

# Thermonuclear hydrodynamics & Type Ia Supernovae

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# Overview

- Basics of SNe Ia
- Theory of ignition and explosion
- Numerical tools and application to the SN case
- 3D simulations of SN Ia explosion
- Study of the ignition process

# What are SNe Ia?

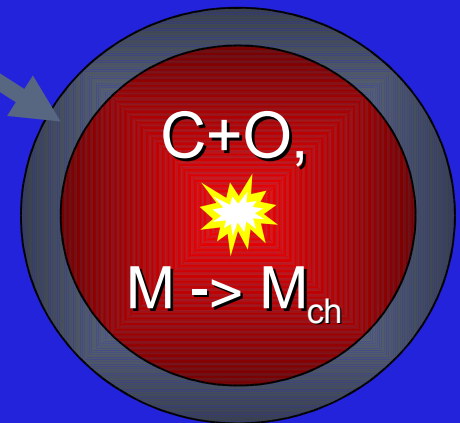
astrophysical events of enormous energy release and brightness

Favored astrophysical model:

thermonuclear explosion of  
a white dwarf star



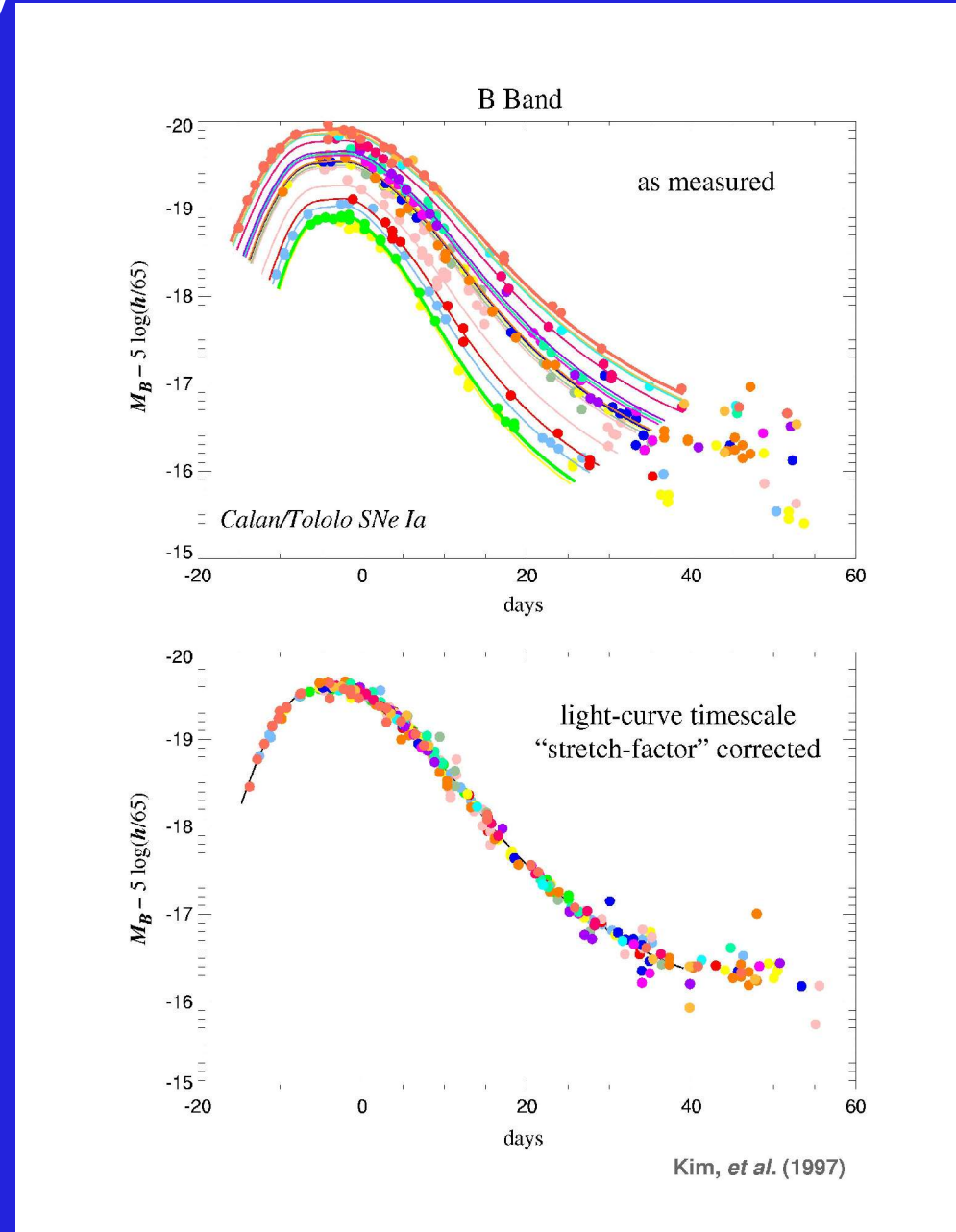
He (+H)  
from binary  
companion



# Why are SNe Ia interesting?

astrophysical relevance: cosmology

- tool for geometrical survey of the universe
  - × brightness
  - × uniformity
  - × empirical calibration
- content of the universe:
  - × 70% dark energy
  - × 30% matter (dark and visible)



