

The Gravitational Wave Background and its Sources

Dark Universe

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- 2 Theory and Detection
 - Linearized General Relativity
 - Detection of GWs & Analysis of signals
 - Detectors & Noise
- 3 Sources
 - Astrophysical
 - Cosmological
- 4 Current Status
- 5 Conclusion
- 6 Literature

Outline

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Introduction

Matter is represented by curvature, but not every curvature does represent matter, there may be curvature "in vacuo".

G. LeMaître

Gravitational waves (GWs) are predicted by General Relativity.

Gravitational wave astronomy is about to become reality as 2nd generation ground-based detectors (LIGO-II) and the first space-detector (LISA) will begin operation in the next decade. These will have a good chance of finding clear signals of GWs from various sources.

GWs are for General Relativity & Cosmology what the Higgs particle is for the Standard Model of Particle Physics:

Their existence is absolutely required, but they are until today not directly confirmed!

Topic of the talk: theoretical aspects of gravitational radiation, possible/imaginable sources and detection of a GW background .

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Linearized Einstein Field Equations

- the exact field equations are

$$G^{\mu\nu} := R^{\mu\nu} + \frac{1}{2}g^{\mu\nu}R = \frac{8\pi G}{c^4}T^{\mu\nu}.$$

- approximate the exact metric $g^{\mu\nu}$ by

$$g^{\mu\nu} \approx \eta^{\mu\nu} + h^{\mu\nu}$$

with the small deviation $h^{\mu\nu} \ll 1$ from the Minkowski metric $\eta^{\mu\nu}$ of flat space

- inhomogeneous wave equation:

$$\square h_{\mu\nu} = -\frac{16\pi G}{c^4}T_{\mu\nu}$$

- similar to the wave equations in electromagnetism, calculate the retarded part of the solution as follows:

$$h_{ret}^{\mu\nu} = -\frac{4\pi G}{c^4} \int d^3y \frac{T_0^{\mu\nu}(x^0 - |\vec{x} - \vec{y}|, \vec{y})}{|\vec{x} - \vec{y}|}$$

General form of GWs

- simplest form of solutions are monochromatic plane waves:

$$h_{\mu\nu} = \text{Re}(a_{\mu\nu} e^{ik_\alpha x^\alpha})$$

- $a_{\mu\nu}$ is a constant amplitude – k_α is the light-like wavevector satisfying

$$k_\alpha k^\alpha = 0 = a_{\mu\nu} k^\nu.$$

- ten independent components, reduced by four due to the second condition
- four unphysical degrees of freedom, "pure coordinate waves"
- GW has two physical degrees of freedom of polarization, transverse to the direction of propagation
- choosing the wave to propagate along the z direction, i.e. $k^\mu = (0, 0, \omega/c, \omega/c)$, only the 4 components a_{xx} , a_{xy} & $a_{yy} = -a_{xx}$ of the amplitude matrix are non-zero

Polarization of GWs

- two linearly independent states of polarization
 - $a_{xx} = -a_{yy}$ $a_{mn} = 0$ otherwise : + polarization
 - $a_{xy} = +a_{yx}$ $a_{mn} = 0$ otherwise : x polarization
- most common choice is "transverse-traceless" gauge (TT) with an amplitude

$$a_{\mu\nu} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & a_+ & a_x & 0 \\ 0 & a_x & -a_+ & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

with zero trace $a^\nu{}_\nu = 0$

- subscripts + & x denote a set of two independent and orthogonal polarization vectors
- $x = +$ rotated clockwise by 45° .

Polarization of GWs

The GWs' metric is

$$ds^2 = (1 + h_{xx})dx^2 + 2h_{xy}dxdy + (1 - h_{xx})dy^2 + dz^2 - c^2 dt^2$$

with

$$h_{xx} = a_{xx} \cos(\omega z/c - \omega t + \phi), \quad h_{xy} = a_{xy} \cos(\omega z/c - \omega t + \psi) \quad \text{with } |\psi - \phi| = \pi/4.$$

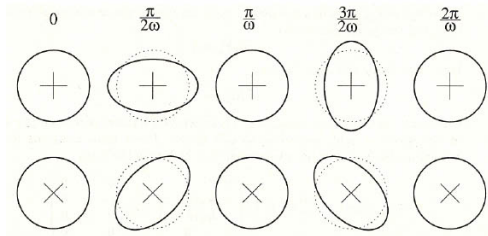
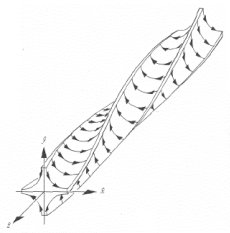
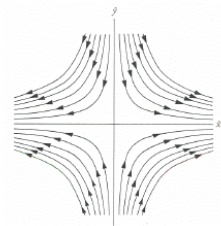


Figure: The two linear polarization states + & x of a gravitational wave.

Force lines



Multipole development of sources

- usually the "Newtonian-quadrupole" approximation is employed, i.e. slow motion & weak fields, only lowest order of contributing multipoles
- EM: $L_{\text{electric monopole}} \sim \dot{Q}$, $L_{\text{electric dipole}} \sim \ddot{d}^2$ and $L_{\text{magnetic dipole}} \sim \ddot{j}^2$
- GW: $\dot{m} = 0$ (conservation of mass), $\ddot{d} = \dot{p} = 0$ (cons. of momentum) and $\ddot{j} \sim \dot{J} = 0$ (cons. of angular momentum) \rightarrow no monopole or dipole radiation [reason: no negative mass charges]
- quadrupole radiation yields lowest-order non-zero contribution:

$$L_{\text{mass quadrupole}} = \frac{1}{5} \langle \ddot{i}^2 \rangle = \frac{1}{5} \langle \ddot{i}_{jk} \ddot{i}^{jk} \rangle$$

- reduced (=trace-free) quadrupole moment is defined as

$$I_{ij} = \int d^3x (x_i x_j - \frac{1}{3} r^2 \delta_{ij}) \rho(x, t).$$

Motion of test particles due to plane gravitational waves *

Geodesic equation of motion:

$$\frac{d^2 x^\alpha}{d\tau^2} + \Gamma_{\mu\nu}^\alpha \frac{dx^\mu}{d\tau} \frac{dx^\nu}{d\tau} = 0$$

Because of $\Gamma_{44}^\alpha = 0$ there exists a solution with $x^a = \text{const.}$ and $x^4 = c\tau$, describing a particle at rest in the coordinate system of the wave, thus apparently uninfluenced!

