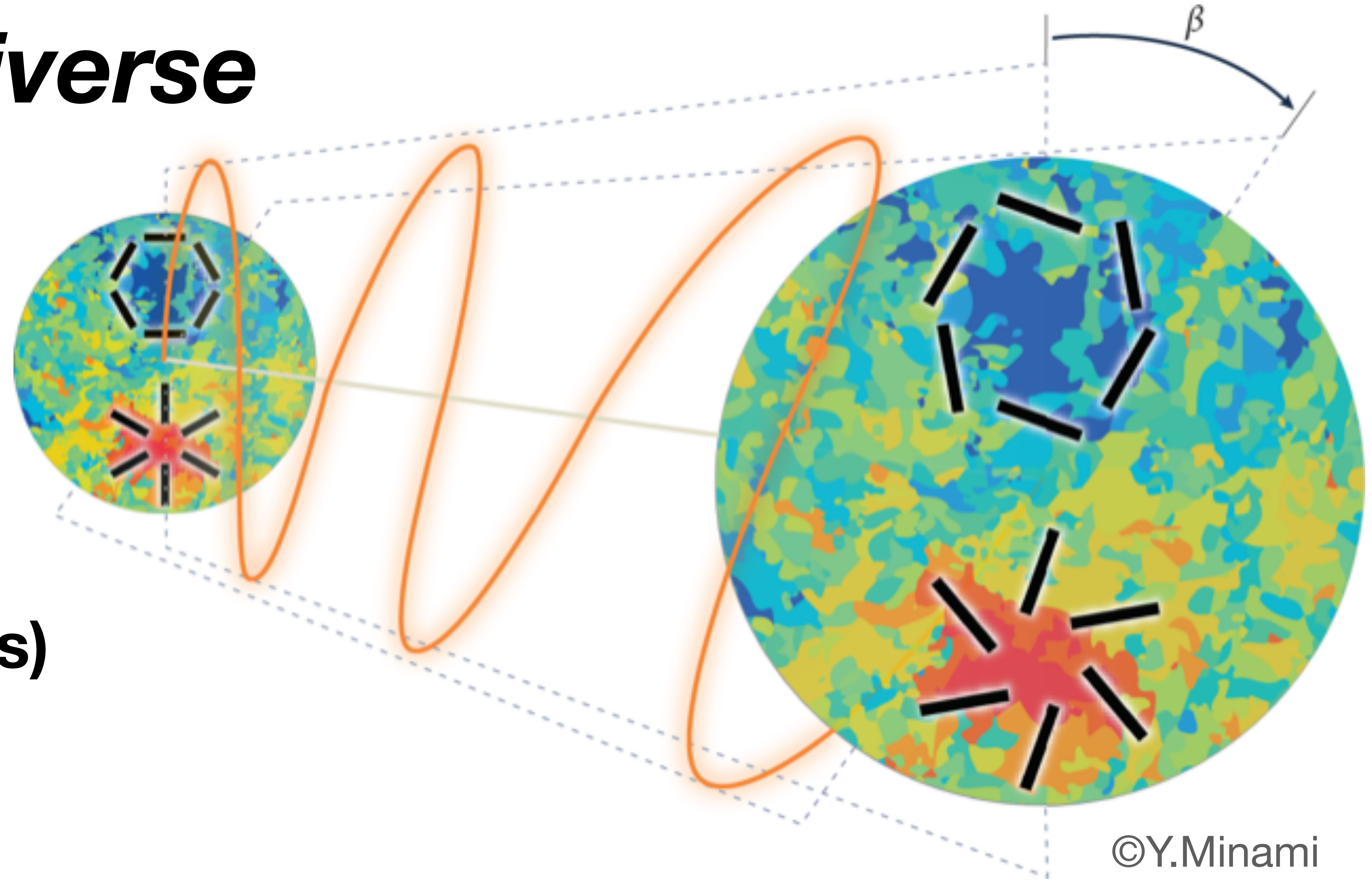
Reference: EK, Nature Rev. Phys. **4**, 452 (2022)

$$I_{\rm CS} = \int d^4x \sqrt{-g}\left(-\frac{\alpha}{4f}\chi F\tilde{F}\right)$$

MAX-PLANCK-INSTITUT FÜR ASTROPHYSIK

# Parity Violation in Cosmology

## *In search of new physics for the Universe*






**Eiichiro Komatsu (Max Planck Institute for Astrophysics)**

***IFPU Focus Week, Trieste***

***May 27, 2024***

# Overarching Theme

## *Let’s find new physics!*

- The current cosmological model (*flat ΛCDM*) **requires** new physics beyond the standard model of elementary particles and fields.
  - What is dark matter (*CDM*)?
  - What is dark energy (*Λ*)?
  - Why is the spatial geometry of the Universe Euclidean (*flat*)?
  - What powered the Big Bang?

# Overarching Theme

## There are many ideas, but how can we make progress?

- The current cosmological model (*flat ΛCDM*) **requires** new physics beyond the standard model of elementary particles and fields.
  - What is dark matter (*CDM*)? => CDM, WDM, FDM, …
  - What is dark energy (*Λ*)? => Dynamical field, modified gravity, quantum gravity, …
  - Why is the spatial geometry of the Universe Euclidean (*flat*)? => Inflation, contracting universe, …
  - What powered the Big Bang? => Scalar field, gauge field, …

Ove

There

***<u>New in cosmology!</u>***

**Violation of parity symmetry may hold the answer to these fundamental questions.**

- The current cosmological model (*flat ΛCDM*) **requires** new physics beyond the standard model of elementary particles and fields.
  - What is dark matter (*CDM*)? => CDM, WDM, FDM, …
  - What is dark energy (*Λ*)? => Dynamical field, modified gravity, quantum gravity, …
  - Why is the spatial geometry of the Universe Euclidean (*flat*)? => Inflation, contracting universe, …
  - What powered the Big Bang? => Scalar field, gauge field, …

# Reference:

Nature Rev. Phys. **4**, 452 (2022)

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Review Article | Published: 18 May 2022

# New physics from the polarized light of the cosmic microwave background

Eiichiro Komatsu

*Nature Reviews Physics* **4**, 452–469 (2022) | Cite this article

***Key Words:***

1. Cosmic Microwave Background (CMB)
2. Polarization
3. Parity Symmetry

https://wwwmpa.mpa-garching.mpg.de/~komatsu/lectures--reviews.html

# Lectures & Reviews

## 2023

- Lecture Slides: **"Parity Violation in Cosmology" [7 x 85 min]**
  - MC Specialized Course, Department of Physics, Nagoya University (June 6–30)
    - The syllabus is available **here**.
    - Reference: "*New Physics from the Polarized Light of the Cosmic Microwave Background*"
      - **Nature Reviews Physics, 4, 452-469 (2022 May 18)**. You can have access to the full text via **this link**. Supplementary information is available **here**.
  - **Lecture 1**: What is parity symmetry? (PDF 3.9 MB; last updated, June 5, 2023)
    - 1.1 Parity
    - 1.2 Vector and pseudovector
    - 1.3 Discovery of parity violation in β-decay
    - 1.4 Helicity
  - **Lecture 2**: Chern-Simons interaction (PDF 1.6 MB; last updated, June 8, 2023)
    - 2.1 Parity symmetry in electromagnetism (EM)

⋮

# Probing Parity Symmetry

## Definition

- **Parity transformation = Inversion of all spatial coordinates**
  - (x, y, z) –> (-x, -y, -z)
- Parity symmetry in physics states:
  - *The laws of physics are invariant under inversion of all spatial coordinates.*
- Violation of parity symmetry = The laws of physics are **not** invariant under…
- Ask "***When we observe a certain phenomenon in nature, do we also observe its mirror image(*) with equal probability?***"
  - (*) "Mirror image" is an ambiguous word. A parity transformation is (x, y, z) –> (-x, -y, -z), whereas a "mirror image" often refers to, e.g., (x, y, z) –> (-x, y, z), where only one of (x,y,z) is flipped.

# Do we also observe this with equal probability?

Note that this is not full parity transformation,
as only one axis is flipped.

# Parity and Rotation

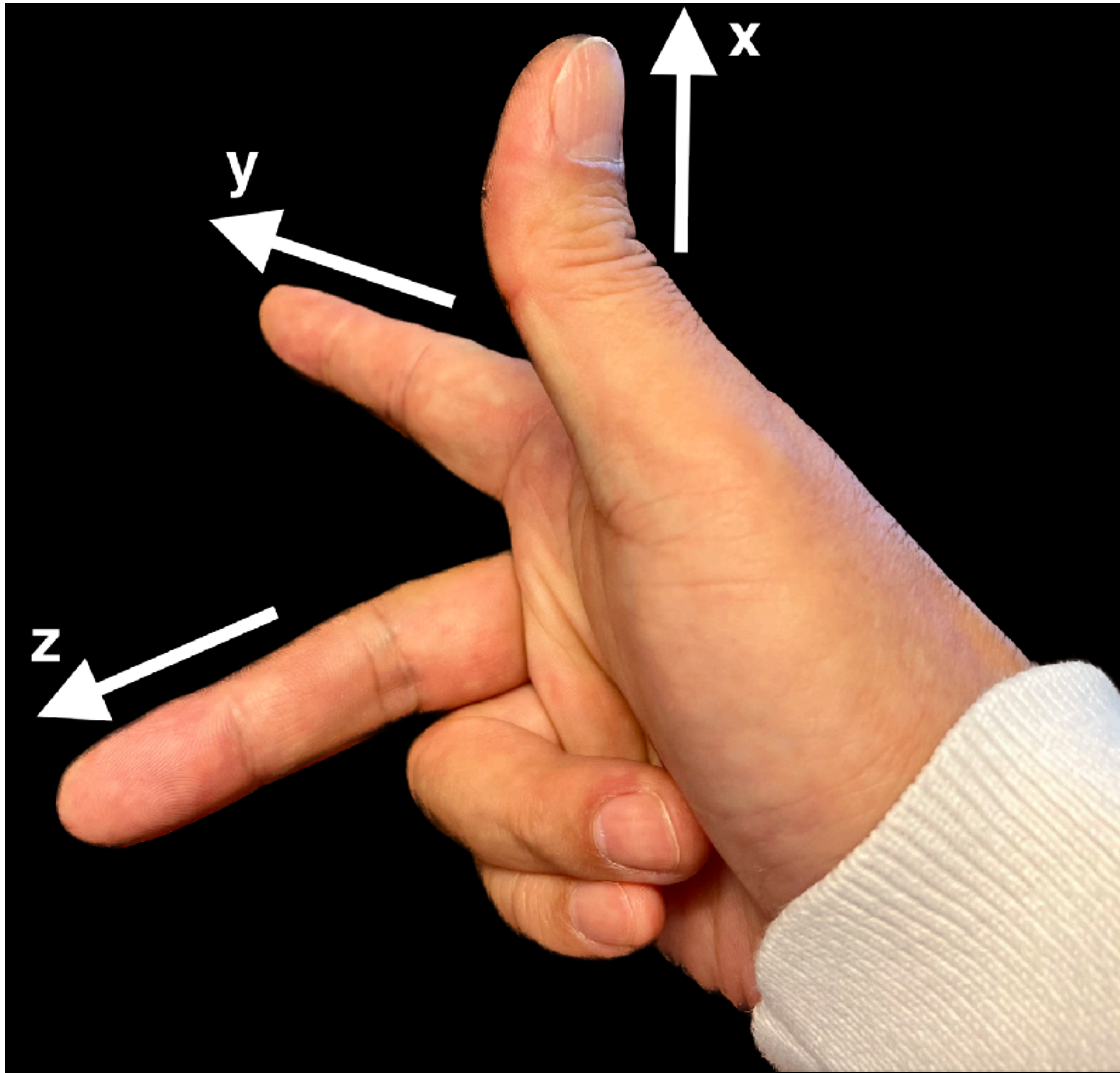
- Parity transformation (***x*** –> -***x***) and 3d rotation (***x*** –> R***x***) are different.
  - R is a continuous transformation and the determinant of R is det(R) = +1.
  - Parity is a discrete transformation and the **determinant is –1**, as

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} \to \begin{pmatrix} -x \\ -y \\ -z \end{pmatrix} = \begin{pmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix}$$

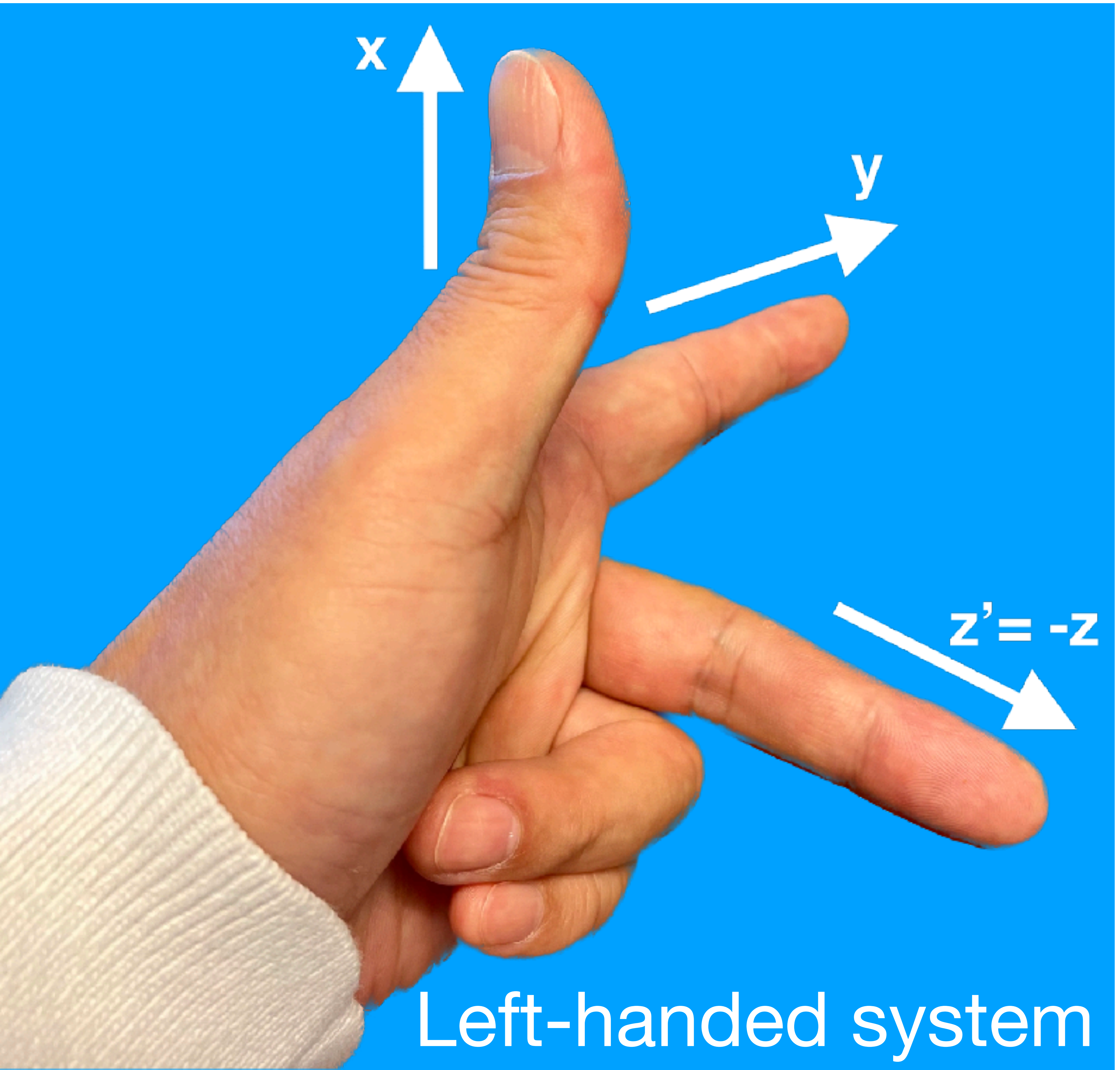
$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} \to \begin{pmatrix} -x \\ -y \\ -z \end{pmatrix} = \begin{pmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix}$$

# Parity = Mirror + 2d Rotation

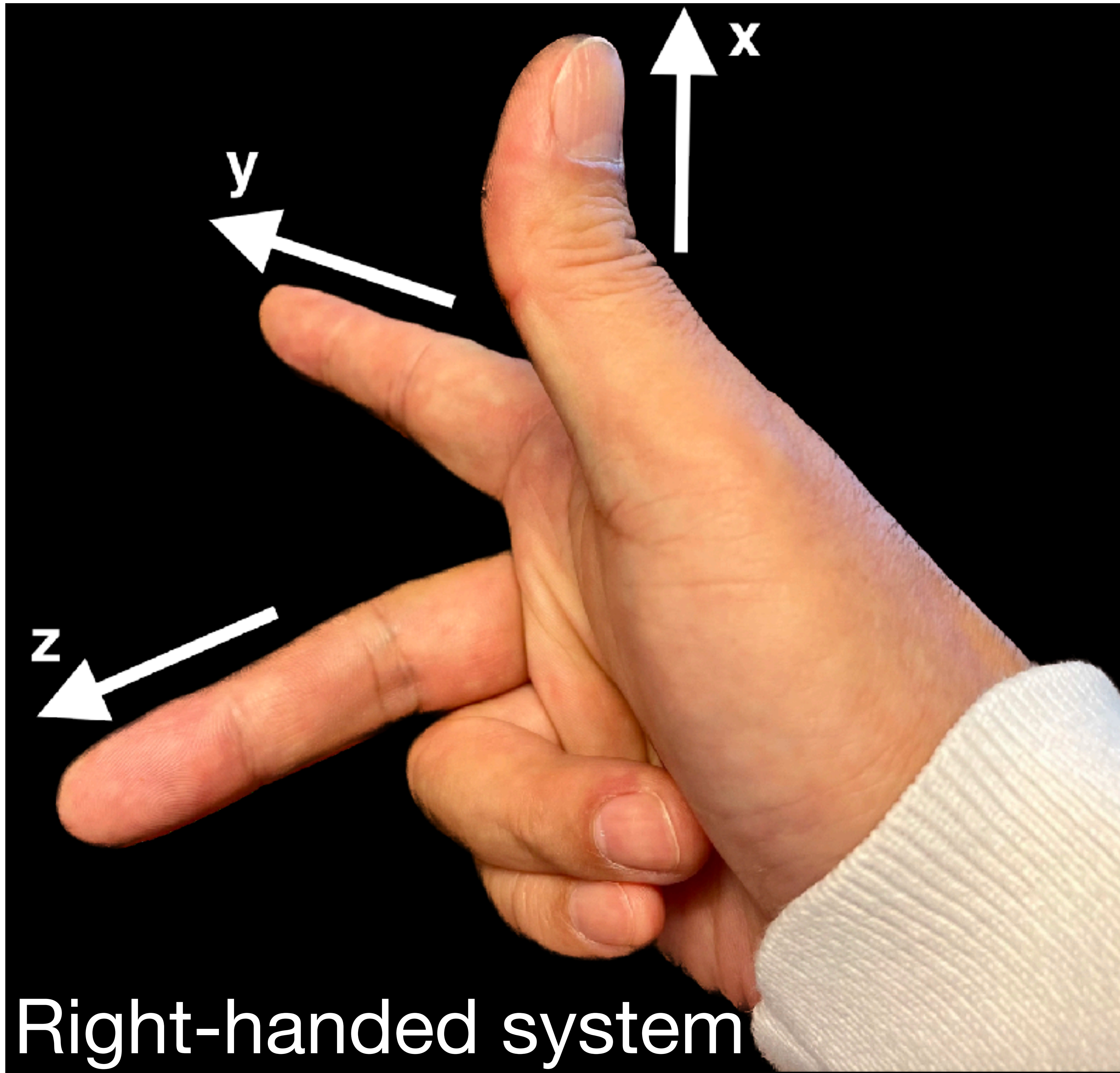
- One may think of parity transformation as a mirror in one of the coordinates (e.g., z –> -z) and **2d** rotation by π in the others.
- Let's demonstrate it!



