Hawking Radiation and Black Hole Evaporation

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Overview

- Classical black hole mechanics
- The quantum vacuum in Minkowski spacetime
- The Unruh effect
- Hawking radiation
- Black hole entropy

The Zeroth Law

- **Zeroth Law of Black Hole Mechanics**
  The surface gravity κ is constant on the horizon of a stationary black hole
- **Zeroth Law of Thermodynamics**
  The temperature T is constant throughout a system in thermal equilibrium

The First Law

- **First Law of Black Hole Mechanics**
  If mass-energy δM carrying angular momentum δJ and zero electric charge falls into a black hole, the change of surface area of the horizon δA is given by

$$\delta M = \frac{\kappa}{8\pi}\delta A + \Omega\delta J$$

- **First Law of Thermodynamics**
  The change of internal energy in a quasistatic process is the sum of the added heat and mechanical work done on the system:

$$\delta E = T\delta S + P\delta V$$

The Second Law

- **Second Law of Black Hole Mechanics**
  (Area theorem by HAWKING, 1971):
  The surface area of the horizon can never decrease,

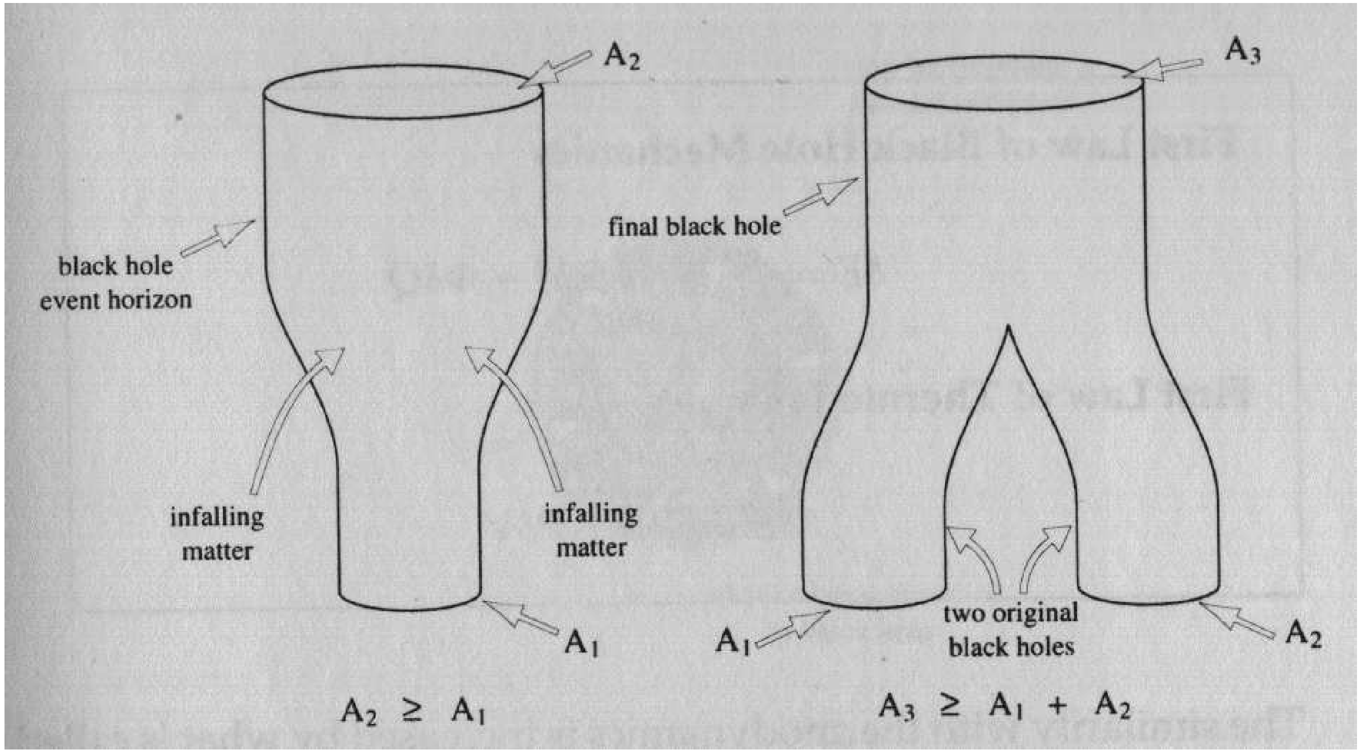
$$\delta A \geq 0$$

- **Second Law of Thermodynamics**
  The total entropy of an isolated system can never decrease:

$$\delta S \geq 0$$

The Area Theorem

The total area of black hole horizons is monotonically increasing with time



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Black Hole Thermodynamics?

