns outwards
- forming of bubbles and turbulences
- explosion itself only a few seconds
- $^{56}\text{Ni} \rightarrow ^{56}\text{Co} \rightarrow ^{56}\text{Fe}$ decay

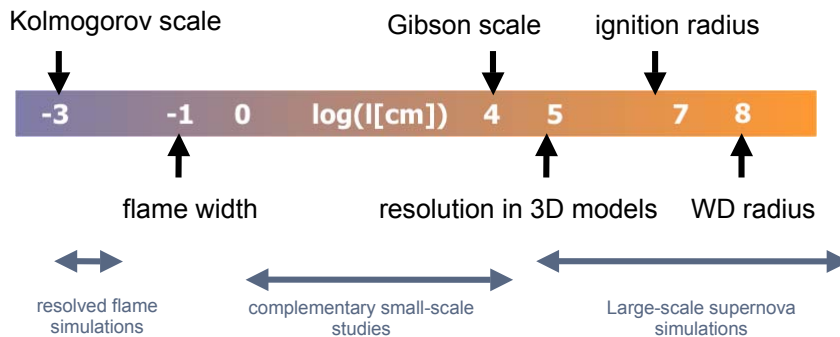
Explosion of supernova



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Length scales in Simulations



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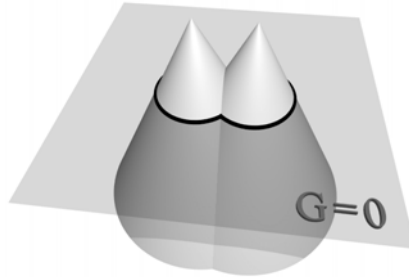
Numerical Methods

- „Large Eddy Simulation“
- Unresolved scales described by sub-grid models
- Level-Set technique:

$$\partial G / \partial t = -D_f \nabla G$$

$$|\nabla G| = 1$$

$$D_f = v_u + s_{\text{tur}}$$



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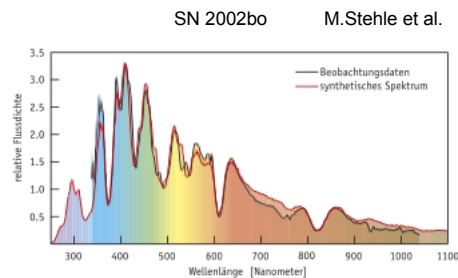
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Model

- „parameter-free“
- initial conditions
- approximations

⇒ good results

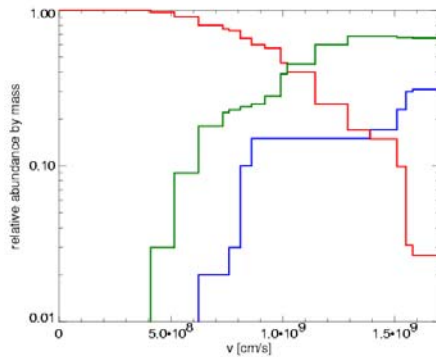
but to less intermediate mass elements



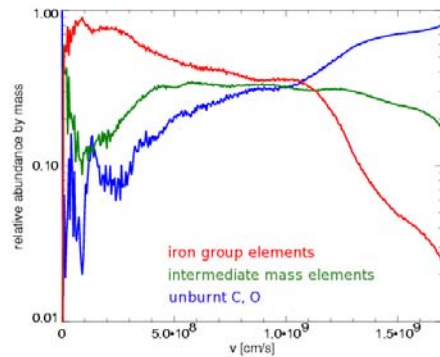
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Abundance of elements



SN 2002bo



SNOB-run

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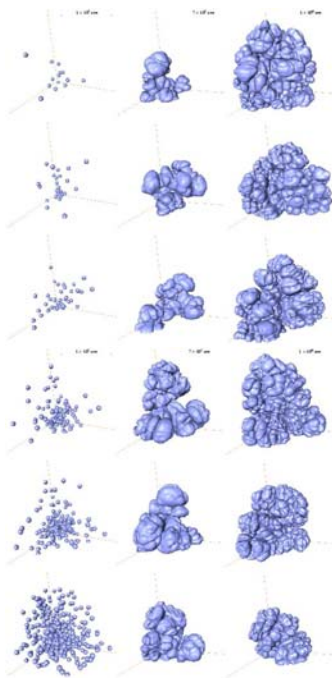
Ignition