# Goals of SN Ia simulations

- reproduce features (energy release, lightcurves, and spectra) of observed nearby SNe Ia
- understand origin of diversity of SNe Ia
- explain peak luminosity-light curve shape relation used to calibrate cosmological distance measurements
- get a handle on the systematical errors

modeling from "first principles" (3D necessary) in conjunction with high quality observations of nearby objects

Scheme for a  
theoretical model of  
SN Ia:



- progenitor model
- ignition model
- explosion model
- other theoretical tools

# The ignition process

The C/O WD grows by accretion from the companion, central density and temperature increase

$\rho_c \sim 2 \times 10^9 \text{ g cm}^{-3}$ : nuclear energy generation exceeds neutrino losses

Convective phase ( $\sim 1000 \text{ yr}$ )  $\rightarrow$  thermonuclear runaway at  $T \sim 10^9 \text{ K}$

# Explosion model

Because of the high sensitivity of the  $^{12}\text{C} + ^{12}\text{C}$  reaction rate on temperature ( $\propto T^{12}$  at  $T \sim 10^{10}$  K), at the conditions of explosive C burning the burning length scale is microscopic.

description of the propagation through the WD

- hydrodynamics: 2 modes:



## deflagration

subsonic flame:  
explosion mediated by  
thermal conduction of  $e^-$

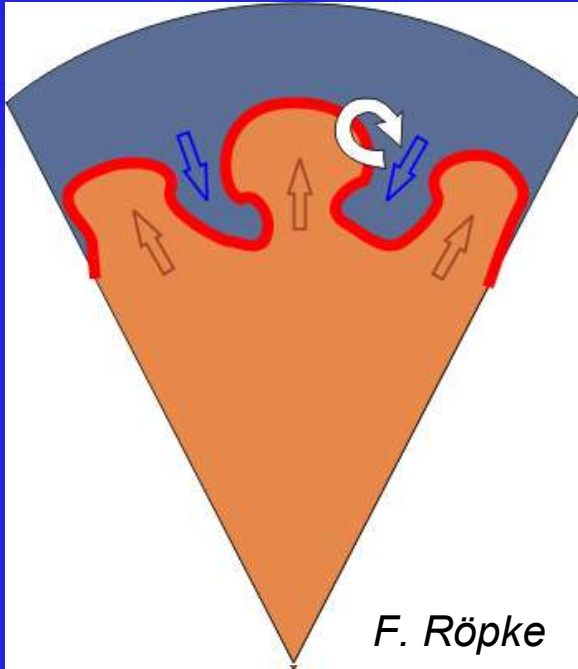
## detonation

(super)sonic  
driven by shock waves

- pure detonation would produce wrong composition of explosion products (Arnett, 1969)
- flame starts out as deflagration
- problem: laminar deflagration flame too slow
- main issue: acceleration of flame propagation

# Instabilities and turbulence

- interaction of flame with turbulence: turbulent combustion
- generic instabilities:



estimate of the Reynolds number around the RT-bubble:

$$Re(l) = \frac{l v(l)}{\eta / \rho}$$

$$L \sim 10^7 \text{ cm}, v_{\text{shear}} \sim 10^7 \text{ cm s}^{-1}$$

$$\rho \sim 10^9 \text{ g cm}^{-3}, \eta \sim 10^9 \text{ g cm}^{-1} \text{ s}^{-1}$$

$$Re \sim 10^{14}$$

- wrinkling of the flame front: flame surface proportional to the net burning rate  $\rightarrow$  propagation is strongly accelerated
- later transition to (supersonic) detonation?

# To sum up, which physical / computational ingredients are necessary for the simulation of SN Ia explosion?

- Equations for the fluid description
- Source terms: gravity, nuclear burning
- Turbulence modeling
- Flame propagation

# Classical motion of a fluid

Simplest hypotheses:

- Continuity assumption (length scales much larger than the mean free path)
- Dissipative effects (species diffusion, heat conduction, fluid viscosity) negligible
- No external source terms

As already explained last week, under these hypotheses the fluid is described by the Euler equations:

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{u}) = 0 \quad (1)$$

$$\frac{\partial \rho \mathbf{u}}{\partial t} + \nabla \cdot (\rho \mathbf{u} \mathbf{u}) + \nabla p = 0 \quad (2)$$

$$\frac{\partial \rho e_{tot}}{\partial t} + \nabla \cdot (\rho e_{tot} \mathbf{u}) + \nabla \cdot (p \mathbf{u}) = 0 \quad (3)$$

