Inflation, Dark Energy, Dark Matter

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Overview

- Inflation
  - Einstein equations with cosmological constant Λ
  - Friedmann universes with Λ, the de-Sitter model
  - Inflation as a solution to some cosmological problems
- Dark Energy (DE)
  - DE and inflation: models for Λ, the Λ problem
  - Theoretical possibilities and experimental results
- Dark Matter (DM)
  - Reasons for assuming DM
  - Candidates for DM, dark-matter searches

Einstein equations revisited

- Einstein equations with cosmological constant

$$\boxed{R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu}} = \boxed{\frac{8\pi G}{c^4} T_{\mu\nu}} + \boxed{\Lambda g_{\mu\nu}}$$

Space-time curvature

Cosmological constant

Matter: source of curvature

- The full equations including Λ follow from a variational principle, using a general Lagrangian restricted only by few basic assumptions:

$$L = (-g)^{1/2} \left(\boxed{R} - \boxed{2\Lambda} \right) + \boxed{L_{matter}}$$

- Interpretation of the Λ term:
  - Additional source of curvature, e.g., by non-zero vacuum energy
  - Dimension of Λ is 1/(length^2)
    => property of space-time at large scales > 1 / (Λ$^{1/2}$)

Cosmological constant vs. ordinary matter

- **Perfect fluid** of density ρ, pressure p, and 4-velocity u.
- Energy-momentum tensor

$$T_{\mu\nu} = (\rho_0 c^2 + p) u_\mu u_\nu - p g_{\mu\nu}$$

- Equation of state links density and pressure, e.g.,

$$p = w \rho$$

$w = 0$, 1/3 for non-rel. matter, radiation

- Adiabatic expansion:
non-relativistic matter: $\rho \propto a^{-3}$
radiation: $\rho \propto a^{-4}$

Cosmological constant

- Energy-momentum tensor

$$T_{\mu\nu} = \frac{c^4}{8\pi G} \Lambda g_{\mu\nu}$$

energy density

$$\rho_\Lambda = \frac{c^2}{8\pi G} \Lambda$$

- Classical Λ acts as a gas with equation of state $w = -1$; but may also comprise contributions with different equation of state
- Λ can be a constant during expansion; but more complex contributions evolve differently
- All contributions to Λ are called "dark energy"

Cosmology with Λ – the Newtonian limit

- $\Lambda > 0$ leads to repulsive instead of attractive gravity.
- Consider the Newtonian limit: the gravitational potential Φ is determined by the sources according to the Poisson equation (including the contribution of pressure and Λ)

$$\Delta \Phi = 4\pi G(\rho + 3p) = 4\pi G(\rho_{matter} + 3p_{matter}) - c^2 \Lambda$$

- A solution to this equation in vacuum is given by a repulsive gravitational force increasing (for $\Lambda > 0$) with distance,

$$F_\Lambda = \frac{1}{3} \Lambda c^2 r$$

- This force is relevant on large scales only.
- Condition for repulsive gravity for more general forms of dark matter:

$$\rho + 3p < 0$$

Cosmology with Λ – Friedmann equations

- Robertson-Walker metric

$$ds^2 = dt^2 - a^2 \left[\frac{dr^2}{1 - kr^2} + r^2 \, d\Theta^2 + r^2 \sin^2 \Theta \, d\Phi^2 \right]$$

- The dynamics of this metric follows from the Friedmann equations

$$\left(\frac{\dot{a}}{a} \right)^2 = \frac{8 \pi G}{3} (\rho + \rho_\Lambda) - \frac{k}{a^2}$$

$$\left(\frac{\ddot{a}}{a} \right) = \frac{-4 \pi G}{3} (\rho - 2 \rho_\Lambda) + 3 p$$

- The equations describe the interplay of the repulsive and attractive forces due to Λ and the fluid, respectively. Positive Λ accelerates the expansion of the universe, negative Λ decelerates it.

- Hubble and deceleration parameter, critical density, relative density

$$H = \frac{\dot{a}}{a}, \;\; q = -\frac{\ddot{a} \, a}{\dot{a}^2}, \;\; \rho_c = \frac{3 H^2}{8 \pi G}, \;\; \Omega = \frac{\rho}{\rho_c}$$

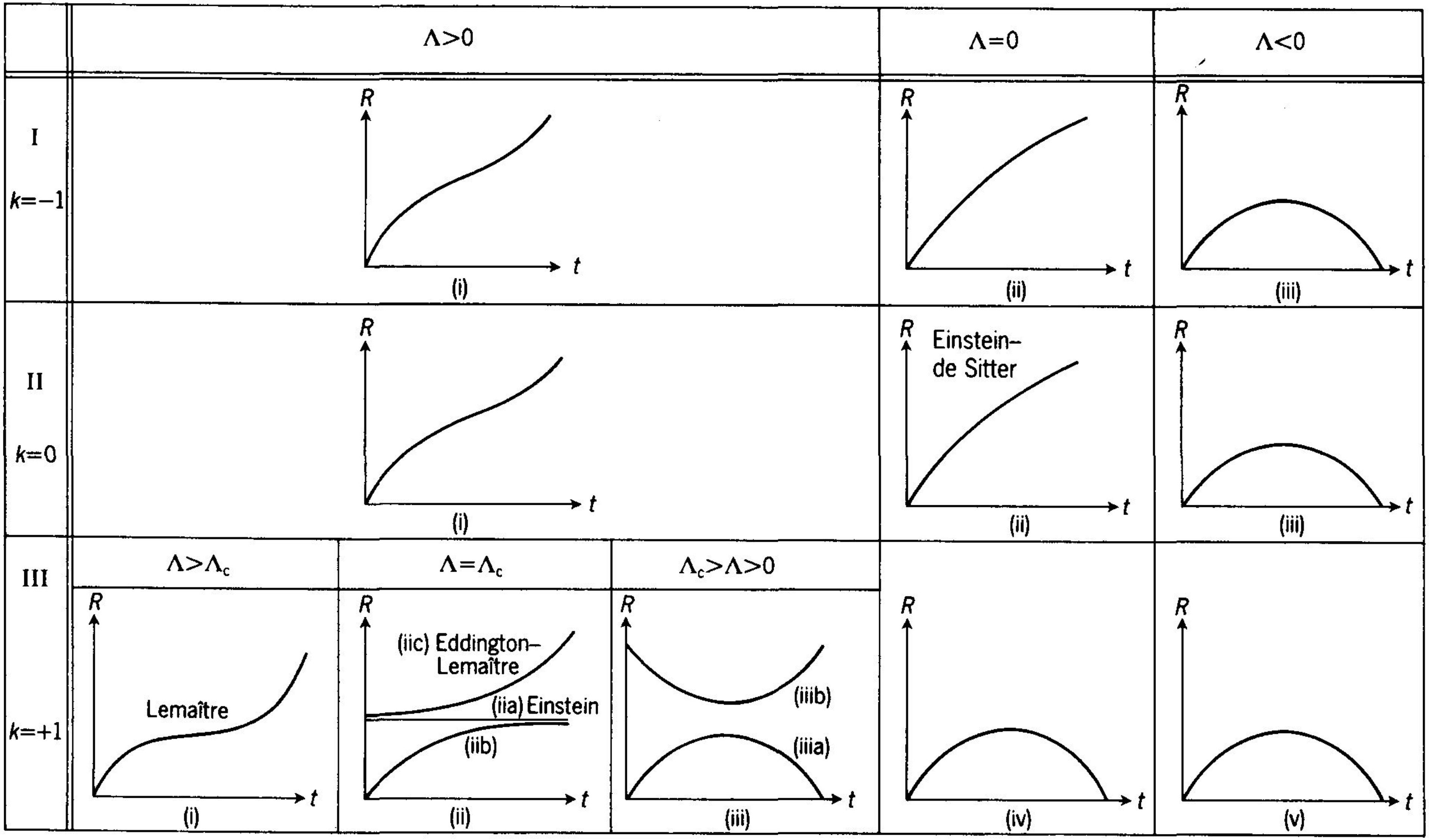
Cosmology with Λ – Friedmann models

- With $\Lambda \neq 0$, the equations admit static, however unstable, solutions with constant a. Depending on the curvature parameter k and the sign and magnitude of Λ, different evolutions of a are possible.
- De Sitter solution: consider a vacuum universe with $\Lambda > 0$, and $k = 0$.
  - Friedmann equations => exponentially increasing scale factor,

$$a(t) = a_0 \exp\left[\left(\Lambda / 3 \right)^{1/2} t \right]$$

  - Approximate solution for a non-vacuum universe if Λ dominates over the matter density and the curvature term.
- Typically, the evolution of the scale factor differs from $\Lambda = 0$ models leading to a modified Hubble law.
  => measurements of Hubble constant and deceleration parameter can, in principle, be used to determine the value of Λ.