- Single ignition in the center
- Multiple ignitions around the center
- Multiple off-center ignitions

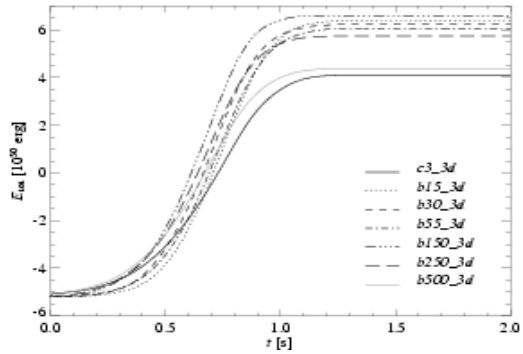
Big influences on produced energy

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Initial Conditions



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Transport mechanism

Reaction zone between fuel and ashes
propagating by

- Deflagration: $v < v_S$ – heat conduction
- Detonation: $v > v_S$ – shock wave

With detonation: to less intermediate mass
elements

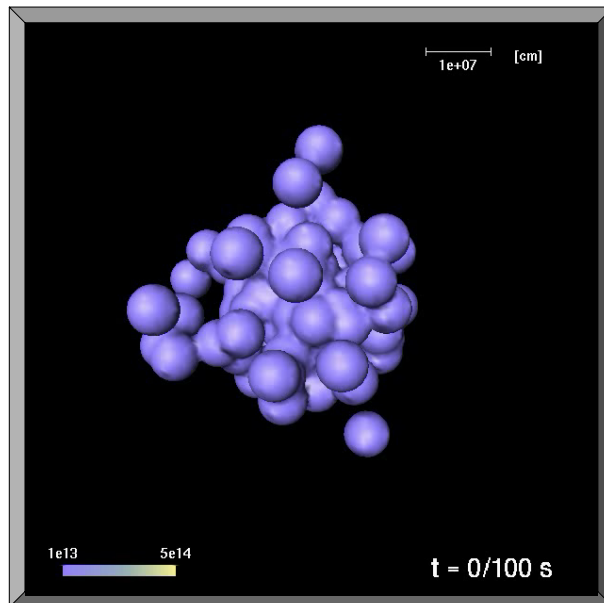
⇒ Deflagration

Flamlet regime

- Rayleigh-Taylor-instability:
Formation of rising bubbles
- Kevin-Helmholtz-instability