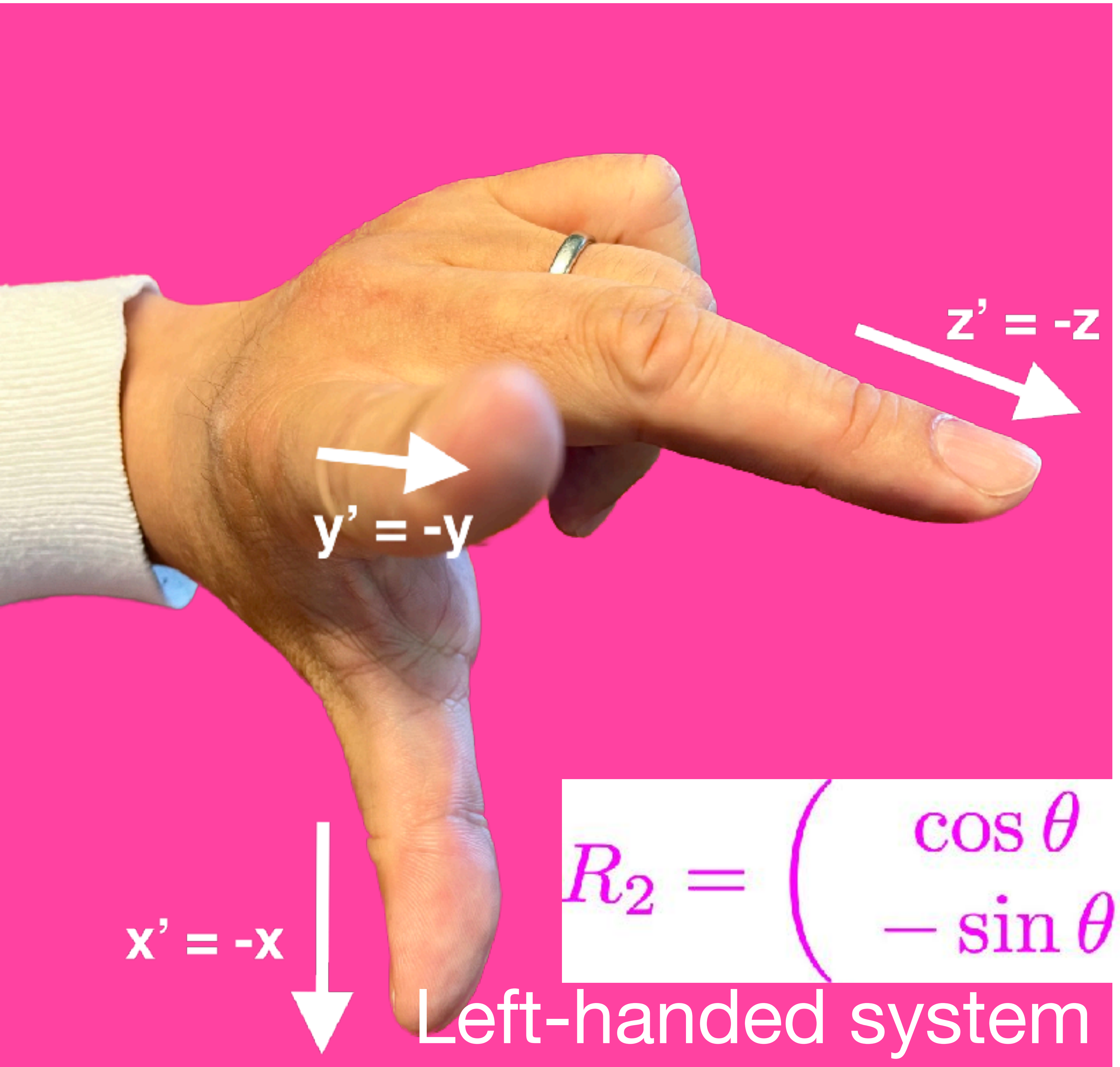
$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} \rightarrow \begin{pmatrix} -x \\ -y \\ -z \end{pmatrix} = \begin{pmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & \boxed{-1} \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix}$$




z -> z' = -z



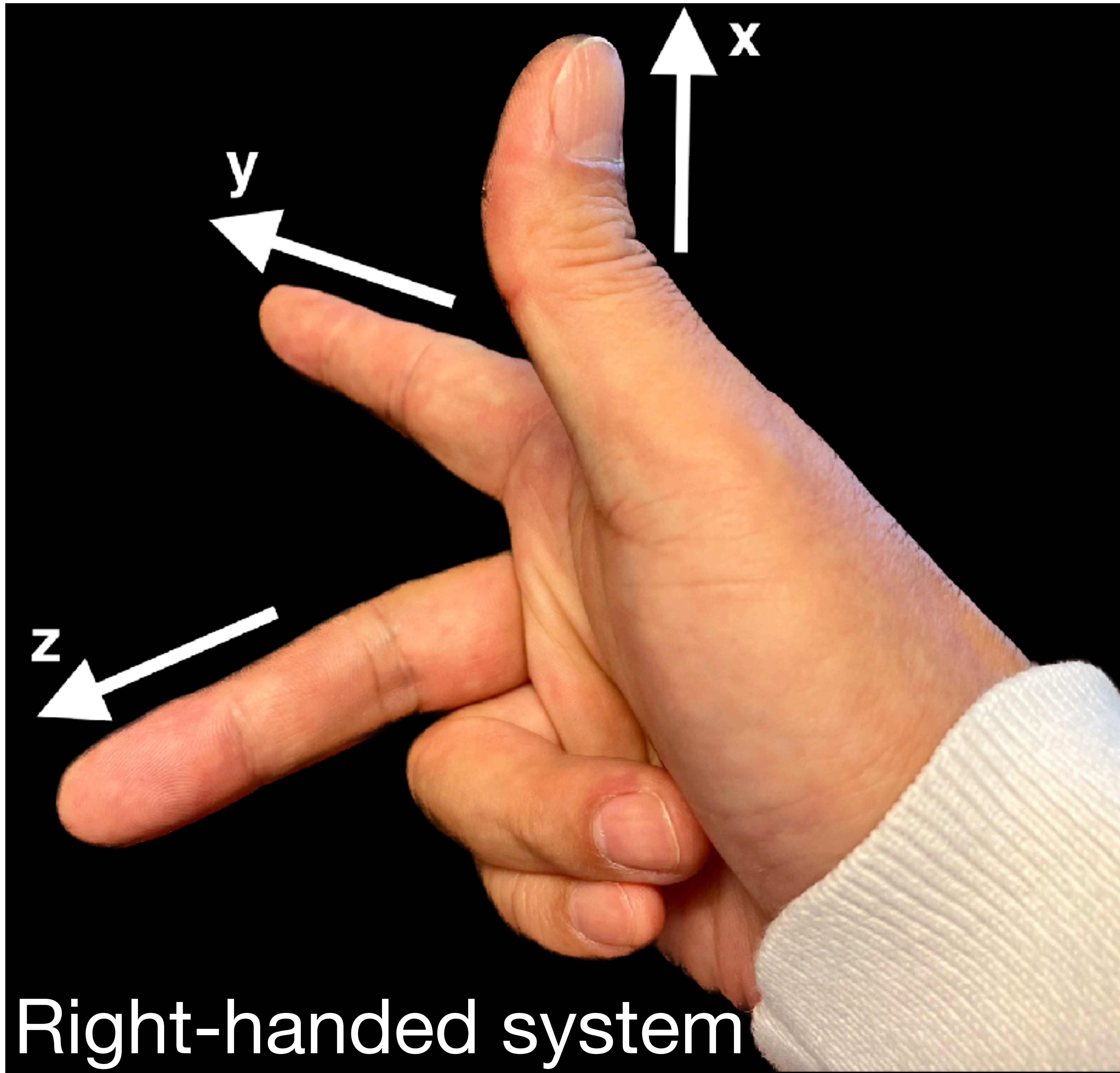
$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} \rightarrow \begin{pmatrix} -x \\ -y \\ -z \end{pmatrix} = \begin{pmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix}$$




**Rotation In x-y**

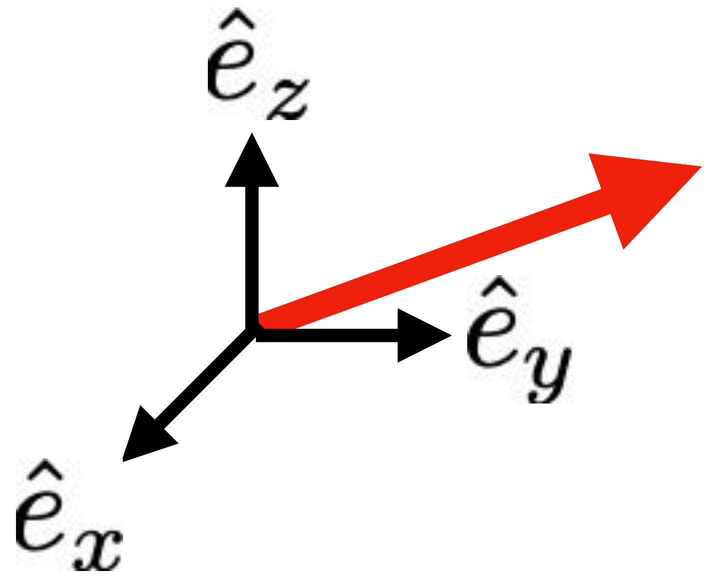
$$R_2 = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}$$

**with θ = π**



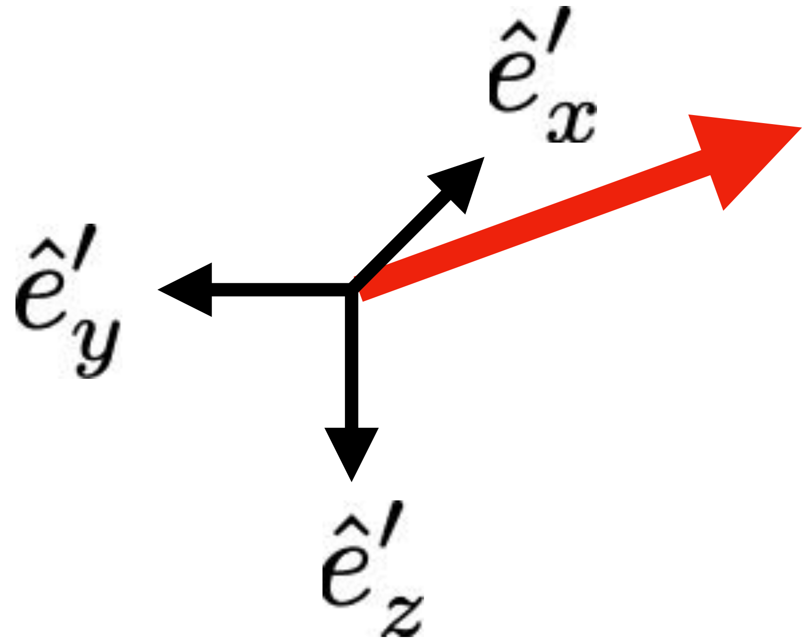
# Parity Transformation: Vector

## E.g., momentum, electric field




$$\mathbf{p} = p_x\hat{e}_x + p_y\hat{e}_y + p_z\hat{e}_z$$

$\hat{e}_i$ is a unit vector.




$$\mathbf{p} = p'_x\hat{e}'_x + p'_y\hat{e}'_y + p'_z\hat{e}'_z$$
$$= -p'_x\hat{e}_x - p'_y\hat{e}_y - p'_z\hat{e}_z$$

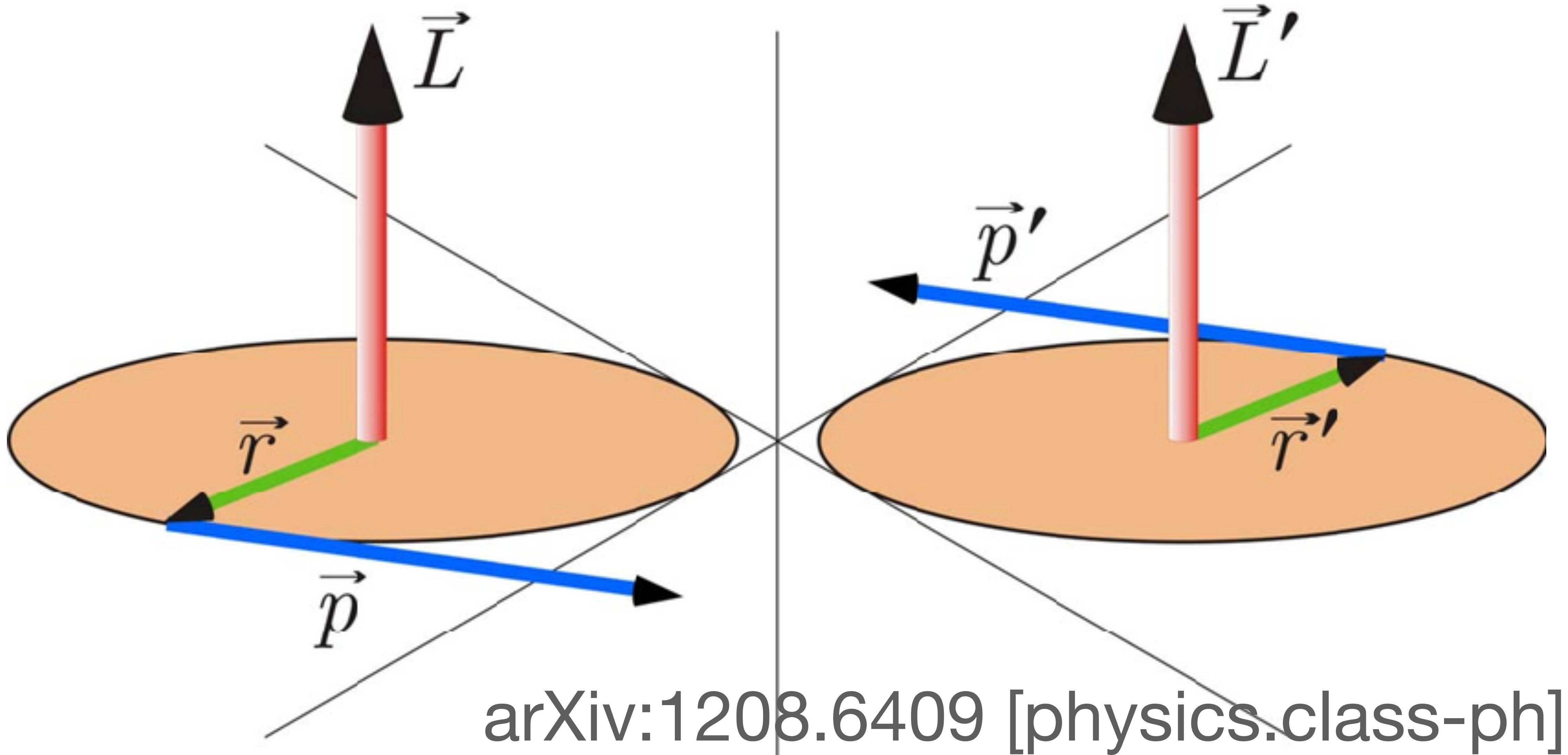
- **p** is the same vector, written using two different basis vectors.
- Therefore, **p**'s components are transformed as $(p'_x, p'_y, p'_z) = (-p_x, -p_y, -p_z)$

# Parity Transformation: Pseudovector

## E.g., angular momentum, magnetic field

- Orbital angular momentum, $\mathbf{L} = \mathbf{r} \times \mathbf{p}$, is a *pseudovector*. Its *components* do **not** change under parity transformation: $(L'_x, L'_y, L'_z) = (L_x, L_y, L_z)$
    - Both $\mathbf{r} = (X, Y, Z)$ and $\mathbf{p} = (p_x, p_y, p_z)$ are vectors whose components change sign. Thus, their products do not change, e.g.,

$$
\begin{aligned}
L'_x &= Y' p'_z - Z' p'_y \\
&= (-Y)(-p_z) - (-Z)(-p_y) \\
&= L_x
\end{aligned}
$$




arXiv:1208.6409 [physics.class-ph]

Key Point

# Parity Transformation: Pseudoscalar

## How to test parity symmetry?

- A dot product of a vector and a pseudovector is a **pseudoscalar**.
  - Like a scalar, a pseudoscalar is invariant under rotation.
  - But, a pseudoscalar changes sign under parity transformation.
- **Experimental test of parity symmetry**: Construct a pseudoscalar and see if the average value is zero. If not, the system violates parity symmetry!
  - *Example*: a dot product of particle A's momentum and particle B's angular momentum: $\mathbf{p}_\mathrm{A} \cdot \mathbf{L}_\mathrm{B}$. Measure this and average over many trials. Does the average vanish, $\langle \mathbf{p}_\mathrm{A} \cdot \mathbf{L}_\mathrm{B} \rangle = 0$?

Letters to the Editor,
Physical Review, 105, 1413 (1957)

# Experimental Test of Parity Conservation in Beta Decay*

C. S. Wu, *Columbia University, New York, New York*

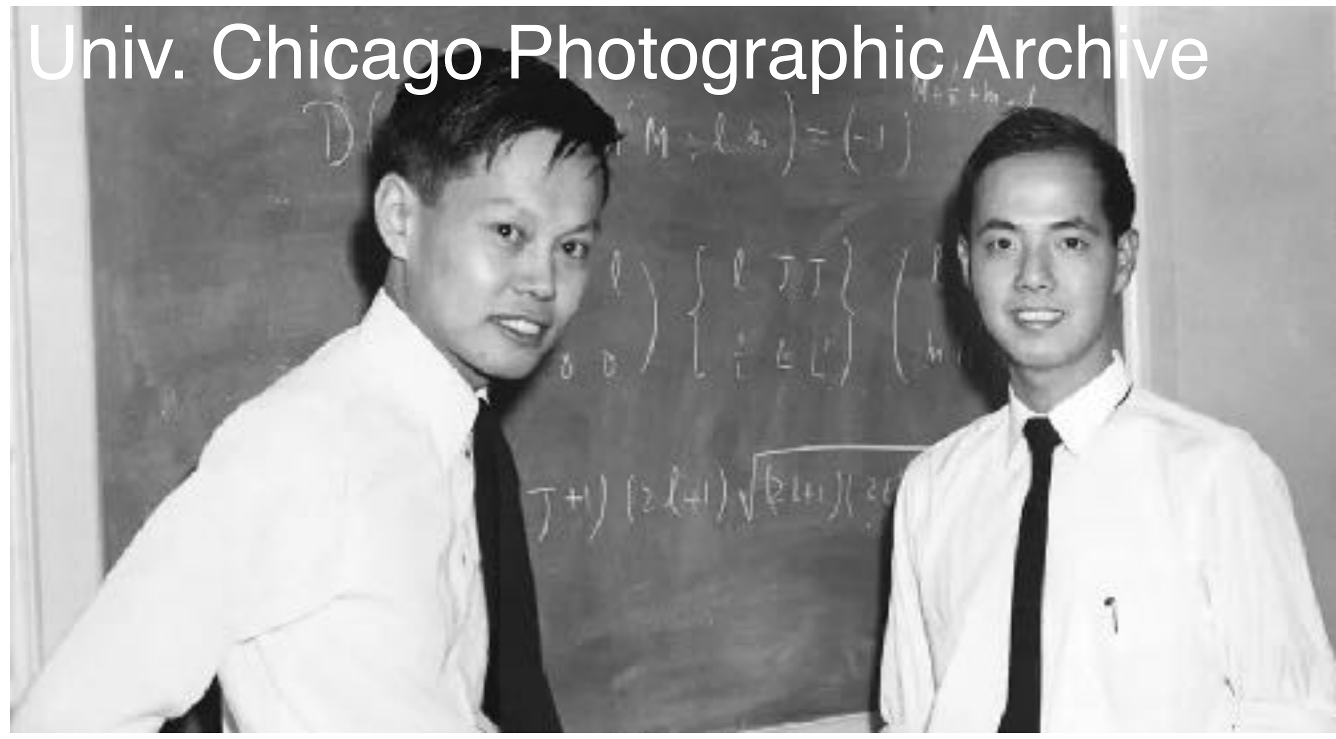
AND

E. Ambler, R. W. Hayward, D. D. Hoppes, and R. P. Hudson,
*National Bureau of Standards, Washington, D. C.*

(Received January 15, 1957)

In a recent paper[1] on the question of parity in weak interactions, Lee and Yang critically surveyed the experimental information concerning this question and reached the conclusion that there is no existing evidence either to support or to refute parity conservation in weak interactions. They proposed a number of experiments on beta decays and hyperon and meson decays which would provide the necessary evidence for parity conservation or nonconservation. In beta decay, one could measure the angular distribution of the electrons coming from beta decays of polarized nuclei. If an asymmetry in the


Chien-Shiung Wu




Chen-Ning Yang  Tsung-Dao Lee

Wu et al. (1957)

# The Wu Experiment of β-decay

**$^{60}Co \rightarrow {}^{60}Ni + e^{-} + \bar{\nu}_e + 2\gamma$**


- Electrons must be emitted with equal probability in all directions relative to **J**, if parity symmetry is respected in β-decay.
  - This was not observed: $\langle \mathbf{p}_\mathrm{e} \cdot \mathbf{J} \rangle \neq 0$. **Parity symmetry is violated in β-decay!**

# Initial reaction

Bildarchiv der ETH-Bibliothek

## Many physicists did not believe it initially.

- To Lee and Yang's theoretical paper on parity violation in β-decay:
  - Wolfgang Pauli said, "*Ich glaube aber nicht, daß der Herrgott ein schwacher Linkshänder ist*" (I do not believe that the Lord is a weak left-hander).
- To Wu's discovery paper:
  - Wolfgang Pauli said, "*Sehr aufregend. Wie sicher ist die Nachricht?*" (Very exciting. How sure is this news?)
- **This was shocking news. The weak interaction distinguishes between left and right!**
- In this talk we ask, "*Does the Universe distinguish between left and right?*" Most scientists answer, "No, of course it doesn't". **That may well be true, but one must at least have a look to be sure!**