Space curvature only influences the *relative acceleration* of two test particles! Going into the local inertial coordinate system of an observer at the origin $(0, ct)$ [via $\tilde{x}^\alpha = x^\alpha + 1/2 f^\alpha{}_\beta(O, ct)x^\beta$] with metric

$$ds^2 = \eta_{\alpha\beta} dx^\alpha dx^\beta - c^2 dt^2 + \text{higher order terms.}$$

A test particle then has a time-varying position with acceleration

$$\frac{d^2 x^\alpha}{dt^2} = \frac{1}{2} \frac{d^2 f^\alpha{}_\beta(O, ct)}{dt^2} x^\beta = -c^2 R^\alpha{}_{4\beta 4} x^\beta$$

$f_{\alpha\beta}$ only has components in the xy-plane, perpendicular to the wave, so this is the plane in which the particles will be accelerated.

Quadrupole formula

common formula for strain amplitude (measured displacement at the place of detection):

$$h(t) = \frac{G}{c^4} \frac{2}{r} \left(\frac{d^2}{dt^2} Q_{ij} \right)_{ret}^{TT}$$

→ only aspherical sources can radiate

Estimation of GW strength and frequency

- Lab GW generation does not work: $L_{GW} \sim 10^{-23} \text{ erg/s} \ll L_0$ for a steel cylinder of radius 1 m, length 20 m and mass 490 tons rotating at 28 rad/s.
- scale for GW to become important: $L_0 = 3,63 \cdot 10^{53} \text{ erg/s} \equiv 1$
- astrophysical sources:
 virial theorem $\rightarrow E_{kin} = -\frac{1}{2} E_{grav} \sim M^2/R$
- characteristic time scale T and frequency f for a system of mass M and size R are

$$f^{-1} = T \sim \frac{R}{\text{mean velocity}} \sim \frac{R}{(M/R)^{\frac{1}{2}}} = \left(\frac{R^3}{M} \right)^{\frac{1}{2}}.$$

- internal power flow is $L_{int} \sim \frac{E_{kin}}{T} \sim \frac{M^2}{R} \left(\frac{M}{R^3} \right)^{\frac{1}{2}} \sim \left(\frac{M}{R} \right)^{\frac{5}{2}}$
- GW luminosity is $L_{GW} \sim L_{int}^2 \sim \left(\frac{M}{R} \right)^5 L_0$
- this is an upper limit of the GW output, which a system will only reach when it is near to its gravitational radius ($M \sim R$).

Estimation of GW strength and frequency - numbers

example:

binary system of two $M = 1.4$ solar mass neutron stars separated by $d = 90 \text{ km}$ at a distance of $R = 15 \text{ Mpc}$ (i.e. in the Virgo cluster)

$$h \approx 10^{-21} \left(\frac{15 \text{ Mpc}}{r} \right) \left(\frac{M}{2.8 M_{\text{sol}}} \right)^2 \left(\frac{90 \text{ km}}{R} \right)$$

$$f \approx 100 \text{ Hz} \left(\frac{M}{2.8 M_{\text{sol}}} \right)^{1/2} \left(\frac{90 \text{ km}}{R} \right)^{3/2}$$

$$\tau \approx 0.5 \text{ s} \left(\frac{M}{2.8 M_{\text{sol}}} \right)^3 \left(\frac{90 \text{ km}}{R} \right)^4$$

By orders of magnitude most radiating objects have similar values of strain, frequency and duration.

Past, Present and Future of GW Detection

- 1916: Einstein publishes his theory of General Relativity as well as the prediction that GWs do exist. He is *not* confident that they would be found.
- 1960s: Joseph Weber builds first aluminium bar resonance detectors and claims to have found GWs signals from SN-bursts.
- 1970s to today: development and construction of interferometric detectors
- future: advanced LIGO (II), enhanced LIGO (III), launch of LISA Pathfinder & LISA

Why Gravitational waves for Astrophysics?

in principle three kinds of radiation are detectable for SNe: electromagnetic, neutrino and gravitational.

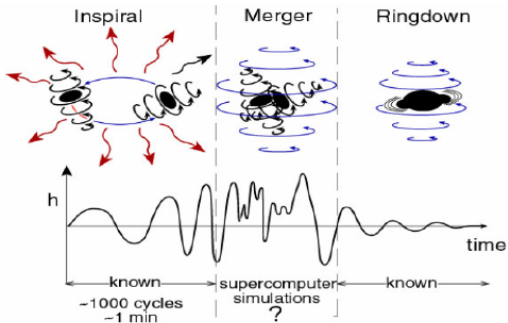
Consider a supernova explosion:

- EM : leaves the star hours after the core collapse
- ν : leave after hundreds of milliseconds
- GW: immediate decoupling from matter following their generation
- we measure *flux* (energy transfer) for both neutrino & EM radiation, which falls of as R^{-2} excluding extra-galactic SN
- we measure *amplitudes* (\leftarrow phase shift of light) of GWs, decaying only linearly R^{-1} with the distance \rightarrow allows probing for extra-galactic objects

Signal types

- 1 burst sources: strong signals of few cycles at characteristic frequencies with a low galactic event rate ($\ll 1$ per year)
- 2 quasi-periodic sources: less strong but much more frequent – model of the temporal structure of signal may make detection not much more difficult than a burst
- 3 background: very weak and noise-like – detection is *only* possible via cross-correlation of several instruments