Kolmogorovs Law

$$v \approx l^{1/3}$$

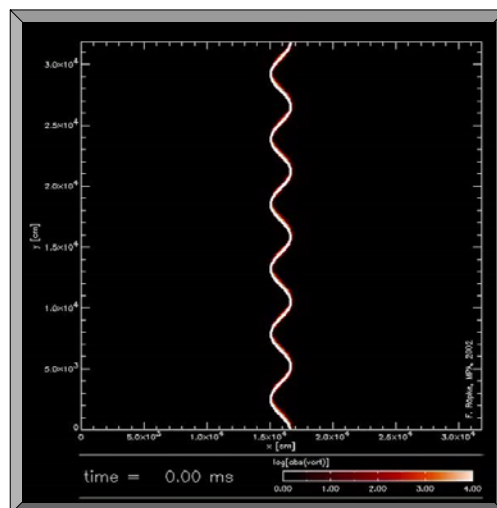


Instabilities

- Instability of thin flames:
Landau-Darrieus-instability

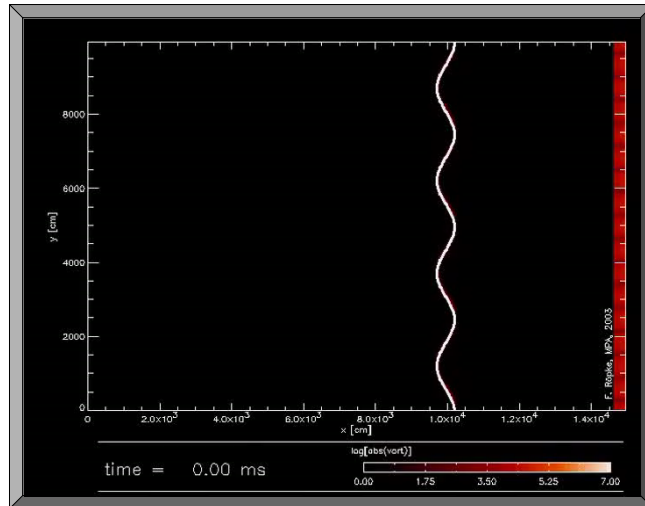
⇒ Stabilization of the flame front

Landau-Darrieus-instability for quiescent fuel



Landau-Darrieus-Instability

for vortical flow



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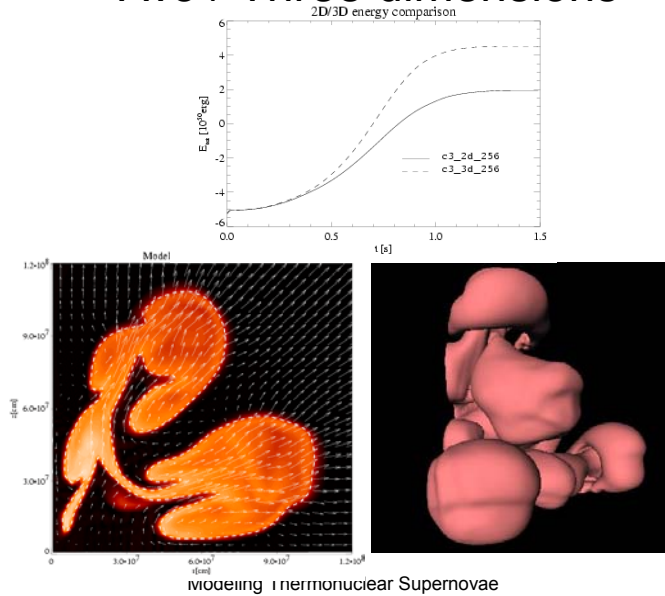
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Turbulence

- Laminar flame speed S_l
 - Above Gibson scale l_g wrinkles increase surface
 - $S_t > S_l$
- ⇒ Turbulent flame speed S_t independent of microphysics

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Two / Three dimensions



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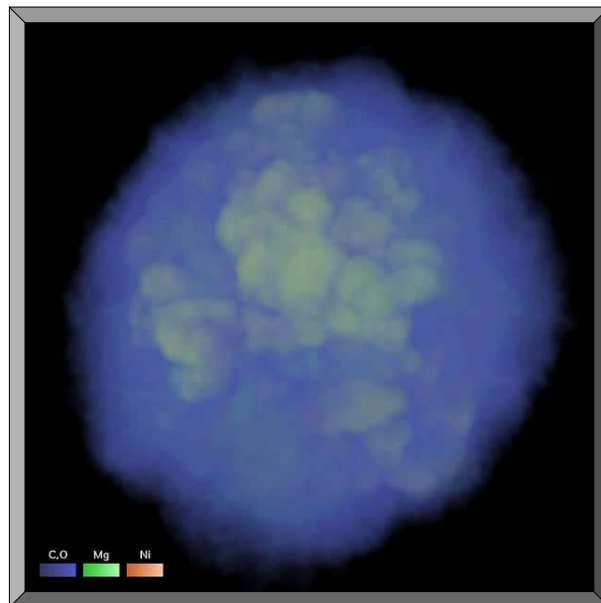
Distributed burning regime

- Critical density $\rho_{\text{DDT}} \approx 10^7 \text{ g/cm}^3$
 - Change of the flame structure
- ⇒ Detonation

