Student Seminar SS04

Nuclear reaction rates: Laboratory vs. Astrophysical Environments

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outline

Nuclear reactions: Notation

A nuclear reaction in which a particle a strikes a nucleus X producing a nucleus Y and a new particle b is commonly symbolised by

$$a + X \rightarrow Y + b$$

or by

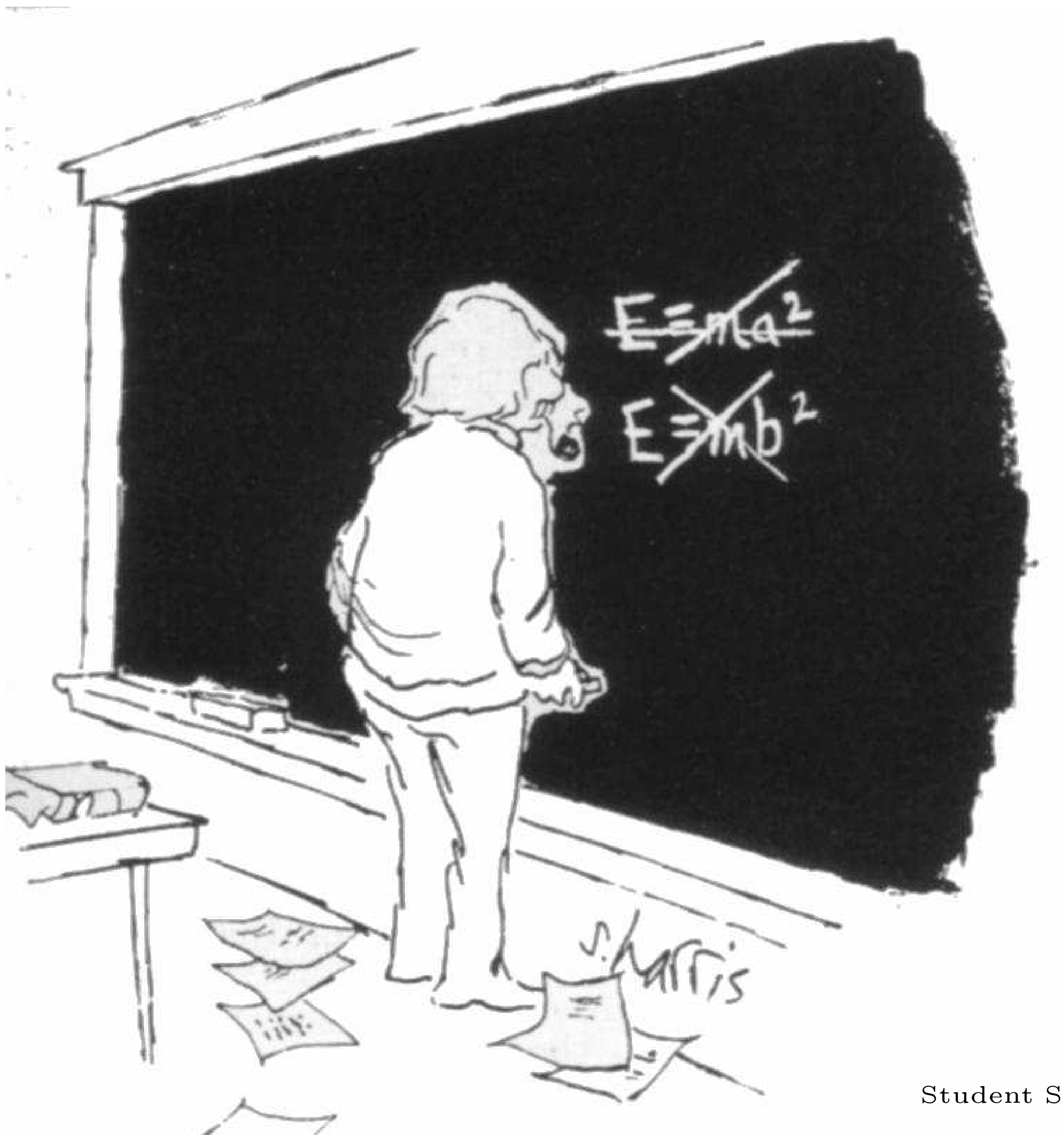
$$X(a, b)Y$$

Nuclear reactions: Examples

For example:

$$p +^{14} N \rightarrow^{15} O + \gamma \quad {}^{14}N(p,\gamma)^{15}O$$

$$^{12}C + d \rightarrow^{13} C + p \quad {}^{12}C(d,p)^{13}C$$


E=ma²
E=mb²
S. Harris

The Q-value

Definition of the Q-value: the amount by which the sum of the rest mass energies of the initial participants of a nuclear reaction *exceeds* the sum of the rest mass energies of all the products of the reaction.

$$X(a,b)Y$$

$$Q = \left[(M_X + M_a) - (M_Y + M_b)\right] c^2$$

exothermic reaction : $Q > 0$

endothermic reaction: $Q < 0$

In principle a exothermic reaction is possible even if the incident particles have *no* kinetic energy!

