Simulating the Universe: Methods and Applications

Neutrino Transport & Core Collapse Supernovae

Advisor: H.-Th. Janka

Michael Lang

13.05.2005

Overview

- Radiation Field and Equation of Transfer
- Radiation-hydrodynamics
- Neutrino Processes
- SN Physics
- Simulation Results
- SN & nucleosynthesis

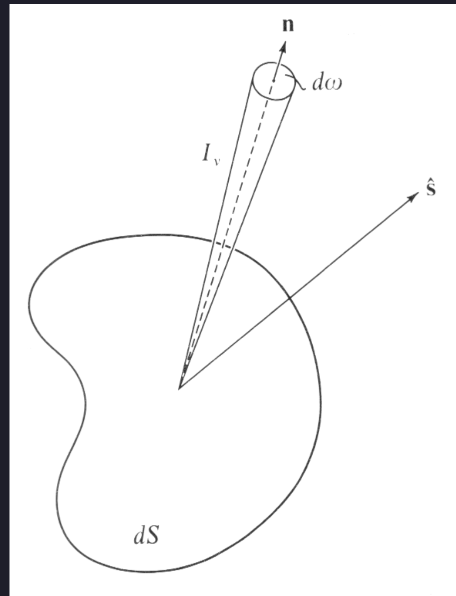
Definition of Intensity

- Specific Intensity

$$\delta E = I(\mathbf{r},\mathbf{n},\nu,t)\, dS \cos\theta \, dt \, d\omega \, d\nu$$

$$dS \cos\theta = \mathbf{n}\cdot d\mathbf{S}$$

$$\mu \equiv \cos\theta$$



[1]

- Photon Picture

$$I(\mathbf{r},\mathbf{n},\nu,t) = (ch\nu)\, f_{\mathrm{R}}(\mathbf{r},\mathbf{n},\nu,t) \quad (1)$$

photon distribution function f_{R} : $f_{\mathrm{R}}(\mathbf{r},\mathbf{n},\nu,t)\, d\omega\, d\nu$ is the number of photons per unit volume at $(\mathbf{r},t)$ propagating with velocity c into a solid angle $d\omega$.

Angular moments

- Mean Intensity and Energy Density:

$$J(\mathbf{r},\nu,t)=\frac{1}{4\pi}\oint I(\mathbf{r},\mathbf{n},\nu,t)\,d\omega, \qquad \varepsilon_{\mathrm{R}}(\mathbf{r},\nu,t)=\frac{4\pi}{c}J(\mathbf{r},\nu,t)$$

- Flux:

$$F(\mathbf{r},\mathbf{n},\nu)=\oint I(\mathbf{r},\mathbf{n},\nu,t)\mathbf{n}\,d\omega$$

- Radiation Pressure Tensor:

$$P_{ij}(\mathbf{r},\nu,t)=\frac{1}{c}\oint I(\mathbf{r},\mathbf{n},\nu,t)n_i n_j\,d\omega=\oint\left[f_{\mathrm{R}}(\mathbf{r},\mathbf{n},\nu,t)cn_i\right]\frac{h\nu n_j}{c}\,d\omega$$

- Angular moments in spherical coordinates

$$\{J,H,K,L,\ldots\}(t,r,\nu):=\frac{1}{2}\int_{-1}^{1}d\mu\,\mu^{\{1,2,3,4,\ldots\}}\,I(t,r,\nu,\mu)$$

Interaction of Radiation with Matter

- Scattering
  - Photon emerges in a new direction with slightly altered energy
  - Depends on radiation field, weak connection with the local thermodynamic properties
- Absorption and Emission
  - Photon is destroyed (*Thermalization*)
  - Photon energy is fed directly into thermal kinetic energy of the gas

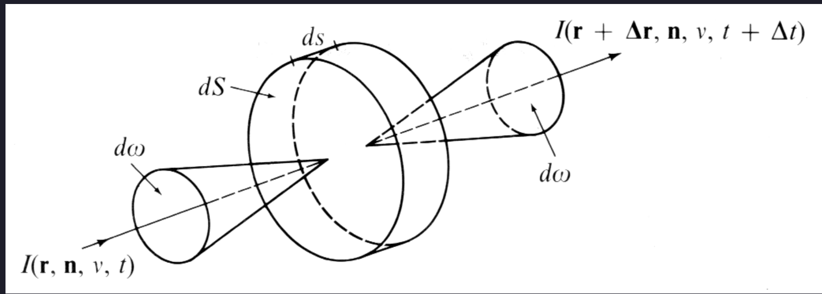
- Extinction Coefficient (opacity) χ

$$\delta E = \chi(\mathbf{r},\mathbf{n},\nu,t) I(\mathbf{r},\mathbf{n},\nu,t)\, dS\, ds\, d\omega\, d\nu\, dt$$

- Emission Coefficient (emissivity) η

$$\delta E = \eta(\mathbf{r},\mathbf{n},\nu,t)\, dS\, ds\, d\omega\, d\nu\, dt$$

The Equation of Transfer



[1]

Derivation:

$$\left[I(\mathbf{r}+\Delta\mathbf{r},\mathbf{n},\nu,t+\Delta t) - I(\mathbf{r},\mathbf{n},\nu,t)\right] dS\, d\omega\, d\nu\, dt$$
$$= \left[\eta(\mathbf{r},\mathbf{n},\nu,t) - \chi(\mathbf{r},\mathbf{n},\nu,t)\, I(\mathbf{r},\mathbf{n},\nu,t)\right] ds\, dS\, d\omega\, d\nu\, dt$$