These is the differential form of the conservation laws for the quantities  $\rho$ ,  $\rho \mathbf{u}$  and  $\rho e_{tot}$ ; since the pressure  $p$  appears in the equations, an additional relation has to be provided, the equation of state (EOS)

$$p = p(\rho, e_i, \mathbf{X}); \quad T = T(\rho, e_i, \mathbf{X}) \quad (4)$$

In SN Ia simulations, the dissipative processes are safely negligible, thus they do not need to be inserted in the Euler equations. Two source terms must be included in this treatment:

**Gravity:** The gravity force modifies the momentum and energy conservation, introducing respectively the terms  $-\nabla\phi$  and  $-\rho \mathbf{u} \nabla\phi$  at the right-hand sides.

$$\nabla^2 \phi = 4\pi G \rho$$

**Nuclear burning**

# Nuclear burning and reactive Euler equations

Basic definitions: mass fraction of the species  $i$

$$X_i = \frac{n_i A_i}{\sum_i n_i A_i} = \frac{n_i A_i}{\rho N_A}$$

Molar abundance of  $i$

$$Y_i = n_i / (\rho N_A)$$

Because of nuclear reactions, the fluid composition changes with time. Following Arnett (1996), the rate of change of  $Y_i$  is

$$\frac{\partial Y_i}{\partial t} + \mathbf{u}(\nabla Y_i) = \dot{R}_i$$

where at the right-hand side is the total reaction rate, which takes into account every reaction producing or destroying  $i$ :

$$\dot{R}_i = \sum_j c_i(j) \lambda_j Y_j + \sum_{i,j} c_i(j,k) \rho N_A \langle j,k \rangle Y_j Y_k + \text{three-body term} \dots$$

where

$$c_i(j) = \pm N_i, c_i(j,k) = \pm N_i / (N_j! N_k!)$$

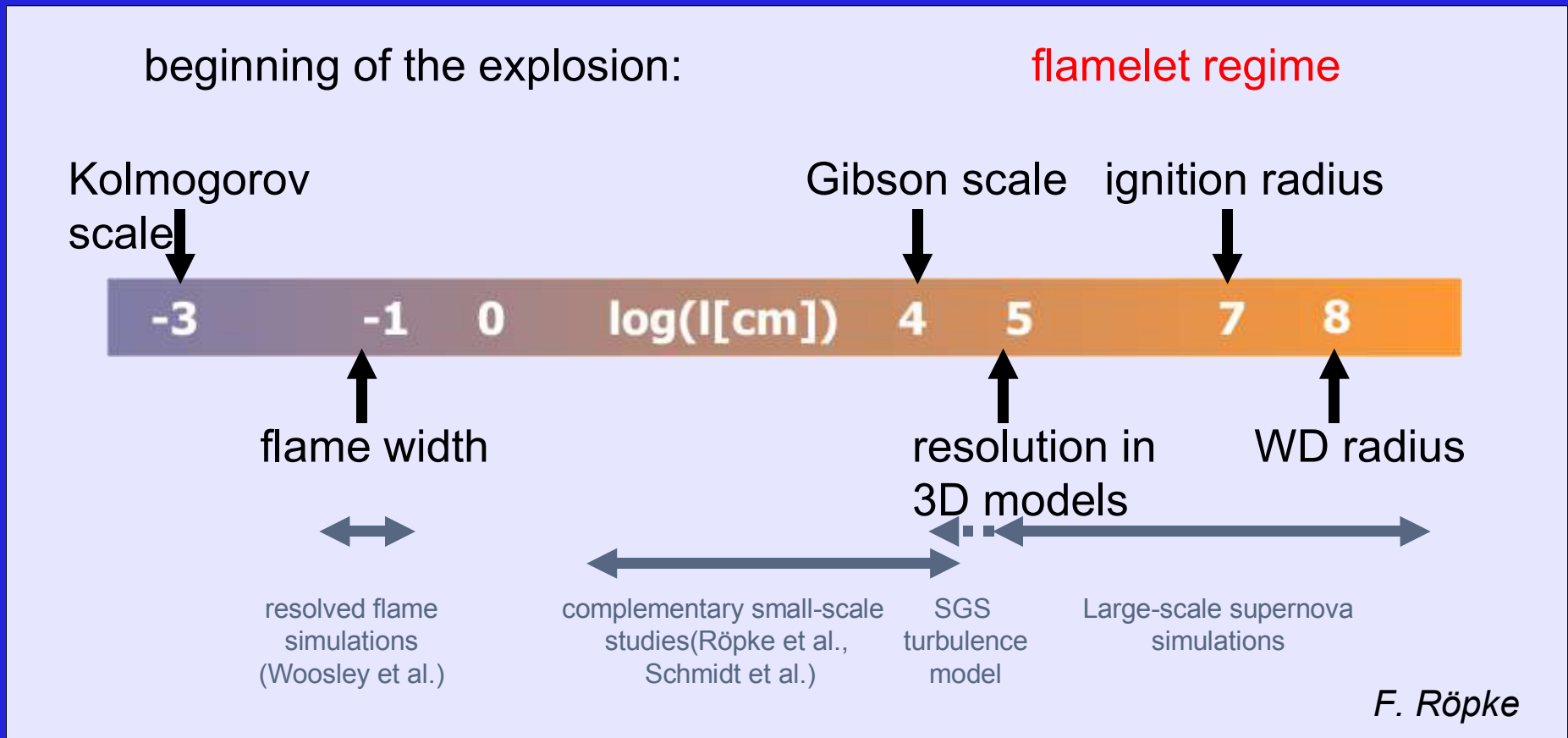
An energy production term must be included at the right-hand side of the Euler equation which expresses the energy conservation; it has the form  $\rho \dot{S}$ , where the energy generation rate is defined by