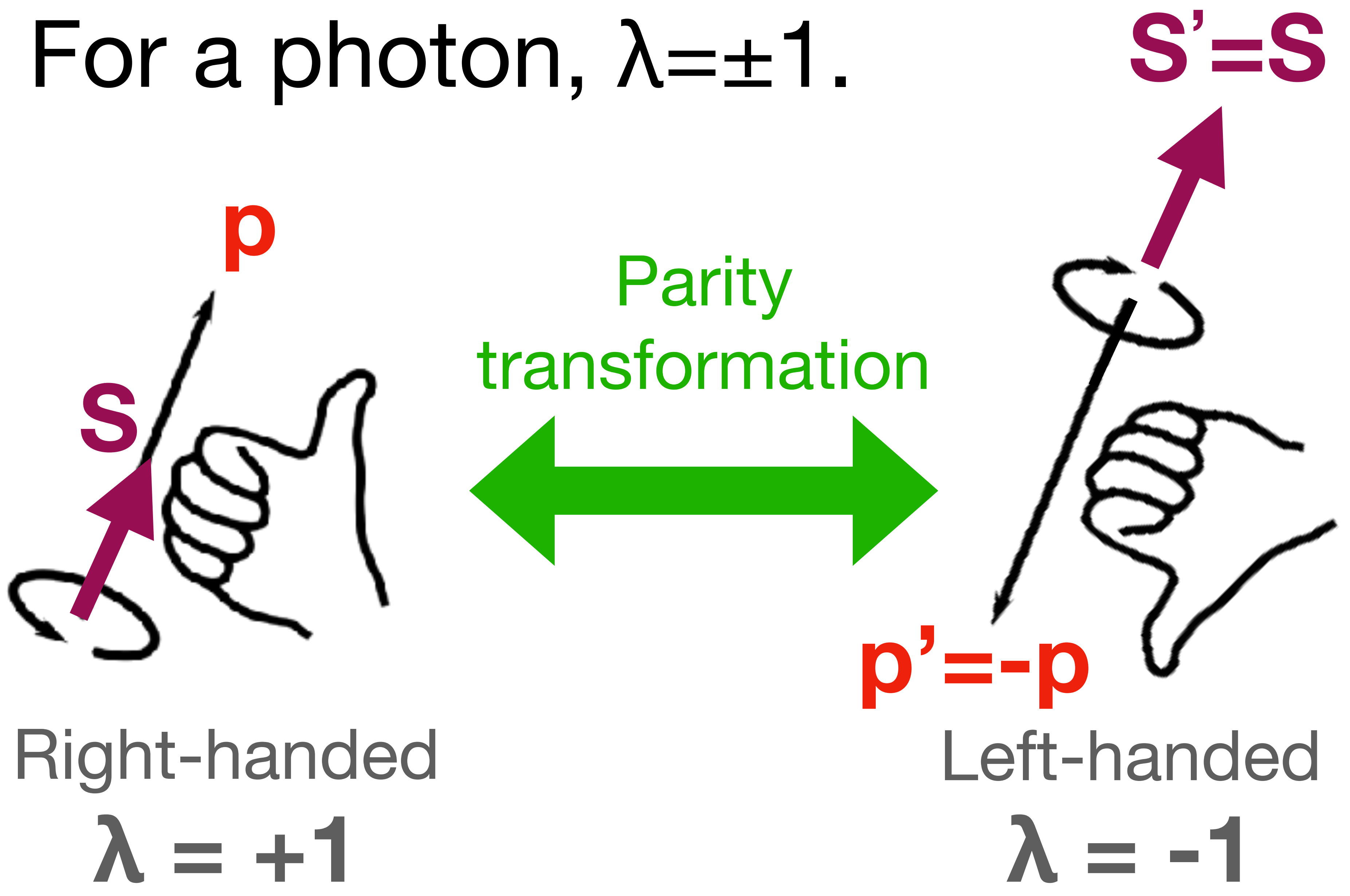
# Helicity is a pseudoscalar

**Party transformation changes "right-handed" to "left-handed" and vice versa**

- For massless particles, we define the "helicity", λ, as

$$\mathbf{S} \cdot \frac{\mathbf{p}}{|\mathbf{p}|} = \lambda\hbar$$

- For a photon, λ=±1.




- **λ is a pseudoscalar** because it is a product of a momentum vector (**p**) and a spin pseudovector (**S**).
  - On the other hand, "scalar", such as $\mathbf{p}^2$ and $\mathbf{S}^2$, does not change sign.
- For a graviton, λ=±2.
- **Asymmetry between λ=±1 and ±2 is the sign of parity violation!**

# Parity Violation in Electromagnetism with

$$I_{\mathrm{CS}} = \int \mathrm{d}^4x \sqrt{-g} \left( -\frac{\alpha}{4f} \chi F\widetilde{F} \right)$$

Throughout this talk, I will assume homogeneity and isotropy of space (invariance under 3d translation and rotation).

# Maxwell's Equations

## In Minkowski space, Heaviside units and *c*=1

$$\nabla \cdot \mathbf{E} = \rho\,, \qquad -\dot{\mathbf{E}} + \nabla \times \mathbf{B} = \mathbf{j}$$
$$\nabla \cdot \mathbf{B} = 0\,, \qquad \dot{\mathbf{B}} + \nabla \times \mathbf{E} = 0$$

- These equations are invariant under Poincaré transformation (spatial translation and rotation and Lorentz boost).

Throughout this talk, I will assume homogeneity and isotropy of space (invariance under 3d translation and rotation).

# Parity-flipping Maxwell's Equations

## In Minkowski space, Heaviside units and *c*=1

$$(-\nabla)\cdot(-\mathbf{E}) = \rho\,, \qquad -(-\dot{\mathbf{E}}) + (-\nabla)\times\mathbf{B} = (-\mathbf{j})$$

$$(-\nabla)\cdot\mathbf{B} = 0\,, \qquad \dot{\mathbf{B}} + (-\nabla)\times(-\mathbf{E}) = 0$$

- These equations are invariant under Poincaré transformation (spatial translation and rotation and Lorentz boost).
- They are also invariant under parity transformation, if **E** and **j** are vectors, ρ is a scalar, and **B** is a pseudovector.

Throughout this talk, I will assume homogeneity and isotropy of space (invariance under 3d translation and rotation).

# Maxwell's Equations in a covariant form

$$\nabla \cdot \mathbf{E} = \rho\,, \qquad -\dot{\mathbf{E}} + \nabla \times \mathbf{B} = \mathbf{j}$$

$$\nabla \cdot \mathbf{B} = 0\,, \qquad \dot{\mathbf{B}} + \nabla \times \mathbf{E} = 0$$

- These equations can be written in a covariant form as

Dual tensor

$$\partial_\nu F^{\mu\nu} = j^\mu \qquad \partial_\nu \tilde{F}^{\mu\nu} = 0$$

$$\mu = 0, 1, 2, 3\,, \quad j^\mu = (\rho, \mathbf{j})\,, \quad \partial_\mu = \partial / \partial x^\mu\,, \quad x^\mu = (t, \mathbf{x})$$

# Antisymmetric Field Strength Tensor, $F_{\mu\nu}$

$F_{\mu\nu} = -F_{\nu\mu}$

$$F_{\mu\nu} = \eta_{\mu\alpha}\eta_{\nu\beta}F^{\alpha\beta}$$ where $\eta_{\mu\alpha} = \mathrm{diag}(-1, 1, 1, 1)$

$$F_{\mu\nu} = \begin{pmatrix} 0 & -E_x & -E_y & -E_z \\ E_x & 0 & B_z & -B_y \\ E_y & -B_z & 0 & B_x \\ E_z & B_y & -B_x & 0 \end{pmatrix}$$

- Equivalently,

$$F^{0i} = E_i$$
$$F^{ij} = \epsilon^{ijk} B_k$$

- Therefore,

$$F^2 \equiv F_{\mu\nu}F^{\mu\nu} = 2(\mathbf{B}\cdot\mathbf{B} - \mathbf{E}\cdot\mathbf{E})$$

This is a *scalar* and is invariant under parity transformation.

# Dual Field Strength Tensor, $\tilde{F}^{\mu\nu}$

$\tilde{F}^{\mu\nu} = -\tilde{F}^{\nu\mu}$

$$\tilde{F}^{\mu\nu} = \frac{1}{2}\epsilon^{\mu\nu\alpha\beta}F_{\alpha\beta}$$

where $\epsilon^{\mu\nu\alpha\beta}$ *Levi-Civita symbol*

$$\epsilon^{\mu\nu\alpha\beta} = \begin{cases} +1 & \text{if } (\mu,\nu,\alpha,\beta) \text{ is even perm. of } (0,1,2,3) \\ -1 & \text{if } (\mu,\nu,\alpha,\beta) \text{ is odd perm. of } (0,1,2,3) \\ 0 & \text{otherwise} \end{cases}$$

$$\tilde{F}^{\mu\nu} = \begin{pmatrix} 0 & B_x & B_y & B_z \\ -B_x & 0 & -E_z & E_y \\ -B_y & E_z & 0 & -E_x \\ -B_z & -E_y & E_x & 0 \end{pmatrix}$$

- Equivalently,

$$\tilde{F}^{0i} = B_i$$
$$\tilde{F}^{ij} = -\epsilon^{ijk}E_k$$

- Therefore,

$$F\tilde{F} \equiv F_{\mu\nu}\tilde{F}^{\mu\nu} = -4\mathbf{B}\cdot\mathbf{E}$$

This is a *pseudoscalar* and changes sign under parity transformation!

# $F\tilde{F}$ in the action?

$$F^2 \equiv F_{\mu\nu}F^{\mu\nu} = 2(\mathbf{B}\cdot\mathbf{B} - \mathbf{E}\cdot\mathbf{E})$$
$$F\tilde{F} \equiv F_{\mu\nu}\tilde{F}^{\mu\nu} = -4\mathbf{B}\cdot\mathbf{E}$$

$$I = -\frac{1}{4}\int d^4x \; F^2 + \int d^4x \; A_\mu j^\mu$$

$$d^4x = dt d^3\mathbf{x}$$

- This action is sufficient to produce all of Maxwell's equations.

- Can we add $\int d^4x \; F\tilde{F}$ to the action?
  - The answer is yes. However, **this is only a surface term**, since $F\tilde{F}$ is a total derivative: $F_{\mu\nu}\tilde{F}^{\mu\nu} = 2\partial_\mu(A_\nu\tilde{F}^{\mu\nu})$ where $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$

Ni (1977); Sikivie (1983); Turner, Widrow (1987)
Carroll, Field, Jackiw (1990)

# $F\tilde{F}$ in the action

## Chern-Simons term

- Consider $I_{\rm CS} = -\frac{1}{4}\alpha \int d^4x\ \theta F\tilde{F}$ with $F\tilde{F} = 2\partial_\mu(A_\nu \tilde{F}^{\mu\nu})$
  - α: a dimensionless constant
  - *θ*: a dimensionless pseudoscalar field
- **This is not a surface term!** Integration by parts gives

$$I_{\rm CS} = \frac{1}{2}\alpha \int d^4x\ (\partial_\mu\theta) A_\nu \tilde{F}^{\mu\nu}$$


https://einstein-chair.github.io/simons2023/

- This is a special case of the so-called *Chern-Simons term(*)*, $p_\mu A_\nu \tilde{F}^{\mu\nu}$ with $p_\mu = \partial_\mu\theta$

*(*) Strictly speaking, Chern-Simons 3-form for an Abelian gauge field is* $A_\nu \tilde{F}^{0\nu} = A_i \epsilon^{ijk}\partial_j A_k$

Adler (1969); Bell, Jackiw (1969); Fujikawa (1979)

# Is there a known example of this term in particle physics?

## Yes, a pion. The ABJ anomaly!


Credit: HiggsTan

- A pion is a composite meson composed of a quark and an antiquark.
  - A neutral pion, $\pi^0$, is composed of either $u\bar{u}$ or $d\bar{d}$, and **is a pseudoscalar**.
    (Chinowsky & Steinberger, 1954)
  - $\pi^0$ is coupled to photons via $l_{CS}$ where
    - $\theta = \pi^0 / f_\pi$ with $f_\pi \sim 184$ MeV (pion decay constant)
    - $\alpha = 2\alpha_{EM} N_c / (3\pi)$ with $N_c = 3$ (the number of quark colors) and $\alpha_{EM} \sim 1/137$ (EM fine structure constant)
- **$\pi^0$ decays into 2 photons via this term, which has been observed.** So, this possibility is not completely crazy!

# Multiple discoveries of $I_{\rm CS} = -\frac{1}{4}\alpha \int d^4x\ \theta F\tilde{F}$

## The presence of this term is well motivated.

- This electromagnetic coupling term has been discovered at least 4 times in the past.
  - **1969**: The ABJ anomaly [Adler, Bell, Jackiw]
  - **1974**: Chern-Simons 3-form [Chern, Simons]
    - Promoted to 4-dimensional theory in 1990 [Carroll, Field, Jackiw]
  - **1977**: Equivalence Principle [Ni]
  - **1983**: Axion electrodynamics [Sikivie]

Carroll, Field, Jackiw (1990)

# Consistency with gauge invariance

**$p_\mu$ cannot be arbitrary**

$$I_{\rm CS} = \frac{1}{2}\alpha \int d^4x \; p_\mu A_\nu \tilde{F}^{\mu\nu}$$

- This action is invariant under the gauge transformation, $A_\nu \to A_\nu + \partial_\nu f$

**if** $\partial_\nu p_\mu - \partial_\mu p_\nu = 0$ Hint: Use integration by parts and the Bianchi identity $\partial_\nu \tilde{F}^{\mu\nu} = 0$

- For example: This implies the presence of a preferred direction in spacetime and violation of Lorentz invariance!
  - $p_\mu$ is a constant vector and not dynamical, or
  - $p_\mu$ is a gradient of a dynamical (pseudo)scalar field, such as $p_\mu = \partial_\mu\theta$.

But see Zhou, Huang, Geng (2023) for a possible way around this in new physics.

# The main goal of this talk

## Let's find new physics!

- We study the cosmological consequence of

$$I_{\rm CS} = -\frac{1}{4}\alpha \int d^4x \ \theta F\tilde{F}$$

- Specifically, we ask if $\theta$ is —
  - responsible for dark matter and dark energy, or
  - active during cosmic inflation.

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$$I_{\rm CS} = -\frac{1}{4}\alpha \int d^4x \; \theta F\tilde{F}$$

- Specifically, we ask if $\theta$ is —
  - responsible for dark matter and dark energy, or
  - active during cosmic inflation.

- More examples:
  - **Non-Abelian gauge fields** [Maleknejad, Sheikh-Jabbari, Soda, Phys. Rept. **528**, 161 (2013)]

$$F\tilde{F} = F^a_{\mu\nu}\tilde{F}^{\mu\nu a}$$

$$F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + g_A \epsilon^{abc} A^b_\mu A^c_\nu$$

  - **Gravitational CS** [Alexander, Yunes, Phys. Rept. **480**, 1 (2009)]

$$R\tilde{R} = R^\beta{}_\alpha{}^{\mu\nu}\tilde{R}^\alpha{}_{\beta\mu\nu}$$

You can have both!

Mirzagholi, EK, Lozanov, Watanabe (2020)

# Correction to Maxwell's equations

## In Minkowski space, Heaviside units and *c*=1

- We now derive the correction to Maxwell's equations from

$$d^4x = dtd^3\mathbf{x}$$

$$I = -\frac{1}{4}\int d^4x \ \left(F^2 + \alpha\theta F\tilde{F}\right) + \int d^4x \ A_\mu j^\mu$$

- Finding the path that gives a stationary point,

$$\partial_\nu F^{\mu\nu} + \alpha(\partial_\nu\theta)\tilde{F}^{\mu\nu} = j^\mu$$

As expected, only the space-time dependence of the $\theta$ field affects Maxwell's equation.

# Correction to the EM wave equation in vacuum

## With the Chern-Simons term

$$\partial_\nu F^{\mu\nu} + \alpha(\partial_\nu \theta)\tilde{F}^{\mu\nu} = 0$$ where $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$

- With $A^0=\phi=0$ in the Lorenz gauge, we find

$$-\Box A^i + \alpha(\partial_\nu \theta)\tilde{F}^{i\nu} = 0$$

$$\Box = \eta^{\alpha\beta}\partial_\alpha\partial_\beta = -\frac{\partial^2}{\partial t^2} + \nabla^2$$

$$A^\mu = \eta^{\mu\alpha}A_\alpha = (\phi, \mathbf{A})$$

$$\ddot{\mathbf{A}} - \nabla^2\mathbf{A} + \alpha\left[-\dot{\theta}(\nabla\times\mathbf{A}) + (\nabla\theta)\times\dot{\mathbf{A}}\right] = 0$$

Correction to the EM wave equation!

Note: **A** is a vector and $\theta$ is a pseudoscalar.

# Helicity basis to probe parity symmetry

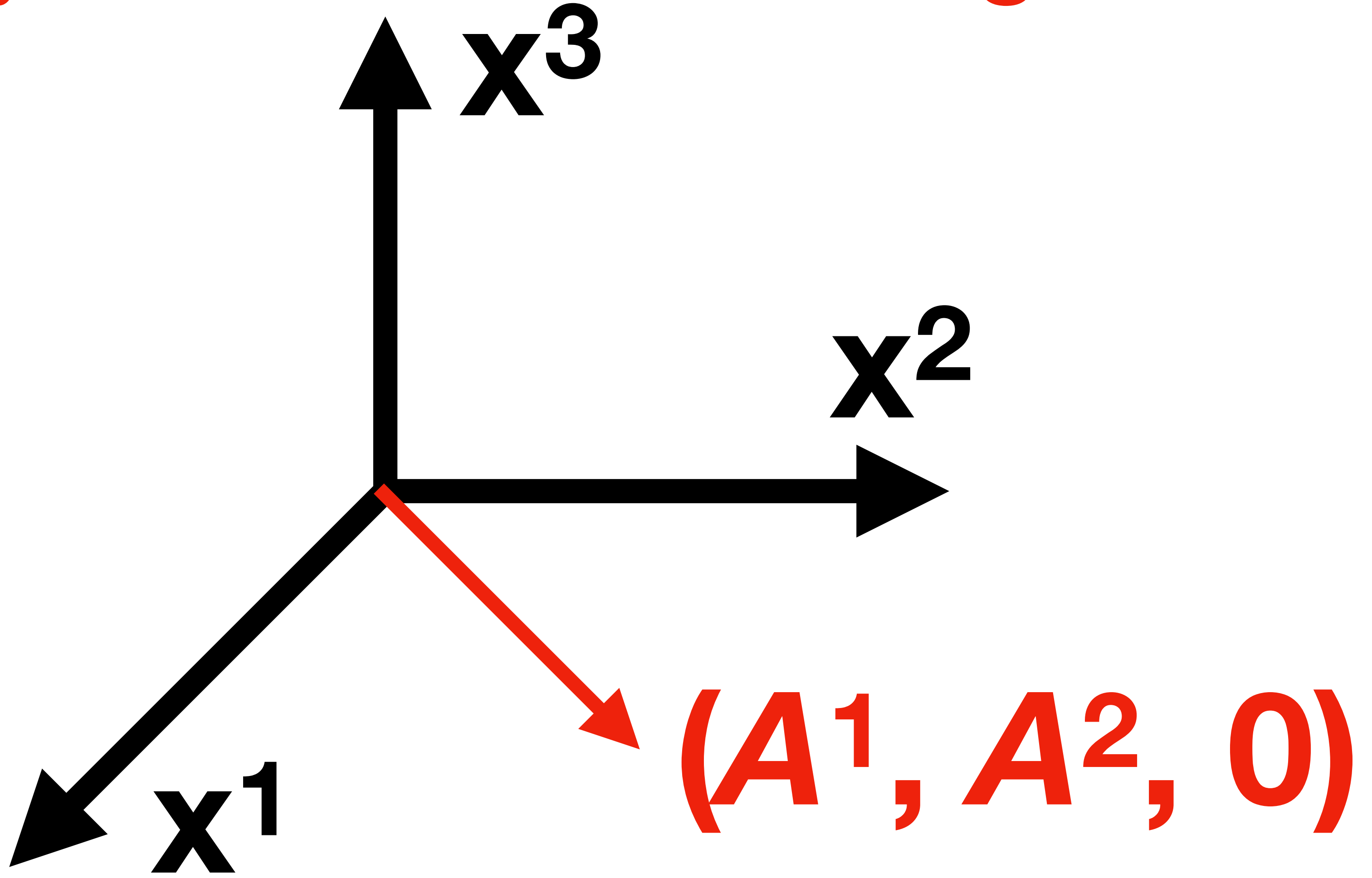
## Going to Fourier space

- Fourier transform of **A**(t,**x**) is $\mathbf{A}(t,\mathbf{x}) = (2\pi)^{-3/2}\int d^3\mathbf{k}\ \mathbf{A}_{\mathbf{k}}(t)e^{i\mathbf{k}\cdot\mathbf{x}}$
  - The EM wave propagates in the direction of **k**. The change in $\mathbf{A}_{\mathbf{k}}$ is perpendicular to **k**.