Equation of Transfer:

$$\left[\frac{1}{c}\frac{\partial}{\partial t} + \frac{\partial}{\partial s}\right] I(\mathbf{r},\mathbf{n},\nu,t) = \eta(\mathbf{r},\mathbf{n},\nu,t) - \chi(\mathbf{r},\mathbf{n},\nu,t)\, I(\mathbf{r},\mathbf{n},\nu,t)$$

Transfer Equation as Boltzmann Equation

- Boltzmann Equation

$$\frac{\partial}{\partial t} f + (\mathbf{v} \cdot \nabla) f + (\mathbf{F} \cdot \nabla_p) f = \left(\frac{Df}{Dt} \right)_{\text{coll}}$$

f: particle distribution function

- For a „gas“ of photons:

$$\mathbf{F} \equiv 0, \qquad \mathbf{v} = c\mathbf{n}$$

- Using equation (1):

$$\frac{1}{c} \frac{1}{h\nu} \left[\frac{\partial I}{\partial t} + c \cdot (\mathbf{n} \cdot \nabla) I \right] = \frac{1}{h\nu} (\eta - \chi I)$$

Moments of the Transfer Equation

- Zeroth order (Energy equation)

$$\frac{4\pi}{c}\frac{\partial J(\mathbf{r},\nu,t)}{\partial t}+\nabla\cdot F(\mathbf{r},\nu,t)=\oint\eta(\mathbf{r},\mathbf{n},\nu,t)-\chi(\mathbf{r},\mathbf{n},\nu,t)I(\mathbf{r},\mathbf{n},\nu,t)\,d\omega$$

- First order (dynamical equation)

$$\frac{1}{c^2}\frac{\partial F_i}{\partial t}+\sum_j\frac{\partial P_{ij}(\mathbf{r},\nu,t)}{\partial x_j}=\frac{1}{c}\oint\left[\eta(\mathbf{r},\mathbf{n},\nu,t)-\chi(\mathbf{r},\mathbf{n},\nu,t)I(\mathbf{r},\mathbf{n},\nu,t)\right]n_i\,d\omega$$

$O(v/c)$ transport equations

$$\left(\frac{1}{c}\frac{\partial}{\partial t} + \beta\frac{\partial}{\partial r}\right)\mathcal{I} + \mu\frac{\partial}{\partial r}\mathcal{I} + \frac{1-\mu^2}{r}\frac{\partial}{\partial \mu}\mathcal{I}$$
$$+ \frac{\partial}{\partial\mu}\left[(1-\mu^2)\left\{\mu\left(\frac{\beta}{r}-\frac{\partial\beta}{\partial r}\right) - \frac{1}{c}\frac{\partial\beta}{\partial t}\right\}\mathcal{I}\right]$$
$$- \frac{\partial}{\partial\epsilon}\left[\epsilon\left((1-\mu^2)\frac{\beta}{r} + \mu^2\frac{\partial\beta}{\partial r} + \mu\frac{1}{c}\frac{\partial\beta}{\partial t}\right)\mathcal{I}\right]$$
$$+ \left((3-\mu^2)\frac{\beta}{r} + (1+\mu^2)\frac{\partial\beta}{\partial r} + \mu\frac{2}{c}\frac{\partial\beta}{\partial t}\right)\mathcal{I} = C\,,$$

$$\left(\frac{1}{c}\frac{\partial}{\partial t} + \beta\frac{\partial}{\partial r}\right)J + \frac{1}{r^2}\frac{\partial}{\partial r}(r^2 H)$$
$$- \frac{\partial}{\partial\epsilon}\left[\epsilon\left(\frac{\beta}{r}(J-K) + \frac{\partial\beta}{\partial r}K + \frac{1}{c}\frac{\partial\beta}{\partial t}H\right)\right]$$
$$+ \frac{\beta}{r}(3J-K) + \frac{\partial\beta}{\partial r}(J+K) + \frac{2}{c}\frac{\partial\beta}{\partial t}H = C^{(0)}\,,$$

$$\left(\frac{1}{c}\frac{\partial}{\partial t} + \beta\frac{\partial}{\partial r}\right)H + \frac{1}{r^2}\frac{\partial}{\partial r}(r^2 K) + \frac{K-J}{r}$$
$$- \frac{\partial}{\partial\epsilon}\left[\epsilon\left(\frac{\beta}{r}(H-L) + \frac{\partial\beta}{\partial r}L + \frac{1}{c}\frac{\partial\beta}{\partial t}K\right)\right]$$
$$+ 2\left(\frac{\partial\beta}{\partial r} + \frac{\beta}{r}\right)H + \frac{1}{c}\frac{\partial\beta}{\partial t}(J+K) = C^{(1)}\,,$$

[5]

- Derived from Lindquist-Equation
- „Newtonian approximation“
  - Order (v/c)
  - flat spacetime

$\beta = v/c$

$C \equiv \eta - \chi I$

$C^{(k)}(t,r,\in) \equiv \frac{1}{2}\int_{-1}^{+1} d\mu\, \mu^k\, C(t,r,\in,\mu)$

Radiation-hydrodynamics

Eulerian, nonrelativistic equations of *hydrodynamics* (Cartesian coordinates):

$$\partial_t \rho + \partial_i(\rho v_i) = 0$$

$$\partial_t(\rho v_k) + \partial_i(\rho v_i v_k + \delta_{ik} p) = -\rho \partial_k \Phi^{\mathrm{Newt}} + Q_{\mathrm{M}_k}$$

$$\partial_t(\rho \varepsilon) + \partial_i(\{\rho \varepsilon + p\} v_i) = -\rho v_i \partial_i \Phi^{\mathrm{Newt}} + Q_{\mathrm{E}} + v_i Q_{\mathrm{M}_i}$$

with $\varepsilon = e + \frac{1}{2} v^2$, Φ^{Newt} Newtonian gravitational potential.