$$\dot{S} = N_A \sum_i \dot{R}_i B_i$$

$B_i$ : binding energy of the nucleus  $i$

So far the theory; how is it applied to the simulations of SNe Ia?

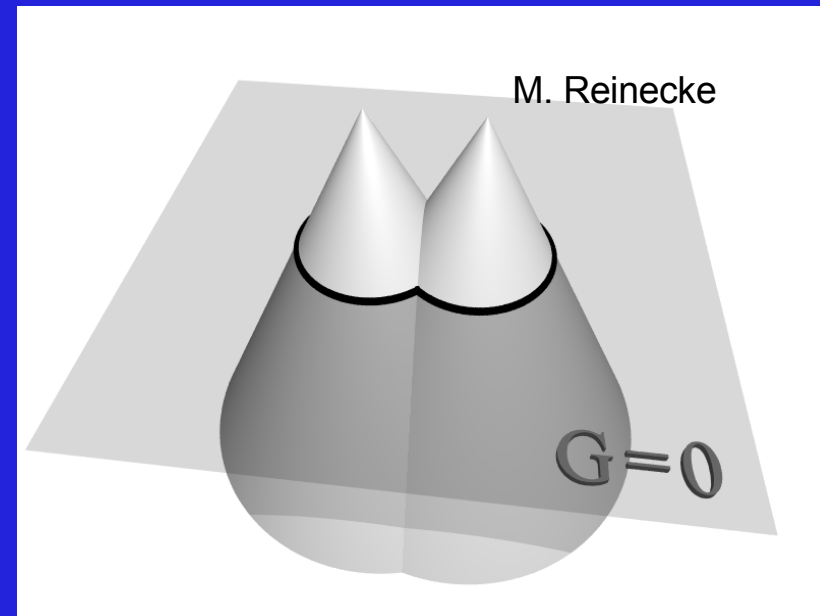
# Relevant length scales in simulations of SN Ia explosions