"Coulomb gauge" $\nabla\cdot\mathbf{A}(t,\mathbf{x}) = 0 \rightarrow \mathbf{k}\cdot\mathbf{A}_{\mathbf{k}}(t) = 0$

- Choose **k** to be on the z(=$x^3$) axis. The helicity states, λ=±1, are given for each Fourier mode by

$$A_{\pm} = \frac{A^1_{\mathbf{k}} \mp iA^2_{\mathbf{k}}}{\sqrt{2}}$$



# Correction to the EM wave equation

## In the helicity basis

$$\ddot{\mathbf{A}} - \nabla^2 \mathbf{A} + \alpha \left[ -\dot{\theta}(\nabla \times \mathbf{A}) + (\nabla\theta) \times \dot{\mathbf{A}} \right] = 0$$

Correction to the EM wave equation!

- If $\theta$ has a time-dependent vacuum expectation value, $\theta$(t,**x**) –> $\bar{\theta}$(t), we find in Fourier space

$$\ddot{\mathbf{A}}_{\mathbf{k}} + k^2 \mathbf{A}_{\mathbf{k}} - i\alpha\dot{\bar{\theta}}(\mathbf{k} \times \mathbf{A}) = 0$$

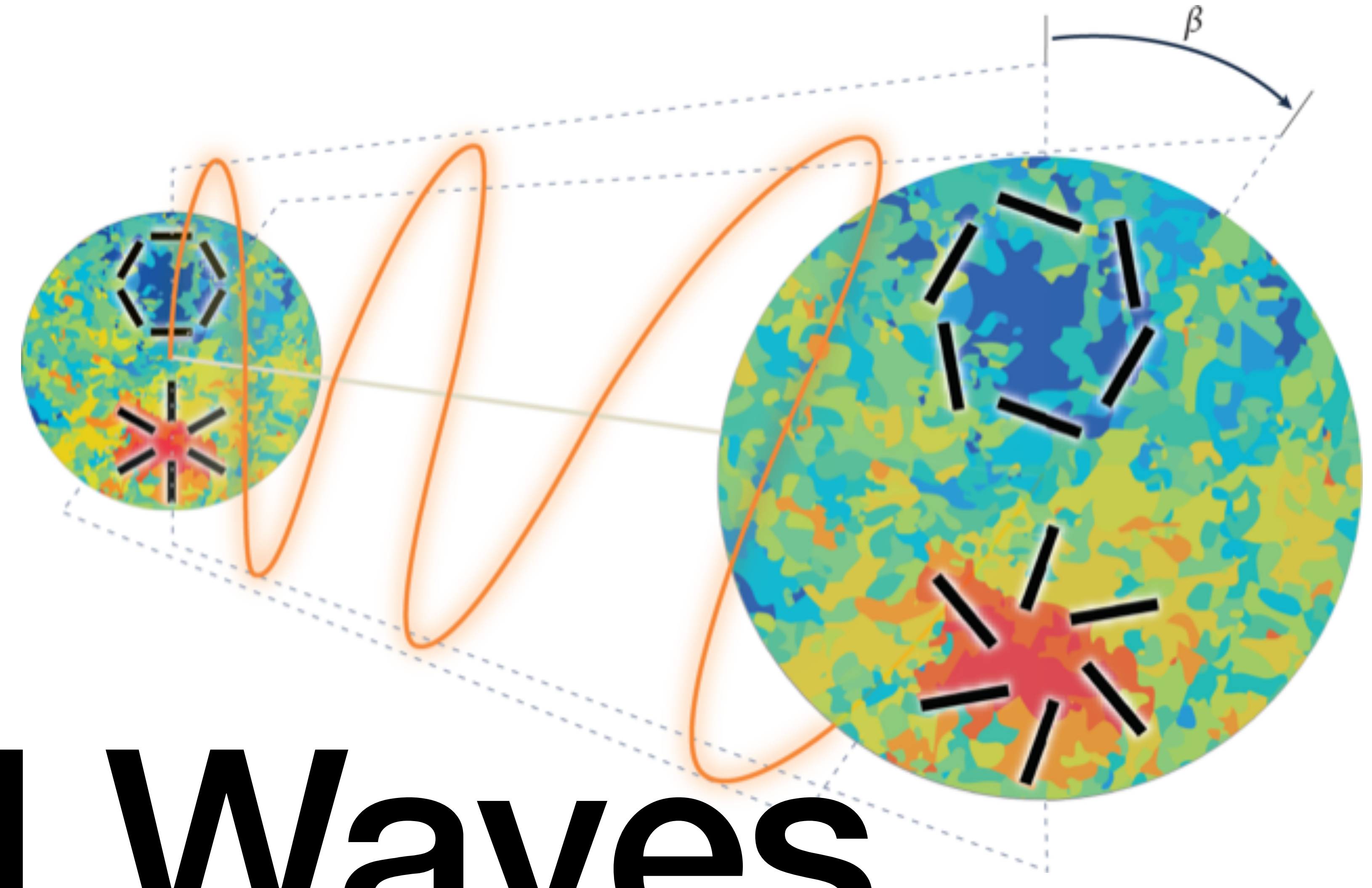
$$\Rightarrow \quad \ddot{A}_{\pm} + \left(k^2 \mp k\alpha\dot{\bar{\theta}}\right) A_{\pm} = 0$$

**Parity violation**

The equation of motion depends on handedness!

## Imagine that space is filled with a pseudoscalar field coupled to photons via the CS term.

$$I_{\rm CS} = \int d^4x \sqrt{-g}\left(-\frac{\alpha}{4f}\chi F\tilde{F}\right)$$




# Parity Violation in EM Waves due to Dark Matter and Dark Energy

Imagine that space is filled with a pseudoscalar field coupled to photons via the CS term.

# Scalar field DM/DE coupled to the CS term

## DM = Dark Matter; DE = Dark Energy

$$I = \int d^4x \sqrt{-g} \left[ -\frac{1}{2}(\partial\chi)^2 - V(\chi) - \frac{1}{4}F^2 - \frac{\alpha}{4f}\chi F\tilde{F} \right]$$

We wrote

$$\theta = \frac{\chi}{f}$$

- $\chi$ is a neutral pseudoscalar field (spin 0).
- Why consider $\chi$ as a good DM/DE candidate?
  - *Why not*? We have an example in the Standard Model: a neutral pion.
  - We expect $\alpha \simeq \alpha_{\mathrm{EM}} \simeq 10^{-2}$ and $f < M_{\mathrm{Pl}} \simeq 2.4 \times 10^{18}$ GeV.
- $\chi$ can be composed of fermions like a pion, or a fundamental pseudoscalar like an "axion" field.

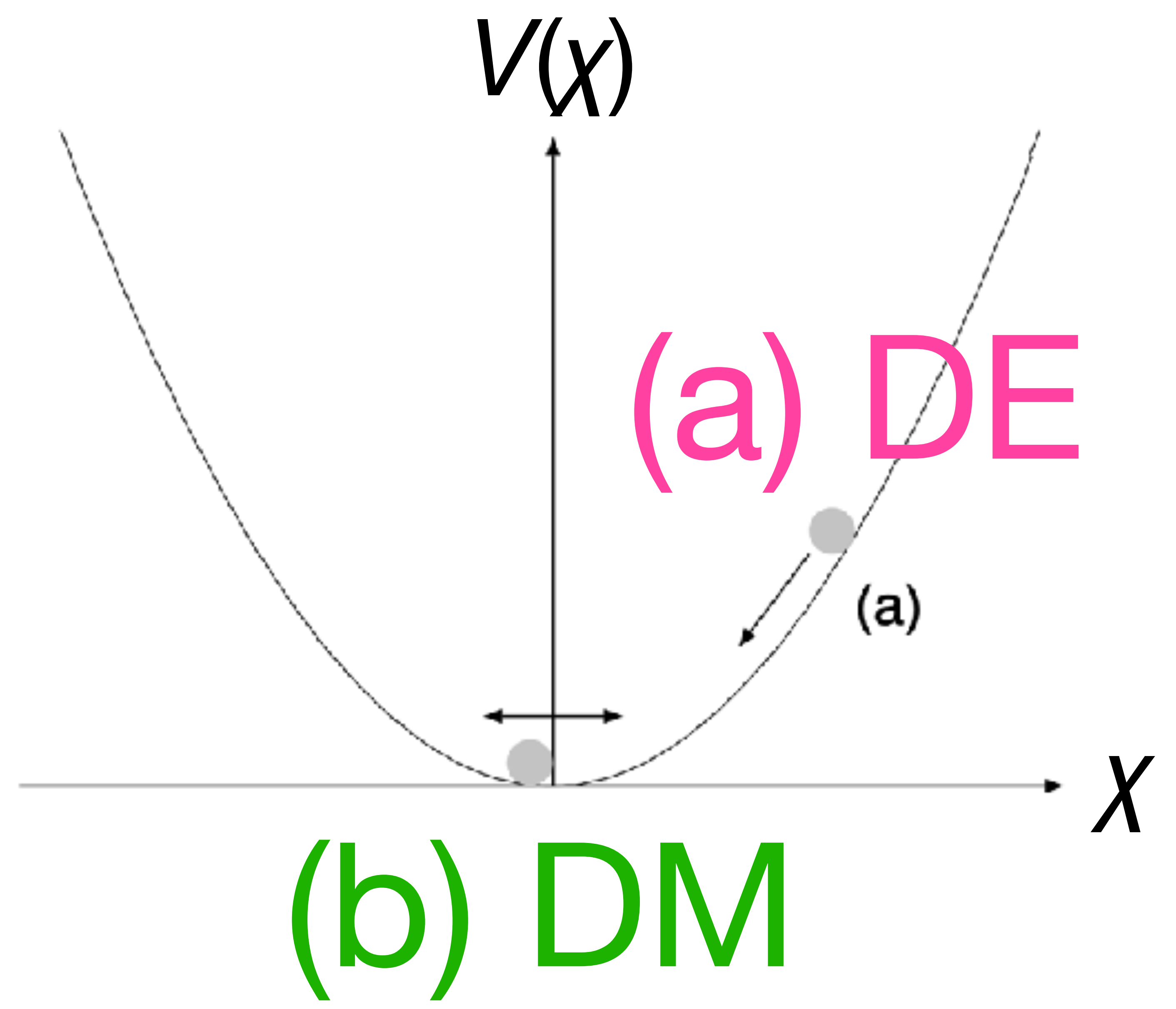
# Distinction between DE and DM

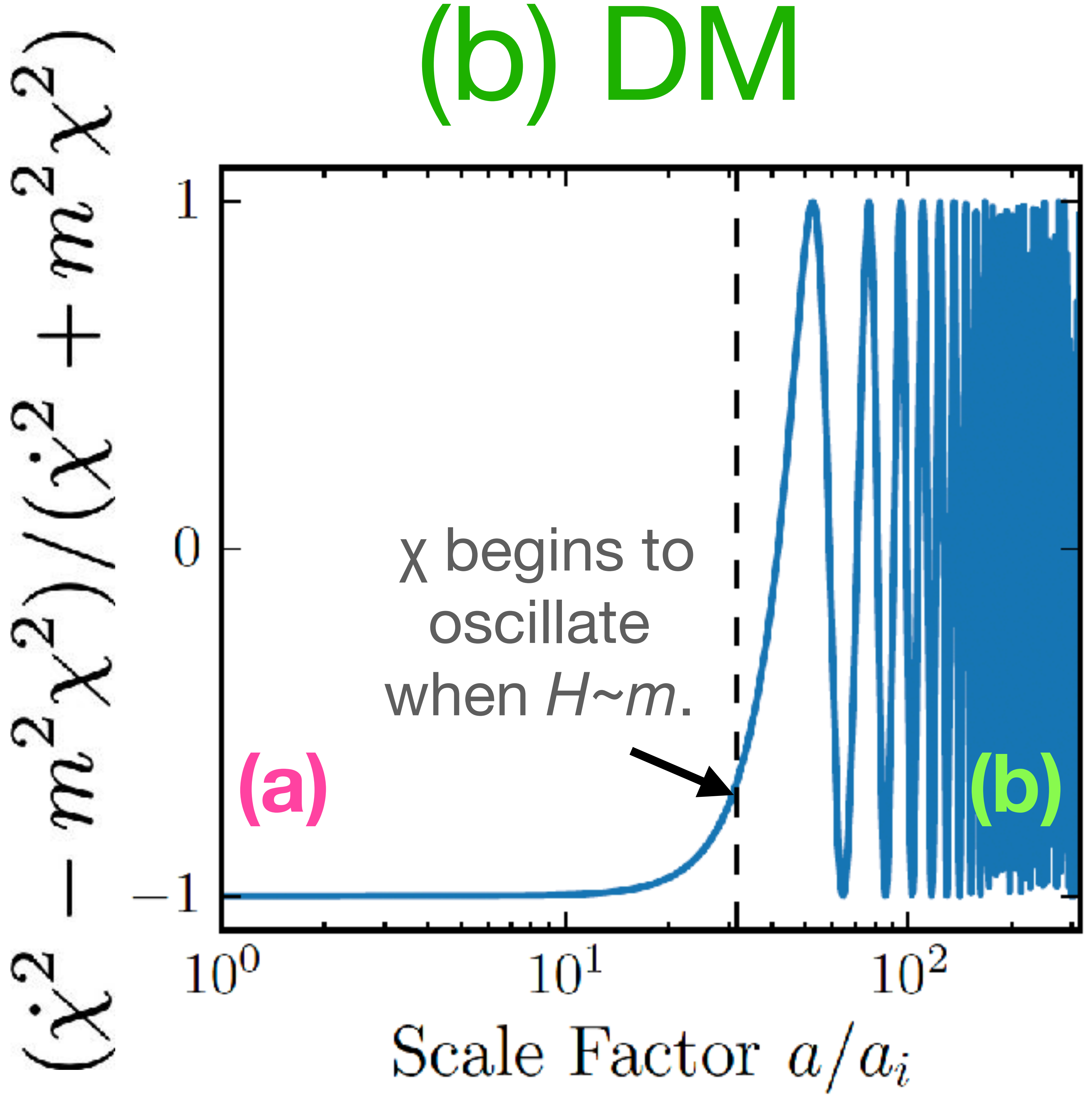
## How small is its mass? Example of $V(\chi) = m^2\chi^2/2$

- The useful criterion is the equation of state parameter, *w*.

$$w = \frac{P}{\rho} = \frac{\langle\dot{\chi}^2\rangle - m^2\langle\chi^2\rangle}{\langle\dot{\chi}^2\rangle + m^2\langle\chi^2\rangle}$$

- $w \simeq -1$: Dark Energy (DE)
  - $m \lesssim H_0 \simeq 10^{-33}$ eV
- $w \simeq 0$: Dark Matter (DM)
  - $m \gtrsim H_0$



# Phase velocity of circular polarization states

## Expanding space, *c*=1

- We write

where $' = \frac{\partial}{\partial \tau} = a\frac{\partial}{\partial t}$

$$A''_{\pm} + \omega_{\pm}^2 A_{\pm} = 0, \quad \omega_{\pm}^2 = k^2 \mp \frac{k\alpha\chi'}{f}$$

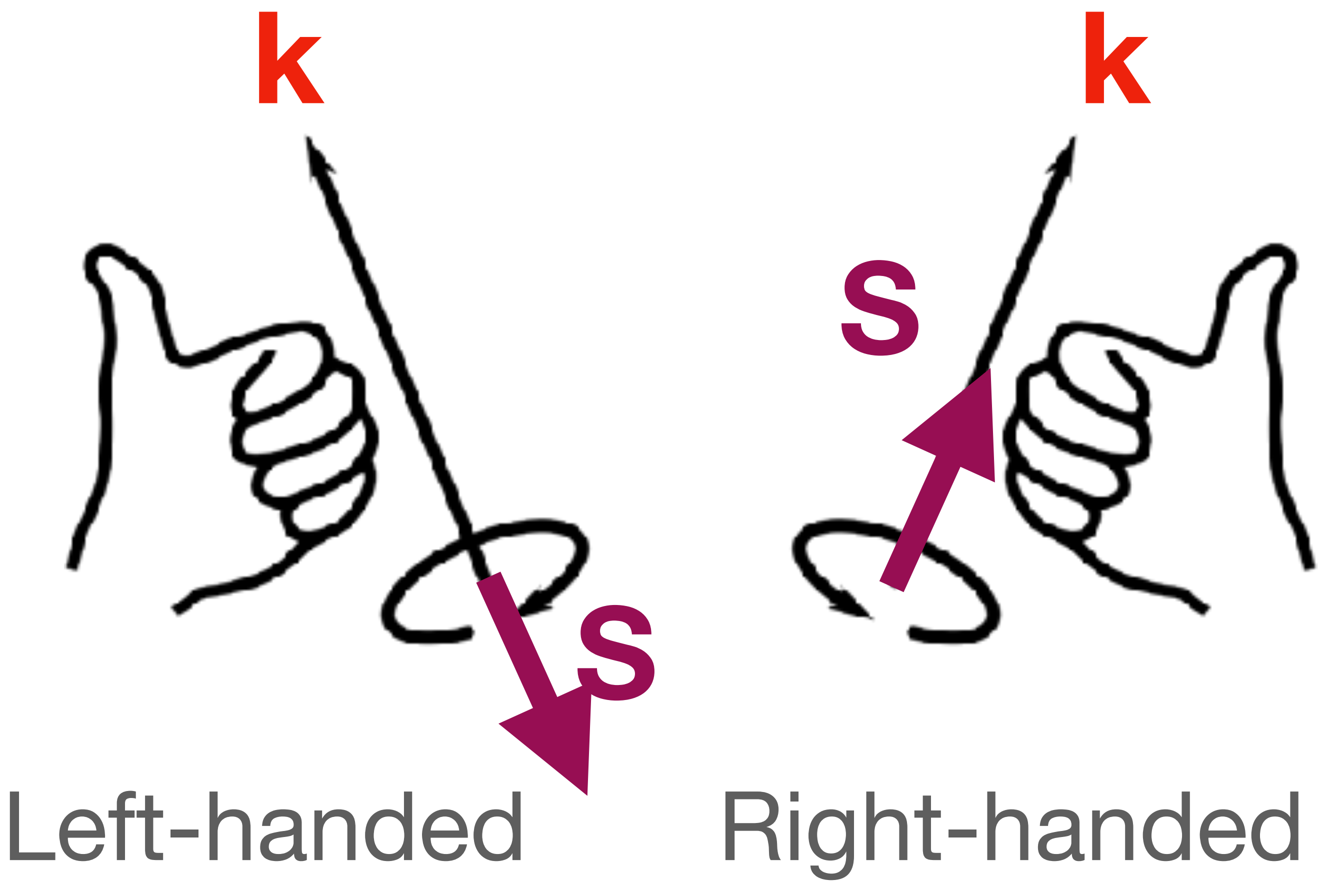
$\tau$ : conformal time

- We work in the limit of $k^2 \gg k\alpha\chi'/f$. This approximation is accurate for the photons we observe today. (However, $\omega_{\pm}^2$ can become negative during inflation!)

- The phase velocity of circular polarization states, $\omega_{\pm}/k$, is

$$\frac{\omega_{\pm}}{k} \simeq 1 \mp \frac{\alpha\chi'}{2kf}$$

| |
|---|
| +: Right-handed state |
| –: Left-handed state |



Carroll, Field, Jackiw (1990); Carroll, Field (1991); Harari, Sikivie (1992)

# Plane-wave (WKB) Solution

## Expanding space, *c*=1

$$A''_\pm + \omega_\pm^2 A_\pm = 0, \quad \omega_\pm \simeq k \mp \frac{\alpha\chi'}{2f}$$

- For $|\omega'_\pm| \ll \omega_\pm^2$, which is satisfied here, an accurate solution is given by

$$A_\pm \simeq C_\pm \frac{\exp\left(-i\int d\tau\ \omega_\pm + i\delta_\pm\right)}{\sqrt{2\omega_\pm}} \quad \sqrt{2\omega_\pm} \simeq \sqrt{2k}$$

We can replace $\omega_\pm$ in amplitude (but not in phase) with $k$.

where $C_\pm$ is the initial amplitude and $\delta_\pm$ is the initial phase.