Conservation equation for the electron fraction:

$$\partial_t(\rho Y_{\mathrm{e}}) + \partial_i(\rho Y_{\mathrm{e}} v_i) = Q_{\mathrm{N}}$$

Neutrino source terms:

$$Q_{\mathrm{E}} = -4\pi \int_0^\infty d\epsilon\, C^{(0)}(\epsilon), \quad Q_{\mathrm{M}} = -\frac{4\pi}{c} \int_0^\infty d\epsilon\, C^{(1)}(\epsilon), \quad Q_{\mathrm{N}} = -4\pi\, m_{\mathrm{B}} \int_0^\infty d\epsilon\, \epsilon^{-1}\, C^{0}(\epsilon)$$

Solution Methods

- Discretisation of the Boltzmann Equation
- Variable Eddington factor method
  - Iterative Solution of moments of the transfer equation
- Diffusion approximation
  - Radiation field is only weakly anisotropic
  - Radiation is treated quasisteady at each instant of time
- Monte Carlo Simulations

Neutrino Processes I

- Thermal Emission
  - Pair annihilation

$$e^+ + e^- \overset{(W,Z)}{\longrightarrow} \nu + \bar{\nu}$$

  - Plasmon decay

$$(\text{plasmon excitation}) \overset{(W,Z)}{\longrightarrow} \nu + \bar{\nu}$$

  - Photoannihilation

$$e^- + \gamma \overset{(W,Z)}{\longrightarrow} e^- + \nu + \bar{\nu}$$

  - Bremsstrahlung

$$e^- + (Z,A) \overset{(W,Z)}{\longrightarrow} (Z,A) + e^- + \nu + \bar{\nu}$$

Neutrino Processes II

- Neutronization
  - Electron capture by nuclei

$$e^- + (Z, A) \xrightarrow{W} \nu_e + (Z-1, A)$$

  - Electron capture by free protons

$$e^- + p \xrightarrow{W} \nu_e + n$$

  - Positron capture by free neutrons

$$e^+ + n \xrightarrow{W} \bar{\nu}_e + p$$

Neutrino Processes III

- Scattering and Absorption
  - Free nucleon scattering

$$\nu + N \xrightarrow{Z} \nu + N$$

  - Coherent scattering by heavy nuclei

$$\nu + (Z, A) \xrightarrow{Z} \nu + (Z, A)$$

  - Electron-neutrino scattering

$$e^- + \nu \xrightarrow{(W,Z)} e^- + \nu$$

  - Nucleon absorption

$$\nu_e + n \xrightarrow{W} p + e^-$$

Core Collapse Supernovae – Progenitors

- $M \geq 8\, M_\odot$
- Central density

$$\rho_c \approx 10^{10}\ \mathrm{g\,cm^{-3}}$$

- Central temperature

$$T_c \approx (8-10) \times 10^9\ \mathrm{K}$$

- Energy source

$$E_b \approx 3 \times 10^{53} \left(\frac{M}{M_\odot}\right)^2 \left(\frac{R}{10\,\mathrm{km}}\right)^{-1} \mathrm{erg}$$

  - $99\% \rightarrow$ ν's
  - $1\% \rightarrow E_{kin}$
  - $0.1\% \rightarrow$ radiation

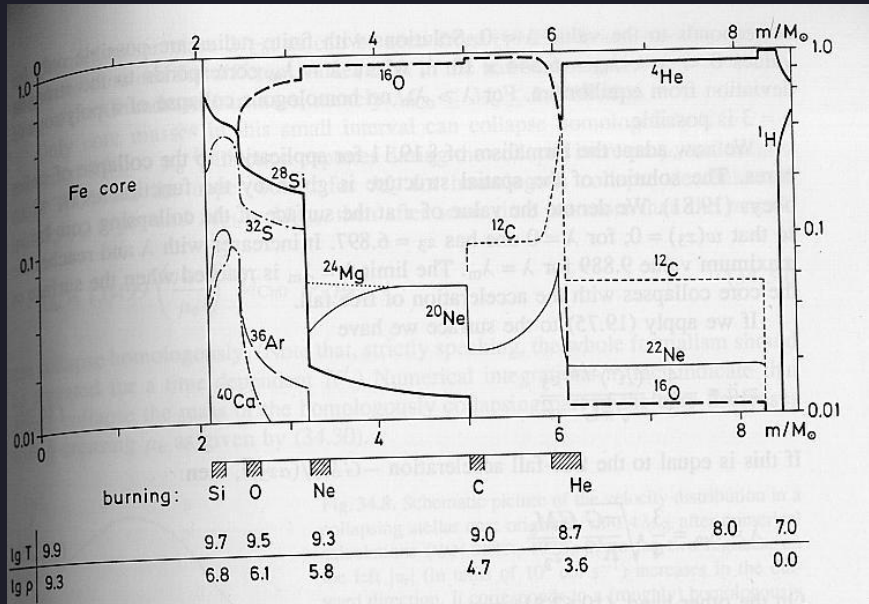
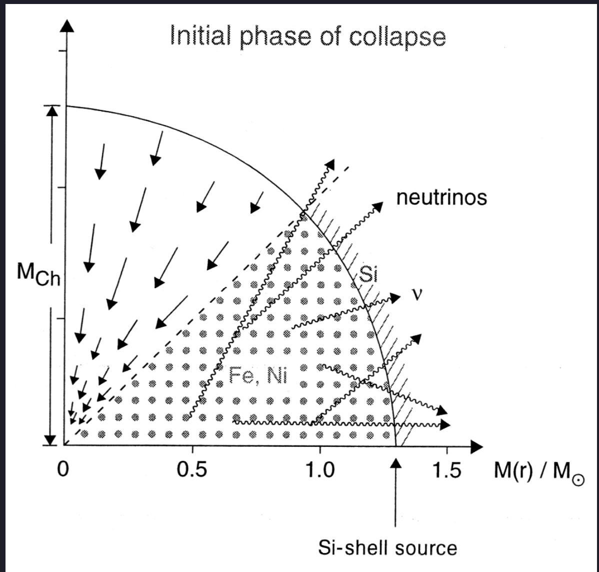
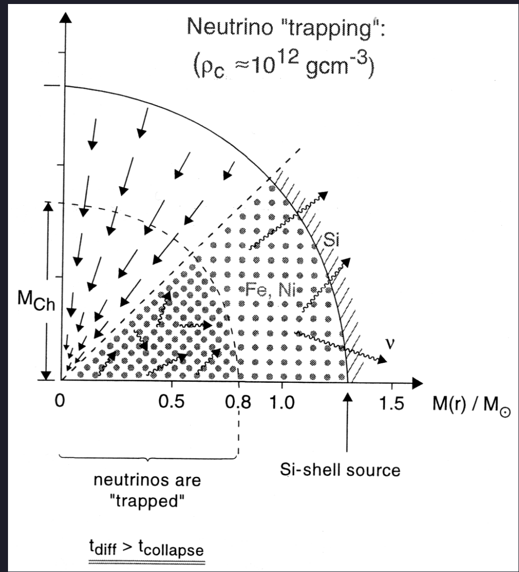