Cosmology with Λ – Friedmann models

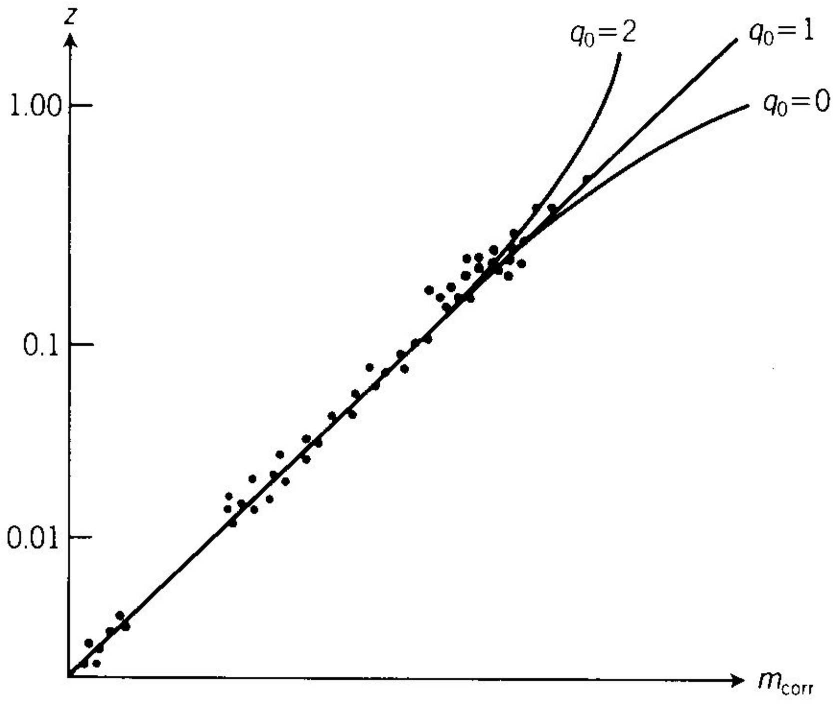


Evolution of Friedmann models with cosmological constant; from d'Inverno (1994)

From models to the real universe

- To determine the cosmic parameters of our universe, one has to measure the Hubble relation by means of independent observations
  - measuring luminosity distances using standard candles such as SNe Ia => determine q_0, thus $(\frac{1}{2}\,\Omega_m - \Omega_\Lambda)$
  - probing the geometry of space-time using CMB measurements => determine the total mass density $(\Omega_m + \Omega_\Lambda)$
  - measuring the matter content (galaxy clusters, large-scale structure) => determine the density of matter (Ω_m)
- From these measurements, one can obtain the densities of matter and Λ and the cosmic equation of state
  - accelerated expansion in a nearly flat universe
  - dark energy makes up for most of the density
  - cosmological constant seems likely