Carroll, Field, Jackiw (1990); Carroll, Field (1991); Harari, Sikivie (1992)

# Cosmic Birefringence

## Rotation of the plane of linear polarization

$$A_\pm \simeq C_\pm \frac{\exp\left(-i\int d\tau\ \omega_\pm + i\delta_\pm\right)}{\sqrt{2\omega_\pm}} \qquad \sqrt{2\omega_\pm} \simeq \sqrt{2k}$$

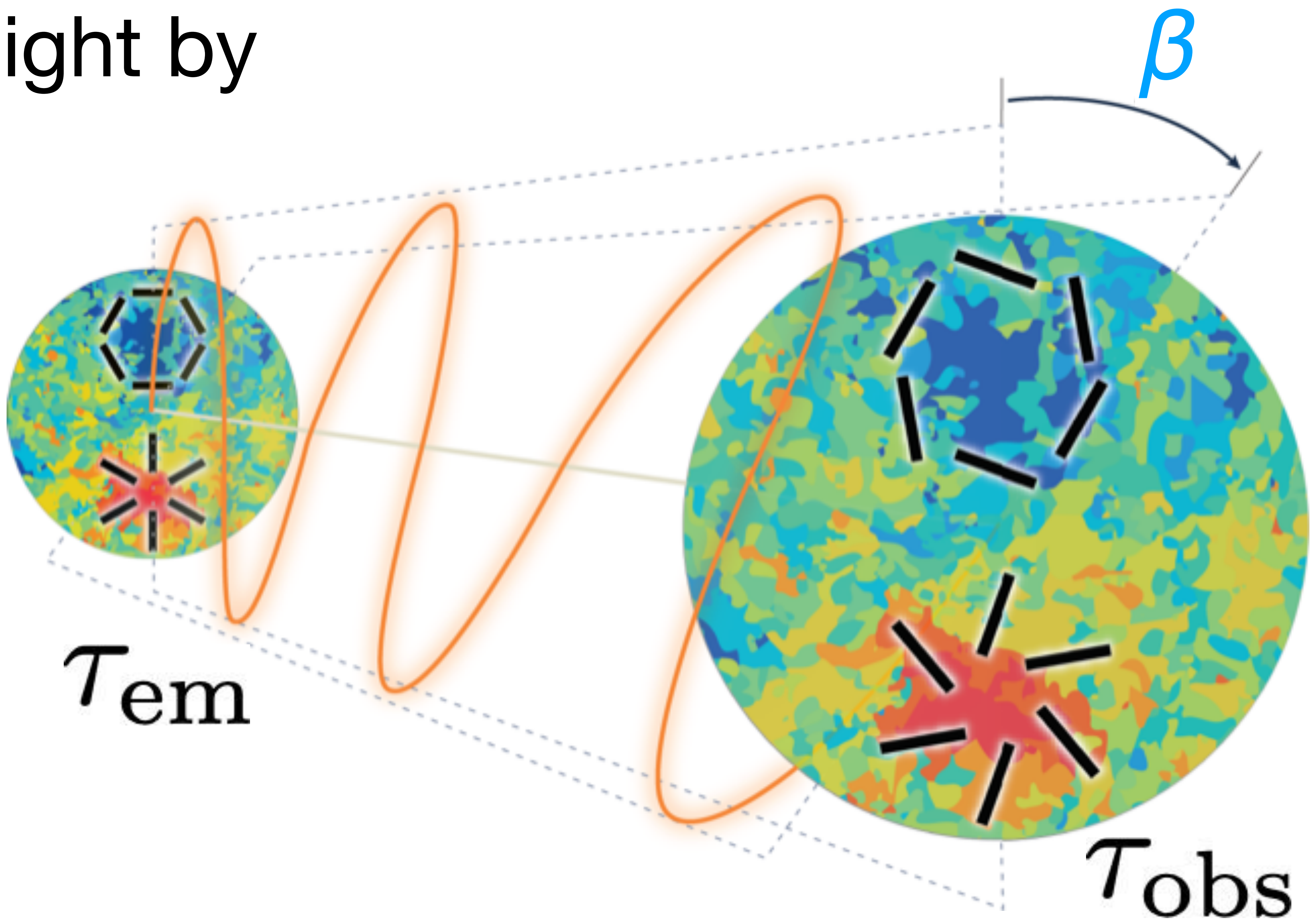
with

$$\frac{\omega_\pm}{k} \simeq 1 \mp \frac{\alpha\chi'}{2kf}$$

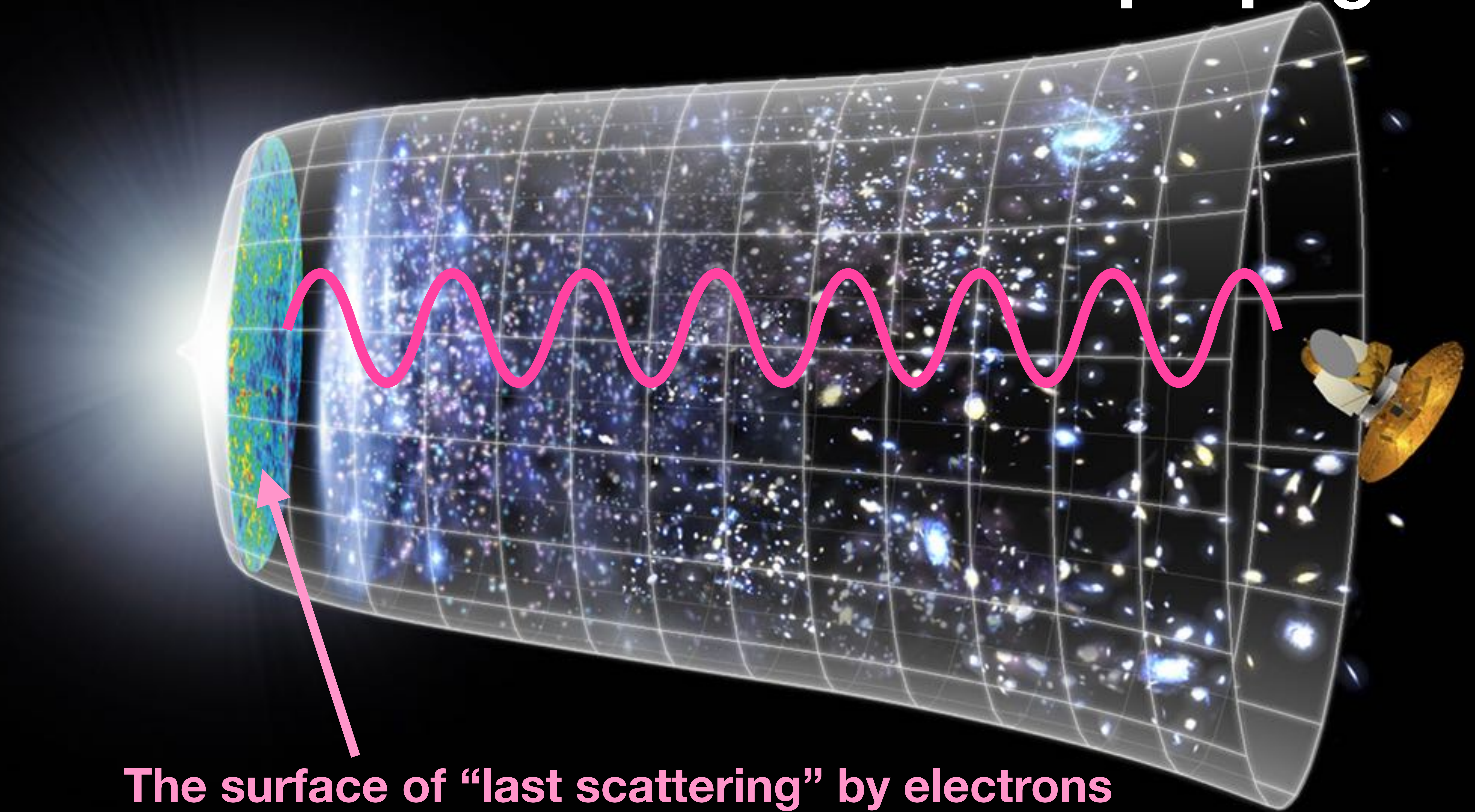
- This **rotates** the plane of linear polarization of light by

$$\beta = -\int_{\tau_{\rm em}}^{\tau_{\rm obs}} d\tau\,(\omega_+ - \omega_-)$$

$$= \frac{\alpha}{2f}\left[\chi(\tau_{\rm obs}) - \chi(\tau_{\rm em})\right]$$



# How does the EM wave of the CMB propagate?



The surface of “last scattering” by electrons

(Scattering generates ***polarization***!)

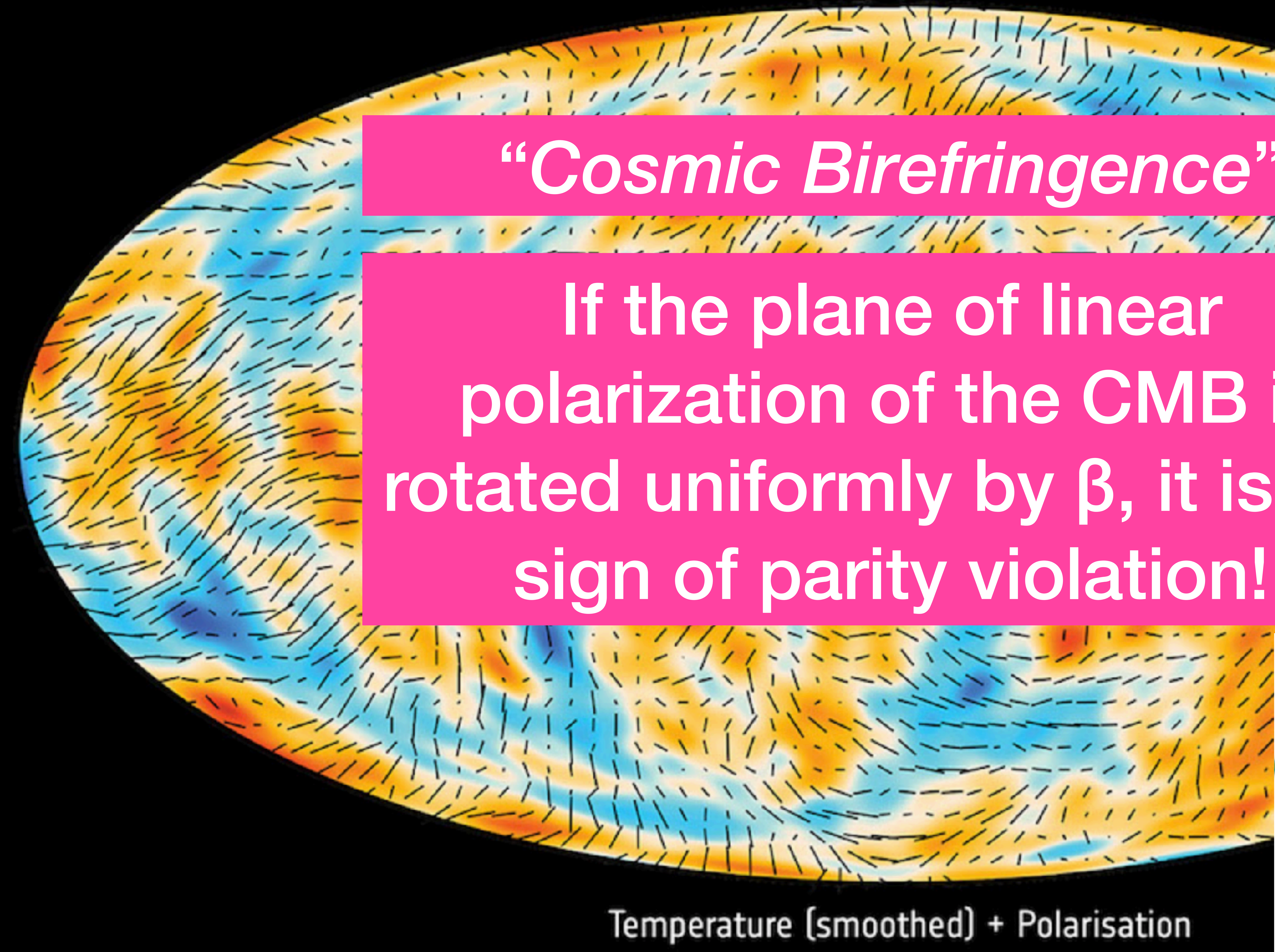
Credit: WMAP Science Team

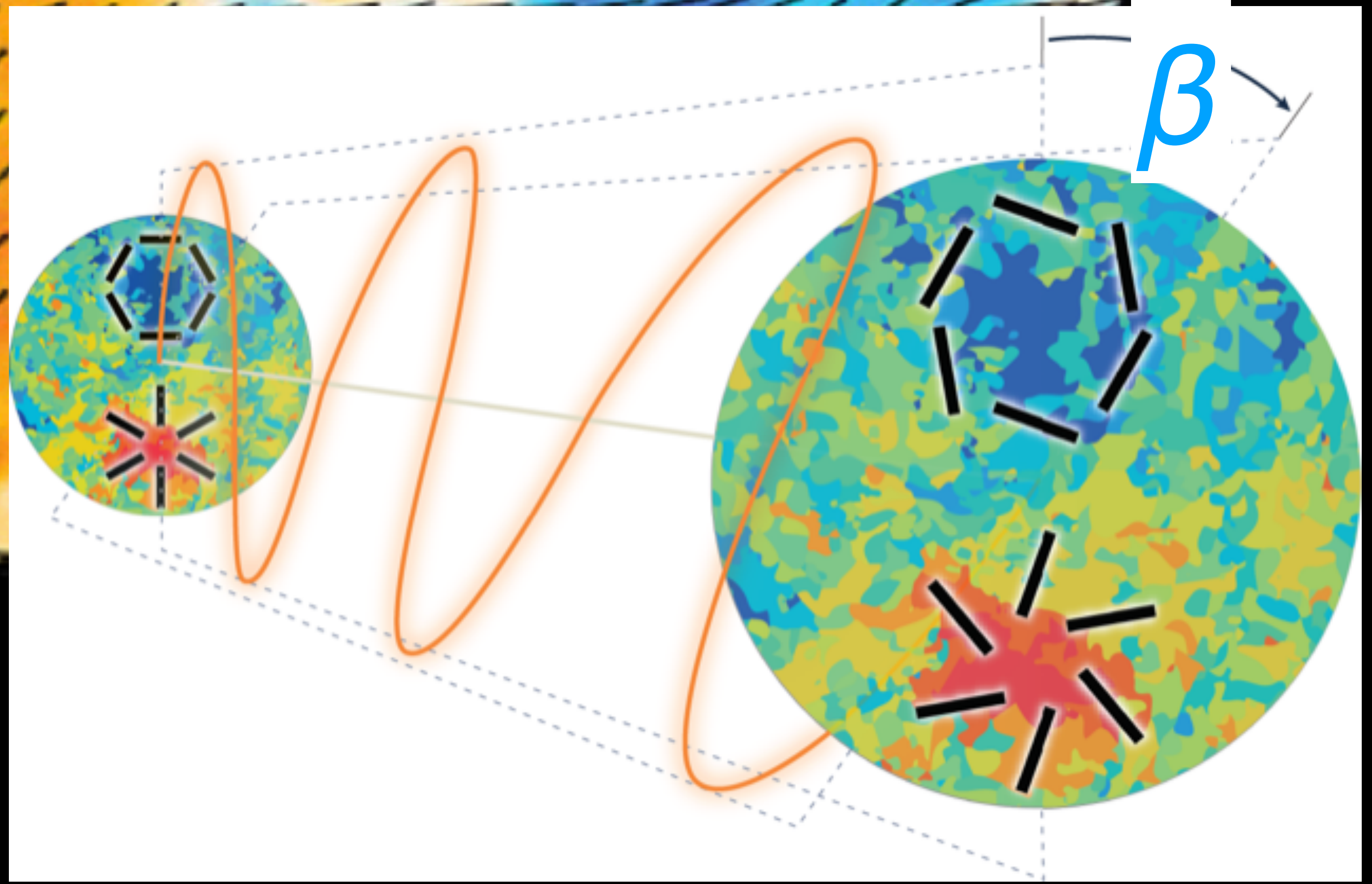
# How does the EM wave of the CMB propagate?

Credit: ESA

## *"Cosmic Birefringence"*

If the plane of linear polarization of the CMB is rotated uniformly by β, it is the sign of parity violation!



Key Point

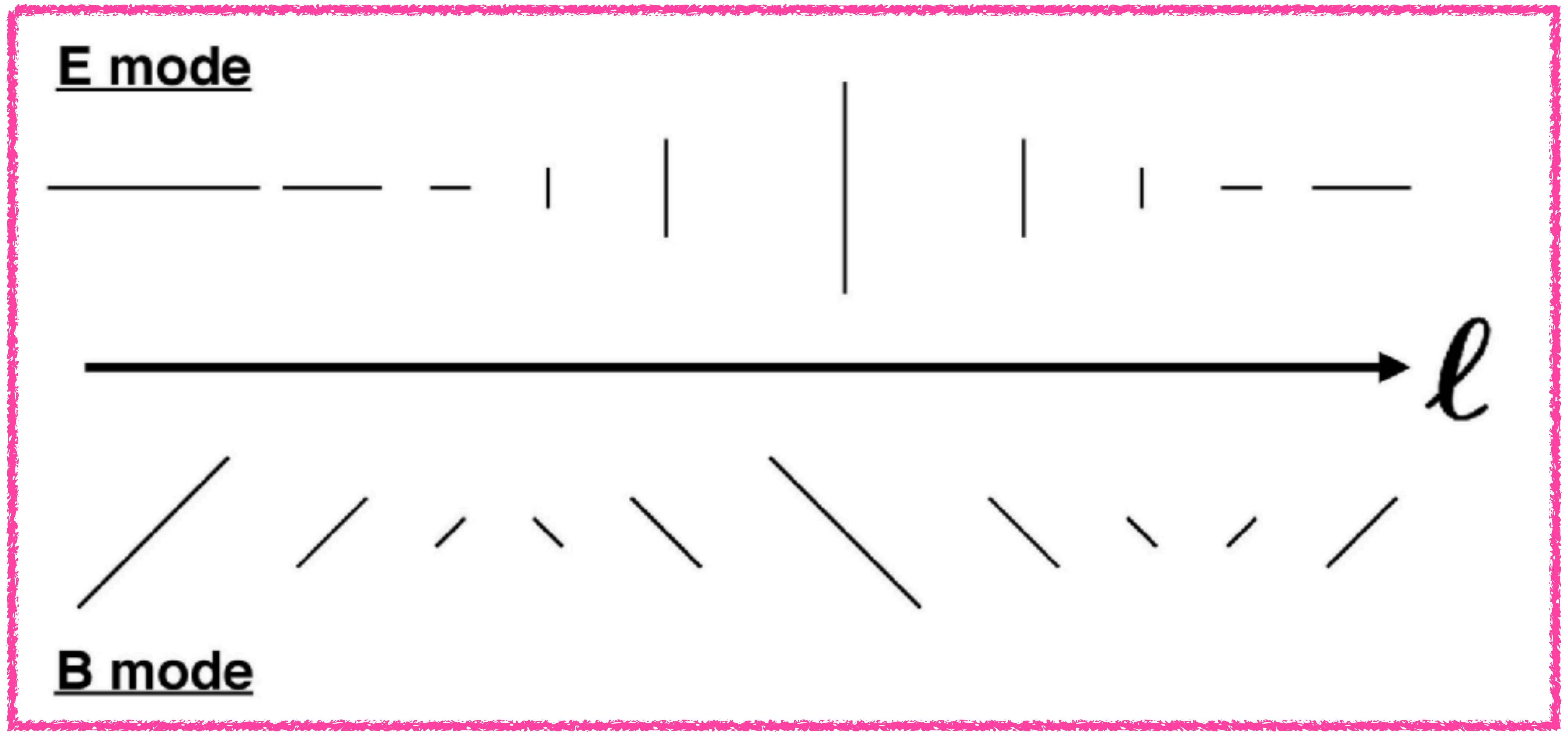
# Pseudoscalar: EB correlation

- The observed pattern of the CMB polarization can be decomposed into eigenstates of parity, called “E modes” and “B modes”.
- E and B modes are transformed differently under the parity transformation. Therefore, the product of the two, **the “EB correlation”, is a pseudoscalar**.
- **The full-sky average of the EB correlation must vanish (to within the measurement uncertainty), if there is no parity violation!**