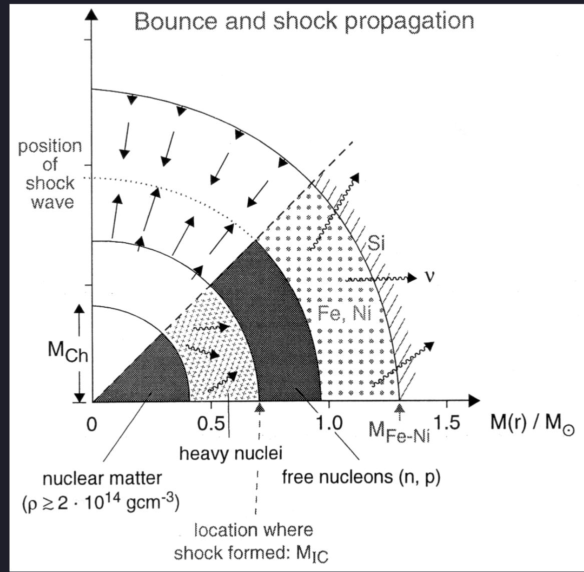
Fig. 34.7. The chemical composition in the interior of a highly evolved model of a $25 M_\odot$ star of population I. The mass concentrations of a few important elements are plotted against the mass variable m. Below the abscissa the location of shell sources and typical values of temperature (in K) and density (in g cm^{-3}) are indicated. (After WOOSLEY, WEAVER, 1986)

[4]

SN – stages of the collapse







[3]

- Deleptonization by e⁻-capture on free p
- ν's can leave the core
- Coherent scattering

- $\rho \geq 3 \cdot 10^{12}$ g cm^{-3}: neutrino trapping
- shocked matter: scattering of free N & absorption on n

- EOS stiffens
- Shock wave
- Photo-disintegration
- Rebound

SNe – Explosion mechanism

- Prompt Explosion
  - $E_{\text{shock}} \approx (4\text{–}10)\times 10^{51}\,\text{erg}$ (enough for explosion!)
  - But: shock wave is damped by photo-disintegration of heavy nuclei into free nucleons
  - Prompt explosion:

$$M_{\text{core}} < 1.35\, M_\odot \text{ and}$$
$$0.8\, M_\odot \le M_{\text{IC}} \le 0.9\, M_\odot$$

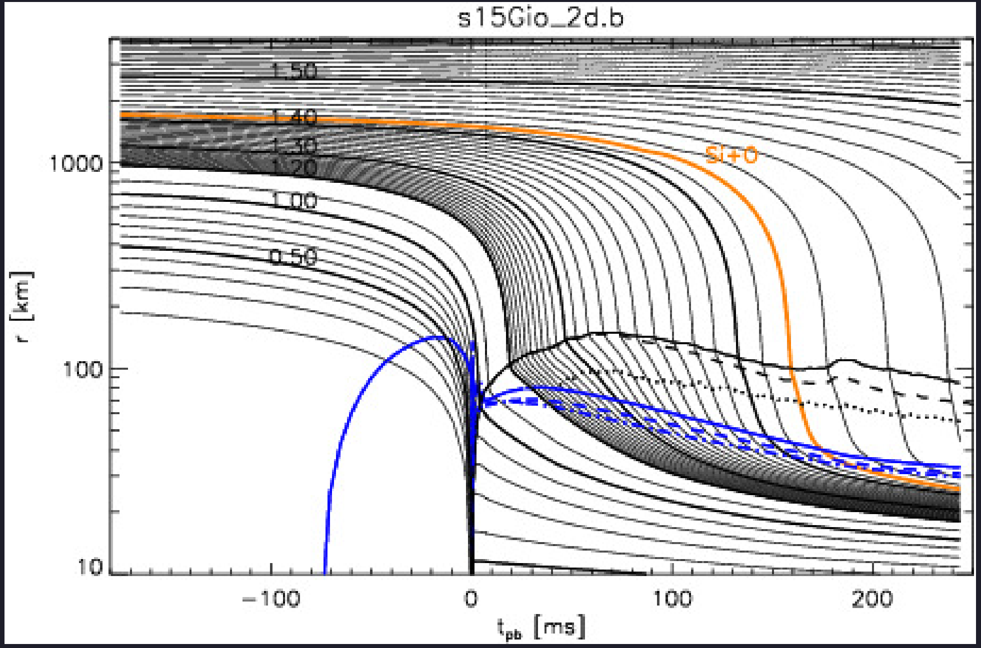
- Delayed Explosion
  - Bethe & Wilson (1985):
    Revival of a stalled supernova shock by neutrino heating
  - Energy deposition:

$$\dot{E} = K(T_\nu)\left[\frac{L_\nu}{4\pi R_m^2} - \left(\frac{T_m}{T_\nu}\right)^2 acT_m^4\right]\text{erg g}^{-1}\,\text{s}^{-1}$$