Signal types: binary inspiral



Credits: Kip Thorne

Waveform for a binary black hole inspiral

Detector signal and analysis

- GW detector output is a time series, including a noise signal $n(t)$:

$$\frac{\Delta L(t)}{L} = s(t) = F^+(t)h_+(t) + F^\times(t)h_\times(t) + n(t)$$

- antenna patterns F^+ & F^\times depend on the frequency and sky location of the source, in the limit of large wavelengths (compared to the detector) simple quadrupole
- representation in Fourier domain as strain amplitude (measured distance change \equiv GW amplitude) density $h(f)$
- $h(f) = \sqrt{S_s(f)}$, with power spectral density $S_s(f) = |\tilde{s}(f)|^2$ and $\tilde{s}(f) = \int_{-\infty}^{\infty} e^{-2i\pi ft} s(t) dt$
- characteristic strain is defined as $h_c(f) = \sqrt{fS_s(f)}$ and is the root mean square in a frequency interval $[f - \Delta f, f + \Delta f]$

Analysis methods*

- GW signal is buried in the detector noise \rightarrow data is random process \rightarrow statistical problem
- basic idea: a signal changes the statistical characteristics of the data x , especially its probability distribution function (pdf):
 signal is absent: $p_0(x)$ or signal is present: $p_1(x)$
- Problem: find a test that is in some (a priori undefined) way optimal

There are several approaches to this:

- Bayesian: assign costs and probabilities to (1) false alarms & (2) false dismissals and minimize the risks
- Minimax: minimize over all rules the maximum of the risks
- Neyman-Pearson: maximize probability of detection (power of test) for a given significance (false alarm prob.)
- \Rightarrow Likelihood ratio test: accept a data s if $\Lambda(s) = \frac{p_1(s)}{p_0(s)}$ is larger than a $\Lambda_{threshold}$, which is a priori unknown but usually large, as not many events are to be expected at current sensitivities

Background

- stochastic background signals are from not discretely resolvable sources
- compare the energy density of the background to that needed to close the universe ρ_{crit} (so $k = 0$)
- standard unit: "energy density in GWs at frequency f per logarithmic frequency interval in units of the cosmic critical density "

$$\Omega(f) = \frac{1}{\rho_{crit}} \frac{d \rho_{gw}}{d \ln f}$$

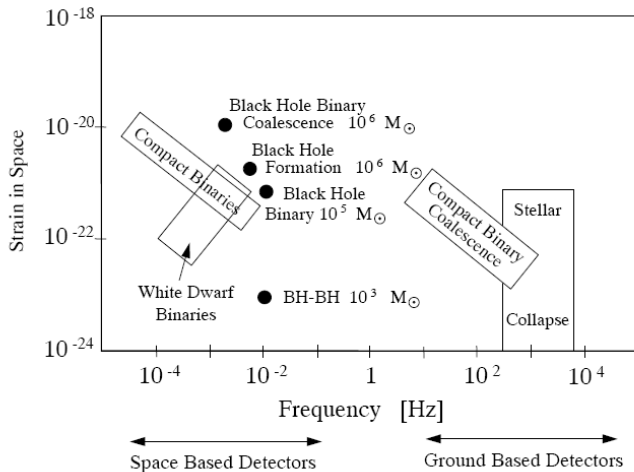
- often seen as $h_{100}^2 \Omega(f)$ with $h_{100} \approx 0.72$ factored out
- relation to strain at a "chirp" frequency and the spectrum:

$$h_c = 3 \cdot 10^{-20} h_{100} \sqrt{\Omega(f)} \frac{100 \text{ Hz}}{f}$$

Differences between EM- and GW- signals & data analysis

- GWs antennas are omni-directional → all-sky searches!
- interferometers are broad-band covering 3 to 4 orders of magnitude in frequency (good for tracking sources with changing frequencies) → higher computational cost
- GWs are tracked *in phase*, in contrast to EM radiation, and SNR is built up by coherently from many wave cycles → number of sources increases as R^3 instead of $R^{3/2}$ – amplitude vs. flux measurement
- antennas are acquiring data for months or even years continuously at rates of MB per seconds → *on-line* searches necessary → need for optimal search algorithms

Overview



Resonance detectors

- resonant bodies, e.g. massive bars of up to several tons, the crust of the Earth and the system Earth-Moon.
- detectable range of frequency: $\sim 1 - 2\text{kHz}$ with a bandwidth of (only!) $\sim 10\text{Hz}$
- favorite source: local asymmetric supernovae – maybe continuous radiation emitted by a neutron star
- currently operated: Univ. of Rome, Padua, Louisiana & Perth
- sensitivities better than 10^{-18} for millisecond pulses



Interferometric detectors

- largest (and most sensitive) class of detectors operating today : ground- and space-based
- operates similar to a Michelson interferometer: due to GWs the length of arms oscillate in time, resulting in a change of intensity measured
- detectable range of frequency: 1 Hz to $\sim 10\text{ kHz}$ at ground, 10^{-4} Hz to 1 Hz in space, with best sensitivity in middle of these ranges
- sensitivity is strongly limited by various noise sources
- examples: German-British GEO-600, LIGO (US), Japanese TAMA-300 & Italian-French VIRGO
- LIGO is largest and consists of 4 km-arm H1 and 2 km-arm H2 interferometers at the Hanford(WA) site and a 4 km-arm L1 in Livingstone, LA

Interferometric detectors

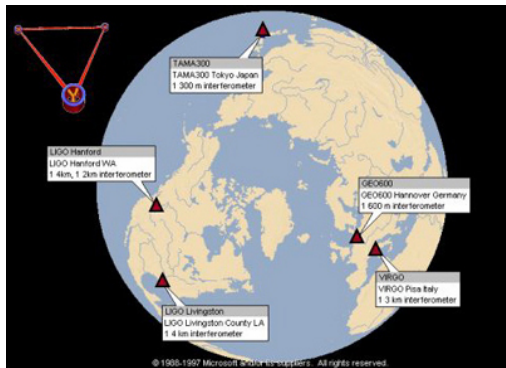


Figure: Location of all ground-based interferometric detectors

Noise Sources

- thermal noise: use of high quality components (silica optic with $Q = 10^7 \rightarrow$ noise of order $10^{-19} m$ at 100 Hz, in the future: sapphire)
- seismic noise (seismic background, traffic, machinery, wind): use of seismic filters
- shot noise (single photons detected \rightarrow phase variance \rightarrow length error): high laser power needed to maximize photon arrival rate

