NSE and Nucleosynthesis in Thermonuclear Supernovae

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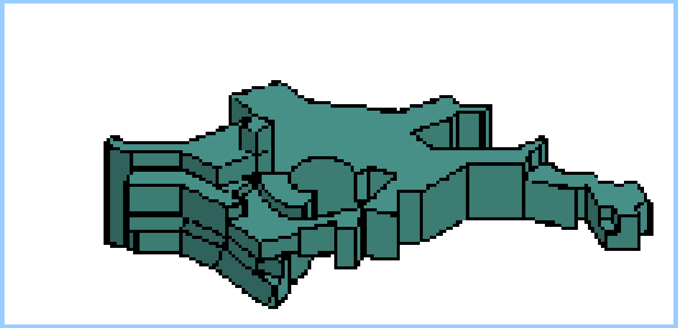


Overview

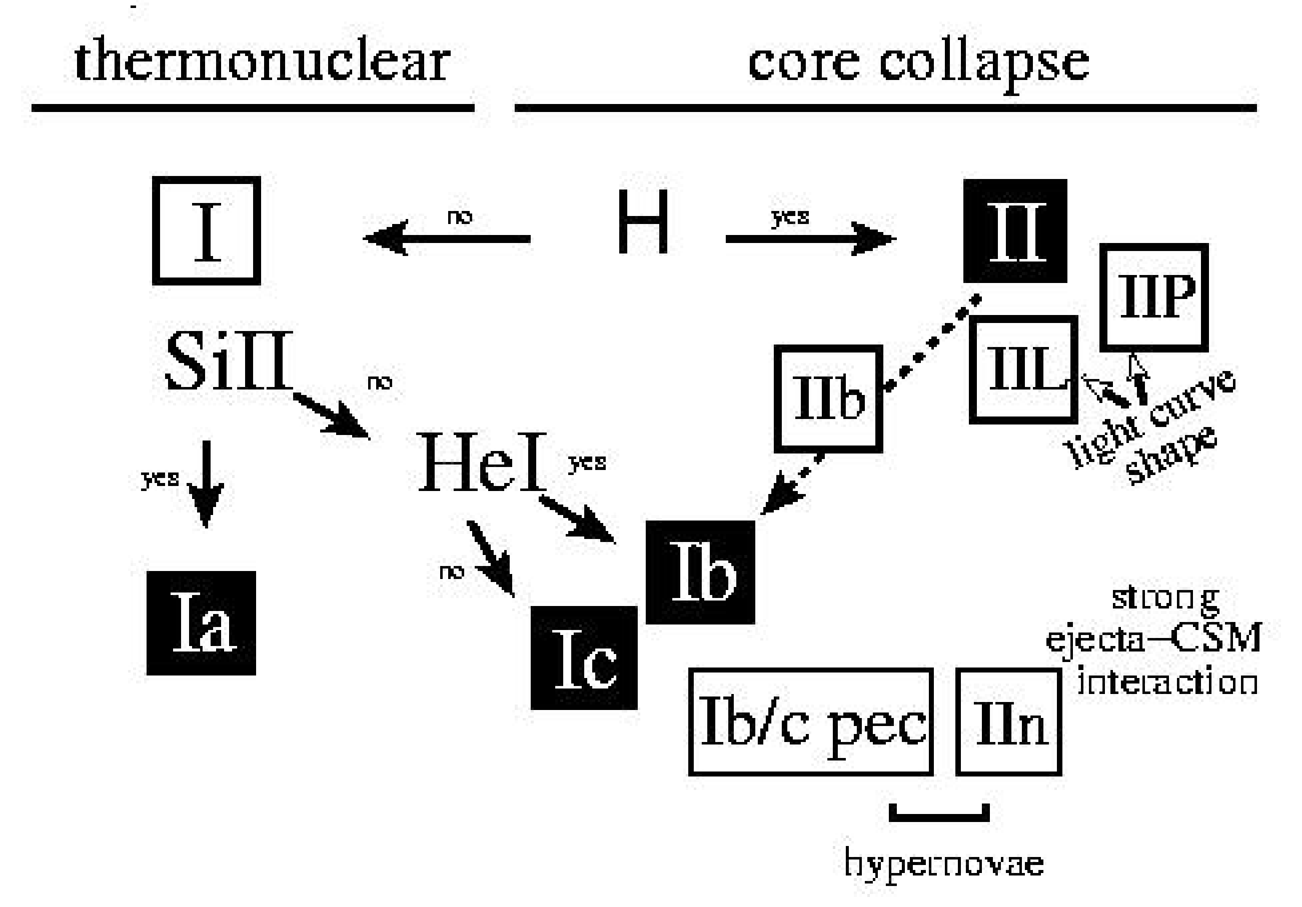
- Thermonuclear supernovae: basic ideas
- Why nucleosynthesis is interesting in this site
- What is NSE?
- Nucleosynthesis in thermonuclear supernovae: tecniques and some results

Thermonuclear supernovae...

... in the following, we will call them shortly

Supernovae of Type Ia (SNe Ia)

Although correct, this name doesn't come from a physical, but from an observational (spectral) classification



Turatto, astro-ph/030117

SN Ia: basic facts

From observational evidences, it is believed that most of SNe Ia are the result of the explosion of carbon-oxygen white dwarfs, that have approached Chandrasekhar mass (~ 1.39 M_{sol}) accreting mass in a binary system.

Other models: sub-Chandrasekhar, double-degenerate scenario...

A thermonuclear runaway is caused by the compressional heating, induced by accretion. Assume $\varepsilon_{nuc} \uparrow$:

In normal stars: $T \uparrow$

$P = \Re/\mu \, \rho T \; \uparrow$

Expansion

Self-regulation

In degenerate stars: $T \uparrow$

$P = k \rho^{5/3}$ remains unchanged!

$\varepsilon_{nuc} \uparrow\uparrow$

$T \uparrow\uparrow$

Thermonuclear runaway

Explosion mechanism

In the currently favored scenario the explosion starts as a deflagration near the WD center.