The Q-value (cont.)

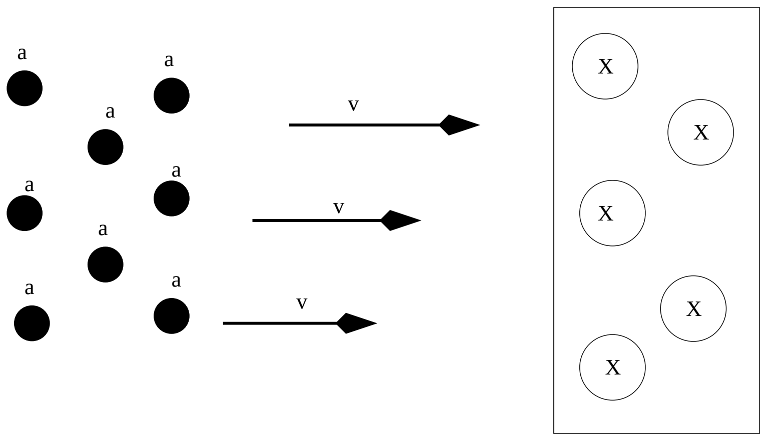
the **laboratory threshold energy** is the energy at which a *endothermic* reaction is energetically possible

$$\begin{aligned} E_{\text{thres}} &> |Q| \\ E_{X,a} - E_{Y,b} &= \left[(M_Y + M_b) - (M_X + M_a)\right] c^2 = -Q \end{aligned}$$

Thus the kinetic energy of the incident particles must be sufficient to

1. penetrate the Coulomb-barrier ($\rightarrow$ Gamow-Peak)
2. exceed the laboratory threshold energy

Cross section

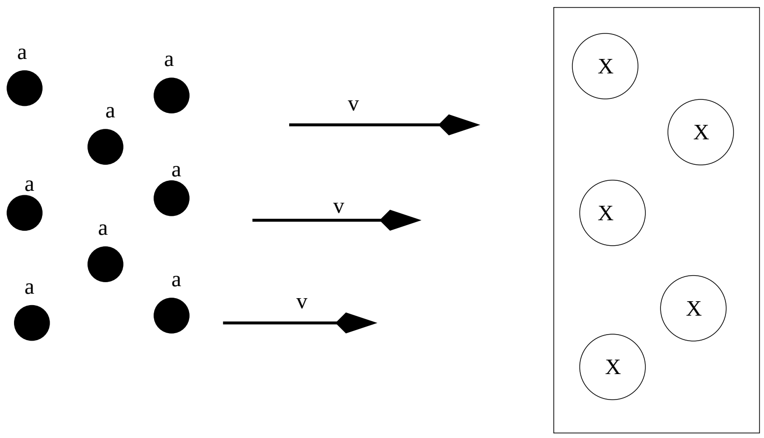


Idealised case: Bombardment of nuclei X with particles a, which have uniform velocity v. Uniform density N_X and N_a

Definition of the cross section:

$$\sigma(\mathrm{cm}^2) = \frac{\text{number of reactions/nucleus } X\text{/unit time}}{\text{number of incident particles/cm}^2\text{/unit time}}$$

Reaction rate



Idealised case: Bombardment of nuclei X with particles a, which have uniform velocity v. Uniform density N_X and N_a

Reaction rate:

$$r_{a,X} \quad = \quad \sigma(\mathrm{v})\mathrm{v}N_aN_X$$

Cross section and reaction rate

with the normalised *relative* velocity distribution $\int \Phi_{a,X} \mathrm{d}v = 1$ we obtain:

$$
\begin{aligned}
r_{a,X} &= (1 + \delta_{aX})^{-1} N_a N_X \int_0^\infty v\sigma(v)\Phi(v)dv \\
&= (1 + \delta_{aX})^{-1} N_a N_X \langle \sigma v \rangle
\end{aligned}
$$

One can show that if the velocity distribution of the incident particles are Maxwellian then the same applies to $\sigma(\mathrm{v})$

Cross section and reaction rate

Transformation in the centre of mass system and separation of translation velocity and relative velocity leads to:

$$
\begin{aligned}
r \;=\; & (1+\delta_{aX})^{-1} N_a N_X 4\pi \left(\frac{\mu}{2\pi k_{\mathrm{B}} T}\right)^{3/2} \times \\
& \int_0^\infty v^3 \sigma(v) \exp\left(-\frac{\mu v^2}{2 k_{\mathrm{B}} T}\right) dv \;,
\end{aligned}
$$

where $\mu = {m_a}^{-1} + {m_X}^{-1}$ is the reduced mass

The Gamow-Peak

Coulomb-barrier: $V = \frac{Z_1 Z_2 e^2}{R} \longrightarrow \; E \sim \mathrm{MeV}$

Astrophysical environments: $kT \longrightarrow \; E \sim 100\,\mathrm{keV}$

How can a significant amount of nuclear reactions proceed, when the Coulomb-potential is to high ?