LIGO 2 noise

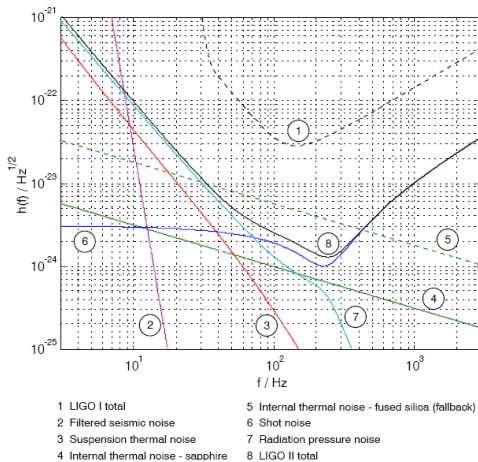


Figure: Noise contributions to the expected sensitivity of the LIGO 1 & 2 interferometer

LIGO noise

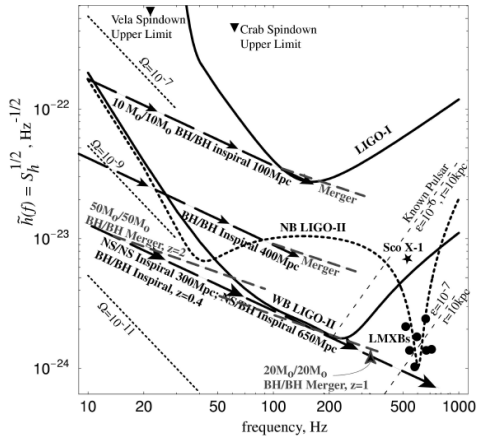


Figure: noise $\tilde{h}(f)$ in several LIGOs as a function of GW frequency f , compared to estimated signal strengths $\tilde{h}_s(f)$

Laser techniques

Optimally, the laser light should be stored for a time comparable to the signal's timescale \rightarrow use of delay lines or Fabry-Perot \rightarrow light does 50-100 bounces in the interferometer, so less laser power is needed.

- Signal Recycling
- Power Recycling

Interferometric detectors

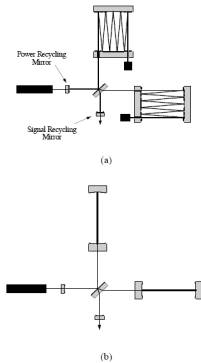


Figure: Michelson interferometer with Power & Signal recycling implemented: (a) with delay lines (b) with Fabry-Perot cavities in the arms

Space-based detectors

- No seismic noise & longer baseline (lower frequencies)
- LISA = Laser Interferometer Space Antenna (planned), NASA & ESA : consists of three spacecraft separated by $5 \cdot 10^6 \text{ km}$ & 20° behind the earth
- BBO = Big Bang Observatory, NASA: follow-up to LISA, frequency range 1-10 Hz, between LISA and ground-based detectors
- DECIGO(planned): Deci-Hertz Interferometer Gravitational-wave Observatory, Japan

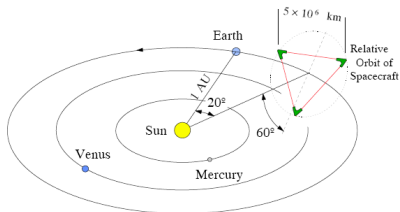
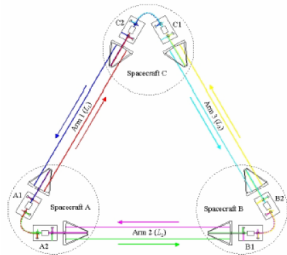
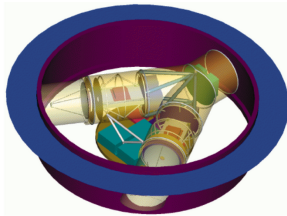


Figure 12: *The proposed LISA detector.*

Space-based detectors



LISA noise

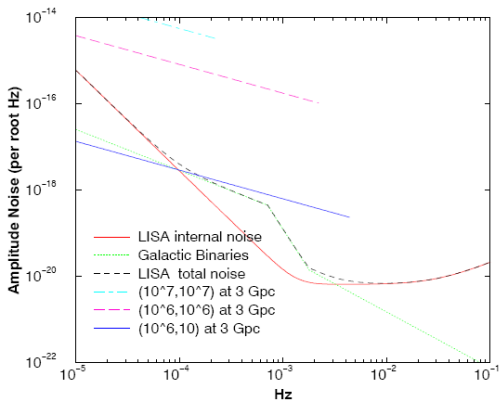


Figure 16: Same as Fig. 15 but for the LISA detector. Note that this figure, in contrast to Fig. 13, uses amplitudes per $\sqrt{\text{Hz}}$. We also plot signals from supermassive black holes. The supermassive BH sources are assumed to lie at a red-shift of $z = 1$ but LISA can detect these sources with a good SNR practically anywhere in the Universe.

Doppler Tracking

- send a radio signal to a spacecraft sending it back to earth
- compare frequencies to measure Doppler shift
- search for perturbations of the distance (possibly) caused by GWs
- roundtrip time ≈ 5800 s
- due to large distances this method is sensitive down to 10^{-6} Hz
- noise source: dispersion from plasma in solar wind and the earth's atmosphere
remedy: use multiband radio transmissions
- Ulysess (Sun) & Cassini (Jupiter) were used in 1992 & 2001 to get a limit on $\Omega_{BBN} < 10^{-5}$

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Overview

As we saw, only systems close to their gravitational radius produce significant amounts of gravitational radiation to be measured in the solar system. Only very compact objects/systems fulfill this requirement; furthermore the system has to be asymmetric (or several objects are involved) and move at speeds comparable to that of light!