> Deflagrations propagate subsonically, with a speed determined on the balance of thermonuclear heating and heat conduction.

Due to the extreme temperature dependence of burning in WD (~ T^{12} at T ~ 10^{10} K) the deflagration zone (“flame”) is less than 1 cm thick.

Deflagration flames are distorted by the Rayleigh-Taylor instability; the flame area is increased and the front is then accelerated. Some models require a transition to detonation.

> Implementation details are beyond the scope of this seminar.

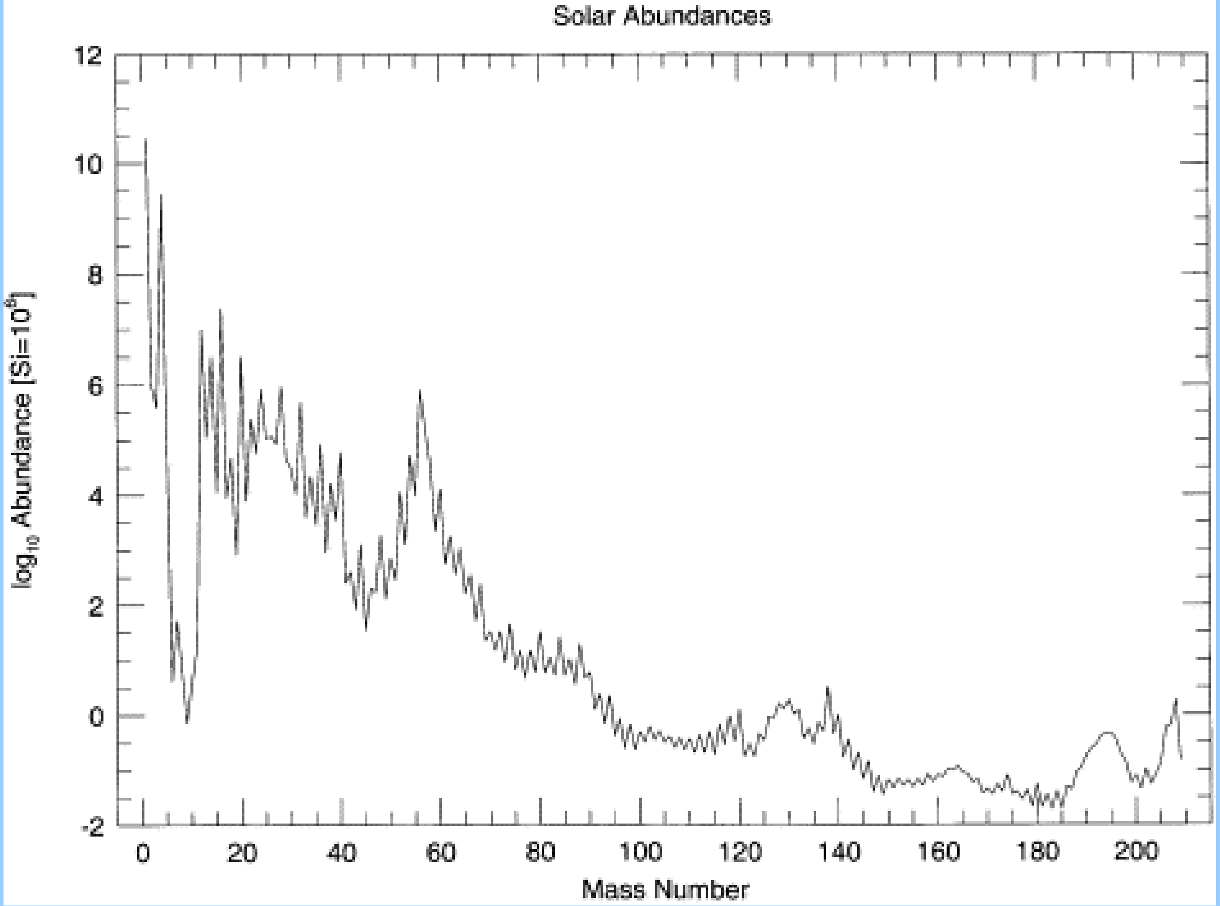
For giving you a better idea: a movie of a 3D explosion (cortesy of F. Röpke)