Solution: quantum mechanical penetration probability of the Coulomb-potential $P \propto \exp\left(-\frac{2\pi Z_1 Z_2 e^2}{\hbar v}\right) = \exp\left(-bE^{-1/2}\right)$

Gamow-Peak (cont.)

Factorise cross section $\sigma(E)$:

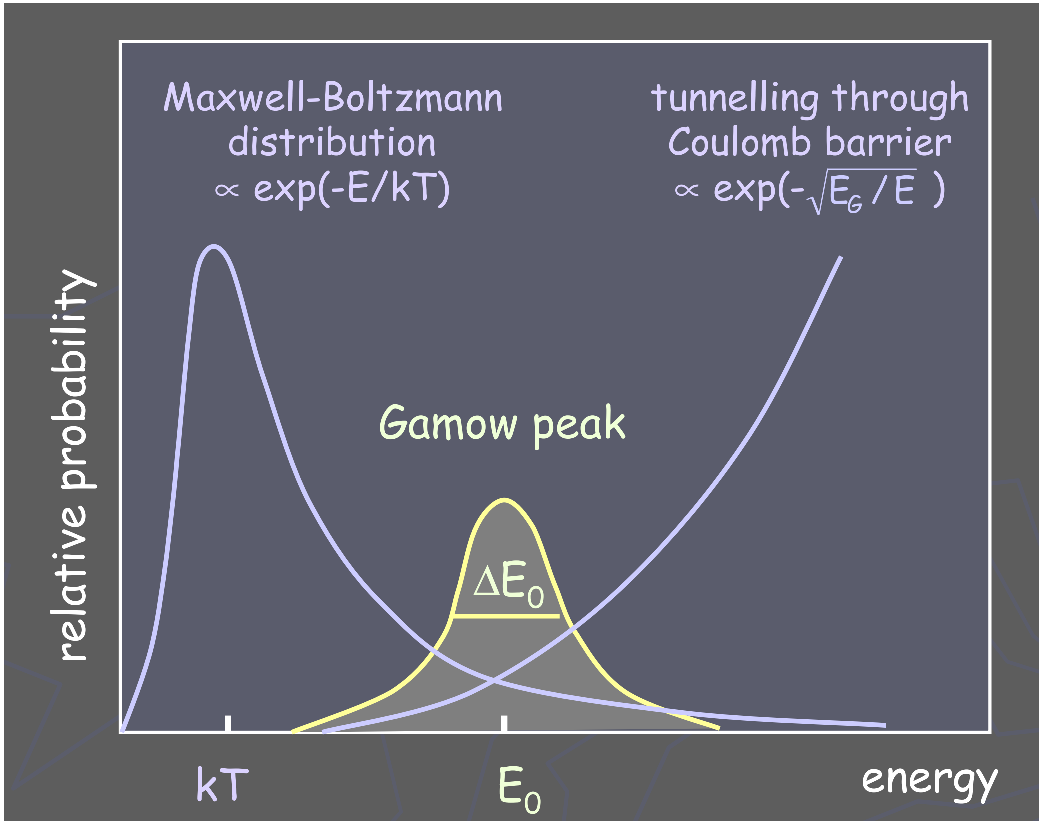
$$\sigma(E) \quad = S(E) \times E^{-1} \times \exp\left(-bE^{-1/2}\right)$$

$S(E)$:	includes everything we do not know about $\sigma(E)$
E^{-1}:	geometrical factor $\propto$ de Broglie wavelength
$\exp\left(-bE^{-1/2}\right)$:	penetration probability

Rewrite the reaction rate as energy dependent function:

$$r_{aX} \quad = \quad (1+\delta_{aX})^{-1} N_a N_X \left(\frac{8}{\mu\pi}\right)^{1/2} (k_\mathrm{B}T)^{-3/2} \times \int_0^\infty S(E) \exp\left(-\frac{E}{k_\mathrm{B}T} - bE^{-1/2}\right) \mathrm{d}E$$

Gamow-Peak (cont.)