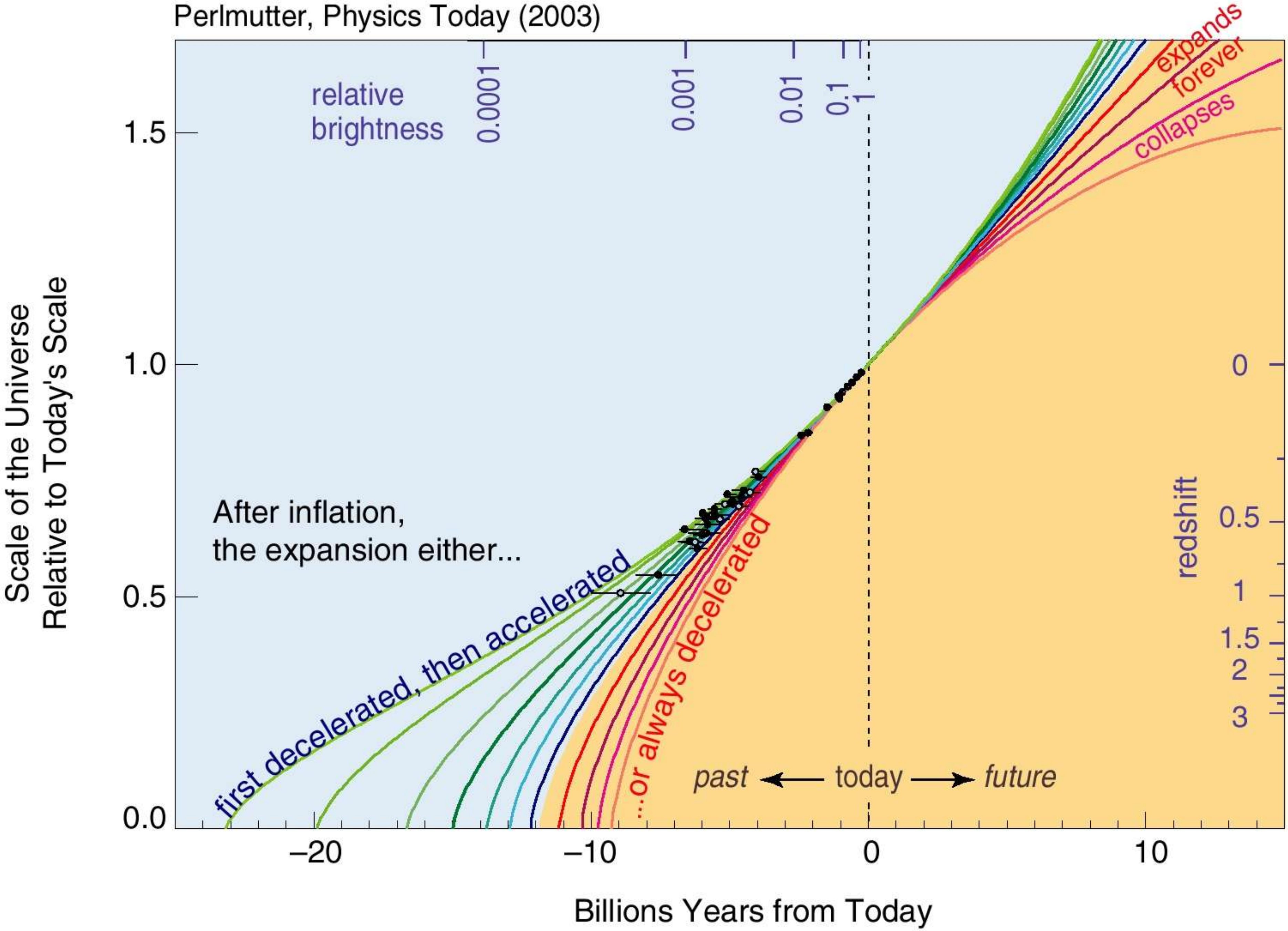
From models to the real universe

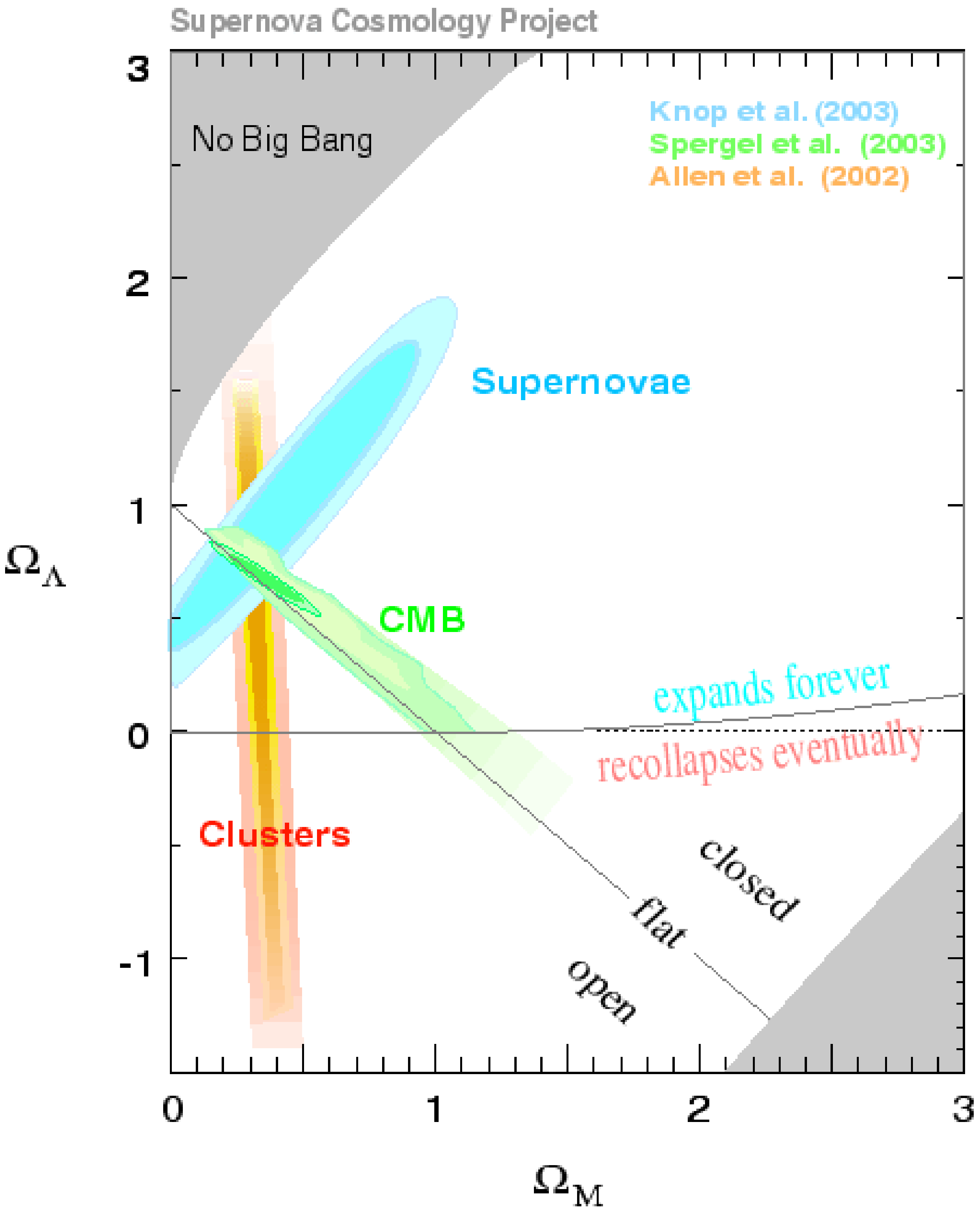


Hubble relation between apparent magnitude and redshift
for different values of the deceleration parameter;
from d'Inverno (1994)

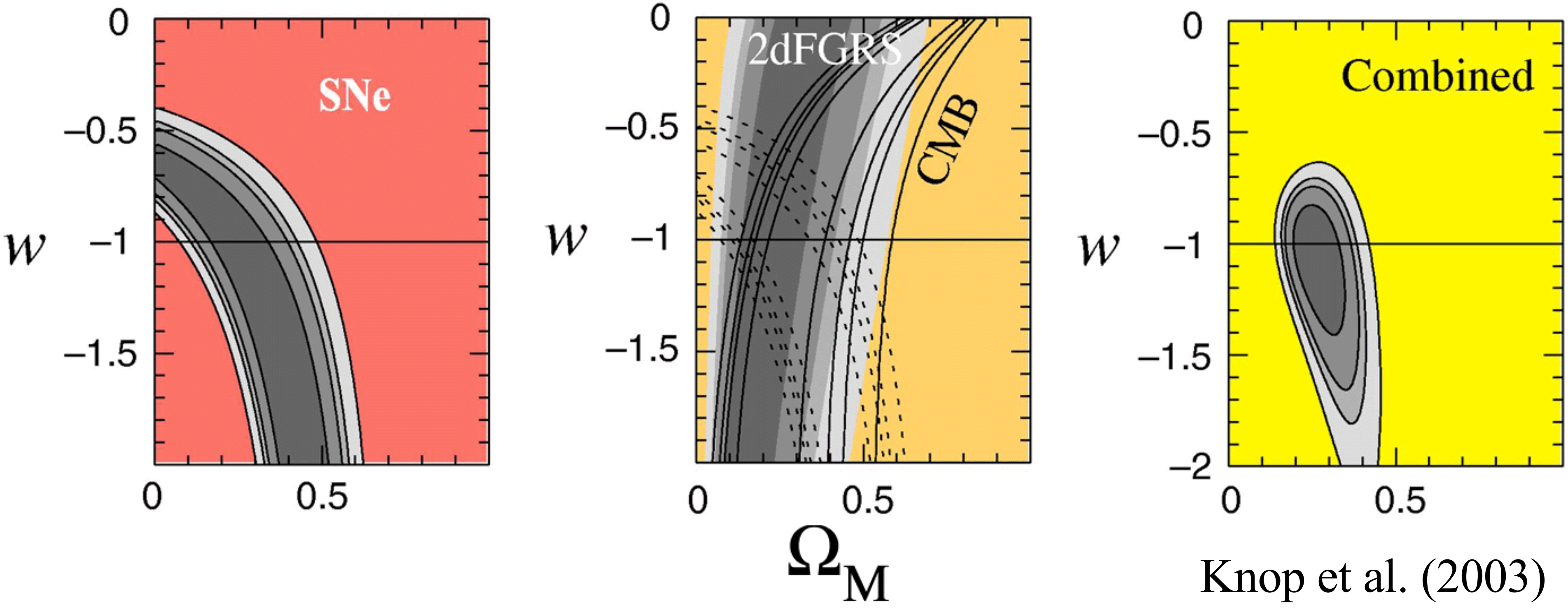
Expansion History of the Universe

Perlmutter, Physics Today (2003)





Knop et al. (2003), results from CMB and clusters added



Knop et al. (2003)

Inflation in our universe

- Inflation is a (de-Sitter-like) phase of accelerated expansion ($\ddot{a} > 0$) triggered by positive dark energy.
- To start inflation, the total energy density must be dominated by Λ; to end it, the vacuum-energy density must decrease below the matter density.
- Could something similar to the de-Sitter scenario also apply to some evolutionary stage of our universe?
  - At present: possibly yes. Until a few billion years ago, the universe has been matter dominated, but due to the different expansion laws for matter and Λ the cosmological constants took over ~ 5 billion years ago, and now is leading to an accelerating expansion.
  - At first glance, Λ cannot play a major role in the early universe since the density of matter and radiation grows as we approach the big bang whereas Λ stays constant, but inflation might also have taken place in the early universe if Λ can evolve with time rather than being constant.

Why early inflation in our universe?