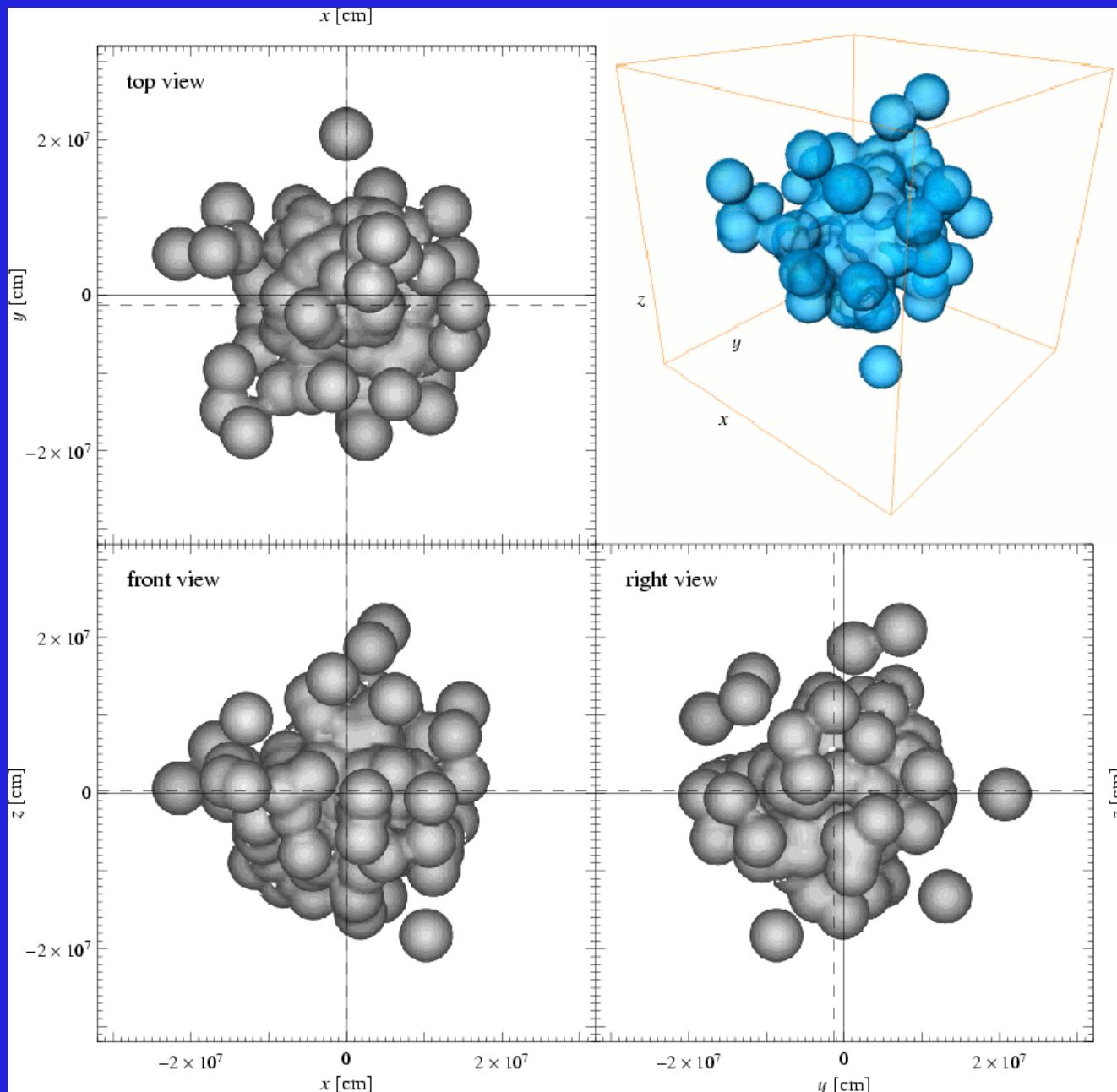
# Numerical tools for the simulation of explosion

explosion model (Reinecke et al., 1999, 2002)

- hydrodynamics: higher order Godunov: PROMETHEUS (Fryxell et al., 1989) implementation of PPM (Colella & Woodward, 1984)
- flame model: WD  $\sim 10^8$  cm  $\rightarrow$  structure of flame  $\sim 1$ mm  
Flame not resolvable, but modeled as a discontinuity between fuel and ashes
- level set method
- turbulence on unresolved scales implemented via sub-grid scale model
- "flamelet regime" of combustion:  
turbulent flame propagation velocity determined from sub-grid scale model
- simplified description of nuclear reactions



# 3D simulation of SN Ia explosion

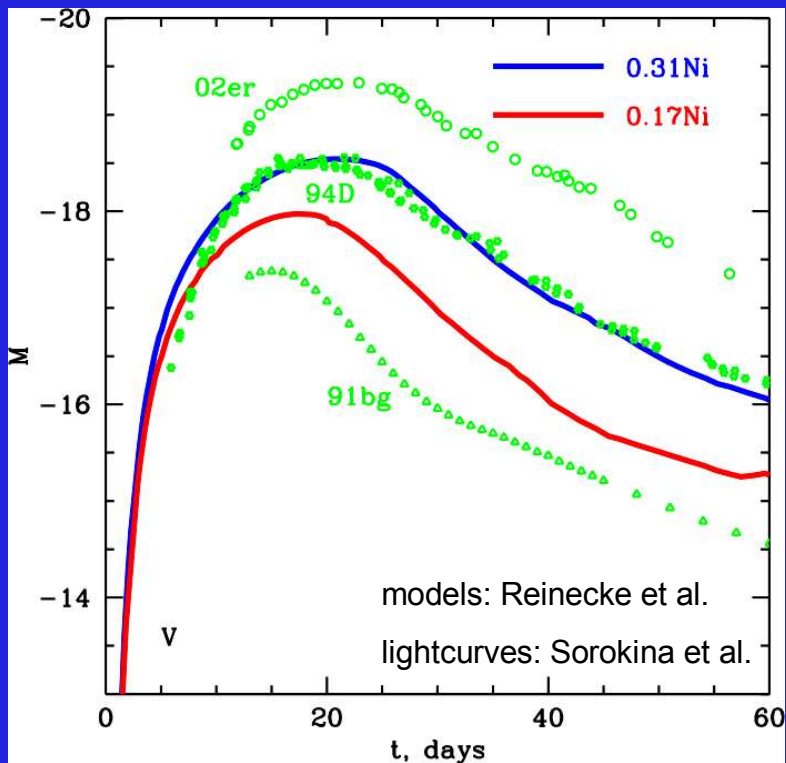


Initial flame displacement, Röpke & Hillebrandt (2005). In the projections, the solid lines denote the WD center, while the dashed lines indicate the center of the flame configuration.