  - Requires high neutrino luminosities

  ⇒ neutrino driven convection

SN Simulations – Explosion?

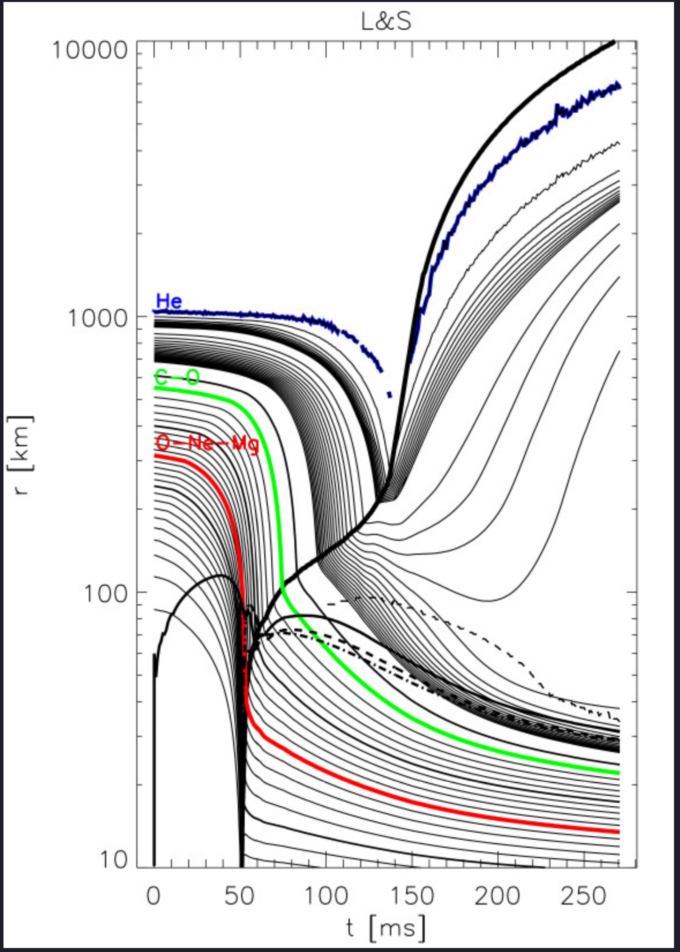


Simulation *without convection* ($15\ M_{\odot}$)

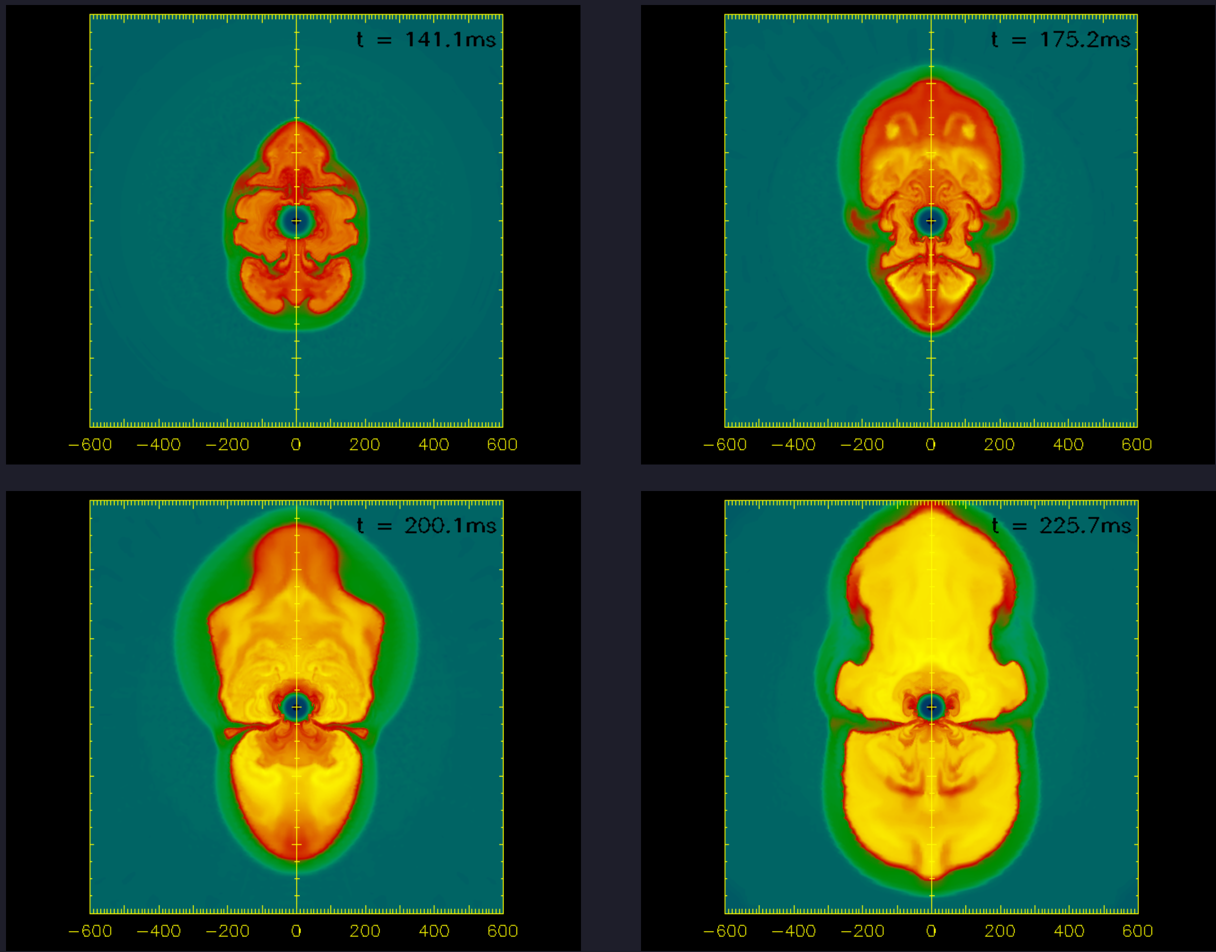
(R. Buras, PhD Thesis)