Problems of the non-inflationary cosmological standard model:

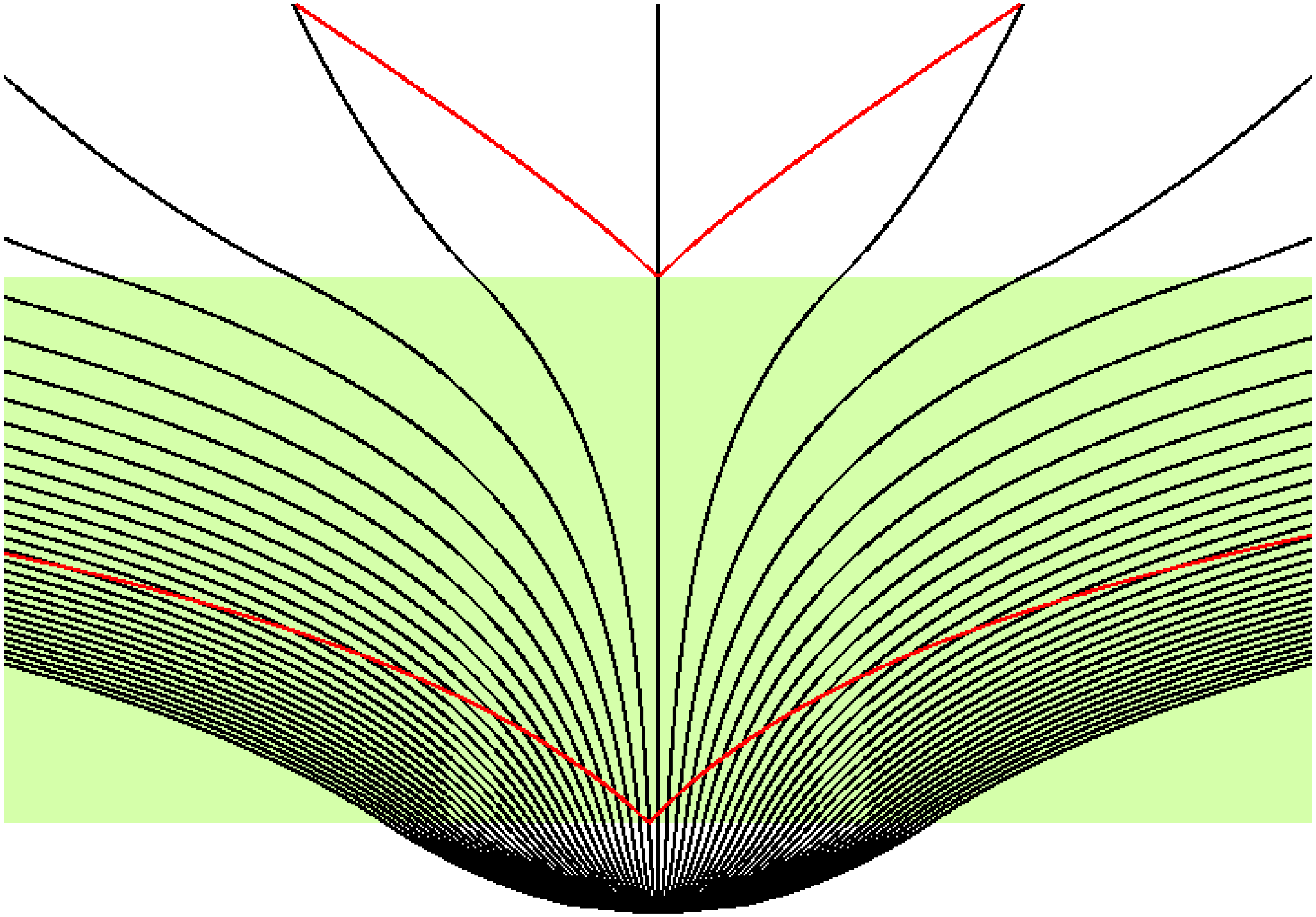
- Observed absence of magnetic monopoles and other heavy particles relic from the hot early phases of the universe
- Nearly exact flatness of space-time: without Λ, $|\Omega - 1|$ increases with time. But now, $\Omega = 1$ to high precision. Thus, even more in the early phases. Why?
- High degree of cosmic isotropy:
  - We see regions in space farther apart from each other than one Hubble length, i.e., with apparently no causal connection. Light emitted at the Big Bang had simply not enough time to travel from one region to another one, providing information to synchrone the physics. Nevertheless, these regions show remarkably equal conditions.

Why early inflation in our universe?

These problems can be solved, or at least alleviated, by the enormous expansion factor during inflation.

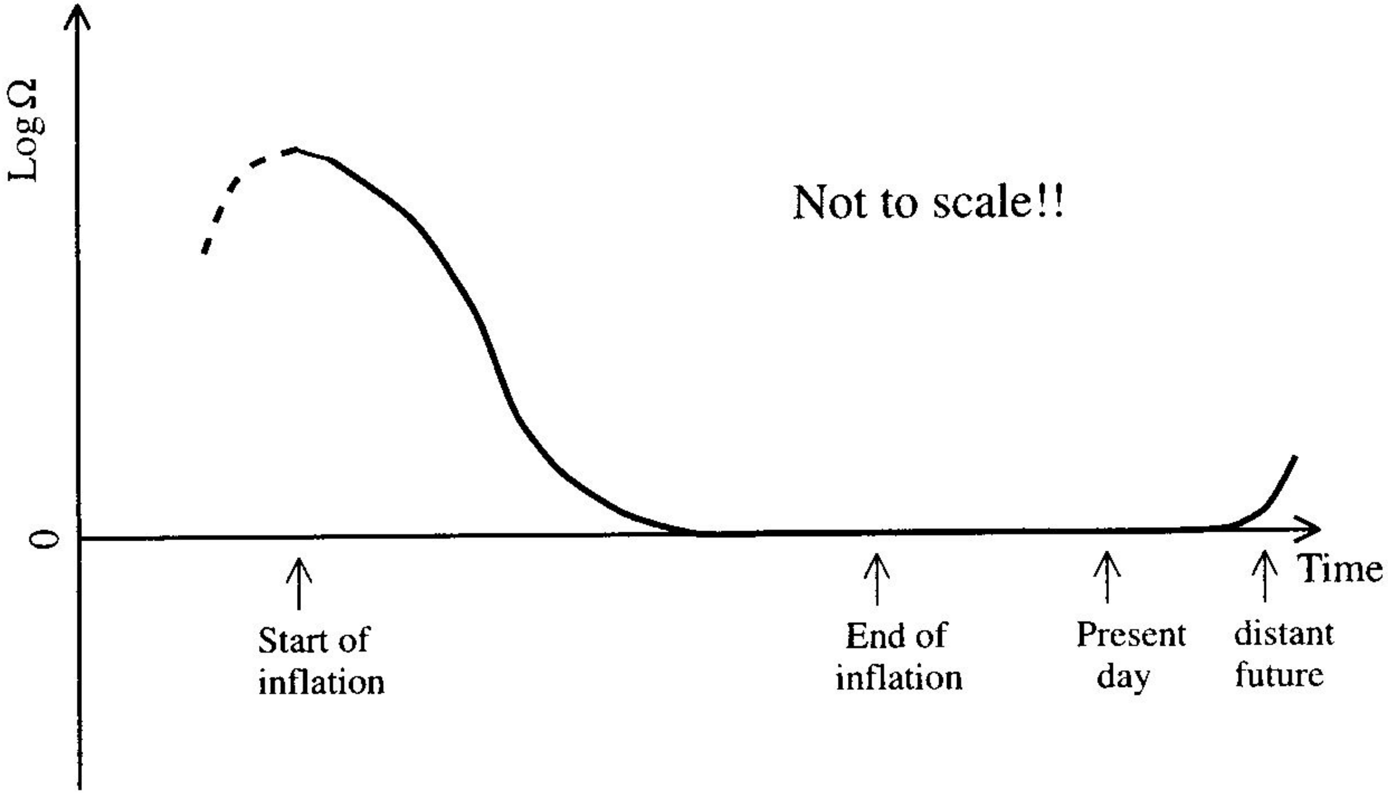
- Comoving horizons decrease during inflation, i.e., a point sees less and less of the universe as distant parts pass through the Hubble radius. The visible universe is only part of one initially causally connected region blown up to super-horizon size during inflation.
- For the same reason, relic particles are diluted
- $|\Omega - 1|$ decreases during inflation, bringing Ω close to 1.
- Quantum fluctuations present in the early universe, or generated during inflation are amplified spatially, and typically, a scale-invariant power spectrum is established. The fluctuations then act as seeds for cosmic structure formation.
  => checking inflation with the help of structure-formation simulations

Why early inflation in our universe?



World lines of matter versus horizon size before, during (green), and after inflation

Why early inflation in our universe?