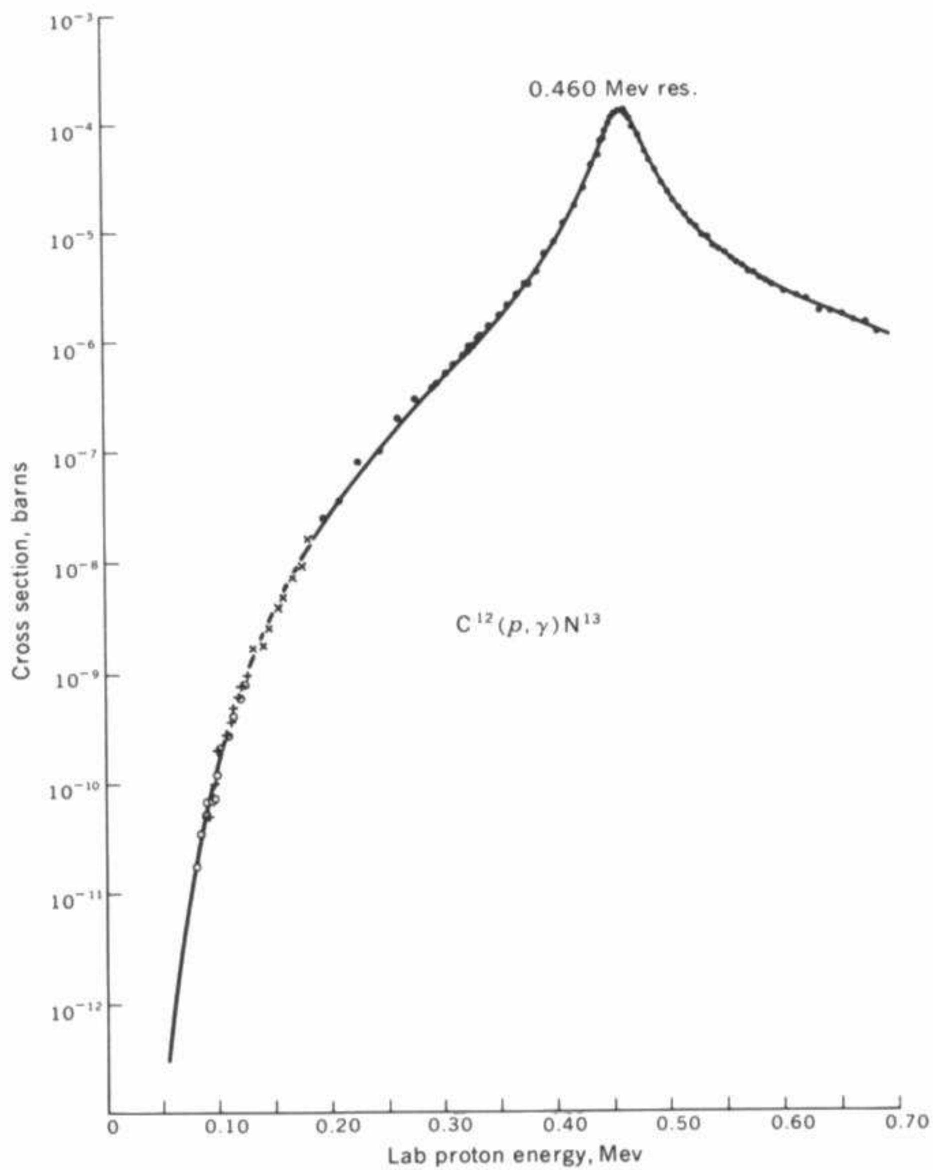
$$\frac{\mathrm{d}}{\mathrm{d}E}\left(\frac{E}{k_{\mathrm{B}}T}+bE^{-1/2}\right)_{E=E_0} = 0 \rightarrow E_0=\left(\frac{bk_{\mathrm{B}}T}{2}\right)^{3/2}$$

The astrophysical S-factor

Recall : $\sigma(E) = S(E) \times E^{-1} \times \exp\left(-bE^{-1/2}\right)$

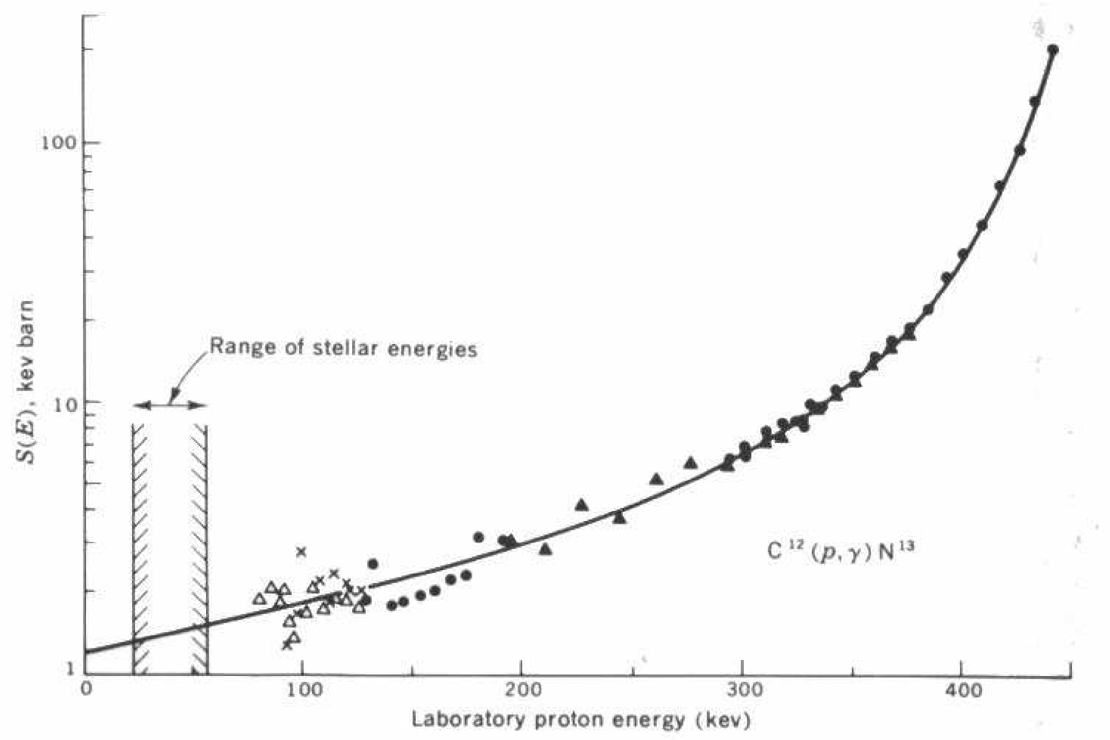
- $S(E)$ is called the astrophysical S-factor
- $S(E)$ must contain all intrinsic nuclear properties of the specific reaction since the other two factors describe only energy dependence
- If no resonance appears: $S(E)$ is often found to be only weakly energy dependent
- No complete theory of nuclei $\longrightarrow$ $S(E)$ from measurements and extrapolation ?