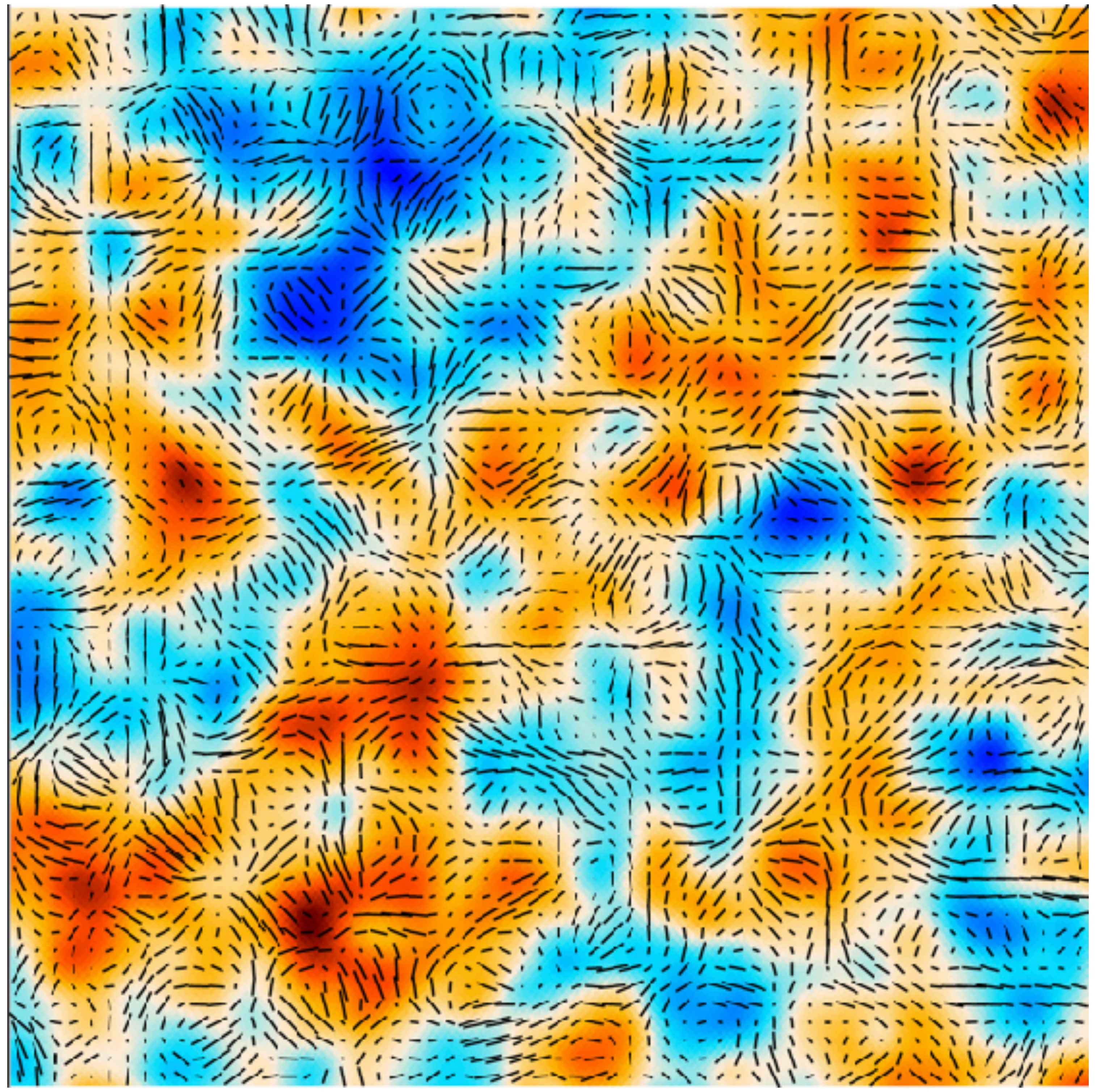
Zaldarriaga, Seljak (1997); Kamionkowski, Kosowsky, Stebbins (1997)

# Parity eigenstates: E and B modes

## Concept defined in Fourier space




This map is dominated by E-mode polarization

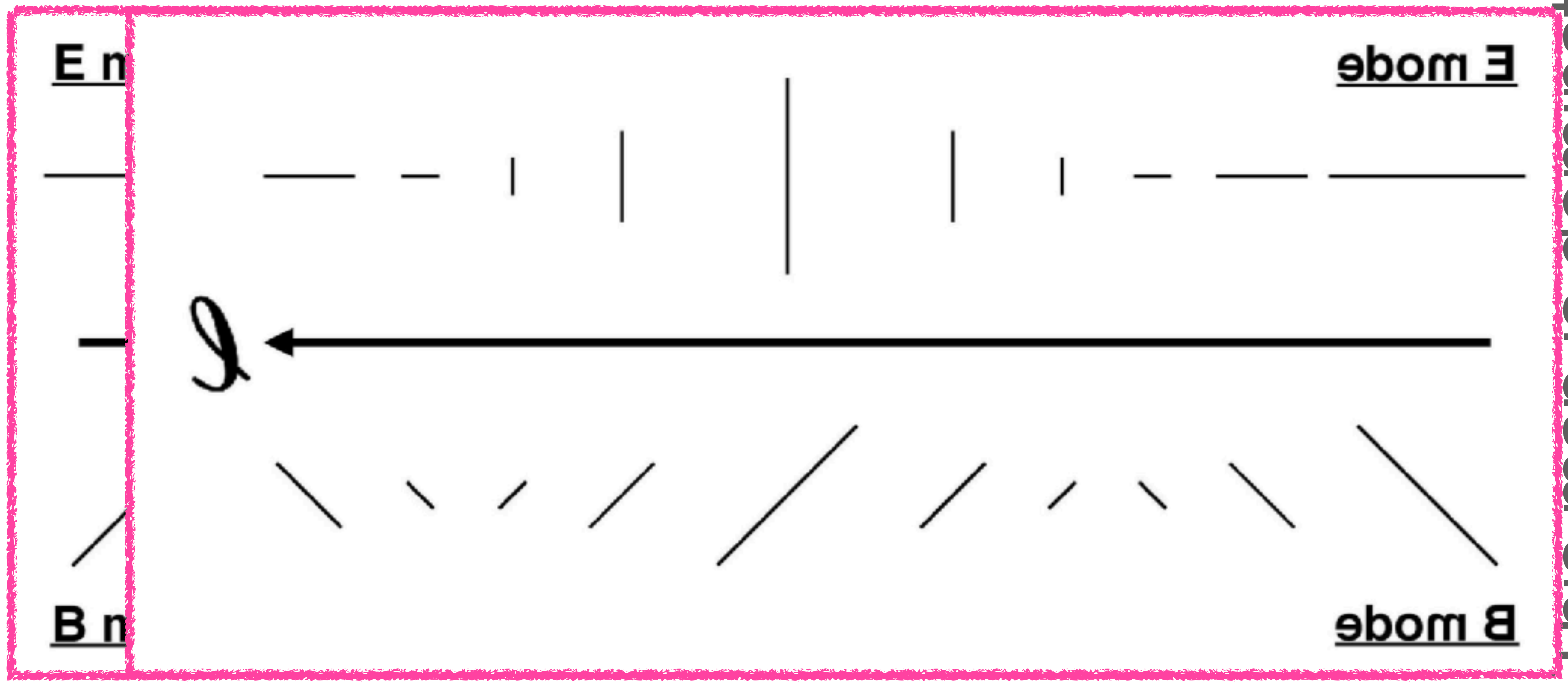


- **E-mode**：Polarization directions are **parallel or perpendicular** to the wavenumber direction
- **B-mode**：Polarization directions are **45 degrees tilted** w.r.t the wavenumber direction

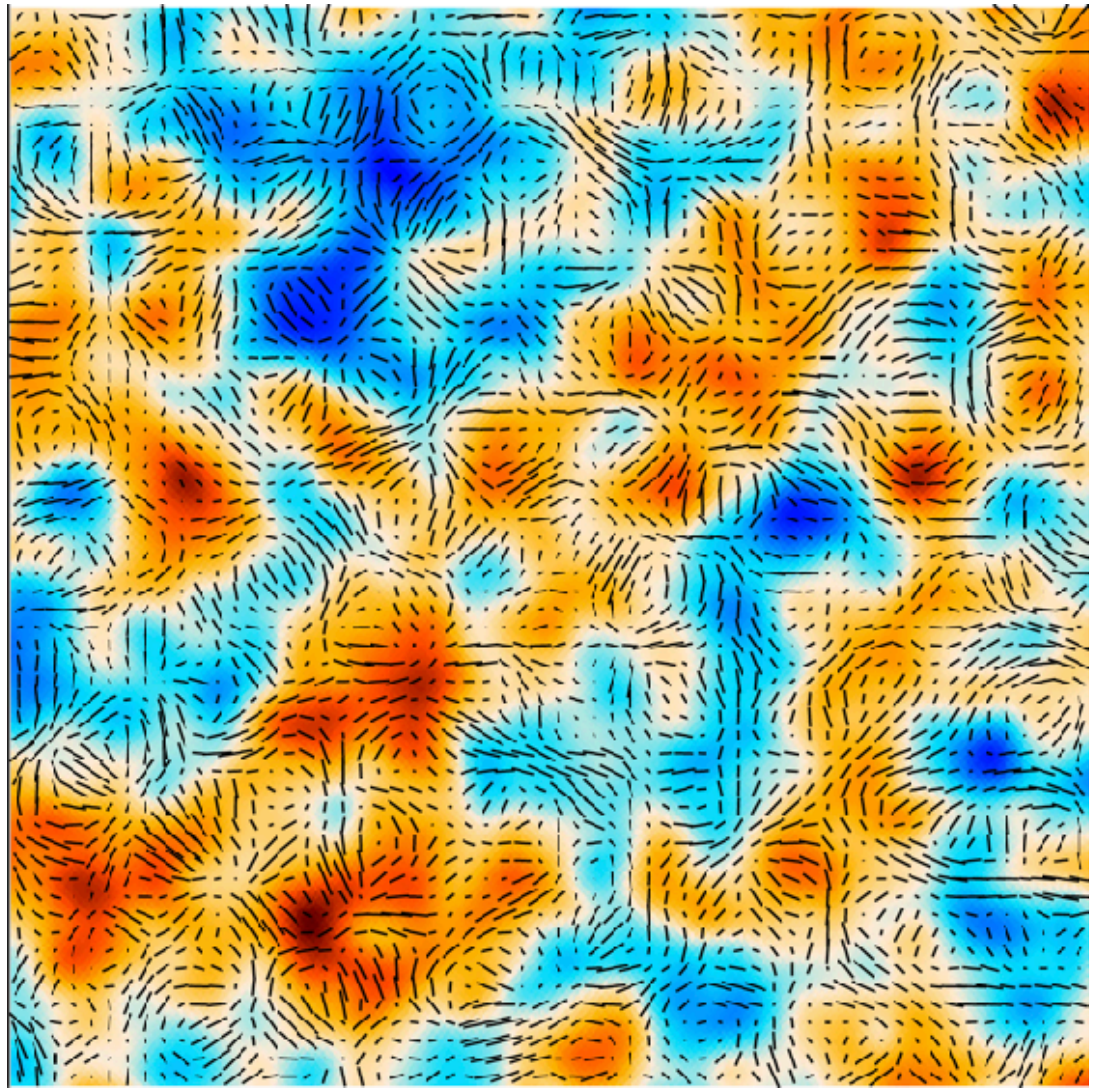
Zaldarriaga, Seljak (1997); Kamionkowski, Kosowsky, Stebbins (1997)

# Parity eigenstates: E and B modes

## Concept defined in Fourier space




This map is dominated by E-mode polarization



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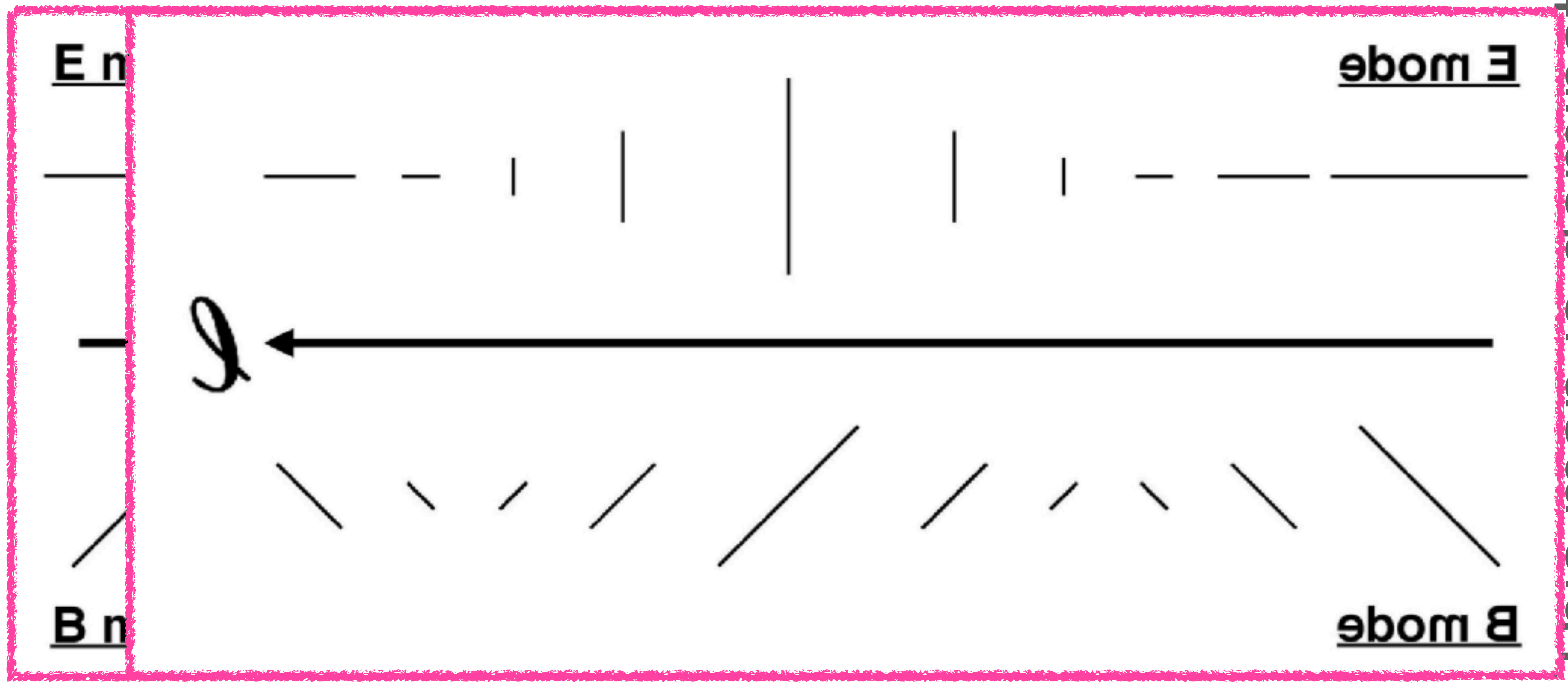
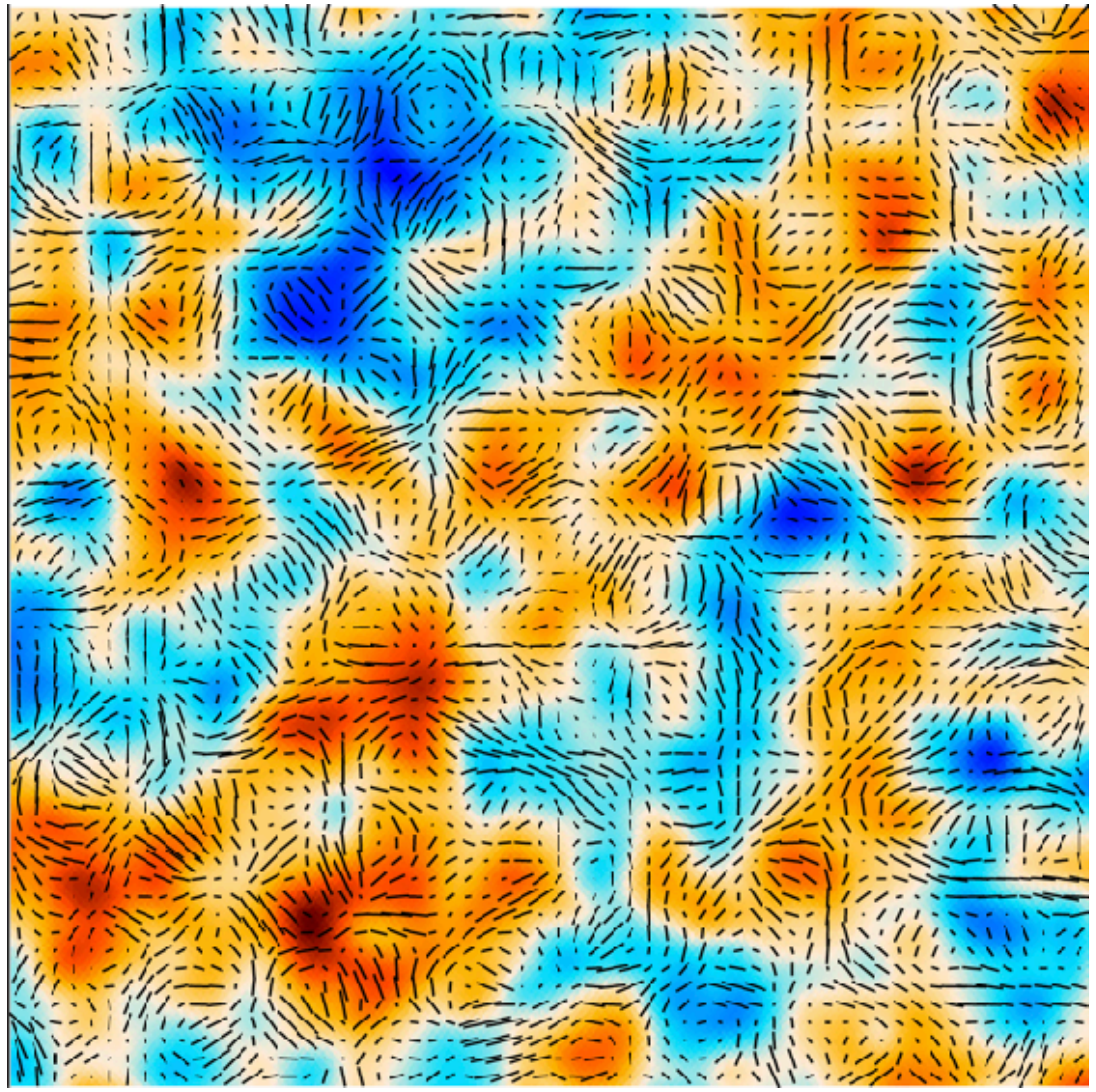
Zaldarriaga, Seljak (1997); Kamionkowski, Kosowsky, Stebbins (1997)

# Parity eigenstates: E and B modes

## Concept defined in Fourier space



This map is dominated by E-mode polarization



$$\langle E_{\boldsymbol{\ell}} E^*_{\boldsymbol{\ell}'} \rangle = (2\pi)^2 \delta_D^{(2)}(\boldsymbol{\ell} - \boldsymbol{\ell}') C_\ell^{EE}$$

$$\langle B_{\boldsymbol{\ell}} B^*_{\boldsymbol{\ell}'} \rangle = (2\pi)^2 \delta_D^{(2)}(\boldsymbol{\ell} - \boldsymbol{\ell}') C_\ell^{BB}$$

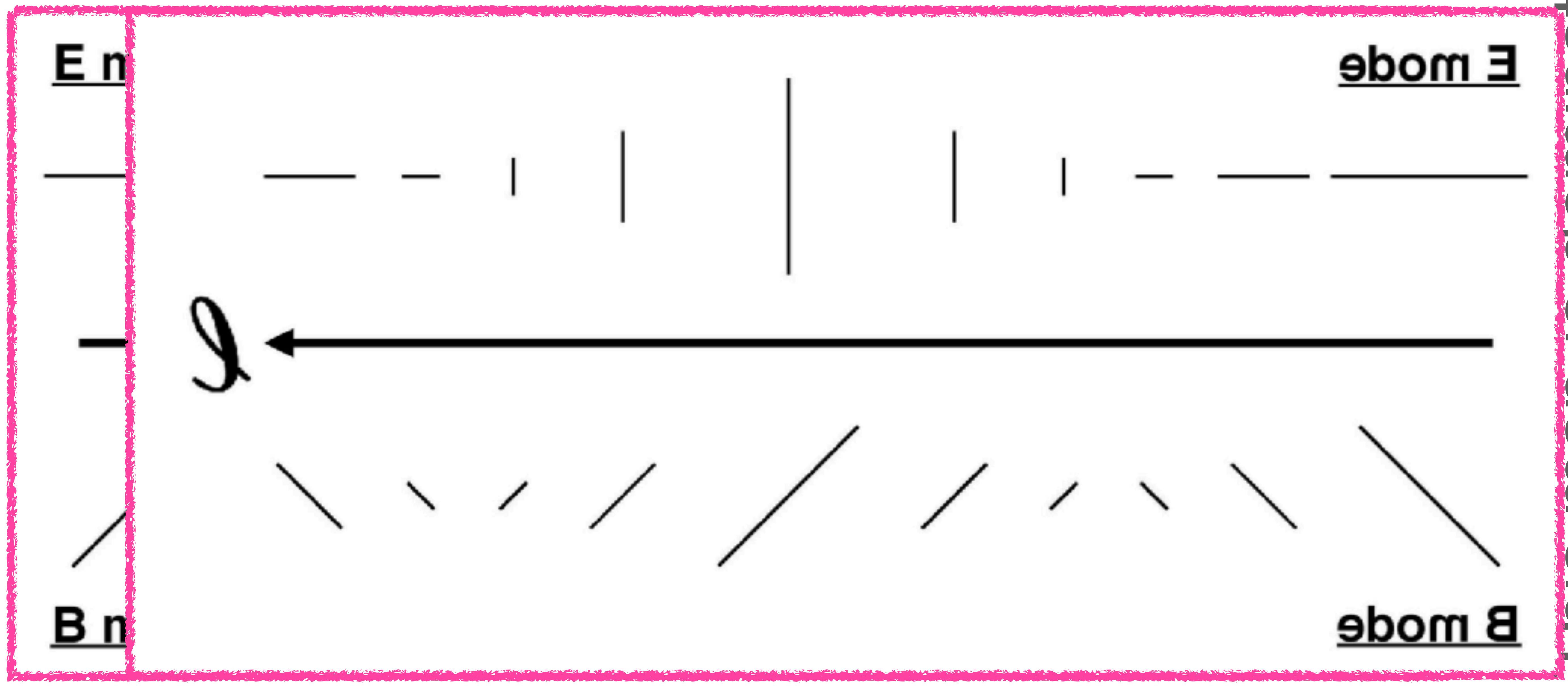
$$\langle T_{\boldsymbol{\ell}} E^*_{\boldsymbol{\ell}'} \rangle = \langle T^*_{\boldsymbol{\ell}} E_{\boldsymbol{\ell}'} \rangle = (2\pi)^2 \delta_D^{(2)}(\boldsymbol{\ell} - \boldsymbol{\ell}') C_\ell^{TE}$$

These are scalars and insensitive to parity violation.