Evolution of the density parameter Ω during inflation;
from Liddle & Lyth (2000)

Dark Energy: possible contributions to Λ

- Dark energy: all kinds of “matter” providing a negative pressure. Putative properties are
  - equation of state with $w < 0$ (cosmological constant: $w = -1$)
  - repulsive gravitational force
  - weak interaction => homogeneous density, does not take part in structure formation
- Possible candidates are:
  - A “bare” cosmological constant. No preferred scale can be inferred.
  - A non-zero vacuum energy provided by vacuum fluctuations of quantum fields => quantum field theory
  - Scalar fields as predicted by, e.g., the theory of supersymmetry (so-called quintessence)
- The total Λ is the sum of these (and possible further) contributions.

Dark Energy: quantum fluctuation of vacuum

- Energy associated with space itself (spontaneous creation and destruction of virtual particles; evidence: Casimir effect). Comparison with classical $p\,\mathrm{d}V$ work yields an equation of state with $w = -1$.
- Quantum fields can be viewed as a set of harmonic oscillators in momentum space. In their ground state (n=0), these oscillators have a non-zero energy

  $$E_n = \hbar\omega(\vec{k})(n+1/2)$$

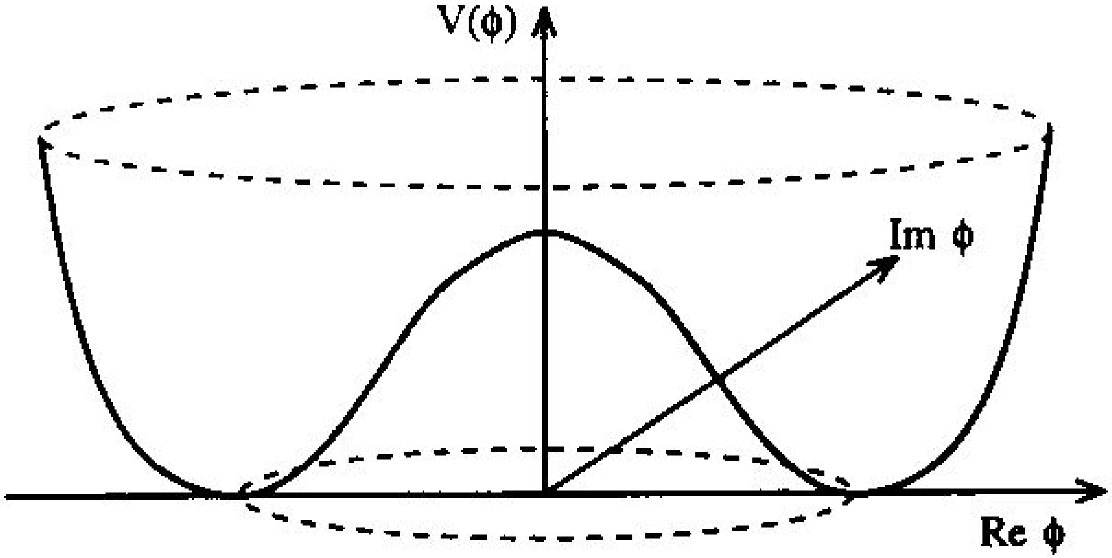
  the total vacuum energy is then given by the sum over all oscillators. The resulting can be transformed into a density of

  $$\rho_V \sim \hbar k_{max}^4$$

  k_{max} is the maximum wave vector of the field, taken to be the energy scale at which QFT fails.
- The Λ problem: take the inverse Planck scale (~10^{19} GeV) for k_{max} => $\rho_V \sim 10^{92}$ gcm^{-3} (!) which is larger than the cosmologically acceptable value by a mere 120 orders of magnitude.

Dark Energy: scalar (spin-0) particles

- Need “matter” with negative pressure: e.g., spin-0 particle, i.e., scalar field
- Consider a scalar field ϕ (“inflaton”) in a potential V(ϕ). Its density and pressure are given by
$$\rho_\phi = 1/2\,\dot{\phi}^2 + V(\phi)$$
$$p_\phi = 1/2\,\dot{\phi}^2 - V(\phi)$$
The inflation condition is fulfilled if $\dot{\phi}^2 < V$
- Analysing the equations of motion, one can derive the conditions for slow-roll inflation:
$$\left(\frac{V'}{V}\right)^2 = \left(\frac{d\ln V}{d\phi}\right)^2 \ll \frac{2}{M_{Pl}^2}$$
$$\frac{V''}{V} = \frac{d\ln V}{d\phi}\frac{d\ln V'}{d\phi} \ll \frac{1}{M_{Pl}^2}$$
- Example fulfilling these conditions: the Mexican-hat potential
$$V = V_0 - 1/2\,m^2|\phi^2| + 1/4\,\lambda|\phi^4|$$
allowing for spontaneous symmetry breaking. ϕ starting at zero and running down to its vacuum expectation value will trigger inflation.



$$V = V_0 - 1/2\, m^2 \left|\phi^2\right| + 1/4\, \lambda \left|\phi^4\right|$$

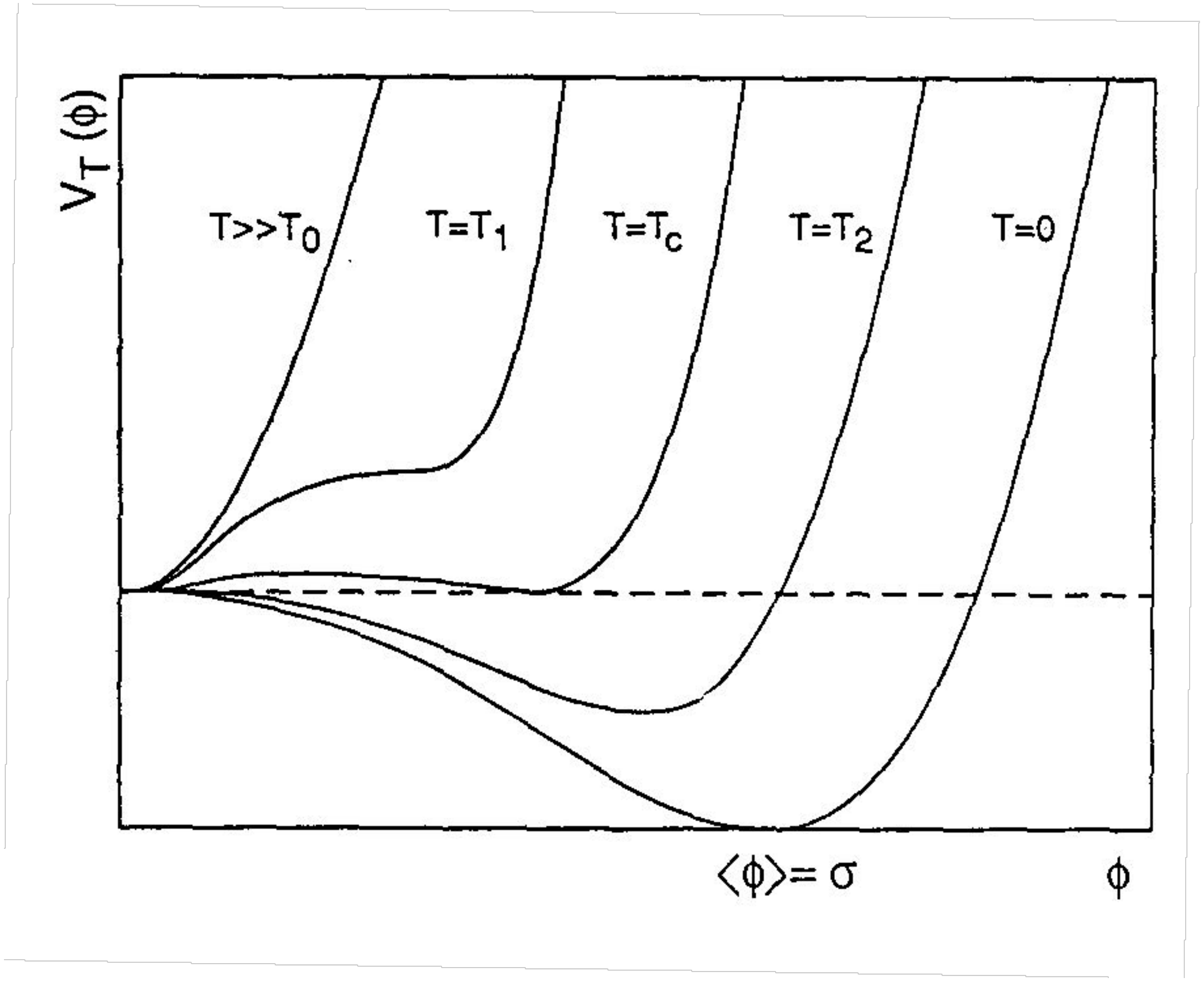
Dark Energy: the inflaton mechanism

- Imagine the scalar field ϕ interacting with some other field ψ, possibly in equilibrium, i.e., $\psi = T$. Then the potential will depend on ψ / T, e.g.:

$$V = V_0 - 1/2\, m^2 \phi^2 + 1/4\, \lambda \phi^4 + 1/2\, \mu \phi^2 \psi^2.$$

  - At high T, the potential has a non-zero minimum V_0 at $\phi = 0$. The inflaton is in a “false vacuum” with non-zero energy
  - T falls below a critical value at which V develops a new minimum V at $\phi \neq 0$. ϕ rolls down the potential, giving rise to slow-roll inflation.
  - After rolling down the potential, the inflaton oscillates about the new vacuum state, later decays, eventually reheating the universe.
- Identify this transition with one of the symmetry-braking phase transitions in the early universe, e.g., GUT symmetry breaking. Then, T is of the order of ~ 10^{15} GeV, and the epoch of inflation is set during the first $10^{-(35...30)}$ s; the universe expands by a factor of ~ $10^{30...45}$.
- More complex models involving, e.g., tunneling through the potential, are required.

Dark Energy: the inflaton mechanism



Possible *T*-dependent inflaton potential;
from Klapdor-Kleingrothaus & Zuber (1997)

Dark Energy: possible scalar fields

- Possible candidates are
  - the Higgs field (electro-weak phase transition at 200 GeV),
  - quark-antiquark bilinears (chiral-symmetry breaking in strong interaction),
  - further particles responsible for unknown phase transitions in the early universe
- Typically, $w > -1$ for these models.
- The Λ problem reappearing: also here, values of Λ that are by far too large in magnitude can be predicted.

Dark Energy: the Λ problem

- Various theories predict values of Λ, but none seems to fit the observed values.
- The individual, apparently uncorrelated contributions to Λ_{tot},

$$\Lambda_{tot} = \Lambda_0 + \Lambda_{QFT} + \Lambda_{scalar} + \ldots$$

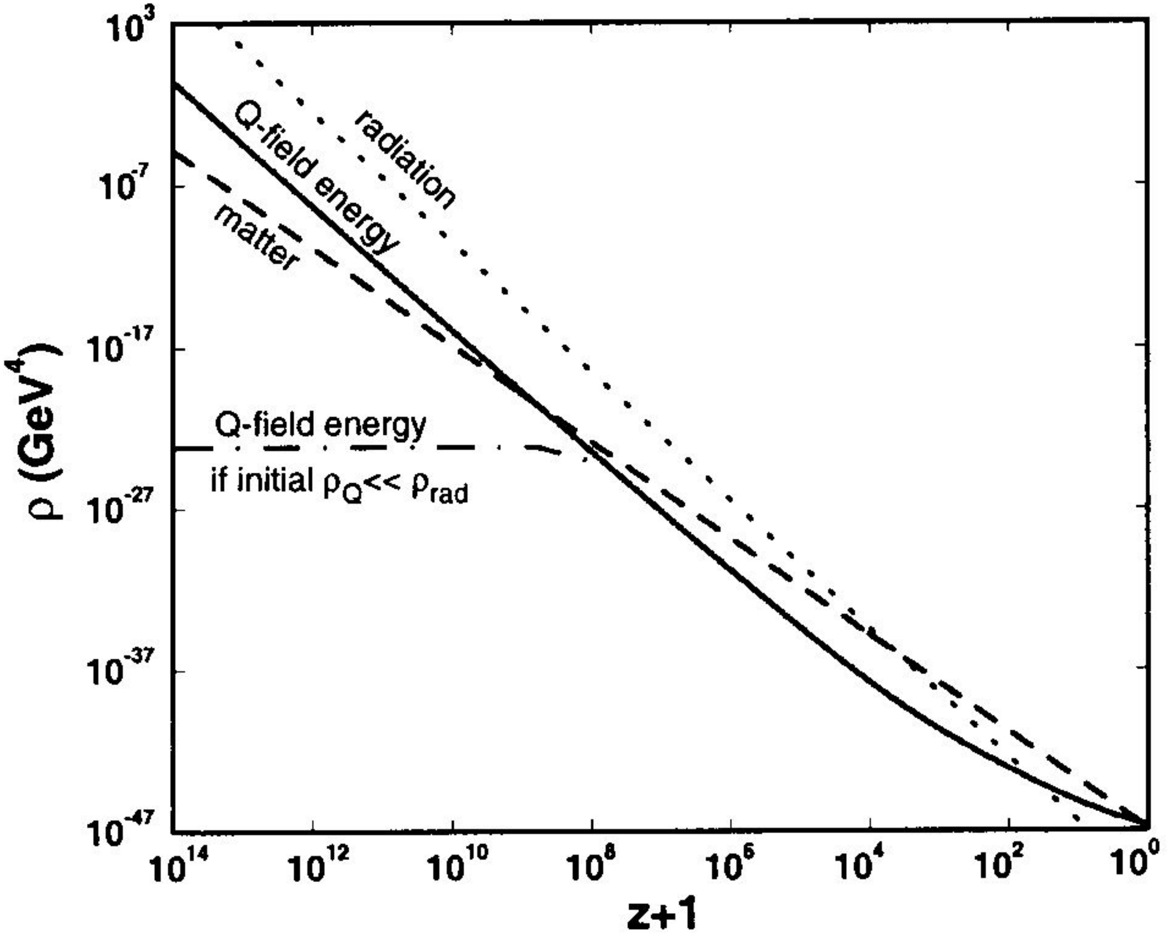
  all by far too large, need to cancel each other nearly exactly. Why at all, and why does Λ start to become important now, and not in the early universe or the farthest future?

- No conclusive solution found yet, but ...

Dark Energy: ways out of the Λ problem

- flat-space-time supersymmetric theories predict vanishing vacuum energy in the SUSY state, but not in current-day SUSY-broken world; non-flat SUSY breaking requires fine-tuning.
- (super-)string-/M-theories may allow for small Λ with SUSY broken
- if is a free parameter, as possible with wormholes, the wave function of the entire universe may have a very pronounced probability maximum at $\Lambda = 0$, i.e., the universe will be dominated by such regions
- tracker fields: certain GUTs predict fields the energy density of which evolve in a way similar to the one of matter and radiation; dark energy tracks the evolution of matter and radiation density in the early phases (radiation-dominated universe) and “freezes out” in the matter-dominated epoch (e.g., k-essence models).