- Analogy to thermodynamics
  - Surface gravity κ is a measure of temperature T
  - Horizon area A is proportional to the entropy S
- Refutations (from the purely classical point of view):
  - ✘ The temperature of a black hole vanishes!
  - ✘ Entropy is dimensionless, the horizon area is a length squared!
  - ✘ The area is separately non-decreasing, whereas only the *total* thermodynamical entropy is non-decreasing!
  - ✘ Numerically, the black hole entropy is vastly larger than the entropy of a star from which the hole could have formed!

The Quantum Vacuum

- The definition of a particle (quantum) depends on the frame of reference
- If the frames of two observers differ only by a Lorentz transformation, then they will agree about the particle content
- If they have relative acceleration, then they will measure different particle numbers!
- The vacuum in Minkowski spacetime appears to be a thermal state when viewed by an accelerating observer (DAVIES, 1975, and UNRUH, 1976)

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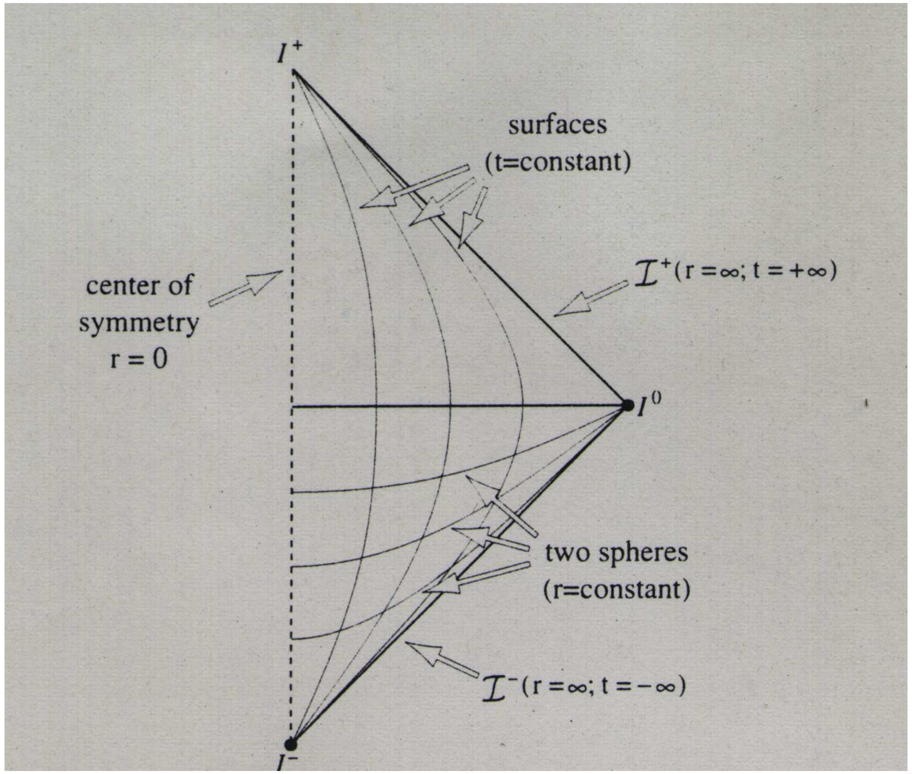
Minkowski Spacetime

Geometric units: $c = 1$, $G = 1$, $k_{\mathrm{B}} = 1$

- $\mathrm{d}s^2 = -\mathrm{d}t^2 + \mathrm{d}r^2 + r^2\mathrm{d}\Omega$
  - $r \to \infty$ ➢ spacelike infinity I^0
  - $t \to +\infty$ ➢ future timelike infinity I^+
  - $t \to -\infty$ ➢ past timelike infinity I^-
- $\mathrm{d}s^2 = -\mathrm{d}u\mathrm{d}v + \frac{1}{4}(u - v)^2\mathrm{d}\Omega$
  - $u = t - r \to \infty$ ➢ future null infinity $\mathcal{I}^+$
  - $v = t + r \to \infty$ ➢ past null infinity $\mathcal{I}^-$

Conformal Minkowski Spacetime

Compactify the manifold to include the boundaries I^-, $\mathcal{I}^-$, I^0, $\mathcal{I}^+$ and I^+