- Inspiralling binaries of neutron stars (NS) and black hole (BH): e.g. two $1.4M_{sol}$ objects traverse the optimal band of LIGO in the last 30 s before collision – match waveform of inspiral and ringdown to get info on physics of the merger
- Collisions, e.g. of BHs, yields $\sim 0.001M_{total}$ in GWs
- Burst Sources
- Periodic sources: e.g. rotating elliptical pulsars, motion of surface material of a NS – EM frequency are well known \rightarrow focus on small bands
- Gamma Ray Bursts: trigger for GWs
- Gravitational collapse: various progenitor stars with different mass/age

Binary Systems

Every inspiraling binary star system will radiate gravitationally with a luminosity $L \propto \frac{M}{d^5}$. The system thus loses energy and also angular momentum and moves continuously faster. The ellipticity and the distance of the partner decrease as well.

The discovery of the first binary pulsar PSR 1913+16 yielded the Nobel Prize to Hulse & Taylor in 1993. PSR 1913+16 is a radio pulsar moving with $\sim 300 \text{ km/s}$ around an invisible companion, a black hole. Using pulsar timing the decreasing orbital period due to GW radiation was measured and was in excellent accord with GR.

→ *Indirect* confirmation of gravitational radiation.

Accretion induced collapse *

- star ejects envelope in a nebula and becomes a cooling white dwarf
- in a binary system, accretion (mass inflow from the companion star) can reheat the dwarf
- upon non-degenerate burning the mass increases until the Chandrasekhar limit is reached and collapse begins
- result: either a SN Ia or a NS
- galactic rate $< 10^{-5}$ per year \rightarrow observation distance 100 Mpc
- emission mechanism:
 - aspherical matter infall
 - instabilities: e.g. rotational = "bar-mode", depending on the amount of rotation $\beta = E_{rot}/E_{grav}$
 - models with low $\beta \sim 0.01$ are studied to "increase" the chance of such instabilities (lower β preferred)

strength of GWs: $h \leq 10^{-24}$ at 100 Mpc and 450 Hz, SNR ~ 3 for LIGO-II
 frequency range: $\approx 60 - 1000$ Hz

Binary Systems: PSR 1913+16

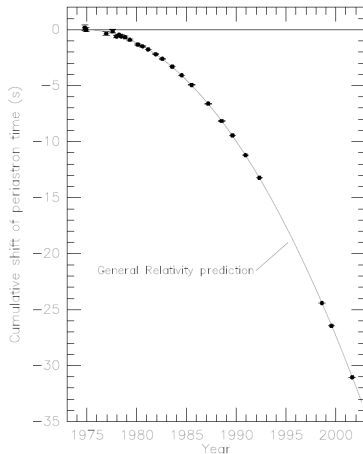
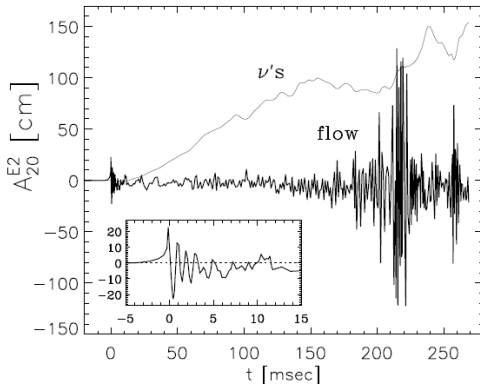


Figure: measured decrease in the orbital period of PSR1913+16 & GR prediction

Gravitational collapse

GWs are produced at all times of a collapse because of aspherical mass motion, convection, rotational and fragmentation instabilities and asymmetric neutrino emissions. BHs can emit due to accretion induced ringing.



Collapse of massive stars

stars between

- 6 and 20-25 M_{sol} : will produce SNe (II, Ib/c) and leave behind a neutron star
- 25 and 40 M_{sol} : some 2 M_{sol} will fall onto the newborn NS \rightarrow black hole formation
- 40-50 M_{sol} : collapse to a BH without a SN
- rotation may create an accretion disk \rightarrow collapsar as GRB engine

rate of galactic core collapse: $\sim 10^{-2}$ to 10^{-1} per year \rightarrow distance of 10 Mpc

strength of GWs: $h = 10^{-24}$ to 10^{-23} at 10 Mpc

frequency range: 60-1000 Hz

A possible stochastic background from these sources may be detectable by a pair of LIGO-II at $h \sim 10^{-27}$ or 2nd generation space detectors.

Collapse of Pop III stars

first generation of stars formed at $z \geq 5$ with masses $\geq 100 M_{sol}$
direct collapse to a BH or first a hypernova explosion
formation rate: $\leq 10^{-3}$ per year in a $10^{11} M_{sol}$ galaxy
→ distance of 50 *Gpc* — probably this rate is much lower
strength of GWs: $h \sim 10^{-23}$ at 10 *Mpc*
frequency range: 10-100 Hz

LIGO-II will probably not be able to detect these events:
not strong enough, redshifted out the frequency window

Overview

Sources possibly contributing to the GW background:

- 1 Relic GWs: a stochastic background (stemming from inflation) akin to the electromagnetic CMB
- 2 Cosmological core collapse SNe of today's and Population III stars
- 3 Population III black hole formation
- 4 ν -driven Gamma Ray Bursts (GRB)
- 5 extra-galactic binary systems
- 6 GW BG in perfect fluid quantum cosmologies (alternative to inflation)
- 7 cosmic strings
- 8 early phase transitions (bubbles wall collisions cause GW)

Individual sources are not identifiable as such because they interfere with each other. Furthermore they are too weak to be detected on their own.

Thermal spectrum?