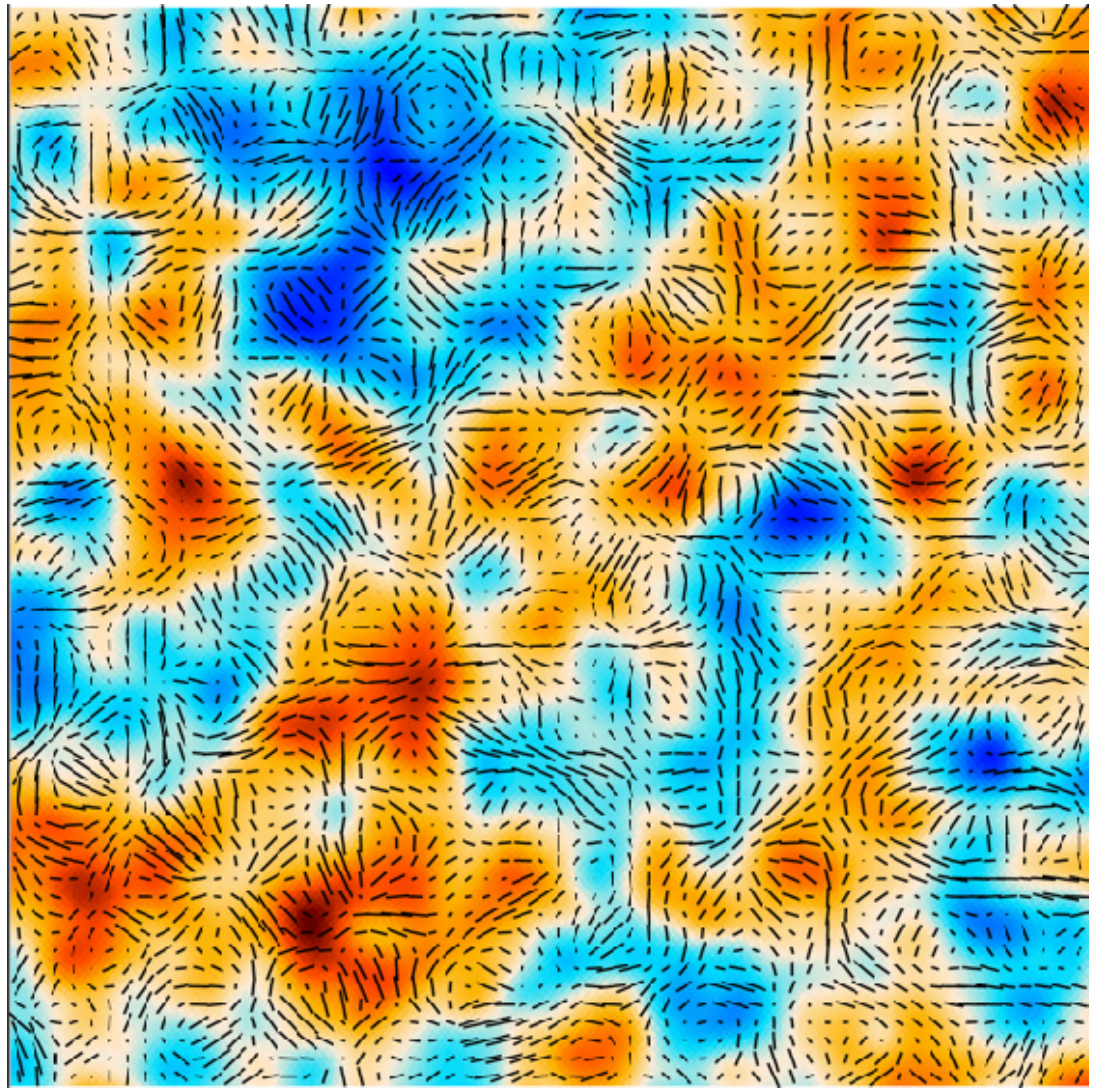
Lue, Wang, Kamionkowski (1999); Feng et al. (2005, 2006)

# Parity eigenstates: E and B modes

## Concept defined in Fourier space




This map is dominated by E-mode polarization



$$\langle E_{\boldsymbol{\ell}} E^*_{\boldsymbol{\ell}'}\rangle = (2\pi)^2\delta_D^{(2)}(\boldsymbol{\ell}-\boldsymbol{\ell}')C^{EE}_{\ell}$$

$$\langle B_{\boldsymbol{\ell}} B^*_{\boldsymbol{\ell}'}\rangle = (2\pi)^2\delta_D^{(2)}(\boldsymbol{\ell} -$$

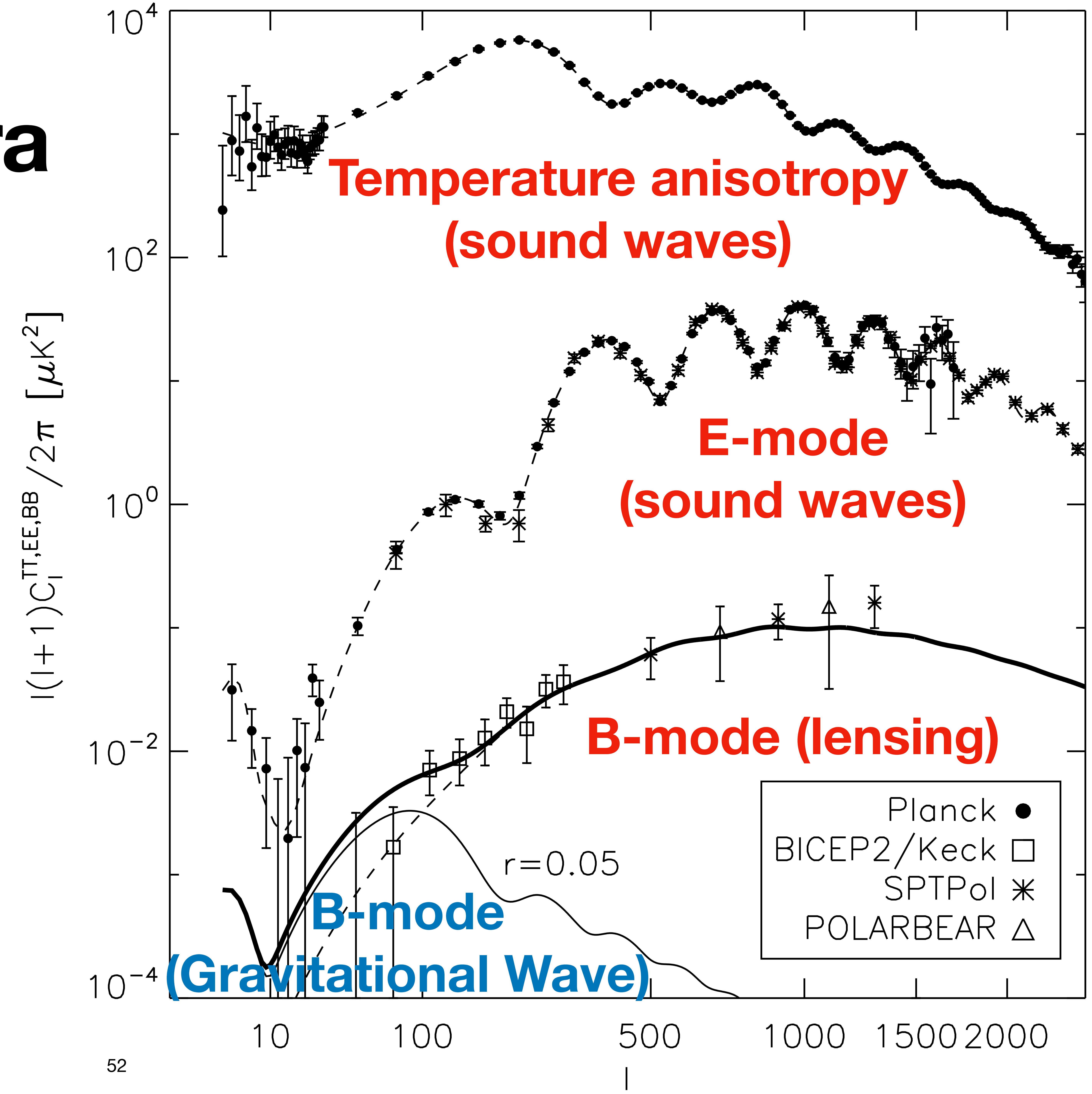
$$\langle T_{\boldsymbol{\ell}} E^*_{\boldsymbol{\ell}'}\rangle = \langle T^*_{\boldsymbol{\ell}} E_{\boldsymbol{\ell}'}\rangle = (2\pi)^2\delta_D^{(2)}(\boldsymbol{\ell}-\boldsymbol{\ell}')C^{TE}_{\ell}$$

The other combinations, <TB> and <EB>, are pseudoscalars and sensitive to parity violation!

# CMB Power Spectra

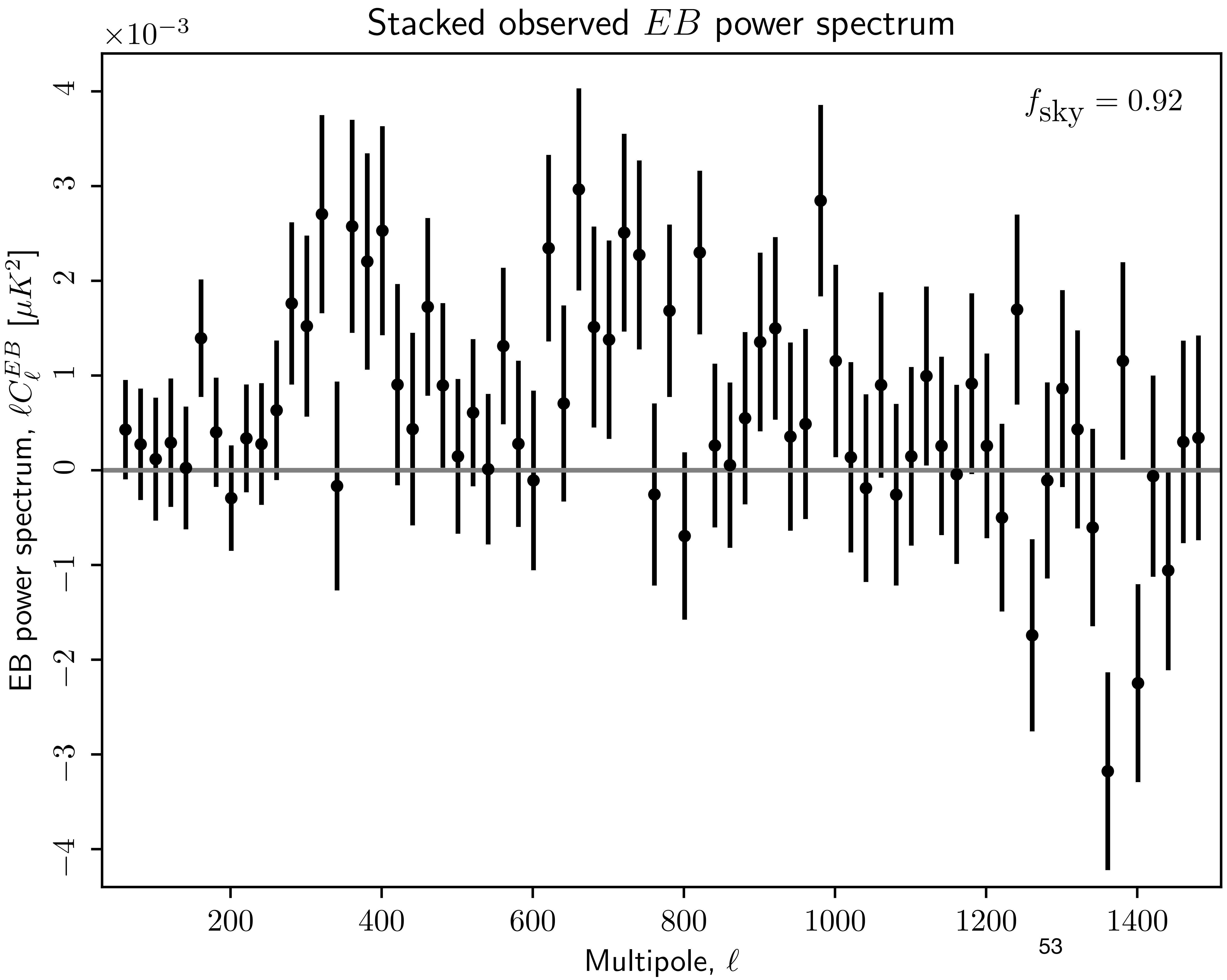
## Progress over 30 years

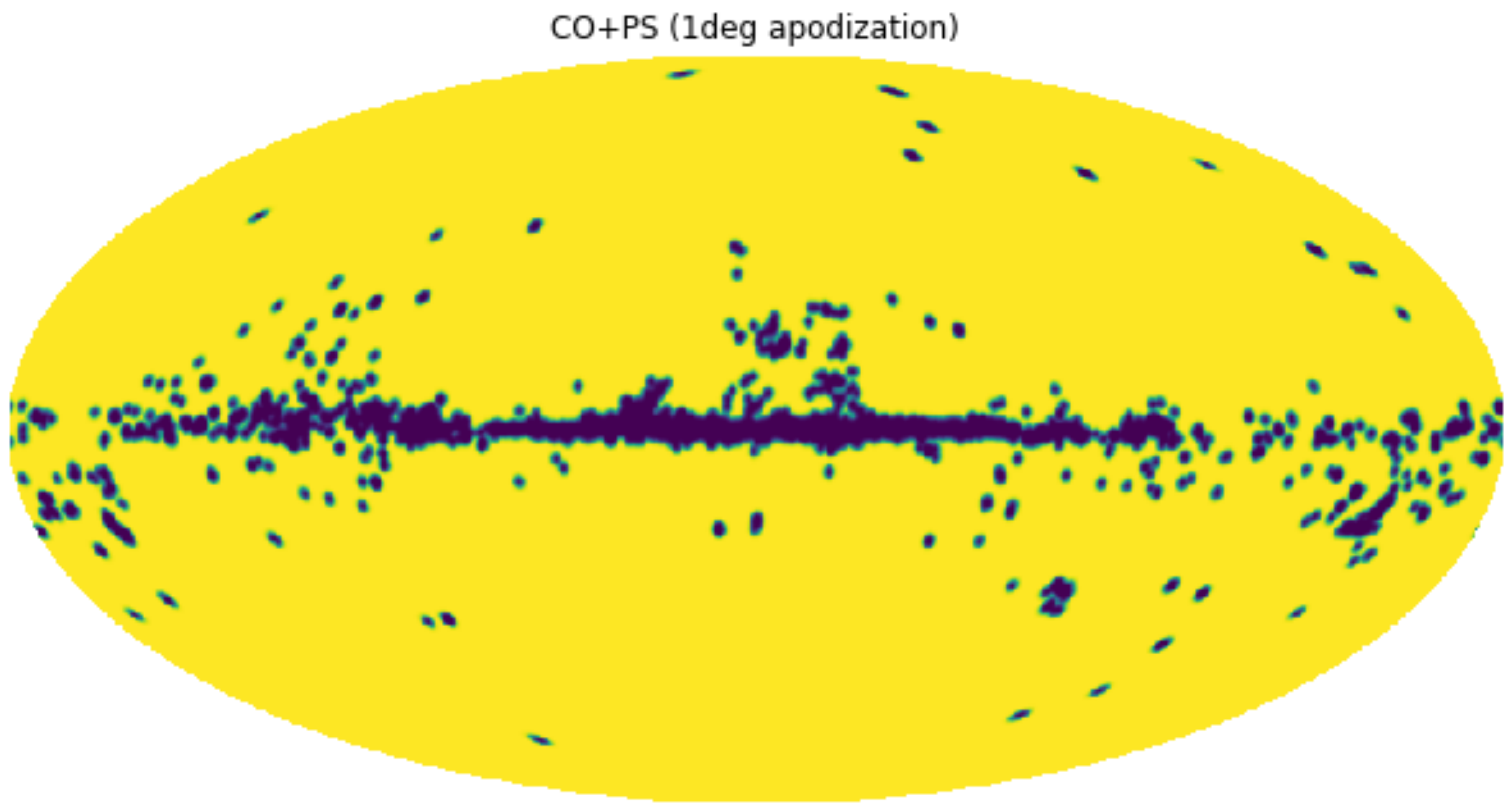
- This is the typical figure seen in talks and lectures on the CMB.
  - The temperature and the E- and B-mode polarization power spectra are well measured.
- **Parity violation appears in the TB and EB power spectra, not shown here.**