Nucleosynthesis in SNe Ia

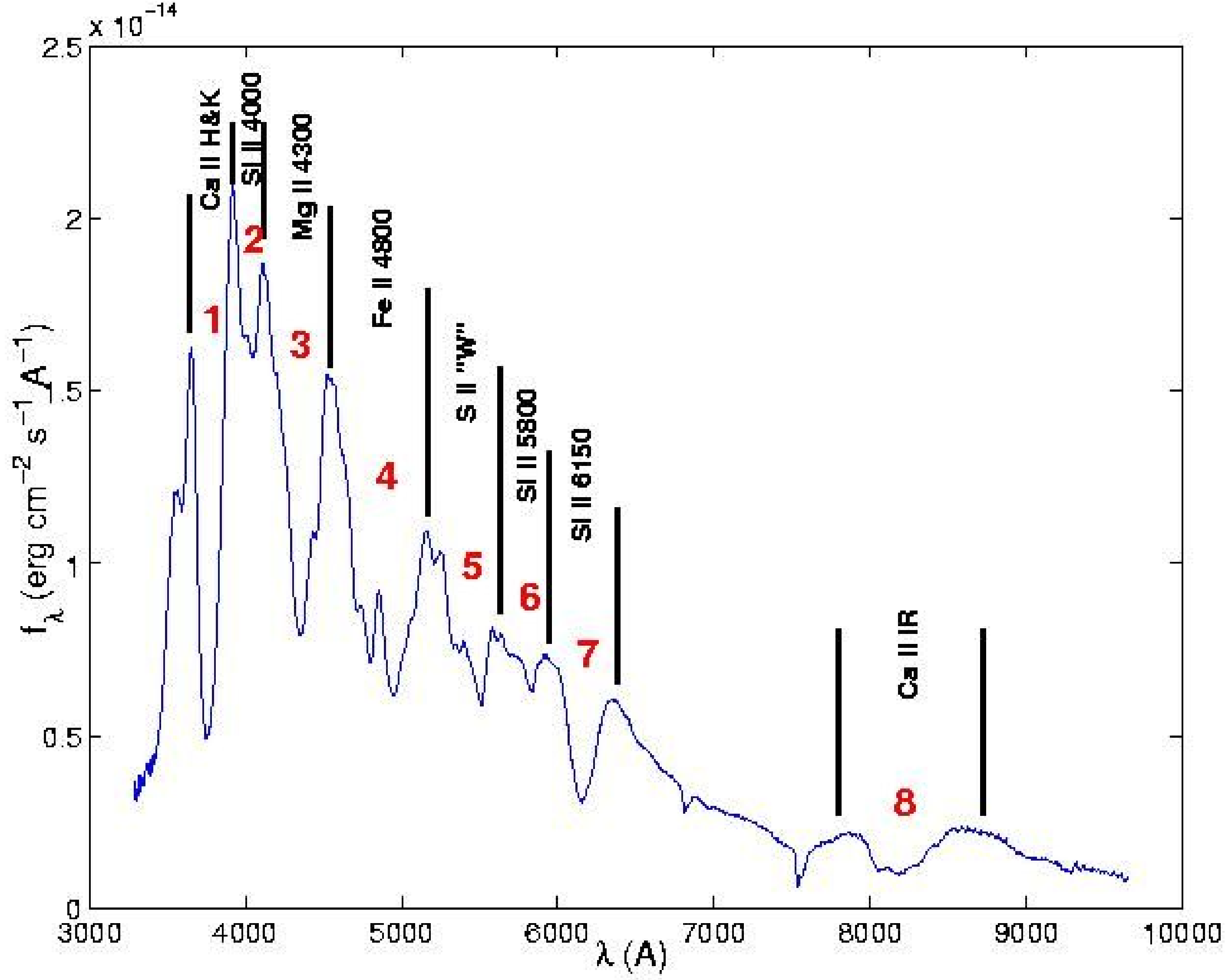
SNe Ia are considered the major Galactic contributors of iron peak nuclei (~ 55%); significant abundances of intermediate mass elements are also produced.

Every detailed study of this framework has to be compared with some observational data:

1) solar isotopical composition



2) spectral features (abundances and velocity dispersion)



From a near-maximum spectrum; G. Folatelli, talk at RTN winter school, Ringberg Castle, 2003.

3) An important constraint: light curves and ^{56}Ni production

Luminosity of SNe Ia is set not by explosion, whose energy goes into expansion, but from decay of ^{56}Ni formed during the nucleosynthesis. This is consistent with the time scales of the light curve.

$^{56}\mathrm{Ni}(e^{-},\nu)^{56}\mathrm{Co}$; $t_{1/2} = 6.10\mathrm{d}$

$^{56}\mathrm{Co} \rightarrow {}^{56}\mathrm{Fe}$ via E.C.(81%) or β^{+} (19%) ; $t_{1/2} = 77.12\mathrm{d}$

Produced nuclei are in excited states, so γ-rays are emitted

One of the most interesting features of SNe Ia:

the homogeneity in peak luminosity. Under the assumption that the energy released on the surface at maximum light equals the energy injected by ^{56}Ni decay (“Arnett's law”), it's possible to evaluate the mass of ^{56}Ni produced in the explosion.

↓

Use of SNe Ia as standardizable candles

Explosion and/or nucleosynthesis models of SNe Ia must account for a ^{56}Ni production (according to 1D models) of about 0.6 M_{sol}

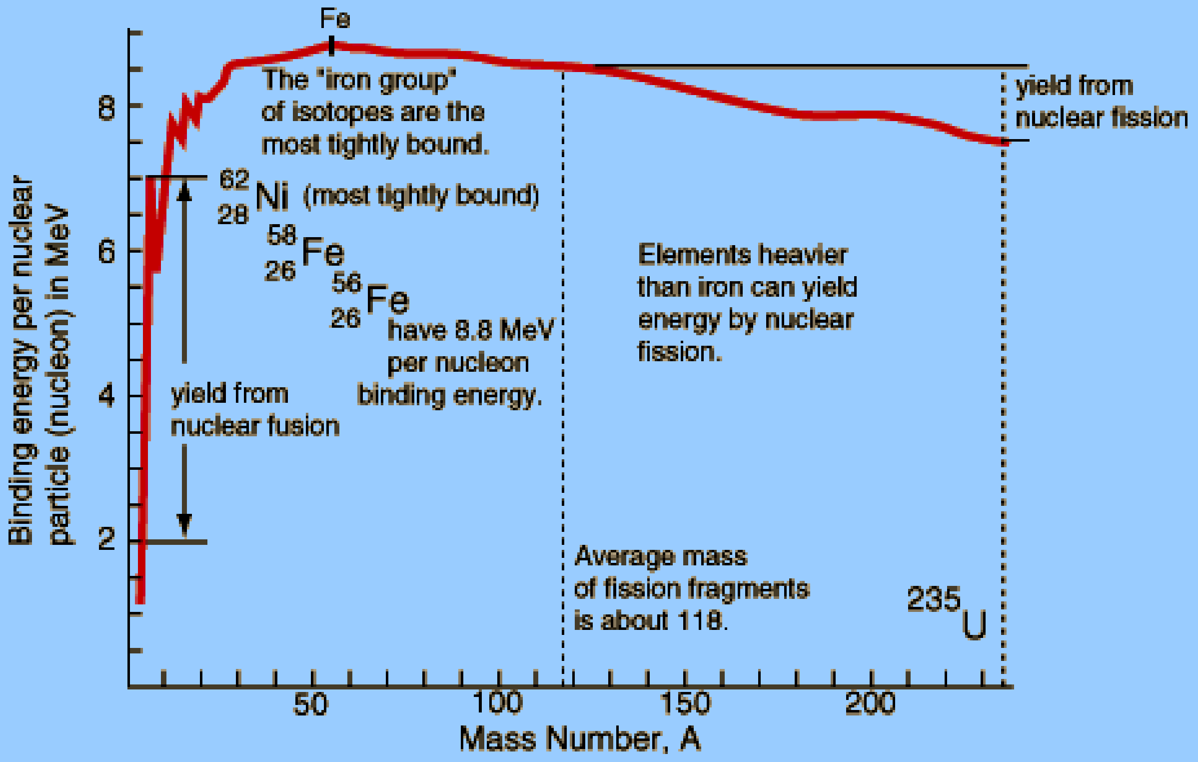
As we will see afterwards in more detail, when the matter is reached by the flame front, its burning achieves the regime of Nuclear Statistical Equilibrium (NSE). Before going on with nucleosynthesis, let's answer the question:

What is the NSE?

A simpler example...

Nuclear burning during hydrostatic stellar evolution (excluding advanced phases and Ne-burning)

- In these phases, burning proceeds by fusion of some fuel, with the production of nuclei with a larger binding energy per nucleon (and subsequent release of nuclear energy). The main flux of nuclear reactions is from lighter to heavier nuclei.



...Si-burning in massive stars:

By analogy with $4\,{}^{1}\mathrm{H} \rightarrow {}^{4}\mathrm{He}$, $3\,{}^{4}\mathrm{He} \rightarrow {}^{12}\mathrm{C}$, $2\,{}^{12}\mathrm{C} \rightarrow {}^{24}\mathrm{Mg}$, $2\,{}^{16}\mathrm{O} \rightarrow {}^{32}\mathrm{S}$

shall we expect $2\,{}^{28}\mathrm{Si} \rightarrow \ldots ?$

No!

The previous description doesn't hold as temperature increases

Si-burning, the correct way

- Some estimates: assume T ~ 4 GK → kT ~ 0.35 MeV
- Separation energy for nucleons (p, n and even α) in nuclei
  $E_{sep} \sim B(A) \sim 8 - 12$ MeV
- Photodisintegration rate: $\lambda_{\gamma} \propto \exp(- E_{sep} / kT)$
- In the tail of the Planckian distribution, $E_{\gamma} \sim E_{sep}$
- Many of the photoejected particles are captured by nuclei with larger E_{sep} → production of more tightly bound nuclei is favoured

Astrophysical sites where NSE is important

- ✗ Necessary requirement: temperature
- ✗ It is not sufficient: timescale constraint

Some examples:

- ✓ Hydrostatic and explosive Si-burning

- ✓ Nucleosynthesis in SNe Ia

  the initial composition is $X(^{12}C) = X(^{16}O) = 0.5$; depending on T(t), NSE conditions can be reached during explosive burning

NSE: definitions and formulae

In the conditions of Nuclear Statistical Equilibrium, strong and electromagnetic reactions are equilibrated by their reverse reactions, but weak reactions are not.

For a nucleus ${}^{A}Z$, composed of Z proton and N = (A – Z) neutrons, in equilibrium with free neutrons, the chemical potential of ${}^{A}Z$ can be expressed in terms of chemical potentials of free nucleons

$$\mu_{Z,A} = Z\mu_P + N\mu_N \qquad (1)$$

μ_i : chemical potential of the particle *i*, defined as the change of energy in the nucleus when a particle *i* is added

For a collection of particles obeying Boltzmann statistics,

$$\mu_i = m_i c^2 + k_B T \ln\left[\rho N_A \frac{Y_i}{G_i}\left(\frac{2\pi\hbar^2}{m_i k_B T}\right)^{3/2} \frac{Y_i}{G_i}\right] \qquad (2)$$

where:

$Y_i = n_i/(\rho N_A)$ nuclear abundance of *i*

$$G_i = \sum_j (2J_j + 1)\exp(-E_j/k_B T)$$

partition function, with a summation over all excited states of spin J_j and energy E_j

Substituting (2) in (1), we obtain...

...the expression for the abundance of ^{A}Z

$$Y(^{A}Z) = \frac{G(^{A}Z)}{2^{A}} \left(\frac{\rho N_{A}}{\theta}\right)^{A-1} A^{3/2} \exp\left(\frac{B(^{A}Z)}{k_{B}T}\right) Y_{n}^{N} Y_{p}^{Z}$$

with $\theta = (m_{u} k_{B} T / 2\pi \hbar^{2})^{3/2}$

Every abundance is expressed as a function of only other two abundances. However, there are two additional constraints:

1) Mass conservation: $\Sigma AY = 1$

2) Charge conservation: $\Sigma ZY = Y_{e}$, where Y_{e} is the electron abundance.

Role of Y_e in NSE

In conditions of thermal equilibrium (*every* reaction balanced by the reverse one) the abundances at equilibrium would be only function of the thermodinamical variables T and ρ.
But *weak* reactions are not balanced, so the nuclear abundances depend also on weak reaction activity, expressed in terms of Y_e.

- A given (T,ρ,Y_e) univocally determines the abundances in the system.

Sometimes, instead of Y_e, the neutron eccess η is used;

$$\eta=\sum_i (N_i-Z_i)\,Y_i=1-2\,Y_e$$

Role of Y_e – II

$$Y(^A Z) = \frac{G(^A Z)}{2^A} \left(\frac{\rho N_A}{\theta}\right)^{A-1} A^{3/2} \exp\left(\frac{B(^A Z)}{k_B T}\right) Y_n^N Y_p^Z$$

At T conditions, typical of nucleosynthesis in SNe Ia, the value of Y_e determines which Fe-peak isotopes dominate.