Example: $^{12}C(p,\gamma)^{13}N$



- The cross section is rapidly changing with the energy!

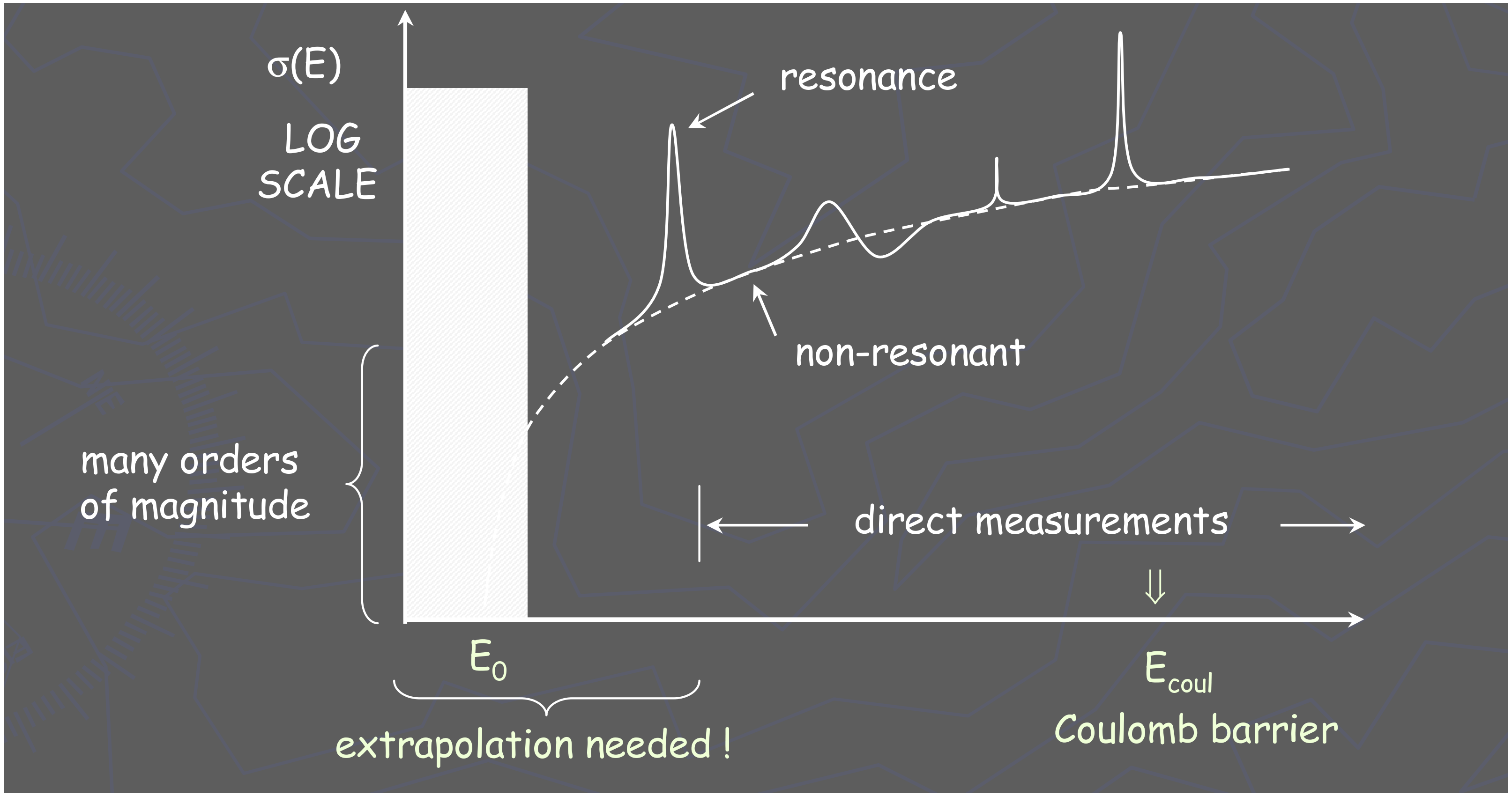
Measuring the cross section (cont.)