Dark Energy: ways out of the Λ problem

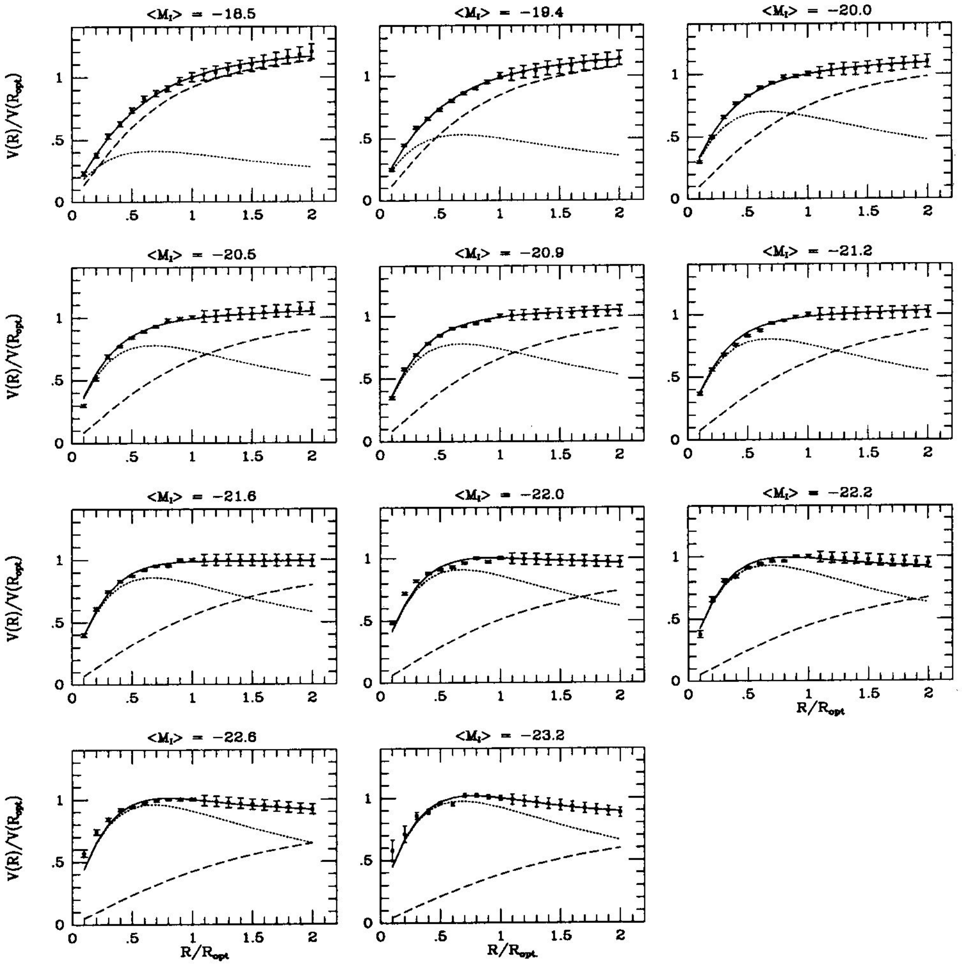


Evolution of the density of matter, radiation, and quintessence tracker field; from Zlatev et al. (1999)

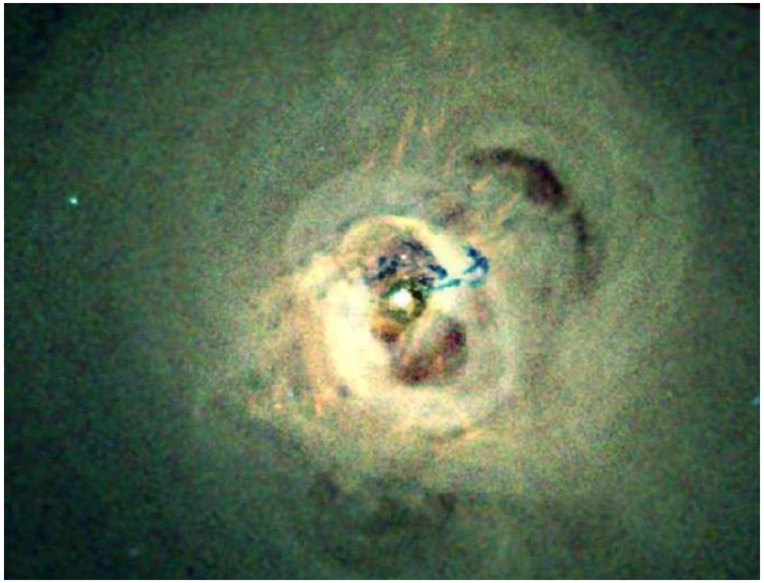
Dark Matter: evidence

- Hints towards DM being needed were found on different scales:
  - too few visible matter in spiral or elliptic galaxies to account for observed dynamics (rotation curves or velocity dispersion)
  - gravitational wells of galaxy clusters, as measured by galaxy velocities or the intra-cluster hot gas, are too deep for visible matter alone
  - cluster masses as inferred from gravitational lensing are much larger than the mass visible in stars and gas
  - coherent motions of galaxy clusters require more mass than oberved
- Theoretical arguments:
  - CMB or SN Ia measurements indicate the presence of DM
  - big-bang nucleosynthesis might require a larger baryonic density than is observed but also rules out that the entire DM is of baryonic nature
  - inflation suggests $\Omega = 1$
  - structure formation works best with DM

Dark Matter: evidence



Typical rotation curves of spirals of different luminosity, and the contributions of disk and halo; from Persic et al. (1999)



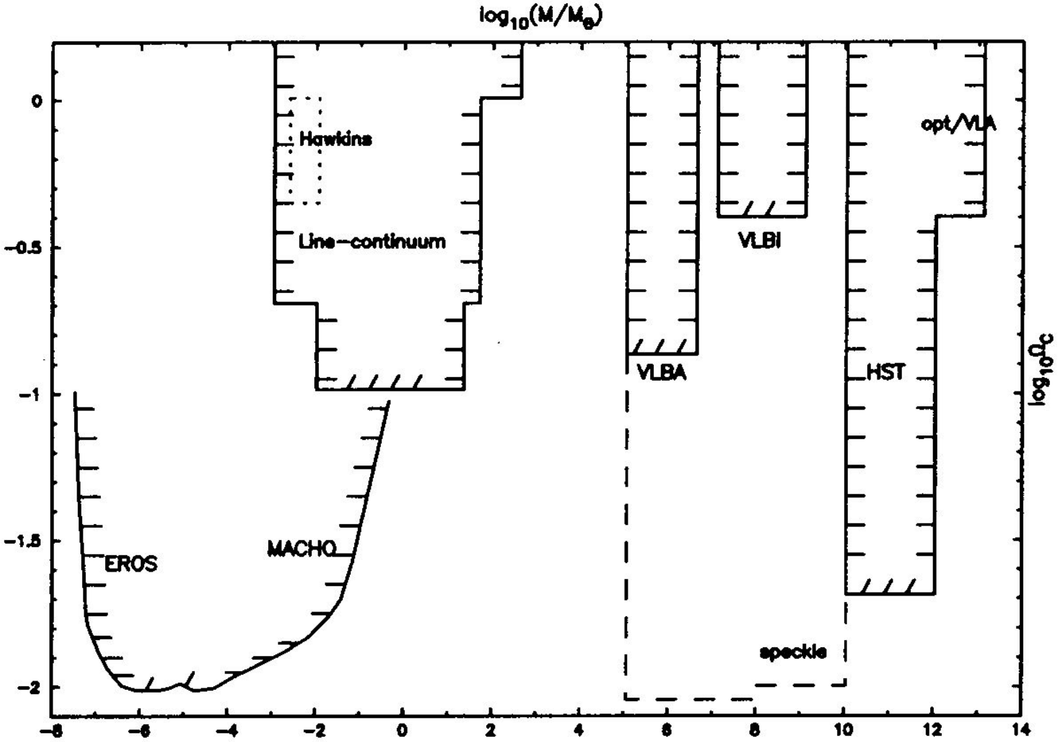
Chandra image of Per cluster
HST image of cluster lensing

Dark Matter: candidates

- Baryonic DM:
  - “snowballs”, sub-planet objects, not gravitationally bound
  - sub-stellar objects, e.g., brown dwarfs
  - low-mass, hence very dim, stars (spectral type M)
  - compact objects, i.e., white dwarfs, neutron stars, black holes
  - cold diffuse gas
- Non-baryonic DM: plenty of models are discussed, from neutrinos to SUSY matter, proposed masses ranging from less than meV up to Planck mass for particles and even more for topological defects. To be dark, the particles have to be interact only weakly.
  - Cold DM (CDM), particles with masses > MeV, e.g., SUSY ...-inos (WIMPs such as the lightest SUSY particle, perhaps the neutralino) or magnetic monopoles, non-relativistic particles
  - Hot DM, particles with masses < keV, e.g., standard neutrinos, relativistic particles, axions