Progenitor ($8\ M_{\odot}$) with ONeMg Core

(F. Kitaura, diploma thesis)

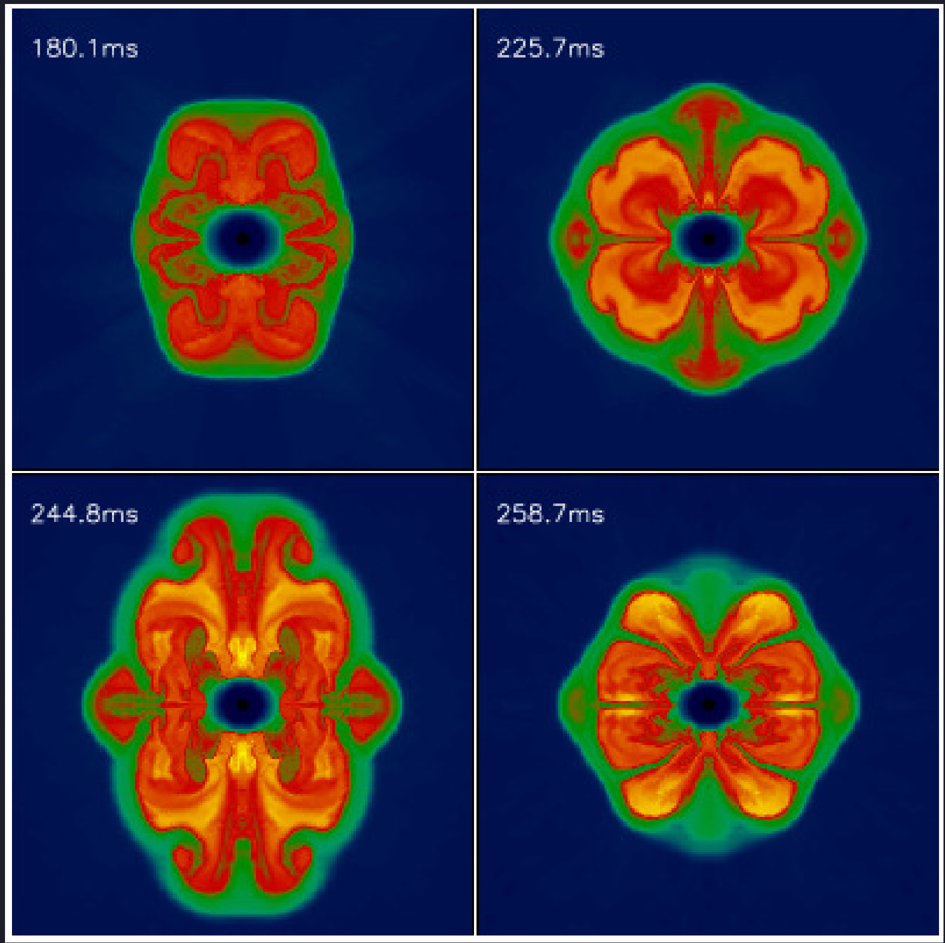


Simulation – 11 $M_\odot$ with convection



(Janka et al.?)

Simulation – $15\ M_{\odot}$ with convection



(Rampp et al.?)

SNe and Nucleosynthesis

- Elements of the progenitor
  - cp. progenitor model
  - $\approx 10\ M_\odot$
- Explosive nucleosynthesis after the shock front
  - ^{56}Ni, ^{58}Ni, ^{44}Ti, …
  - e. g. SN1987A: $M\,(^{56}\mathrm{Ni}) \approx 0.07\ M_\odot$
- Neutrino-driven winds

Neutrino-driven Winds

- Long term cooling of the nascent neutron star via neutrinos (~ 1 min)
- Winds could be the site where the r-process occurs.