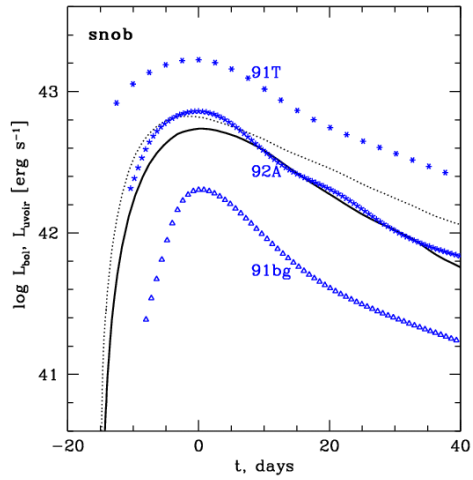
provides more intermediate-mass elements

Some parameters

- White dwarf $r \approx 1500$ km
 $m = M_{\text{Chan}} \approx 1.4 M_{\text{sun}}$
- density $\rho \approx 10^9 \dots 10^7$ g/cm³
- laminar flame speed $S_l \approx 10^7 \dots 10^4$ cm/s
- flame thickness $\delta \approx 10^{-4} \dots 1$ cm
- Kolmogorov scale $l_k \approx 10^{-4}$ cm
- Gibson scale $S_l = v(l_g)$



Light Curves



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References

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1 General

Classification of Supernovae

Supernovae are classified by the occurrence of their spectral lines

I : no hydrogen lines

Ia : strong silicon lines

Ib,c : weak/strong helium lines

II : hydrogen lines

and originate from

Ia : thermonuclear explosion of white dwarfs

Ix&II : collapsing heavy stars

The type Ia Supernovae are so important, because most of them (even if there are some outliers) have homogeneous spectral properties and light curves (85%), but also dispersion in their peak luminosities. So they might be used as standard candles, or as some other kind of distance indicators, to determine the expansion rate of the universe and geometrical structure, like we heard last week. It is necessary to have a good knowledge about the explosion, in order to determine systematic differences and the correlation between the luminosity and the shape of light curve.

Here we have an example for the light curve of a supernovae in the red and blue band. While the red one is more or less similar at different Supernovae, the blue one is what we use for comparison.

2 Model

Supernovae Ia are brightly burning, so they have to get much energy out of other processes, like the fusion of light elements. While there is no clear knowledge about the progenitor and several candidates, there are only two preferred models, which more or less satisfy the constraints. Those are

- the absence of hydrogen and the presence of a silicon absorption line, which means the progenitor should contain no or only less hydrogen and some nuclear processes are going on.
- homogeneous light curves need a homogeneous class of progenitors
- the shape of the light curve indicates that it has to be a compact star and we have the radioactive decay of ^{56}Ni (nickel) to ^{56}Co (cobalt) and following to ^{56}Fe (iron)

this all leads us to a white dwarf.

Since there are also significant differences between Supernovae Ia, there may be not only one class of progenitor.

Some Possibilities are a white dwarf in a binary system accreting mass from the companion star (single-degenerate-scenario) or the merging of two white dwarfs (double-degenerate).

Most appropriate systems of binary white dwarfs have a mass less than the Chandrasekhar mass and also it is difficult to say, how the merging of two stars with likely different conditions (mass, angular momentum) may lead to the same burning conditions. So this model does not fit the regular Ia Supernova, but it may be considered for some special ones, like the SN1991T, which was extremely bright.

The single-degenerate scenario can be either a white dwarf at Chandrasekhar mass, or one with a mass less than that. The Sub-Chandrasekhar mass white dwarf has accumulated a layer of helium and can be brought to explosion by a detonation in this layer. The outcomes fit the subluminous supernovae.

The preferred model is the thermonuclear disruption of a single white dwarf at Chandrasekhar mass. It has some advantages, apart from the absence hydrogen lines.

They provide very homogeneous starting conditions by

- a similar amount of energy

The white dwarfs accrete mass from their companion star, until they reach a critical mass, called Chandrasekhar mass. With the absorbed mass, the density is growing until fusion begins. So the explosion takes place at a fix mass.