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QFT in a Nutshell

- Massless scalar quantum field ϕ in 1+1 Minkowski spacetime
- Field equation $\partial_\mu \partial^\mu \phi = -\phi_{,tt} + \phi_{,rr} = 0$
  ⇨ plane-wave solutions $f^{\pm}_{\omega} = \mathrm{e}^{-\mathrm{i}\omega(t \mp r)}/\sqrt{2\omega}$
- Wave-packets following outgoing null rays onto $\mathcal{I}^+$:

$$\phi = \int \mathrm{d}\omega \, [\mathfrak{a}^+_\omega f^+_\omega + (\mathfrak{a}^+_\omega)^\dagger (f^+_\omega)^*]$$

  - $(\mathfrak{a}^+_\omega)^\dagger$ and $\mathfrak{a}^+_\omega$ are, respectively, *creation* and annihilation operators
  - Number operator for outgoing particles of frequency ω is $\mathfrak{N}^+_{\mathrm{inert}} = (\mathfrak{a}^+_\omega)^\dagger \mathfrak{a}^+_\omega$

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Inertial vs. Accelerated Observer

- Basis modes in the inertial frame $f_\omega^+ = \mathrm{e}^{-\mathrm{i}\omega u}/\sqrt{2\omega}$, $f_\omega^- = \mathrm{e}^{-\mathrm{i}\omega v}/\sqrt{2\omega}$
- For a *Rindler* observer moving with const. acceleration a, the null coordinates transform to u' and v'
- The transformation from the inertial to the accelerated frame is given by $\mathrm{d}s^2 = -\mathrm{d}u\mathrm{d}v = -\mathrm{e}^{a(v'-u')}\mathrm{d}u'\mathrm{d}v'$
- The Rindler observer measures different time $t' = \frac{1}{2}(u' + v')$ and length $r' = \frac{1}{2}(v' - u')$ and has different basis modes $g_\omega^+ = \mathrm{e}^{-\mathrm{i}\omega u'}/\sqrt{2\omega}$, $g_\omega^- = \mathrm{e}^{-\mathrm{i}\omega v'}/\sqrt{2\omega}$

$$\phi = \int \mathrm{d}\omega \left[\mathfrak{b}_\omega^+ g_\omega^+ + (\mathfrak{b}_\omega^+)^\dagger (g_\omega^+)^*\right]$$

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The Unruh Effect

- vacuum state $|0(\mathcal{I}^-)\rangle$, $\forall\omega : \mathfrak{a}^-_\omega|0(\mathcal{I}^-)\rangle = 0$
  - ☞ There are no in/outcoming particles seen by an inertial observer
- Number operator for outgoing modes in the accelerated frame is $\mathfrak{N}^+_{\mathrm{Rindler}} = (\mathfrak{b}^+_\omega)^\dagger \mathfrak{b}^+_\omega$
- For the vacuum state $|0(\mathcal{I}^-)\rangle$, the Rindler observer finds

$$\langle 0(\mathcal{I}^-)|\mathfrak{N}^+_{\mathrm{Rindler}}|0(\mathcal{I}^-)\rangle = \frac{1}{\mathrm{e}^{2\pi\omega/a} - 1}$$

  - ☞ The Rindler observer beholds a flux of thermal radiation propagating to $\mathcal{I}^+$
  - ☞ The temperature of the vacuum is proportional to acceleration:

$$T = \hbar a/2\pi$$

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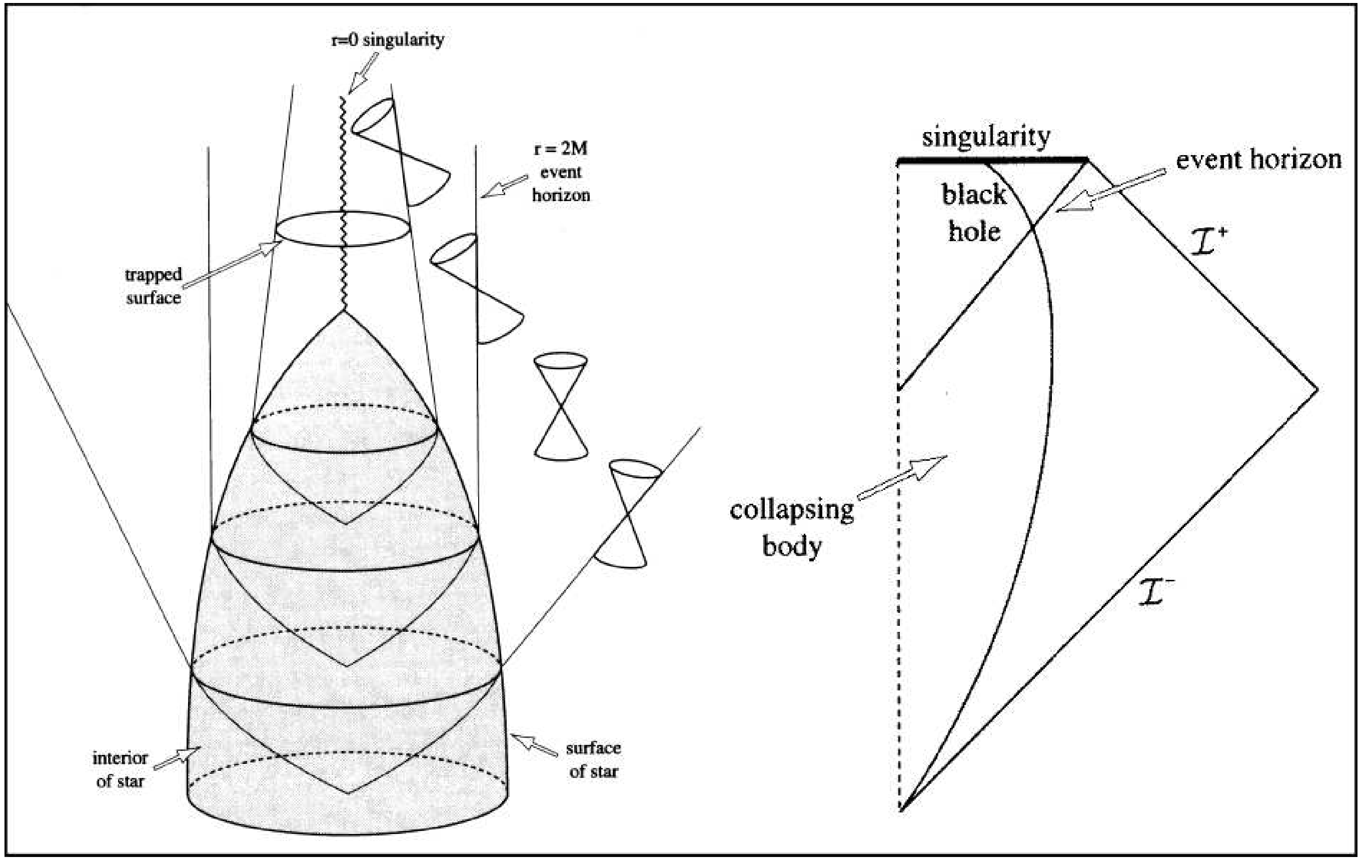
The Collapsing Star Spacetime

A black hole is a region in spacetime which is not in the past of of $\mathcal{I}^+$



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QFT in Curved Spacetime

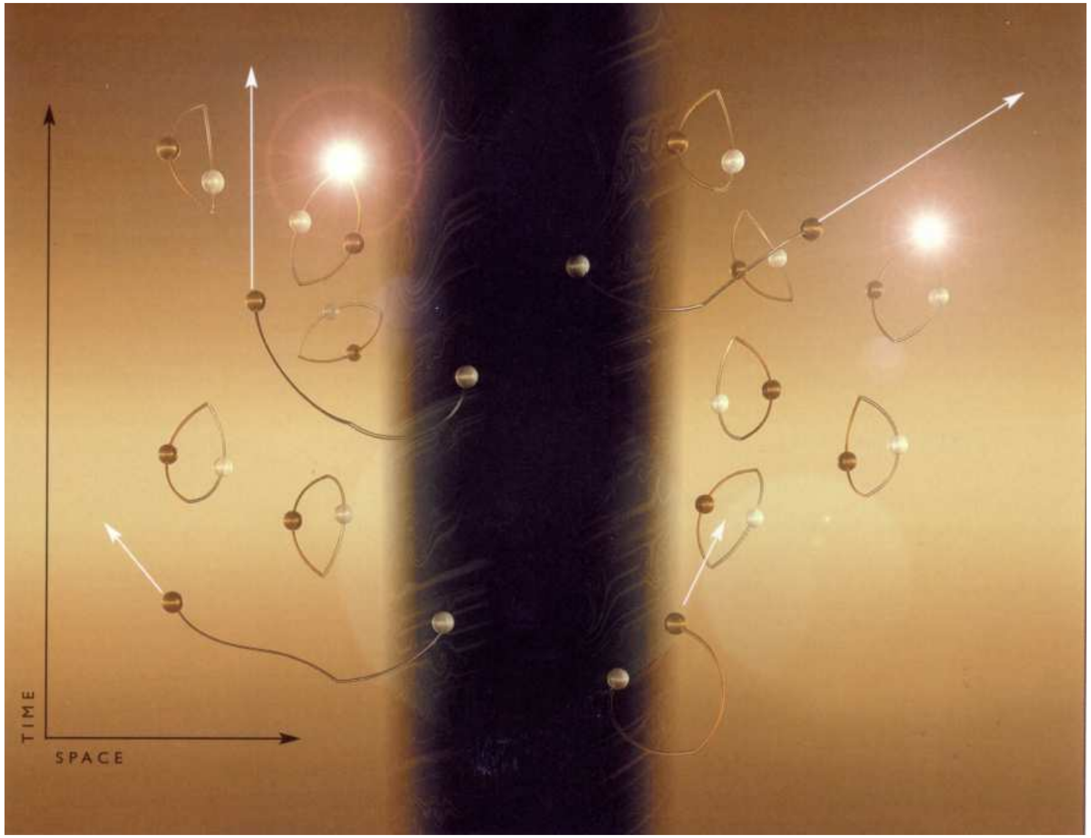
- Use a prescribed background geometry with metric $g_{\mu\nu}$:

$$g^{\mu\nu}\partial_\mu\partial_\nu\phi = 0$$

- HAWKING (1973) studied a scalar field ϕ propagating in the background spacetime around a star collapsing to a black hole
  - Towards $\mathcal{I}^-$, the spacetime is nearly Minkowskian
  - Positive frequency modes are partially converted into negative frequency modes near the horizon (pair creation)
  - Whereas positive frequency modes scatter off the horizon, negative frequency modes are absorbed
  - Assuming the state $|0(\mathcal{I}^-)\rangle$ in the remote past (the *Boulware state*), HAWKING found that in the future of the collapse, there is a flux of thermal particles to $\mathcal{I}^+$

Pair Creation Near the Event Horizon

Virtual pair creation close to the horizon can result in a real particle escaping to $\mathcal{I}^+$ which carries away a fraction of the black hole mass



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Hawking Radiation

- A black hole produces thermal radiation with $T = \frac{\hbar}{2\pi}\kappa$
- The temperature is proportional to the surface gravity $\kappa = \frac{1}{2}f'(R_{\rm H}) = \hbar/8\pi M$, where $f(r) = 1 - 2M/r$
- The wavelength of Hawking radiation is $\lambda_{\rm H} = 8\pi^2 R_{\rm H}$
- T vanishes in the classical limit $\hbar \to 0$
- $a = \kappa/f^{1/2}(r)$ is the local acceleration of a stationary observer
- Hawking radiation is a consequence of the state near the horizon being the vacuum as viewed by free-fall observers

Black Hole Evaporation

- Negative frequency (energy) particle flux into $\mathcal{H}^+$ diminishes the black hole mass
- The the rate of positive energy emission to $\mathcal{I}^+$ is $\propto M^{-2}$
- Black hole life time is of the order M^3
  - $(M/M_{\rm P})^3 T_{\rm P} \sim 10^{-28} M^3$ [cgs]
  - A solar mass black hole ($M \sim 10^{33}$ g) emits thermal radiation of roughly 10^{-11} eV and lives about 10^{54} times the age of the universe
  - Only *primordial* mini black holes could be observed if they were around

Bekenstein-Hawking Entropy

- There is entropy associated with a black hole horizon:

$$S_{\mathrm{BH}} = \frac{A_{\mathrm{H}}}{4\hbar}$$

- The unit of horizon area is $L_{\mathrm{P}}^2 \sim 10^{-66}\mathrm{cm}^2$
- For a stellar mass black hole, $S_{\mathrm{BH}} \sim 10^{78}$ which is about 10^{21} times the entropy of a collapsing star from which the hole could have formed
- Generalised second law: The sum of BH entropy and the entropy outside black holes can never decrease

$$\delta(S_{\mathrm{outside}} + S_{\mathrm{BH}}) \geq 0$$

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The Meaning of Black Hole Entropy

- In a large self-gravitating system (the Universe) gravitational collapse can occur
  ⇨ The notion of entropy has to be extended
- HAWKING: extra quantum uncertainty
  - S_{BH} corresponds to the maximum information lost in the BH
  - Information about quantum states can not be restored
- PENROSE: complementary quantum uncertainty
  - Loss of information corresponds to shrinking of phase space volume as trajectories converge towards singularities
  - This is counter-balanced by quantum measurements in which information and, thus, phase space volume is regained

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More to Read...

- S. HAWKING, *Quantum Mechanics of Black Holes*, Scientific American, January 1977
- S. HAWKING and R. PENROSE, *The Nature of Space and Time*, Princeton University Press, 1996
- T. JACOBSON, *Introductory Lectures on Black Hole Thermodynamics*, **www.glue.umd.edu/~tajac/BHTlectures/lectures.ps**
- J. TRASCHEN, *An Introduction to Black Hole Evaporation*, **gr-qc/0010055**

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