^{56}Ni : Z = 28, N = 28, Z/A = 0.5
^{54}Fe : Z = 26, N = 28, Z/A ~ 0.48

$$B(^{54}\text{Fe} + 2\text{H}) < B(^{56}\text{Ni}) < B(^{54}\text{Fe})$$

When Y_e is ~ 0.5, charge conservation requires to have ^{54}Fe + 2H , ^{56}Ni production is favoured
With decresing Y_e, ^{54}Fe production is favored

In general, the most abundant isotopes are the most bound ones for which Z/A ~ Y_e

Further comments on abundance formula

$$Y(^{A}Z)= \frac{G(^{A}Z)}{2^{A}}\left(\frac{\rho N_{A}}{\theta}\right)^{A-1} A^{3/2}\exp\left(\frac{B(^{A}Z)}{k_{B}T}\right) Y_{n}^{N} Y_{p}^{Z}$$

Let's make a comparison with the expression of the abundance evolution, without the condition of NSE

$$\dot{Y}_{i}=\sum_{j} c_{i}(j)\lambda_{j} Y_{j}+\sum_{i,j} c_{i}(j,k)\rho N_{A}<j,k>Y_{j} Y_{k}+three-bodyterm...$$

where:

$$c_{i}(j)=\pm N_{i}, c_{i}(j,k)=\pm N_{i}/(N_{j}!\, N_{k}!)$$

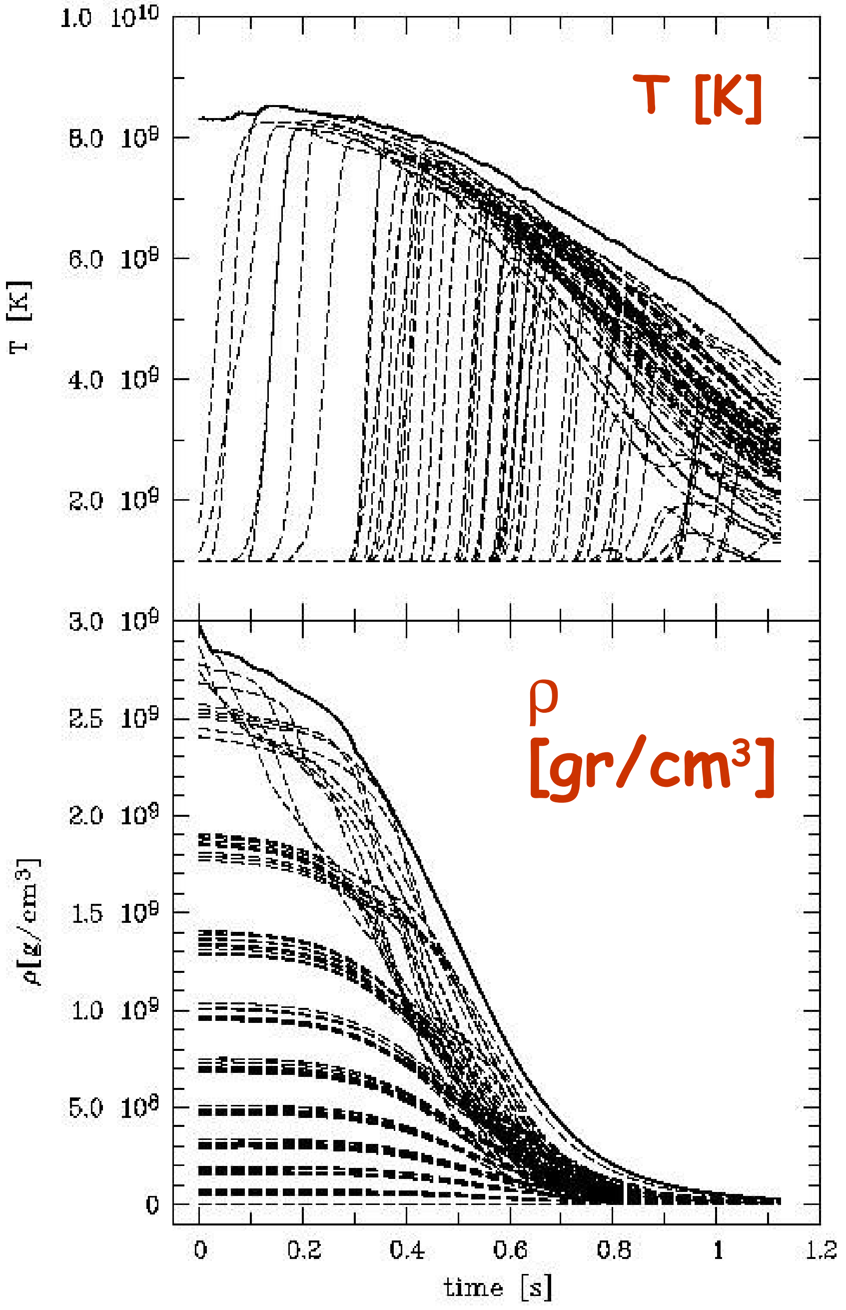
- Computationally, NSE abundances are much easier to be calculated.
- In situations with very large reaction fluxes, almost balancing each other, a NSE network can give more accurate results than a nuclear reaction network.
- From a operational point of view, NSE formula requires the knowledge of binding energies and partition function, quantities that are better known than many reaction rates.
- Limit to the use of NSE networks: in many astrophysical sites, the NSE condition is not valid during the whole burning process.

Burning conditions in SNe Ia and NSE

Timescale ~ 1 s → NSE is achieved at $T > 6 \quad 10^9$ K

Explosive nucleosynthesis: $T > 1.5 \quad 10^9$ K

Although NSE is reached for part of the burning process, a detailed study of nucleosynthesis requires the solution of a complete nuclear reaction network.



(Travaglio et al, in preparation)

Implementation of nucleosynthesis in the study of SNe Ia

The study of nucleosynthesis has, in general, two aims:

- ✗ Evaluation of nuclear energy released by the process;
- ✗ Abundance calculation.

Two approaches are possible:

1) follow at the same time the two processes, solving the reaction network together with the other equations (e.g. Euler equations) and with

$$\epsilon = -\sum_i \dot{Y}_i N_A \Delta M_i$$

2) follow them in two steps: first calculating energy generation (and variations of T and ρ as a function of time) and then using the input of T(t) and ρ(t) in the reaction network calculation.