- Nucleosynthesis is sensitive to:
  - Mass-loss rate
  - Dynamic timescale
  - Entropy per baryon
  - Electron fraction

$$\dot{M} = 4\pi r^2 \rho \upsilon$$

$$\upsilon \frac{d\upsilon}{dr} = -\frac{1}{\rho}\frac{dP}{dr} - \frac{GM}{r^2}$$

$$\dot{q} = \upsilon\left(\frac{d\varepsilon}{dr} - \frac{P}{\rho^2}\frac{d\rho}{dr}\right)$$

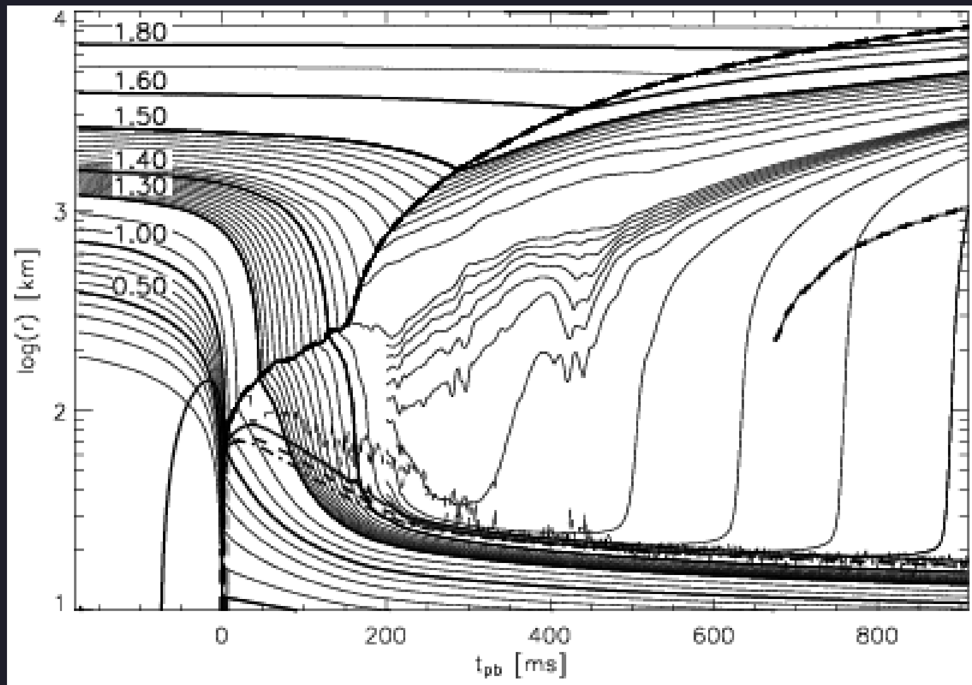
- Basic neutrino reactions

$$\nu_e + n \leftrightarrow p + e^-$$

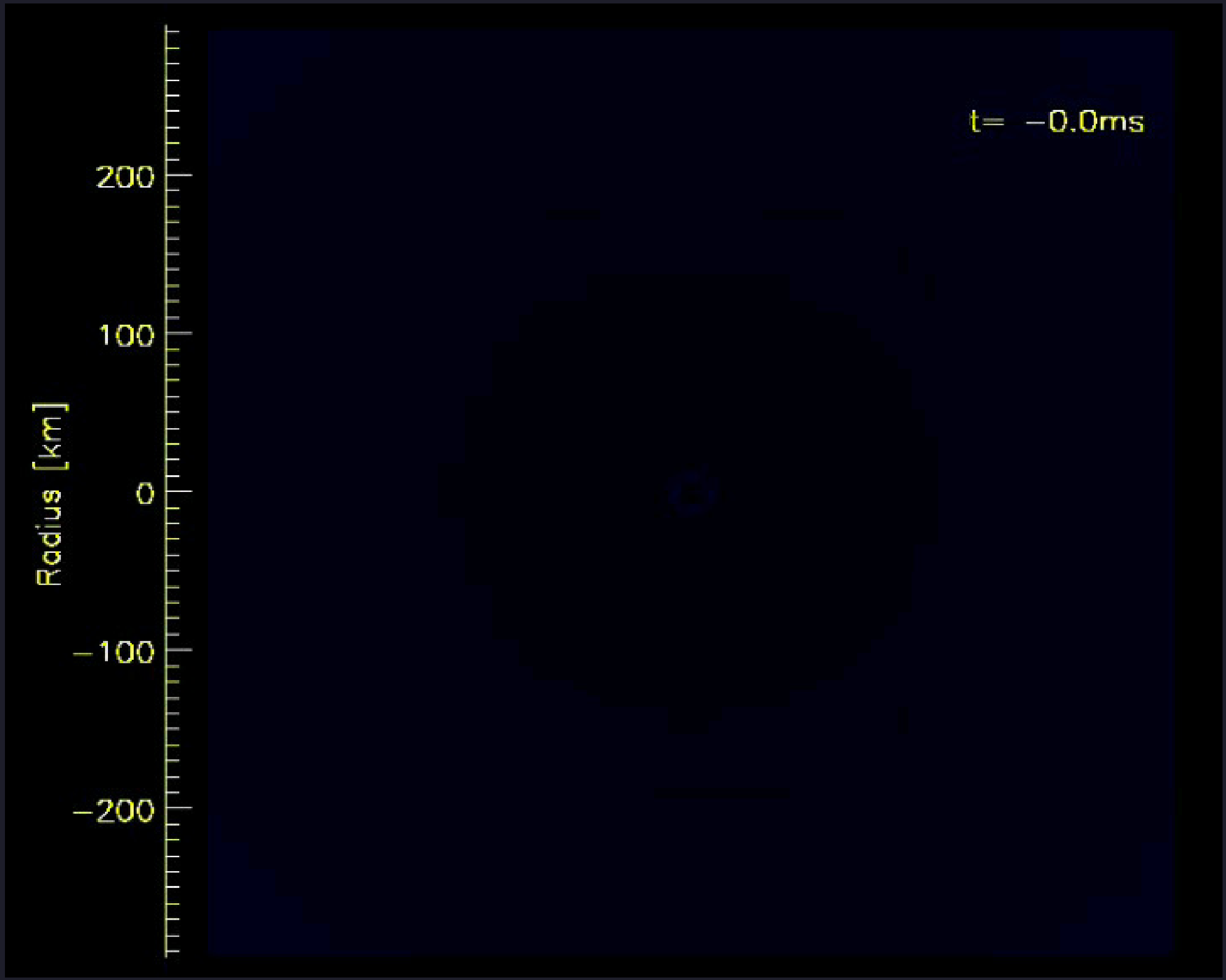
$$\bar{\nu}_e + p \leftrightarrow n + e^+$$

$$\upsilon \frac{dY_e}{dr} = \lambda_{\nu_e n} + \lambda_{e^+ n} - \left(\lambda_{\nu_e n} + \lambda_{e^+ n} + \lambda_{\bar{\nu}_e p} + \lambda_{e^- p}\right) Y_e$$