- a similar development of the explosion

The white dwarfs have a similar composition and so the same amount gets burned and we get similar lines for calcium (Ca), silicon (Si), magnesium (Ma) and sulfur (S) at maximum and for iron (Fe) in a later phase

Principle of the Supernova-explosion

The White Dwarfs consisting of carbon (C) and oxygen (O) accretes mass from his companion star, until it reaches the Chandrasekhar-mass. Here density and temperature are high enough to fuse the carbon and oxygen - so we have a flame ignited. Due to heat conduction the flame burns its way from the center outwards. This leaves hot and light burnt material behind, while we got cold and dense fuel in front of the flame. This causes the forming of bubbles, which ascend into the fuel, which in turn leads to turbulences. Those turbulences enlarge the flame surface and so burning rate, which finally leads to an explosion.

The explosion itself does not take much time and the remains should cool down fast, instead of illuminating for several weeks. This can be explained by the decay of ^{56}Ni (nickel) to ^{56}Co (cobalt) and following to ^{56}Fe (iron) which takes 80 to ninety days.

3 Simulation

Describing the explosion, we have a very big range of different time- and length scales

- accretion of matter: some billion years
- explosion: about two seconds
- radius of the white dwarf: thousands kilometer
- size of the flames: less than a millimeter

So one has to regard on the parts, accretion, ignition and explosion individually.

3.1 Numerical methods

In principle one has to solve equations of hydrodynamics and those describing the fusion of carbon and oxygen. Additional one has to consider the energy transport through heat conduction and turbulences. In principle are these the same equations like in a simulation of an Otto-engine. To simplify the equations of hydrodynamics, we make the problem discrete and map it onto a grid. So you get a coupled set of nonlinear algebraic expressions instead of differential equations, which can be solved easier. In newer simulations the grid expands with the white dwarfs. This is called **LARGE-EDDY-SIMULATION**. But everything on small scales, which cannot be resolved is described by numerical models. Those sub-grid objects should have only little effects on large scale flame propagation.

The flame can be seen as a sharp discontinuity between the fuel and the ashes and is modeled using the **LEVEL-SET** technique. Here the moving fronts are associated with the zero level of a distance function G . This requires knowledge of the flame propagation velocity, which is composed of the advection of the flame in the flow and its self-propagation due to burning. The flame front is described by the G -equals-0-Level.

The models should be parameter-free, which means, once they are build, they should only depend on physical starting conditions, like composition of the white dwarf and flame ignition. As composition we choose the same ratio for carbon and oxygen and approximate

- that we have fully ionized plasma
- we can represent the flames by a sphere with perturbations or bubbles

There were numerous simulations with different initial conditions in 2 and 3 dimensions. The results agree very good with the observations, besides that to less carbon and oxygen are burned and so to less intermediate mass elements are produced. Here we have the observed and calculated spectra of a supernova, which fit quite good.

Here are the observed occurrences of elements in comparison with the simulated ones, and we see, iron is good, but to much carbon and oxygen and to less intermediate mass elements.

Now we look in detail at the explosion.

3.2 ignition

As the white dwarf grows to the Chandrasekhar mass the central density is increasing and the thermonuclear fusion starts. During the next thousand years the burning conditions change and due to convection and temperature fluctuations a thermonuclear fuse leads the formation of a flame.

The flame ignition is one of the free parameters and turns out to be very influential on the energy production. So we have several different choices. A single ignition in the center, multi-spots around the center and under certain conditions, burning bubbles can rise some hundred kilometer and can be responsible for off-center ignition.

Multiple ignitions around the center seem to increase the strength of the explosion, while a single ignition in the center leads to nearly no instabilities, because of the high symmetry. Here we have some examples of flame ignitions, and we see, having too many ignitions has the effect, that flames merge again and the behavior is similar to less spots and the advantage of a large surface is lost.