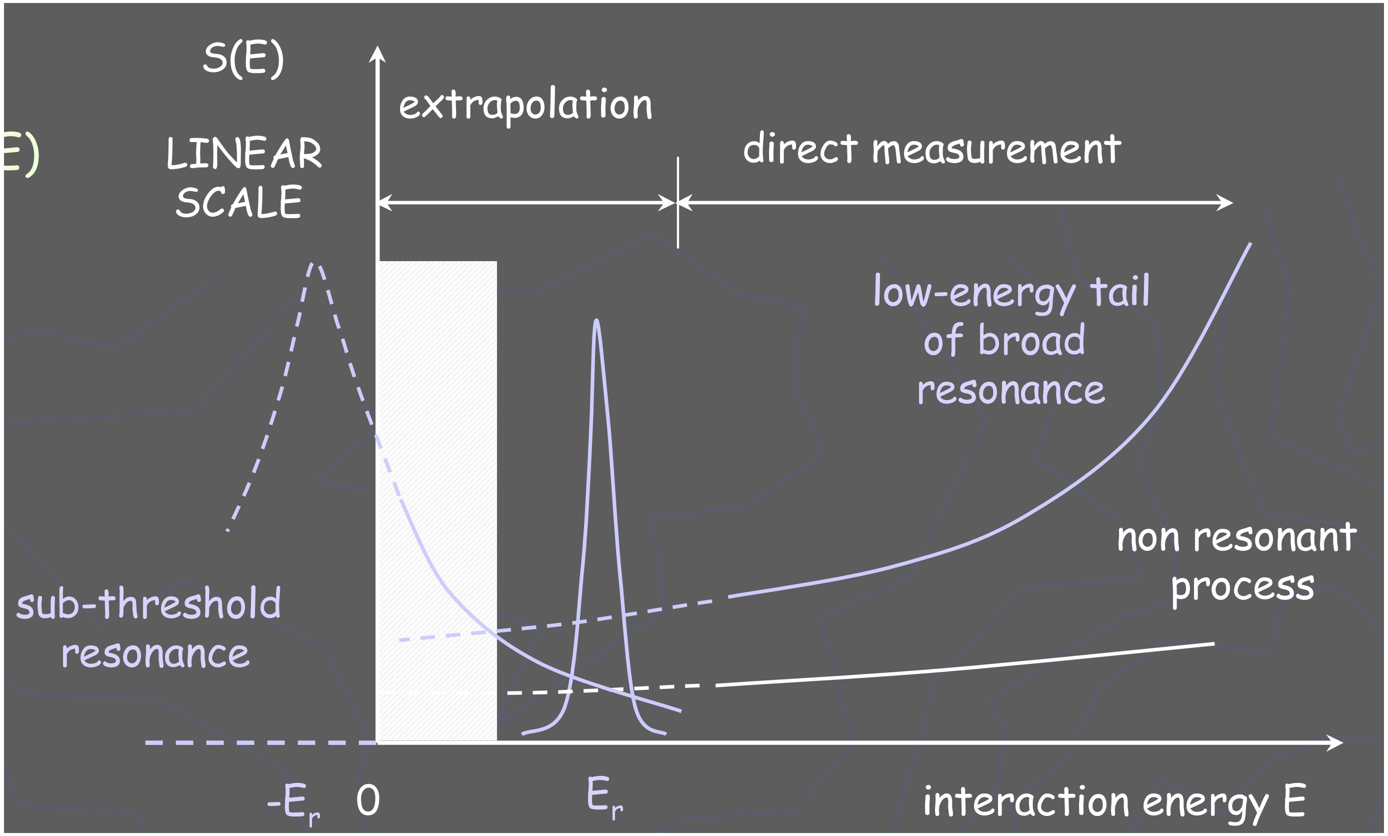
Recall: $S(E) = \sigma(E) \times E \times \exp\left(-bE^{-1/2}\right)$

$S(E)$ a slowly varying function of energy $\longrightarrow$ extrapolation more safely !

Resonances I



Resonances II



Electron screening

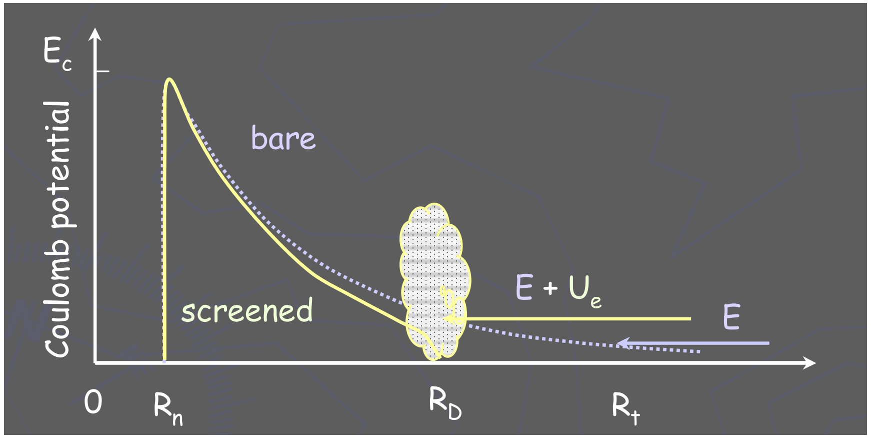
- Laboratory: Interaction between **ions** (projectiles) and **atoms or molecules** (target)
- (Stellar plasma: **Ions** surrounded by **electron cloud**