# Main results of 3D, pure deflagration simulations

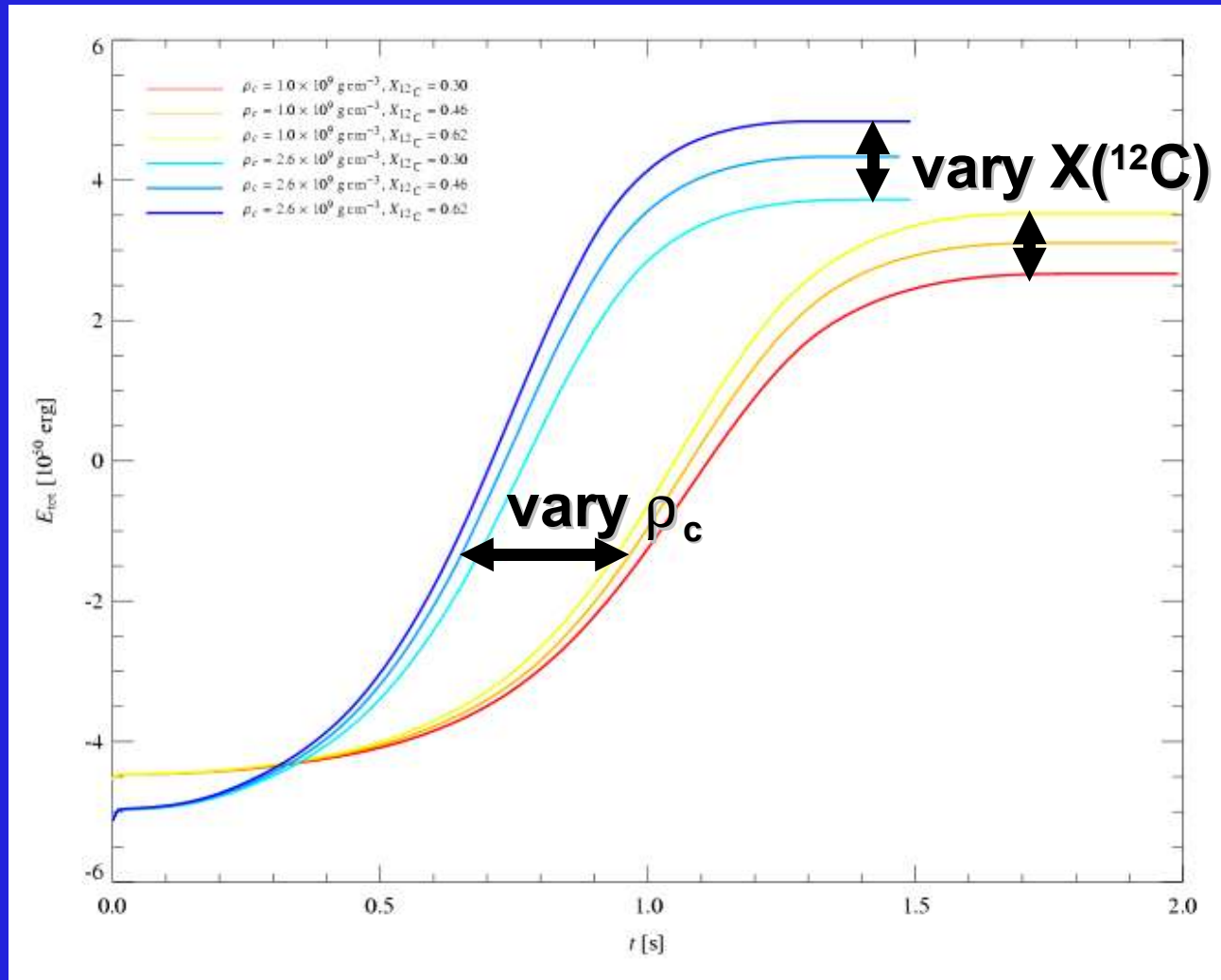
- the outcome is a successful explosion, though the energetic output is rather on the weak side of the range in SNe Ia;
- the derived maximum velocities of the ejecta are in the range of spectral observations ( $1.0 - 1.5 \cdot 10^9 \text{ cm s}^{-1}$ );
- a possible shortcoming is the presence of unburned C and O at low velocity, albeit it cannot be completely ruled out by observations.