3.3 explosion

The fuel can reach a critical temperature T_C , at which the burning proceeds instantly compared to the fluid motion. So when the fuel exceeds this temperature, a reaction zone forms between burned and unburned material. The nuclear burning is confined to this thin layer that either propagates as a subsonic flame or as a supersonic detonation.

deflagration moves subsonically. The burning front consists of a diffusion zone, which heats the fuel, and a thin reaction layer, where the fuel is consumed and energy is generated. The flames are strongly affected by turbulent fluctuations

detonation The heat of the burning products is high enough to form a shock wave that ignites the fuel by compressional heating. So the burning front propagates by shock heating and moves supersonically, which means unburned material cannot expand before it's burned. This cannot occur above critical density.

If there is only a prompt detonation, which consumes the whole star at the speed of sound, there would be produced only iron and nearly no intermediate mass elements. Therefore it should at least start with deflagration.

So in this FLAMELET REGIME we have a burning front moving subsonically and leaving behind hot burned material. The buoyancy of this hot burned fluid leads to the so called RALEIGH-TAYLOR-INSTABILITY. Small surface perturbations grow until they form bubbles or mushrooms that float upwards, while spikes of cold, dense fluid fall down. This increases the surface area and thereby the fuel consumption.

Secondary instabilities (KEVIN-HELMHOLTZ) coming from the velocity shear along the bubbles, lead to turbulent fluctuations in the range from large bubbles ($\approx 10^7$ cm) to Kolmogorov scale ($\approx 10^{-4}$ cm). This cannot be resolved, so one has to fall back again to statistical approximations.

An estimation for the cascade of velocity fluctuations is given by KOLMOGOROV'S LAW, which describes the mean velocity $v \approx l^{1/3}$ of turbulent eddies of size l in the case of isotropy and statistical stationarity.

Here we see the burning front, first as ignition bubbles and then propagating and forming mushrooms, with growing surface.

The laminar flame speed S_l describes the flame propagation as a function of density and fuel composition, assuming that the unburnt material is at rest. To determine accurate values, numerical solutions of hydrodynamical equations are needed. Approximating the flame as discontinuity moving with the laminar speed, one sees, that thin flames are linearly unstable, this is called the LANDAU-DARRIEUS-INSTABILITY. Accordant to this instability, perturbations grow, until a web of structures is formed and stabilizes the front, so this does not in general lead to turbulences. As a result a strong acceleration of the burning front is avoided.

Here we have the burning front moving through quiescent fuel and the instabilities grow until they form a settled structure.

Here the burning front moves through turbulent fuel. We get no calm front, but it also does not collapse and rebuilds constantly.

The GIBSON SCALE describes the scale, where turbulent velocities are so small, compared with the burning speed of the flames, that the flames burn through the eddies, before they can change their shape. If we move above the Gibson scale, where the velocity is much faster than the laminar flame speed, turbulence begins to wrinkle and deform the flame at large scales, which increases the flame surface, which again means the turbulent flame speed S_t becomes larger than the laminar speed S_l .

This means, if the turbulence is strong enough, it becomes independent of the laminar speed and therefore of the microphysics of burning and diffusion. This is called the CORRUGATED FLAMELET REGIME.

Here the comparison of the flame in two and three dimensions. In a three dimensional simulation we get more surface than in two dimensions and so we have more energy released.

Sometimes it can be observed in terrestrial experiments that turbulent deflagrations crosses over to detonation. Due to the expansion, the fuel density drops and this broadens the flame width. So it is plausible that at some later point, with small density and slow flamelets, the flame structure may change significantly.

The mechanism which triggers the transition is still unclear, but end of the flamelet regime and the start of the DISTRIBUTED BURNING REGIME would occur at densities about $\rho_{DDT} \approx 10^7 \frac{\text{g}}{\text{cm}^3}$. Neither nuclear burning nor turbulent mixing can here be properly described under simplifications. It seems that the laminar flame structure is disrupted by by turbulence and forms various different reaction zones. Since we have to less carbon and oxygen burned to intermediate-mass elements in the deflagration, this speculation would provide us the right amount at high velocities.

Some parameters of the white dwarf and the explosion

We have the white dwarfs somewhere at the end of the explosion, and the distribution of the elements, while Magnesium represents the intermediate mass elements and Nickel the iron group nuclei.

And at last the whole explosion of the white dwarf from ignition and flame propagation and the transition to the detonation.