This method is called post-processing

Post-processing in SNe Ia – first step

In the context of SN Ia explosion, energy generation is important, because the velocity of the propagating flame depends on it.

The implementation of energy generation in SN Ia simulation is minimal, because of computational cost. Let's refer to the multi-D calculations performed with PROMETHEUS:

- No reaction network is used

- At the flame arrival, matter is istantaneusly burned to
  "a NSE mixture of ^{56}Ni and α" if $\rho > 5.25 \quad 10^7$ gr cm^{-3}, or to
  "^{24}Mg" if $5.25 \quad 10^7 < \rho < 10^7$ gr cm^{-3}.

The energy release is calculated accordingly.

Post processing calculation of nucleosynthesis: two possibilities

Definition: Lagrangian coordinates are fixed to a given parcel of fluid, but move in space.

1) Lagrangian 1D calculations: a lot of works on SN Ia nucleosynthesis (e.g. Thielemann et al. 1986 and 2003, Iwamoto et al 1999, Brachwitz et al. 2000) follow this approach, mostly taking thermodynamical data from the explosion model W7 (Nomoto et al. 1984).

2) In the last part of this seminar some results, based on another approach (Lagrangian multi-D with particles), will be presented. They will appear on a forthcoming paper by Travaglio, Hillebrandt, Reinecke & Thielemann.

Lagrangian approach in multi-D calculations

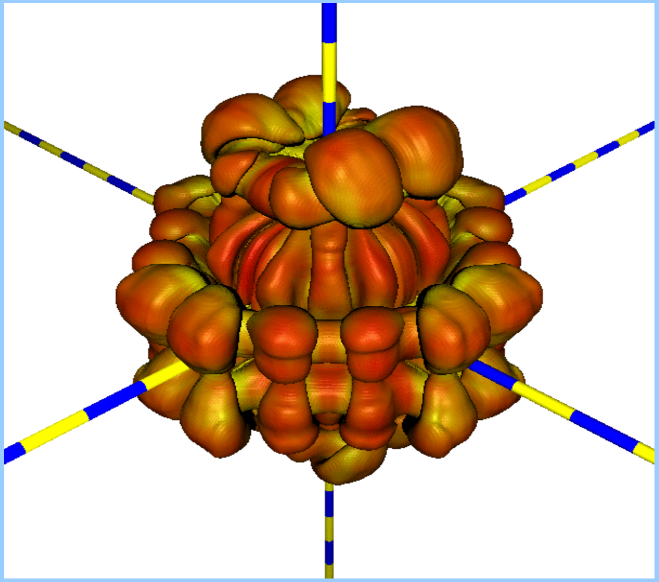
In order to perform Lagrangian multi-D nucleosynthetic calculations, a “Lagrangian component” is added to the Eulerian (i.e. fixed grid) scheme of SN Ia simulation, in the form of tracing particles passively advected with the flow.

Each of these “particles” records its T and ρ history, which is used in the following post-processing calculation.

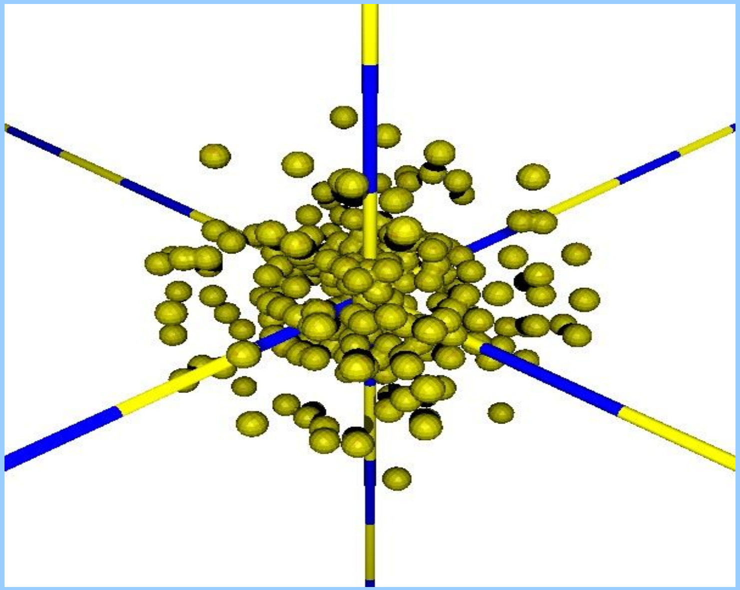
A more direct explanation: a movie of a 3D particle distribution
(courtesy of C. Travaglio)

Details about the method

- Models of ignition: 2D and 3D models with ignition at center or in off-centered bubbles.