# Further studies



↑ Synthetic light curves

Effect of varying C/O ratio and central density, Röpke 2005 →



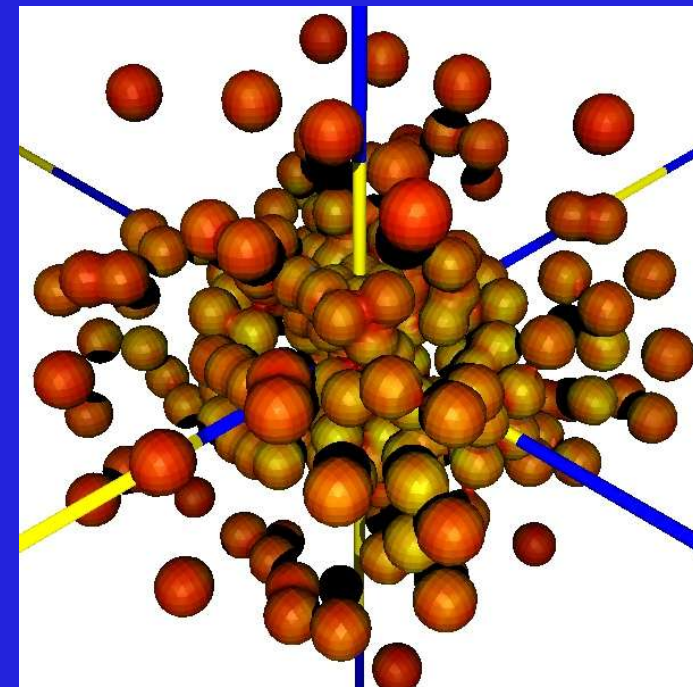
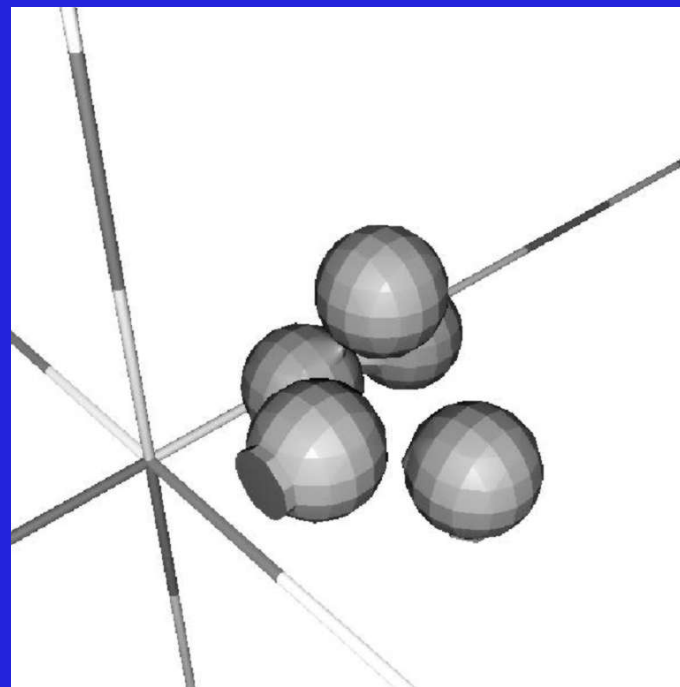
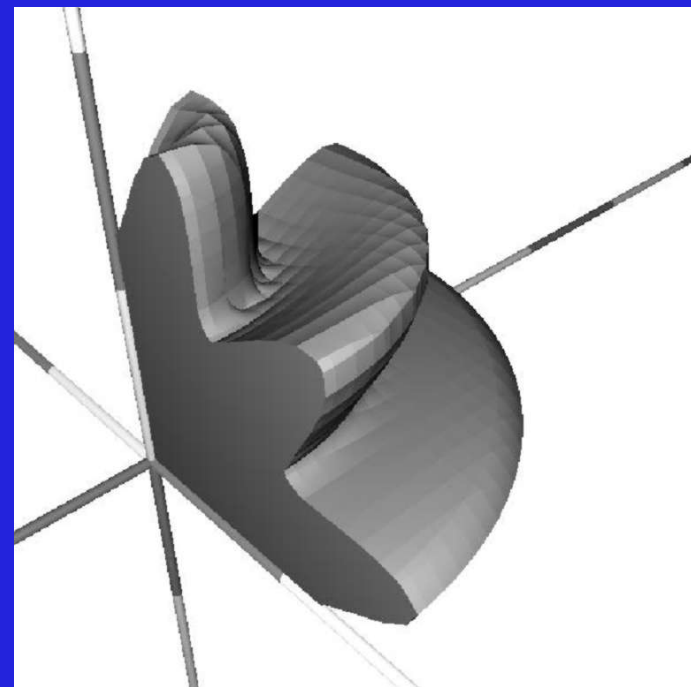
Other studies: i.e. nucleosynthesis (post-processing), effect of metallicity, ignition process

Further technical issues: nuclear burning, SGS model, DDT

# The ignition process of thermonuclear flames

It is important for investigating the initial flame position

Some examples of different possibilities:



Reinecke, Hillebrandt,  
Niemeyer: 2002, A&A  
**386**, 936

Reinecke, Hillebrandt,  
Niemeyer: 2002, A&A  
**391**, 1167

Travaglio, Hillebrandt,  
Reinecke, Thielemann:  
2004, A&A **425**, 1029

Previous analytical studies indicate that the ignition is driven by floating “bubbles”, generated as temperature fluctuations in the convective core.