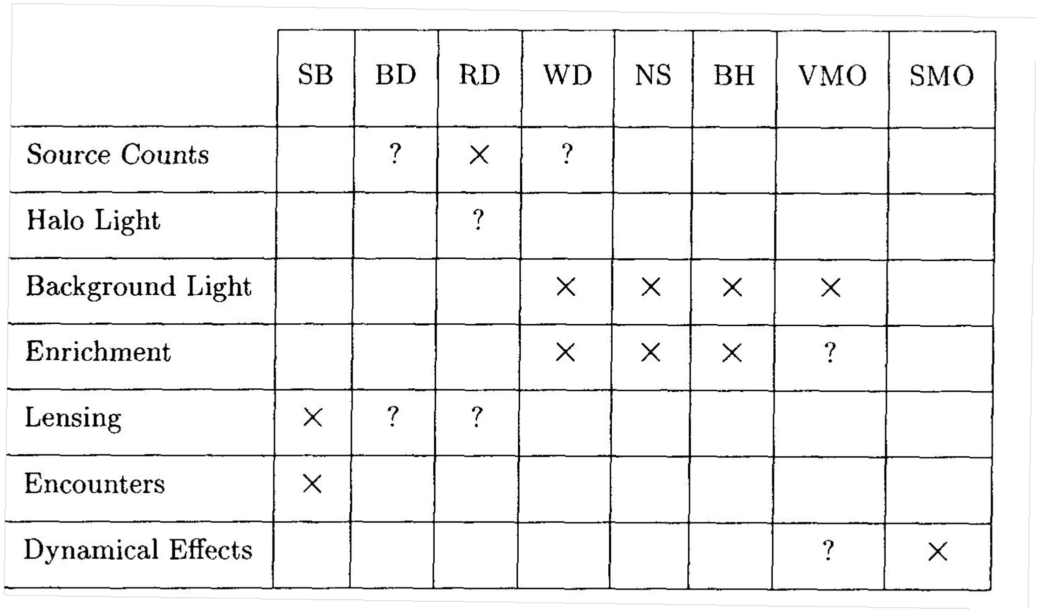
Dark Matter: searching for baryonic DM

- gravitational lensing
  - (macro-)lensing: aberration of stellar background light (e.g., Einstein rings) due to foreground masses leading to multiple images; sensitive to masses > 10^5 solar masses
  - microlensing: brightening of a background star as a DM object passes by, has to be distinguished from intrinsic variability of the star; sensitive to masses < 10^3 solar masses
- Further constraints can be obtained from number counts, from the dynamical impact of these objects, from their contribution to background light or to nucleosynthesis
- Constraints on MACHOs (massive compact halo objects)
  - large fraction of brown dwarfs (> 10%) can be ruled out in our galaxy, an even lower bound can be placed on low-mass stars
  - white dwarfs, neutron stars, stellar-mass black holes can be ruled out
  - very-massive and super-massive objects are also not very likely



Limits on the density of baryonic DM of different mass; from Carr (2000)

Viability of different baryonic DM models. Models excluded due to number counts, observations of background light, chemical evolution, lensing events, or dynamical reasons. From Carr (2000). (SN: snowballs, BD/RD/WD: brown/red/white dwarfs, NS: neutron stars, BH: black holes, VMO/SMO: very massive/supermassive objects)



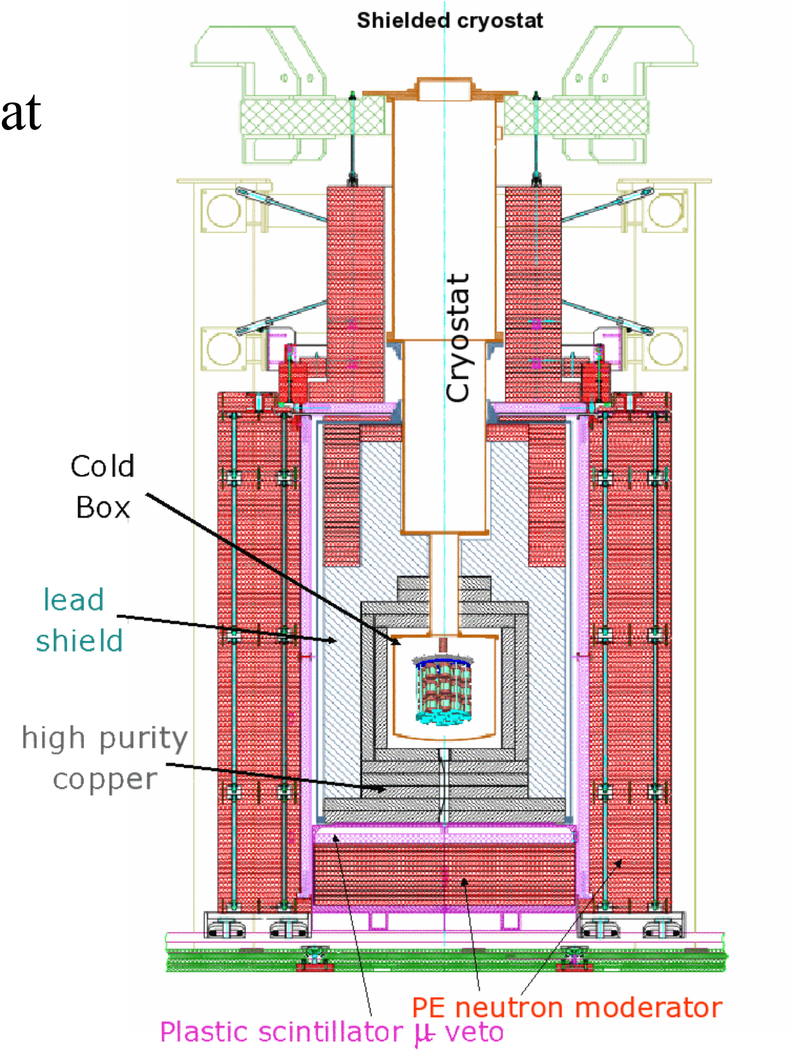
	SB	BD	RD	WD	NS	BH	VMO	SMO
Source Counts		?	×	?				
Halo Light			?					
Background Light				×	×	×	×	
Enrichment				×	×	×	?	
Lensing	×	?	?					
Encounters	×							
Dynamical Effects							?	×

Dark Matter: searching for non-baryonic DM

- Constraints from lab experiments
- Constraints from astrophysics: DM in the sun might affect its structure
- indirectly by measuring DM decay products such as ν, γ, p^-, e^+ emitted at sites where the DM particles accumulate (sun, earth, galaxy centre, ...)
  - high-energy γ telescopes on earth or in space
  - neutrino detectors such as Cherenkov telescopes
  - cosmic-ray telescopes measuring the spectra of CR p^-, e^+
- direct detection with various kinds of detectors: measuring the nuclear recoil of DM-nucleus scattering events
  - via ionisation or scintillation in NaI, Ge diodes; problem: low ionisation efficiency ("conventional" detectors)
  - via phonons in cryogenic detectors (near 100 % efficiency)
  - general problem: very low cross sections, very low signal vs. background

Dark Matter: searching for non-baryonic DM

The CRESST cryostat



Dark Matter: current status of non-baryonic DM

- DAMA claims detection of a 60 GeV WIMP, not (yet?) confirmed by other experiments; future experiments may decide
- limits on DM from γ, ν flux from the galactic centre can be obtained, but only next-generation experiments can give conclusive evidence.
  - possible, but problematic, interpretation of γ source near the galactic centre detected by EGRET as DM annihilation
- lower bounds on DM masses from synchrotron radiation emitted by secondary $e^{-/+}$ in the galactic magnetic field
- high-energy ν flux from the sun or earth predicted by some DM models may be probed with IceCube, AMANDA, ANTARES (~ 10 events/year)
- e^+ excess in cosmic rays at ~ 10 GeV the result of DM annihilation? SUSY DM requires clumping to reconcile data, but other kinds fit better

Summary: Concordance Cosmology

- Composition of the universe:
  - universe is nearly flat, the total density being critical
  - $\Omega_{\text{baryion}} = 0.04$, $\Omega_{\text{matter}} = 0.26$, $\Omega_{\Lambda} = 0.74$, equation of state $w = -1$
    - non-baryonic dark matter of still unclear nature is required
    - dark energy, possible a cosmological constant
- History of the universe
  - early phase of inflation (~ during first 10^{-30} s) seems very likely
  - various symmetry-breaking phase transitions
  - baryogenesis, nucleosynthesis
  - decoupling of matter and radiation
  - structure formation
  - accelerated expansion as Λ becomes more important than matter

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