Neutrino-driven Winds



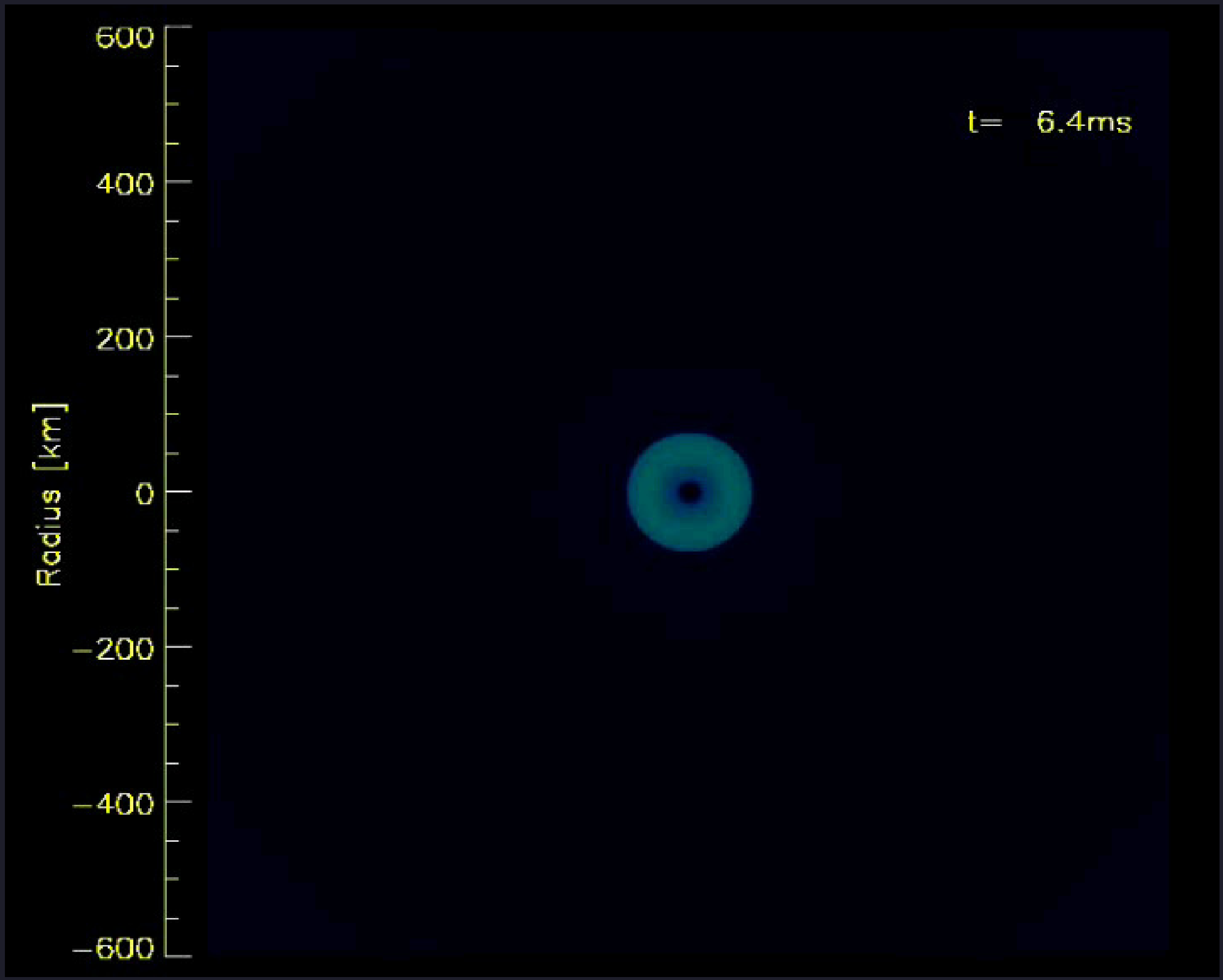
Simulation Results



R. Buras, MPA

$15\ M_{\odot}$

Simulation Results



11 $M_\odot$

References

Textbooks

1. Mihalas, D.: *Stellar Atmospheres,* Freeman & Company, Univ. Chicago Press (1987)
2. S. Shapiro, S. Teukolsky: *Black Holes, White Dwarfs and Neutron Stars,* John Wiley & Sons, New York (1983)
3. R.J. LeVeque et al.: *Computational Methods for Astrophysical Fluid Flow,* Springer, Berlin (1998)
4. R. Kippenhahn, A. Weigert: Stellar structure and Evolution, Springer (1994)

Papers

5. M. Rampp, H.-Th. Janka, A&A 396 (2002) 361 (astro-ph/0203101)
6. H. A. Bethe, J. R. Wilson, ApJ 295 (1985) 14
7. Y.-Z. Qian, S. E. Woosley, ApJ 471 (1996) 331

8. B. Müller: *Supernova Explosionen (Typ Ib, Ic, II)* (talk, unpublished)