The dynamics of such bubbles is therefore a powerful tool for the study of SNe Ia ignition.

Simulations of the bubble evolution have been performed using the FLASH code (cf. Fryxell et al. 2000). Details:

- A single bubble is set at rest in the computational domain;
- 2D Cartesian geometry, in plane parallel approximation;
- Only a small part of the WD ( $5 \text{ km} \times 20 \text{ km}$ ) is simulated;
- Initial WD model (provided by S.E. Woosley):  $T_c = 7 \times 10^8 \text{ K}$ ,  
 $\rho_c = 2.4 \times 10^9 \text{ gr cm}^{-3}$ ;
- Constant gravitational field;
- Minimal reaction network: 7 alpha isotopes.

# Parameter study

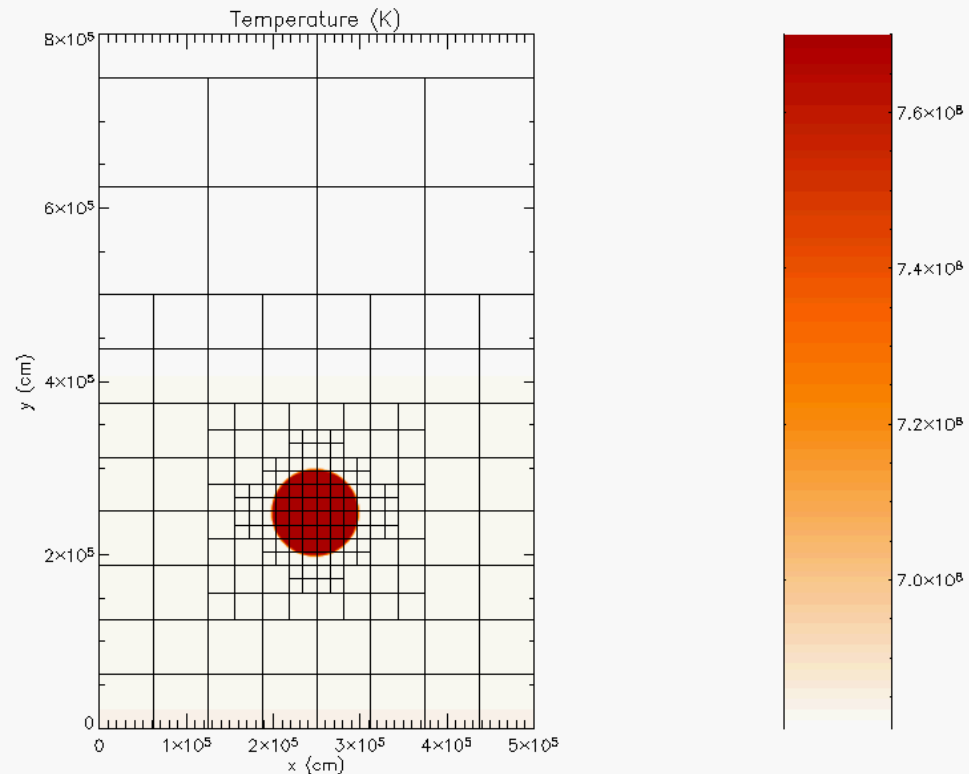
Parameters:

Distance from the center - Diameter - Temperature

Is there any favoured combination of these parameters?

What are the time scales to runaway?

What can we infer about the initial distribution of the ignition points?



time = 0.000 ps  
number of blocks = 264  
AMR levels = 5

# Ignition scenarios

$$g_{eff} \approx \frac{4}{3} \pi G \rho r \left( \frac{\Delta \rho}{\rho} \right) = \frac{4}{3} \pi G \rho r \delta_p \left( \frac{\Delta T}{T} \right)$$

**Bubble diameter D:** larger values favoured in simulations, but turbulent motions disrupts very large (> 1 km; Woosley et al. 2004) bubbles. Estimate:  $D \leq 1$  km.

**Central distance R:** smaller values (50 – 100 km) are favoured, because of smaller gravitational acceleration and larger background T.

**Bubble temperature T:** its role is more complex.

From the single bubble simulations, the most favored ignition model is the **multi-spot scenario**, with the ignition points distributed in the WD core up to  $R \sim 200$  km and according to the convection pattern before the runaway.

# References

- About theory of SNe Ia: Hillebrandt, W. & Niemeyer, J. C. 2000, *ARA&A*, **38**, 191
- About reactive Euler equations: Arnett, D. 1996, *Supernovae and nucleosynthesis* (Princeton University Press).
- ...