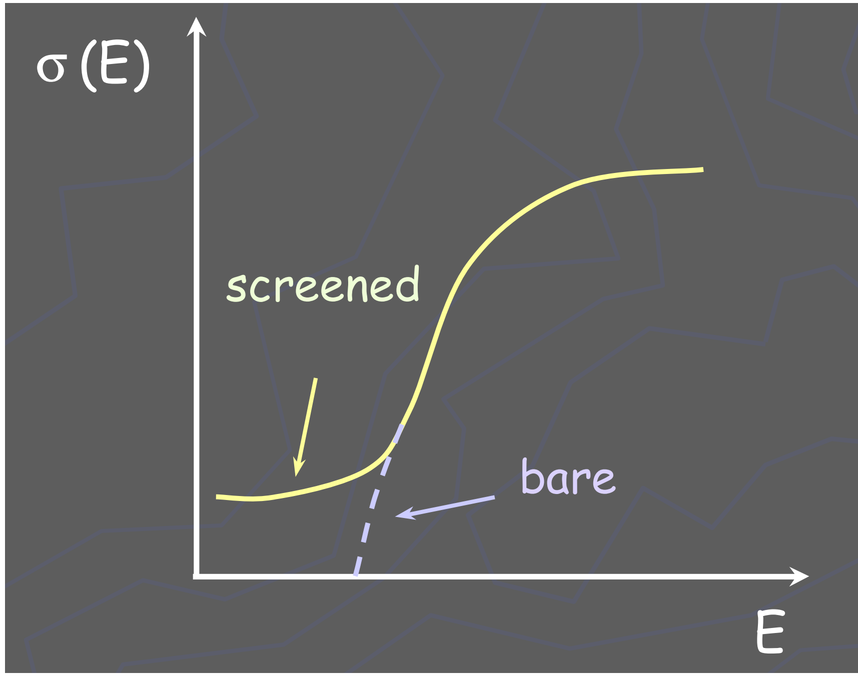
$\Longrightarrow$ at low energies electron screening effects become important

$\Longrightarrow$ knowledge of electron screening effects are important for

astrophysical nuclear reaction models

Electron screening (cont.)





$$f_{\mathrm{lab}}(E) = \frac{\sigma_s(E)}{\sigma_b(E)} \geq 1$$

The Trojan Horse Method

The astrophysical relevant process

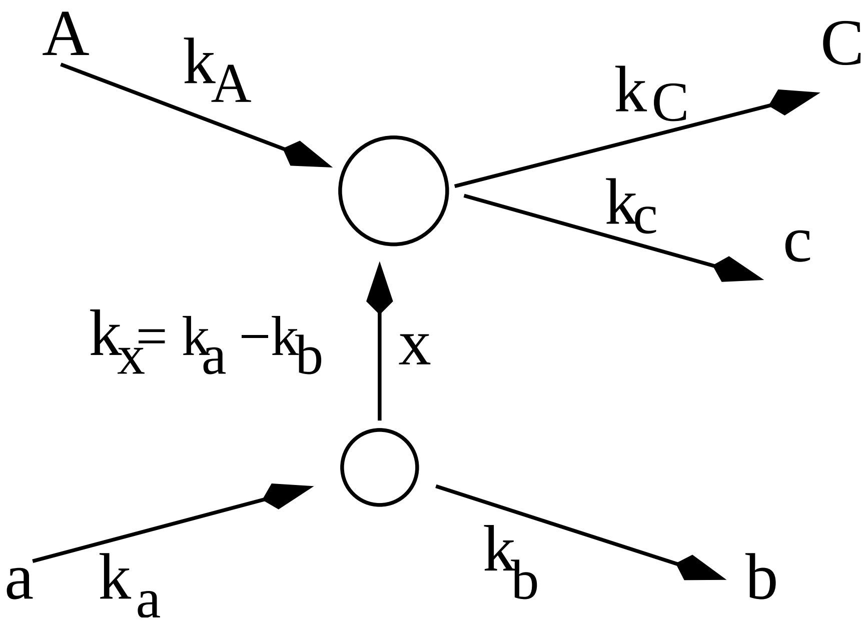
$$A + x \;\; \rightarrow \;\; C + c$$

is studied via the reaction

$$A + a \;\; \rightarrow \;\; C + c + b$$

where the nucleus a ("Trojan Horse") is clusterised as $b + x$, and assumed to break-up into two clusters x and b.