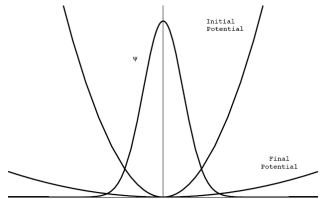
- thermal spectrum of gravitons from time of decoupling from primordial plasma
- for dimensional reasons $T_{decoupling} \approx M_{Planck}$
- today's temperature $T_g = (3.91/106.75)^{1/3} \cdot 2.75 K \approx 0.9 K$ or smaller
- frequency peak is in the GHz range \rightarrow not detectable.
- probably gravitational radiation did not have enough time to reach thermal equilibrium: the GW radiation has never existed

Relic GWs - Creation via parametric amplification

- ① Imagine a ground state ($N_i = 0$) quantum harmonic oscillator in an initial potential U . The uncertainty principle states that the oscillator is not truly at rest, $E_i = \frac{1}{2}\hbar\omega_i$, and executes zero-point motion about it's classical rest position.
- ② If U changes adiabatically ($t_f - t_i = T \gg 2\pi/\omega_i$), the final energy will be $E_f = \frac{1}{2}\hbar\omega_f$, where work is going into changing the potential energy and "quantum work" $E_f - E_i$ for changing the oscillators energy. No quanta are created: $N_f = 0$
- ③ If U changes in a non-adiabatically way ($T \ll 2\pi/\omega_i$), the oscillator cannot "follow". If $\omega_f \gg \omega_i$ the final energy will be $E_f = \frac{1}{2}E_i = \frac{1}{4}\hbar\omega_i \approx \hbar\omega_f(\frac{1}{4}\frac{\omega_i}{\omega_f})$ and thus a number of quanta $N_f = \frac{1}{4}\frac{\omega_i}{\omega_f}$ has been created.

Relic GWs - Inflation

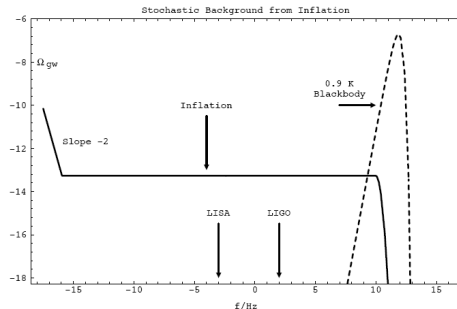
- The polarization states of a GW can be Fourier decomposed into spatial harmonics, i.e. a set of many oscillators. These can couple to the gravitational field of the surrounding universe, because the Einstein equations are nonlinear.
- As Inflation is *not* adiabatic but takes only a very short time, the frequency of the GW modes drop very quickly, resulting in a high occupation number.
- Although this is $\sim \hbar$ only, the perturbations are macroscopic and a characteristic GW spectrum should be measurable today.
- Relic GWs allow exploration of the very early universe down the Planck era 10^{-22} s after the Big Bang.



Relic GWs - Constraints

- CMB isotropy: Sachs-Wolfe effect describes relation between temperature fluctuations and GWs \rightarrow limits on GWs at *very low* frequencies
- pulsar timing: GW changes the assumed perfect clock of the pulsar by shifting the pulses –

$$h_c = \Delta t/t = \mu\text{sec}/8\text{years} < 10^{-14} \rightarrow \Omega(10^{-8}\text{Hz}) < 10^{-8}$$
 is rather a bound for cosmic strings
- Big-Bang nucleosynthesis: weak upper bound



Core collapse SNe

- expected rate of SNe in the visible universe: 1 per second (!)
- $\approx 99\%$ of released energy of 10^{53} erg goes into neutrinos, 1% into kinetic energy and 10^{-4} into EM radiation
- frequency range: $10^{-4} - 10^4 \text{ Hz}$ \rightarrow ground- and space-based interferometers necessary
- GW generating processes:
 - anisotropic ν -emission
 - convection \rightarrow mixing on large scales & development of inhomogeneities
 - rotation

In principle all three kinds of radiation are detectable for BG SNe, but only GWs do not get redshifted too much.

Core collapse SNe

GW emission due to anisotropic ν -emission:

$$h(t) = \frac{2G}{D} \int_{-\infty}^{t-D} d\tau L_{\nu}(\tau) q(\tau)$$

energy density in GWs:

$$\Omega_{gw}(f) = \frac{16\pi^2 D^2}{15G_N \rho_c} \int_0^{\infty} dz \frac{R_{SN}(z)}{1+z} |dt/dz| ((1+z)f)^3 |\tilde{h}(f_z)|^2$$

where $R_{SN}(z)$ is the SN rate and $\tilde{h}(f)$ the strain amplitude of one SN at distance D .

One then needs a model for cosmic star-formation rate as well as the strain spectrum from core-collapse simulations to estimate the energy density. Power of GW background is uncertain by several o.o.m. because R_{SN} is not very well known, neither is the asymmetry parameter $q \rightarrow$ better simulations! The inflationary BG is probably not masked by SN BG.

Core collapse SNe

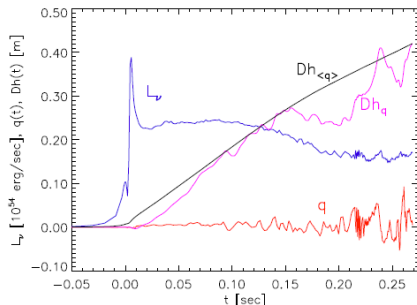


FIG. 2: Neutrino luminosity $L_\nu(t)$, anisotropy $q(t)$, and GW strain $h(t)$ times distance from anisotropic neutrino emission only, see Eq. (1), for model s15r of Ref. [6], as functions of time after bounce. We also show the GW strain $h(t)$ times distance from anisotropic neutrino emission, using the average anisotropy $\langle q \rangle = 0.45\%$.

Core collapse of Pop III stars

- hypothetical population of first stars:
masses of $300M_{sol}$, $z \approx 15$ and the spectrum of ordinary SNe
- two extremes of the rate of Pop III SNe give a range of observed events
from $\frac{1}{5}s^{-1}$ to $4 \cdot 10^{-4}s^{-1}$
- background could be dominated by Pop III stars in the range
 $[10mHz, 1kHz]$, while ν s are redshifted to undetectable energies!