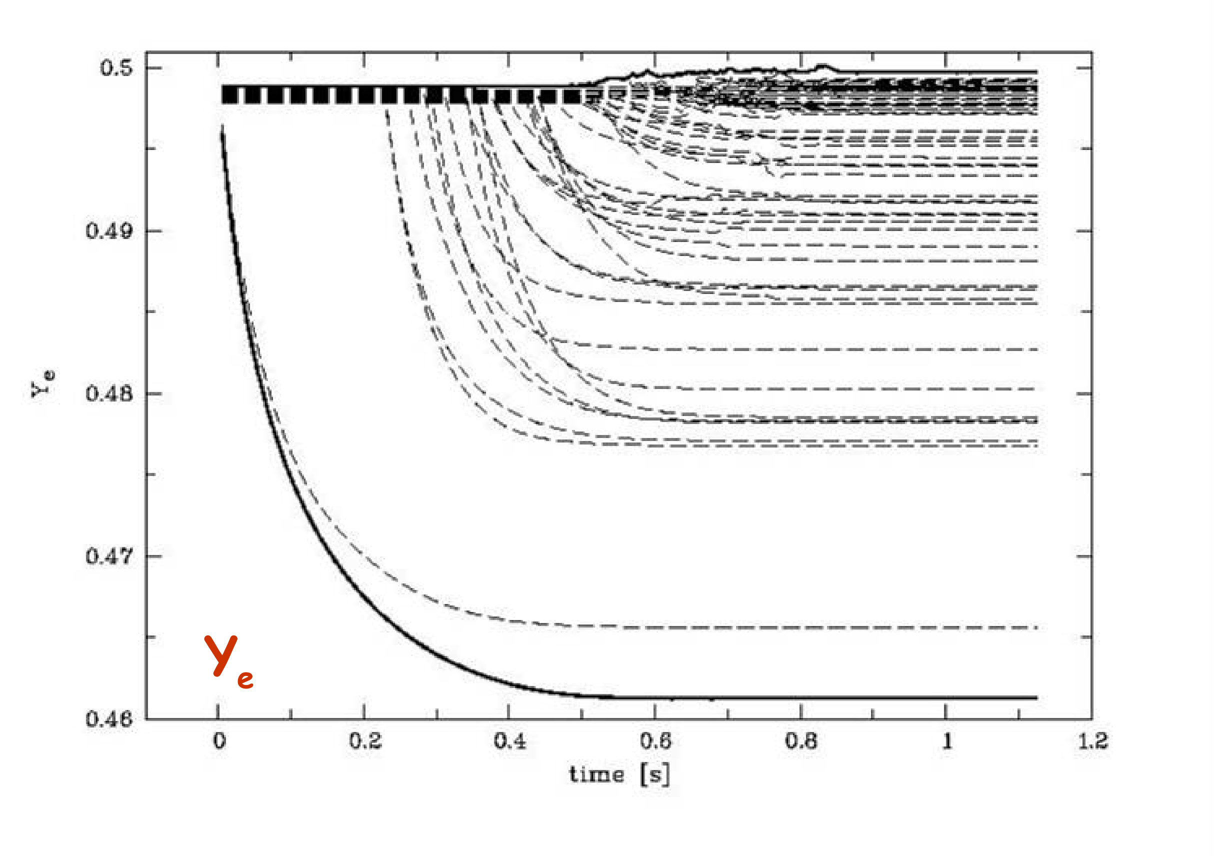
Centrally burned model, after 0.6 s, grid size 256^3 zones



Initially burned material, <u>30 ignition bubbles</u>, grid size <u>768^3 zones</u> (1 ring = 100 km)

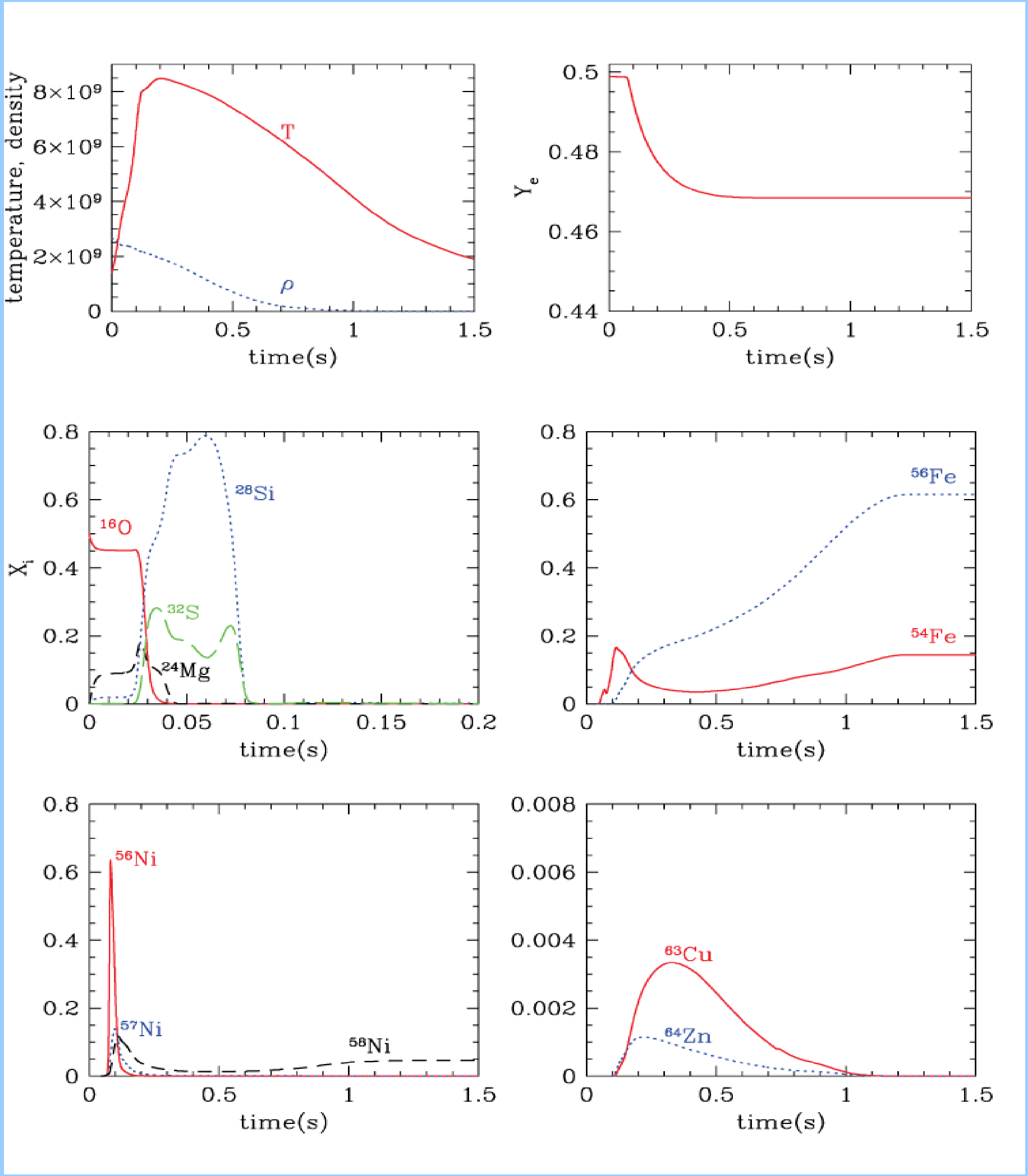
Details about the method – II

- Number of particles: $27^3 = 19683$
- Nucleosynthesis network: 383 species, up to ^{98}Mo.
- Y_e distribution in the markers:



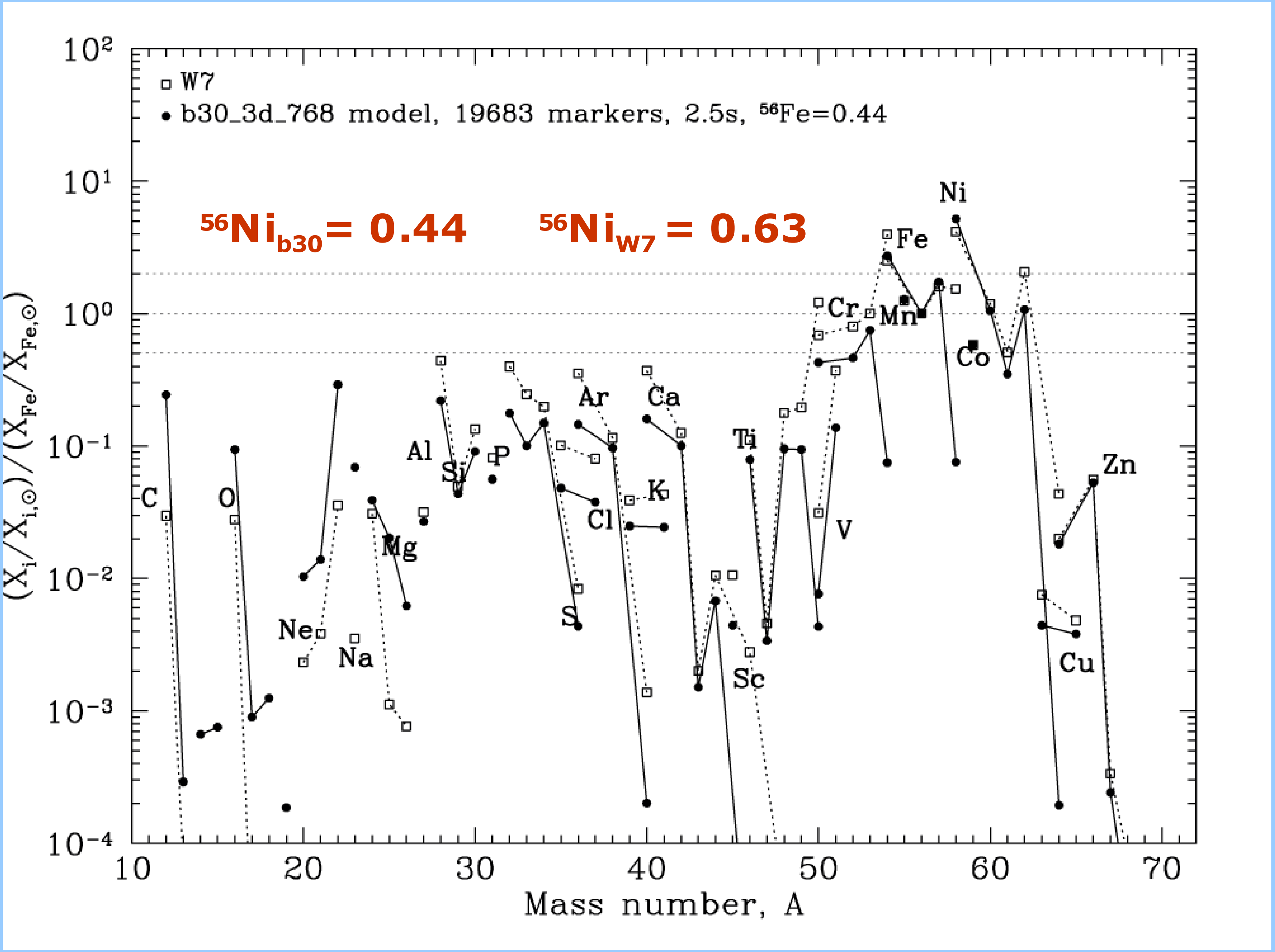
Example of a nucleosynthesis calculation

Case of a marker initially located in the innermost dense zone of the white dwarf



Pay attention! This time scale is different from the other ones!

Nucleosynthetic yields



$^{56}\mathrm{Ni}_{b30} = 0.44$ $^{56}\mathrm{Ni}_{W7} = 0.63$

Nucleosynthesis: summary of results

- ✗ Produced abundances are sensitive to ignition process: presented results refer to the supposed “best” model (ignition in 30 bubbles).

- ✗ General consistency with standard (W7) nucleosynthesis, but:

  - ✓ ^{56}Ni production: 0.42 M_{sol} (in the W7-based nucleosynthesis, 0.59 M_{sol});

  - ✓ Final amount of unburned material: too high, with respect to W7 and observations.

- ✗ Another interesting analysis: dependence on C / O ratio (Röpke & Hillebrandt 2004)

References

- About SN Ia explosion:

Hillebrandt & Niemeyer, ARA&A 38 (2000) 191.

- About NSE:

Hix & Thielemann, J. of Comp. and Appl. Math. 109 (1999) 321.

Clayton, Principles of Stellar evolution and Nucleosynthesis, par. 7.1 and 7.2

- About SNe Ia nucleosynthesis:

Thielemann, Nomoto & Yokoi, A&A 158 (1986) 17

Iwamoto et al., ApJ Suppl. 125 (1999) 439

Brachwitz et al., ApJ 536 (2000) 934

Travaglio, Hillebrandt, Reinecke & Thielemann, coming soon...