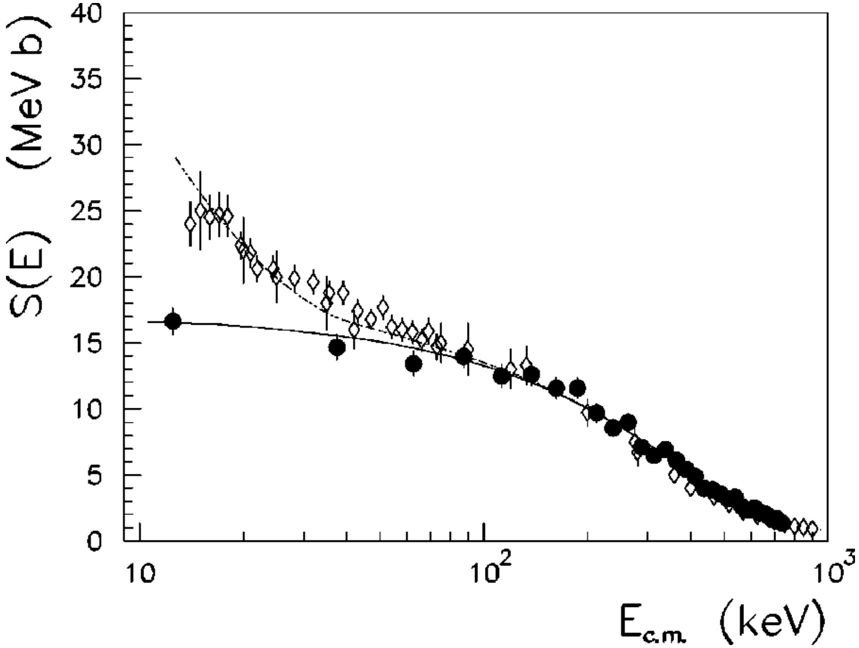
Trojan-Horse (cont.)



The momentum distribution of the ”Horse” is studied, in order to extract information of the desired two-body reaction.

Trojan-Horse (cont.)

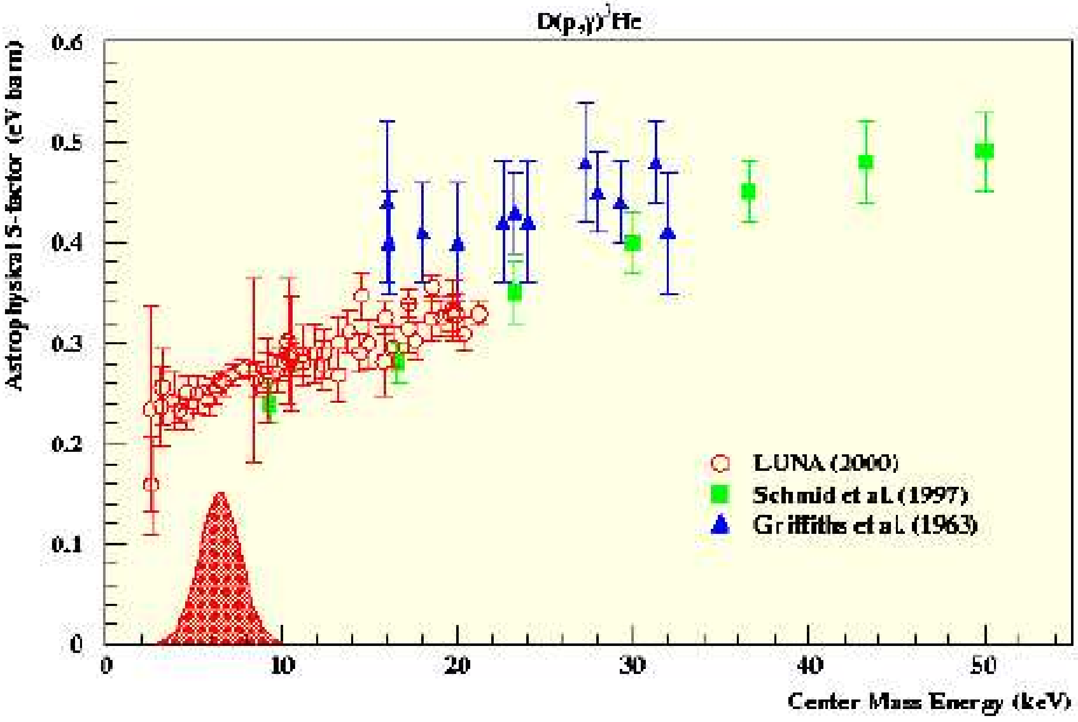
Example: the reaction $^{6}Li(d,\alpha)^{4}He$ via the reaction $^{6}Li(^{6}Li,\alpha\alpha)^{4}He$



Spitaleri et. al, Phys. Rev. C 63 (2001)

The LUNA-Experiment

- first experiment to meassure in the energy range of the Gamow-Peak



$$D(p,\gamma)^3He$$

Literature

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- Williams, ”Nuclear and particle physics”, Oxford Science Publications
- Langanke & Assenbaum,”Effects of Electron Screening on Low-Energy Fussion Cross Sections, Z. Phys. A,**327**
- Baur & Typel, ”Theory of the Trojan-Horse Method”,nucl-th:/0401054
- Spitaleri et. al, ”Trojan-Horse method applied to $^{2}H(^{6}Li,\alpha)^{4}He$ at astrophysical energies