Core collapse of Pop III stars

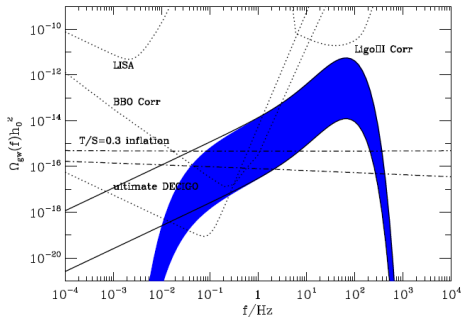


FIG. 8: GW background for our standard PopIII core collapse case and event rates $4.4 \times 10^{-4} \text{ s}^{-1} < R_{\text{III}} < 0.2 \text{ s}^{-1}$. For the blue shaded band the source spectrum shown in Fig. 3 was cut off at $f \leq 1 \text{ Hz}$ and in this sense does not rely on the zero-frequency limit. The solid lines are obtained from continuing the source signal by the zero-frequency limit.

ν -driven Gamma Ray Bursts (GRB)

- powerful asymmetric jets with enormous emissions of high energy neutrinos
- anisotropic energy flows driven by the neutrino-antineutrino-annihilation
- a single GRB at 10kpc (Galactic centre) should be detectable by LISA
- GRB GW background will be far below inflationary background levels

ν -driven Gamma Ray Bursts (GRB)

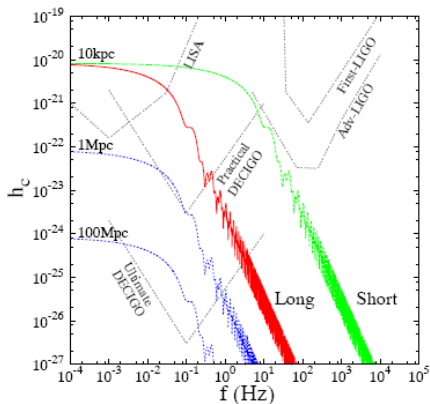


Figure The characteristic amplitude of GWs from a single GRB at the the Galactic centre (solid line). The “Long” duration t_f is 10 sec and “Short” is 0.1 sec (long-dashed line). On the other hand, the short-dashed lines represent the “Long” duration GRB at 1Mpc (upper) and 100Mpc (lower).

ν -driven Gamma Ray Bursts (GRB)

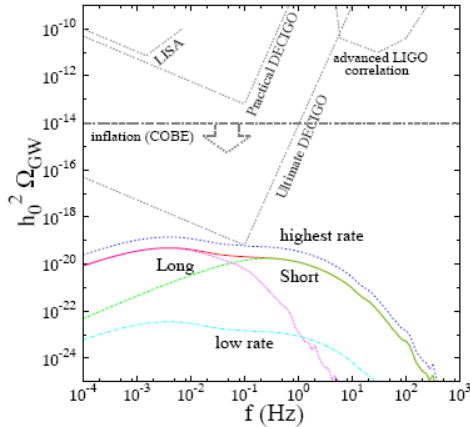


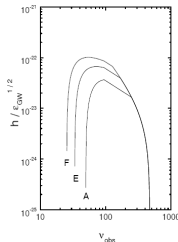
Figure: A Gammy-Ray-Burst GW background is undetectable.

Formation Pop III black holes

Depending on the efficiency of GW production ϵ_{GW} one gets an amplitude for this process:

$$h_{BH} = 7.4 \cdot 10^{-20} \epsilon_{GW}^{1/2} \frac{M_{remnant}}{M_{sol}} \frac{r_z}{1Mpc}^{-1}$$

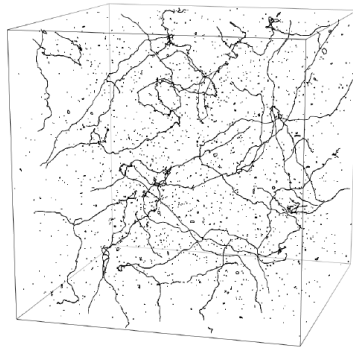
This might be detectable ($S/N > 1$) by LIGO-II for maximum ϵ_{GW} , and probably with LIGO-III.



BG amplitude as function of frequency and efficiency rate

Cosmic strings

- one-dimensional (in space) string-like objects formed during a phase transition
- network of strings, oscillating relativistically under their own tension
- primary decay mechanism: emission of GW



Cosmic string network

Predictions

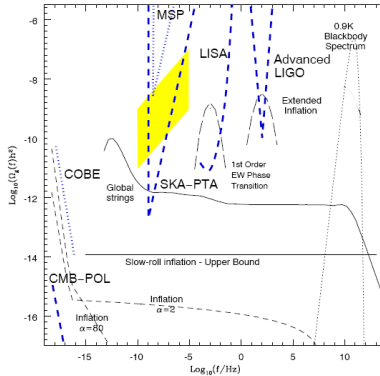


Figure : Predicted gravitational wave backgrounds. The top of the figure contains current and predicted limits placed on the gravitational wave background from millisecond pulsar timing, LISA and advanced LIGO.

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- 2 Theory and Detection
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 - Detectors & Noise
- 3 Sources
 - Astrophysical
 - Cosmological
- 4 Current Status**
- 5 Conclusion
- 6 Literature

Status - GW bursts

LIGO & TAMA – Joint search for GW bursts [gr-qc/0507081]

"No gravitational-wave candidates were observed, and we place an upper bound of 0.12 events per day on the rate of detectable millisecond-duration gravitational-wave bursts with at least 90% confidence. Simulations indicate that our network has a detection efficiency of at least 50%(90%) for narrow-band signals with root-sum-square strain amplitude greater than approximately $2 \times 10^{-19} \text{Hz}^{-1/2}$ ($10 - 18 \text{Hz}^{-1/2}$) in the frequency band $700 - 2000 \text{Hz}$."

Advantage of joint coincidence search: twice as much data as LIGO alone → stronger limits – low background rate of less than 1 per 40 years

Disadvantage: network sensitivity is limited by the least sensitive detector at each frequency

Status - Stochastic GW background

LIGO – Upper Limits on a Stochastic Background of Gravitational Waves
[astro-ph/0507254]

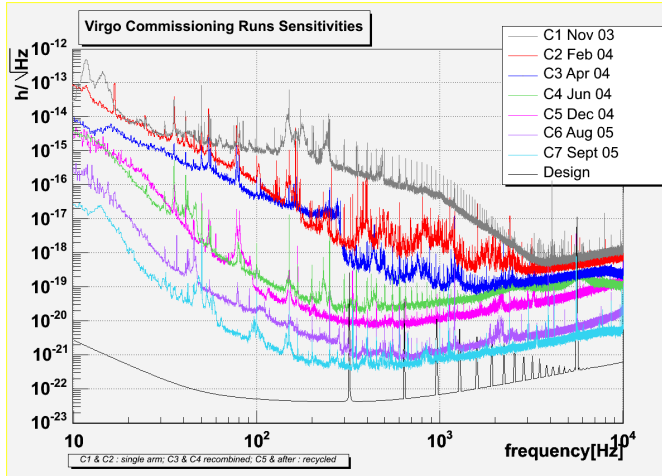
“The estimates for Ω_α are entirely consistent with no stochastic background, within the sensitivity of the measurement. Furthermore, the cross-correlation spectrum shows no distinct features, and the $\approx 26,000$ values of Y_I (..), follow the expected normal distribution.”

LISA & LIGO status

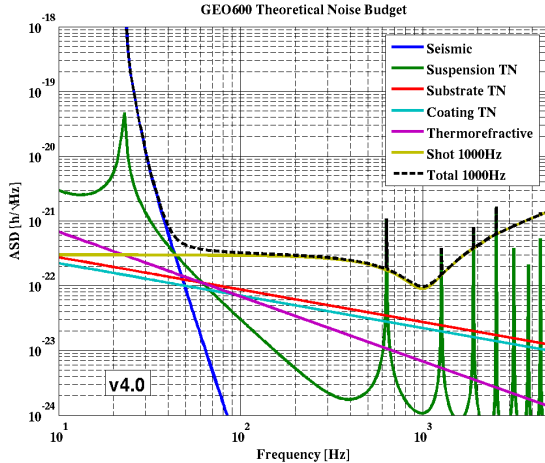
- LISA Pathfinder will test the general concepts and technologies needed for LISA
- launch scheduled for 2007, duration: 6 - 12 months
- LISA will launch in 2015
- LIGO-II (advanced LIGO) will begin start observations in 2013

Neil Cornish concludes in "LISA and Beyond
Amazing things are possible for large sums of money.

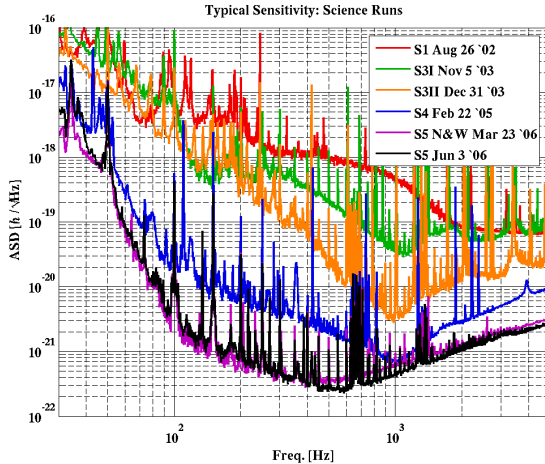
VIRGO Sensitivity curves



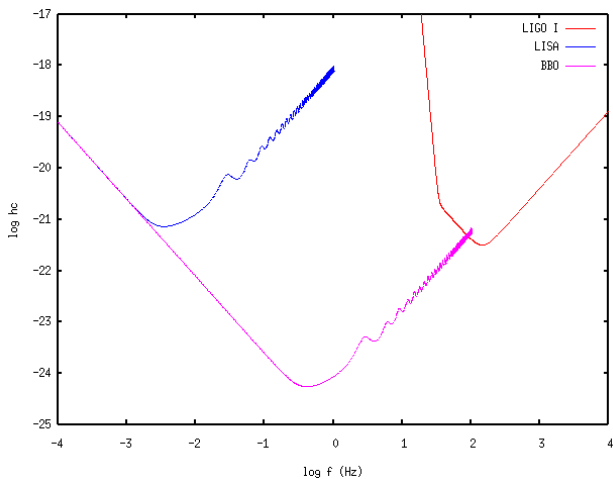
Recent GEO Sensitivity curves



Recent GEO Sensitivity curves



LIGO I - LISA - BBO



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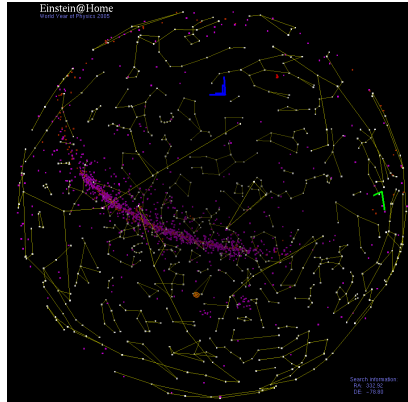
Conclusion

- Gravitational wave searches are a highly complementary to electromagnetic and neutrino experiments:
Long range and immediate decoupling from the producing matter are their big advantages, allowing looks deep into all stellar processes.
- Physical models can be constrained by "Signal Inversion"
- Direct confirmation of a GW signal is to be expected in the next 5-10 years.

Einstein@home

If your CPU is lazy most of the time you may have it search for GW in data from LIGO & GEO600 for Einstein@Home:

<http://www.boinc.de/einstein.htm> or <http://einstein.phys.uwm.edu> .



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Literature

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- Grishchuk et al., "Gravitational Wave Astronomy in Anticipation of First Sources to be Detected", astro-ph/0008481
- J. Jordan & N. Cornish, "The Status of Gravitational Wave Astronomy "
- B. Allen, "The stochastic gravity-wave background: sources and detection", gr-qc/9604033

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- A. Buonanno et al., "Stochastic Gravitational Wave Background from Cosmological Supernovae" ,astro-ph/0412277
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<http://www.livingreviews.org/Articles/Volume3/2000-3hough/>
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