# This is the EB power spectrum (WMAP+Planck)

## Nearly full-sky data (92% of the sky)



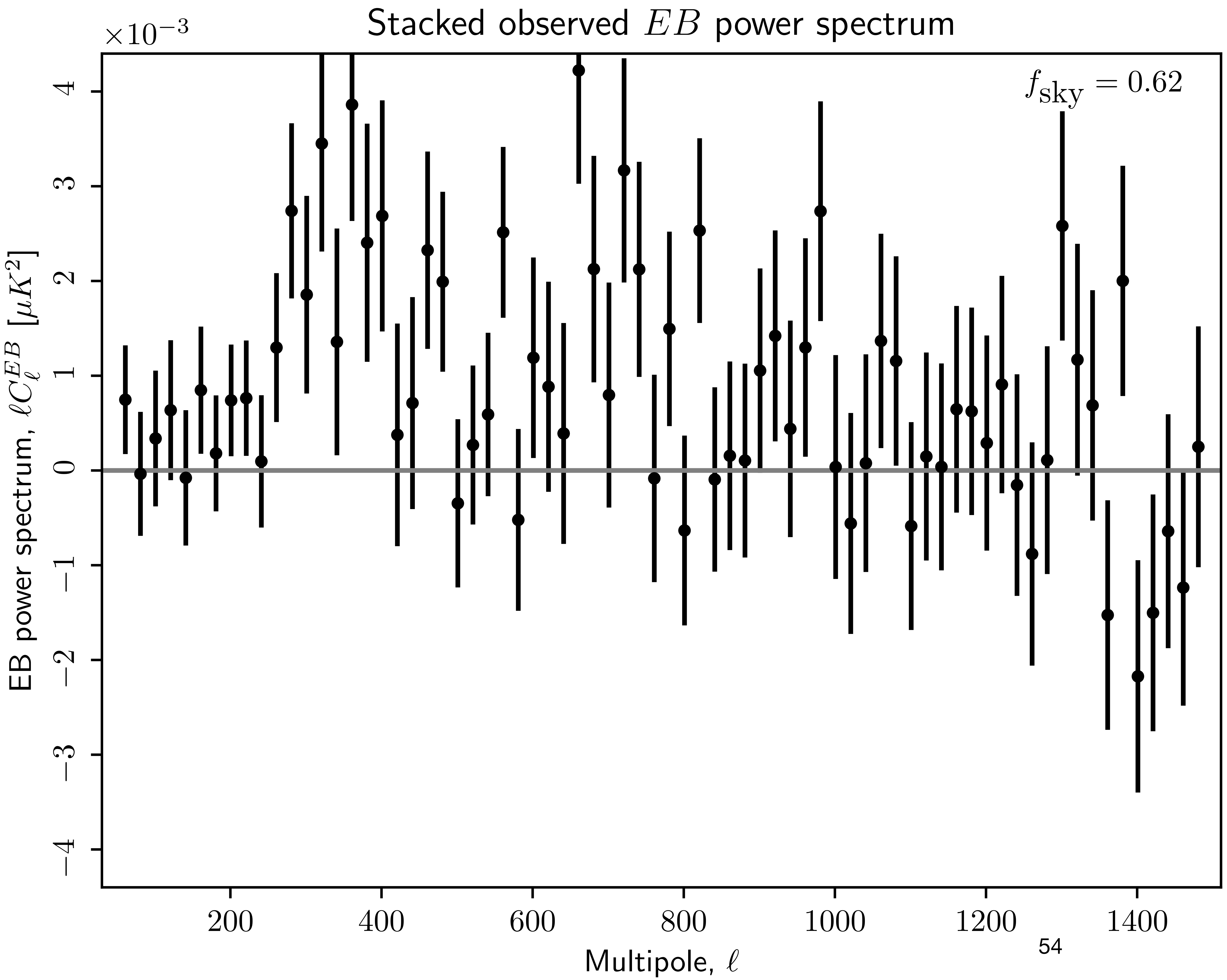



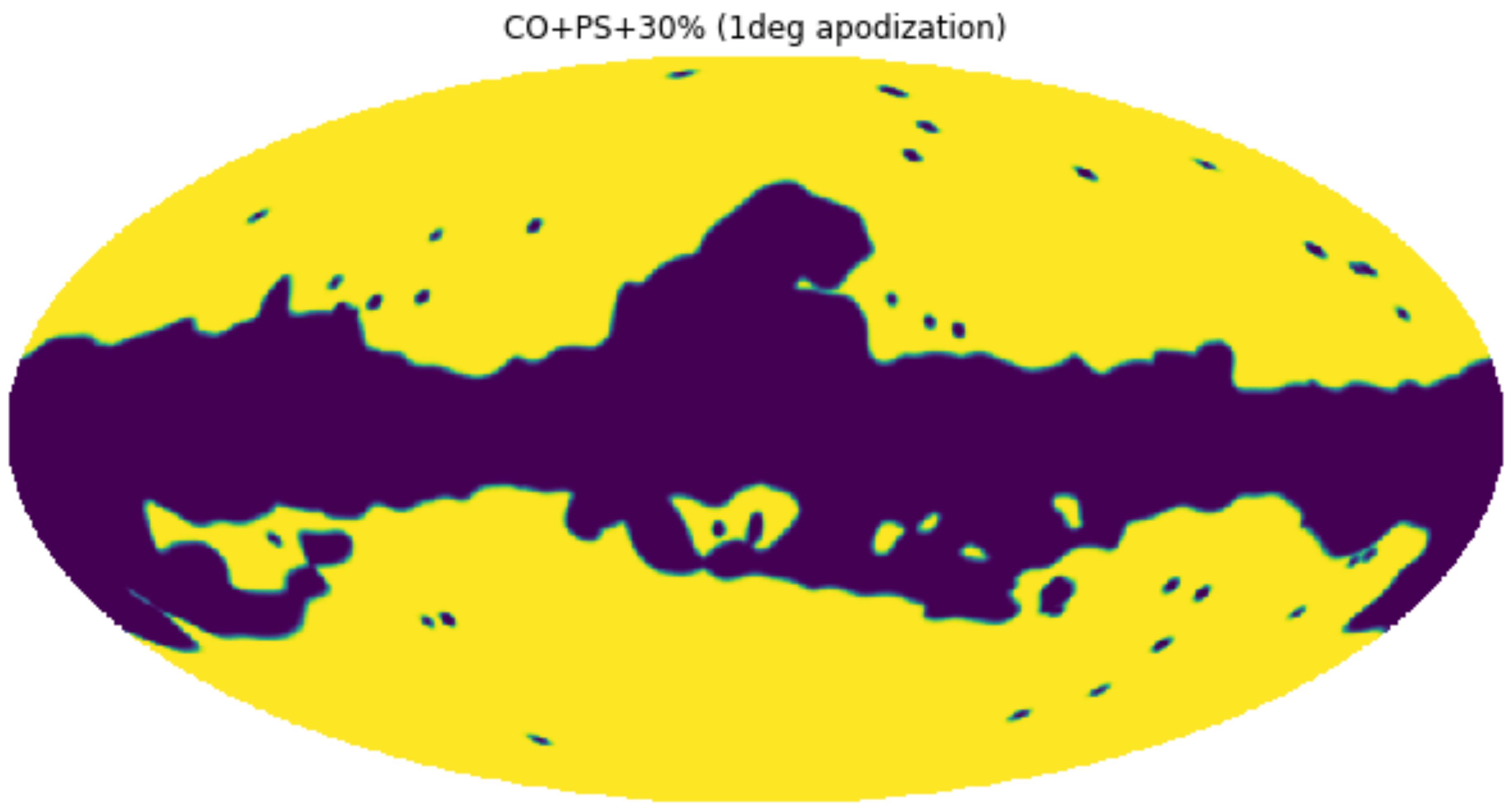

- $\chi^2 = 125.5$ for DOF=72
  - Unambiguous signal of something!

Eskilt, EK (2022)

# This is the EB power spectrum (WMAP+Planck)

## Galactic plane removed (62% of the sky)



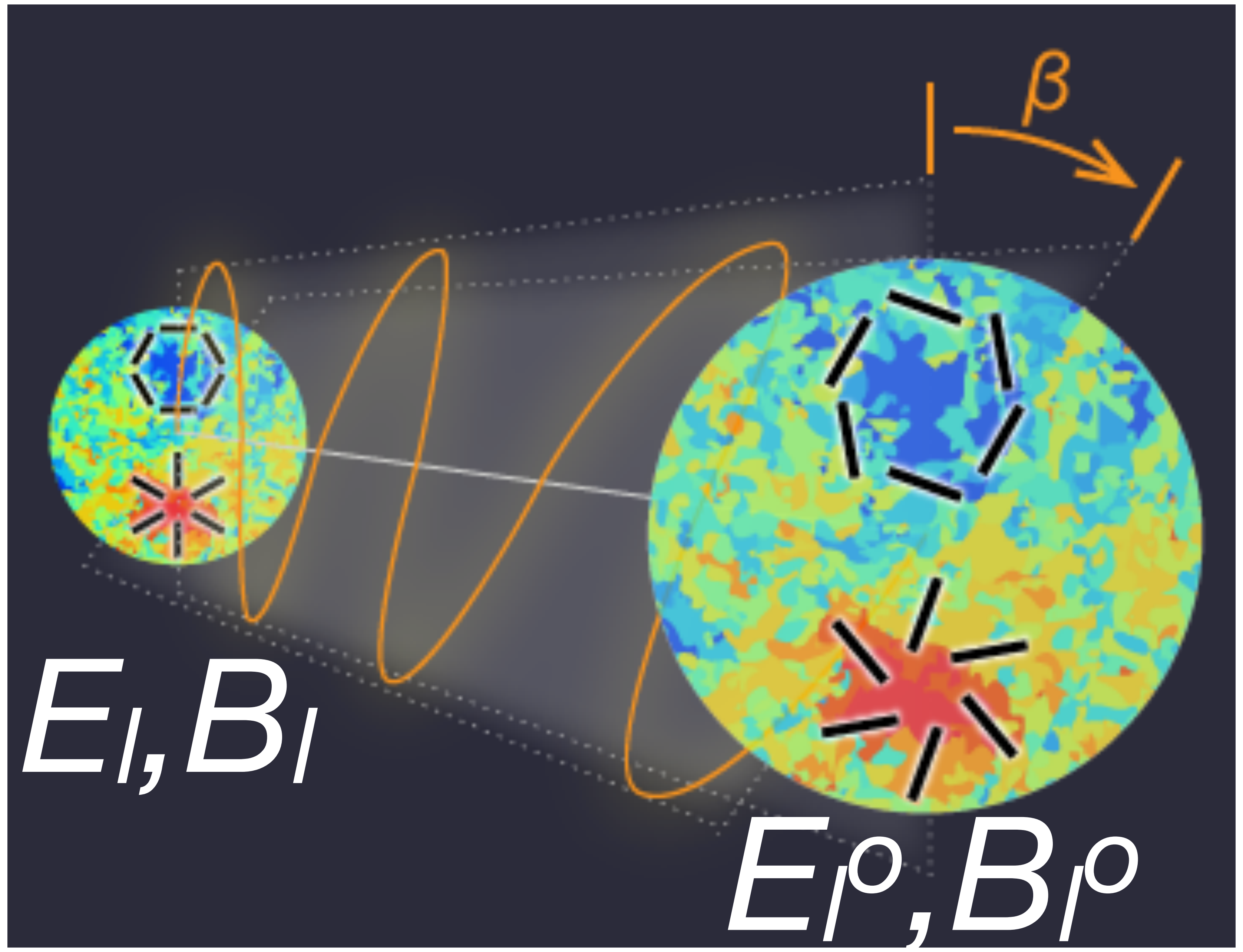

- $\chi^2 = 138.4$
  - The signal exists regardless of the Galactic mask. This rules out the Galactic foreground.

Lue, Wang, Kamionkowski (1999); Feng et al. (2005, 2006)

# E-B mixing by rotation of the plane of linear polarization

- Observed E- and B-mode polarization, $E_l^o$ and $B_l^o$, are related to those before rotation as

$$E_\ell^{\rm o} \pm iB_\ell^{\rm o} = (E_\ell \pm iB_\ell)e^{\pm 2i\beta}$$




- which gives

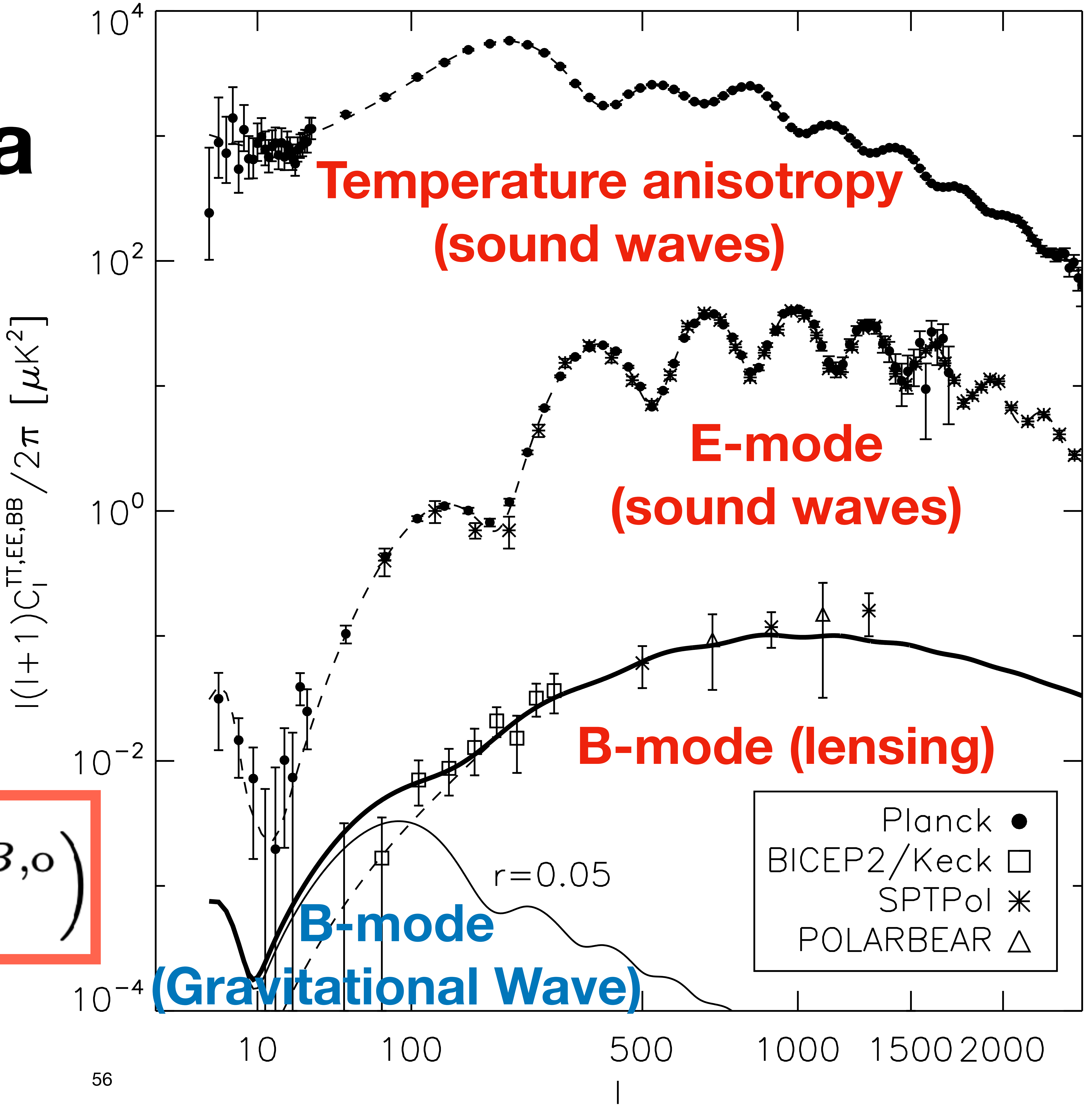
$$E_\ell^{\rm o} = E_\ell \cos(2\beta) - B_\ell \sin(2\beta)$$

$$B_\ell^{\rm o} = E_\ell \sin(2\beta) + B_\ell \cos(2\beta)$$

# CMB Power Spectra

- Rotation of the plane of linear polarization **mixes** E and B modes.
- Therefore, the EB correlation will be given by the difference between the EE and BB correlations.
- Observed EE is much greater than BB. We expect EB to look like EE!

$$C_\ell^{EB,\mathrm{o}} = \frac{\tan(4\beta)}{2}\left(C_\ell^{EE,\mathrm{o}} - C_\ell^{BB,\mathrm{o}}\right)$$

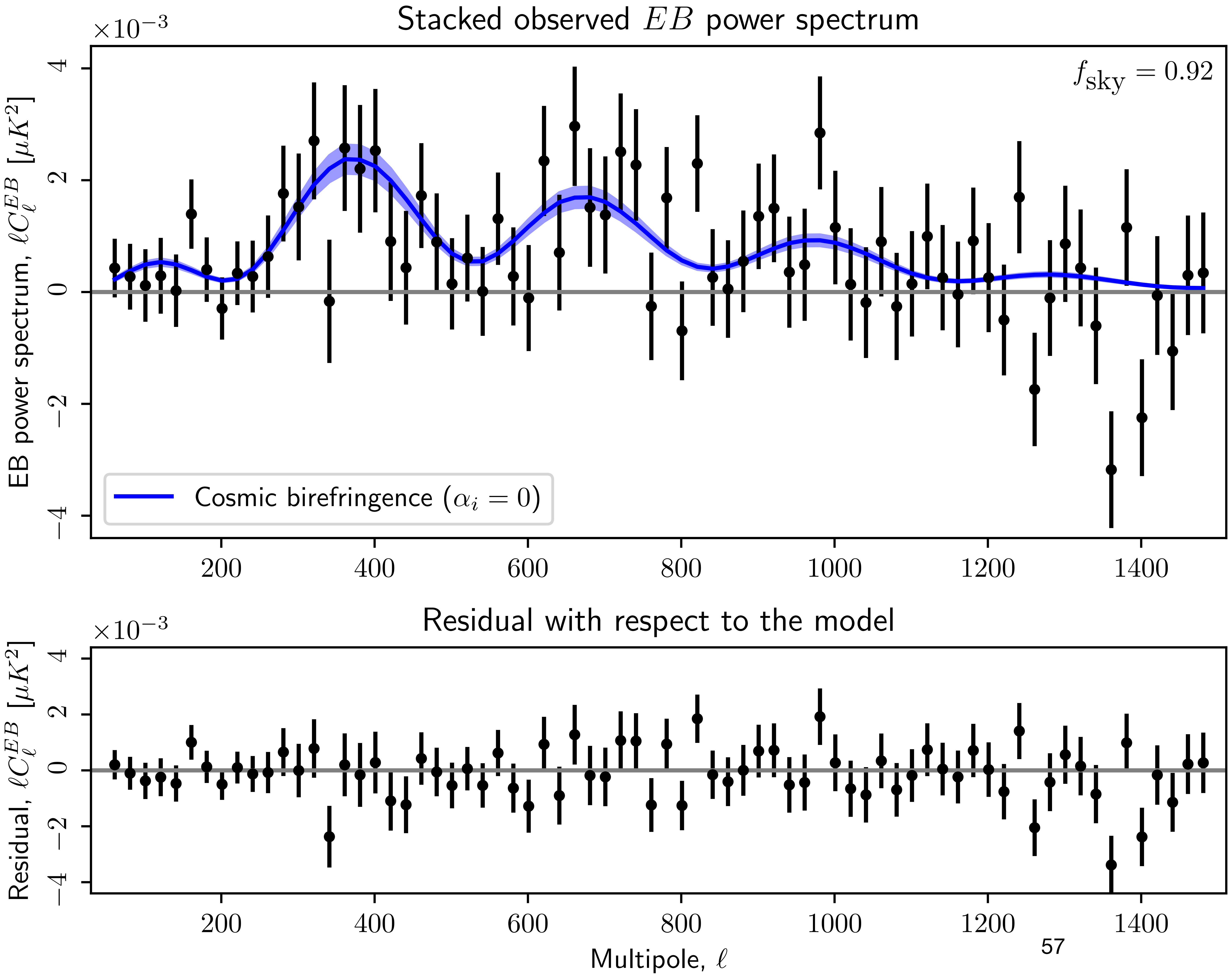


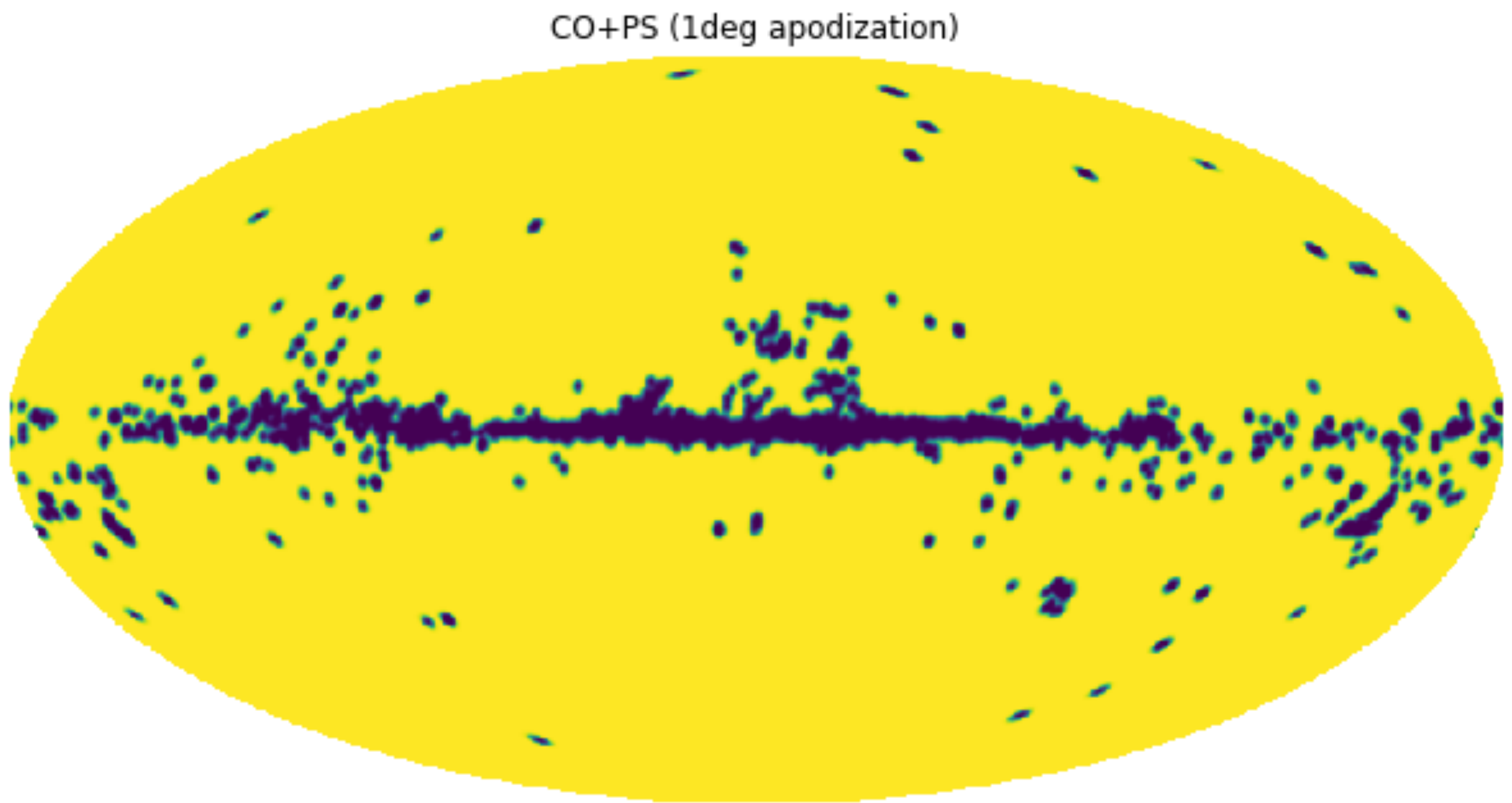
Eskilt, EK (2022)

# Cosmic Birefringence fits well(?)

$$C_\ell^{EB,o} = \frac{\tan(4\beta)}{2}\left(C_\ell^{EE,o} - C_\ell^{BB,o}\right)$$

## Nearly full-sky data (92% of the sky)



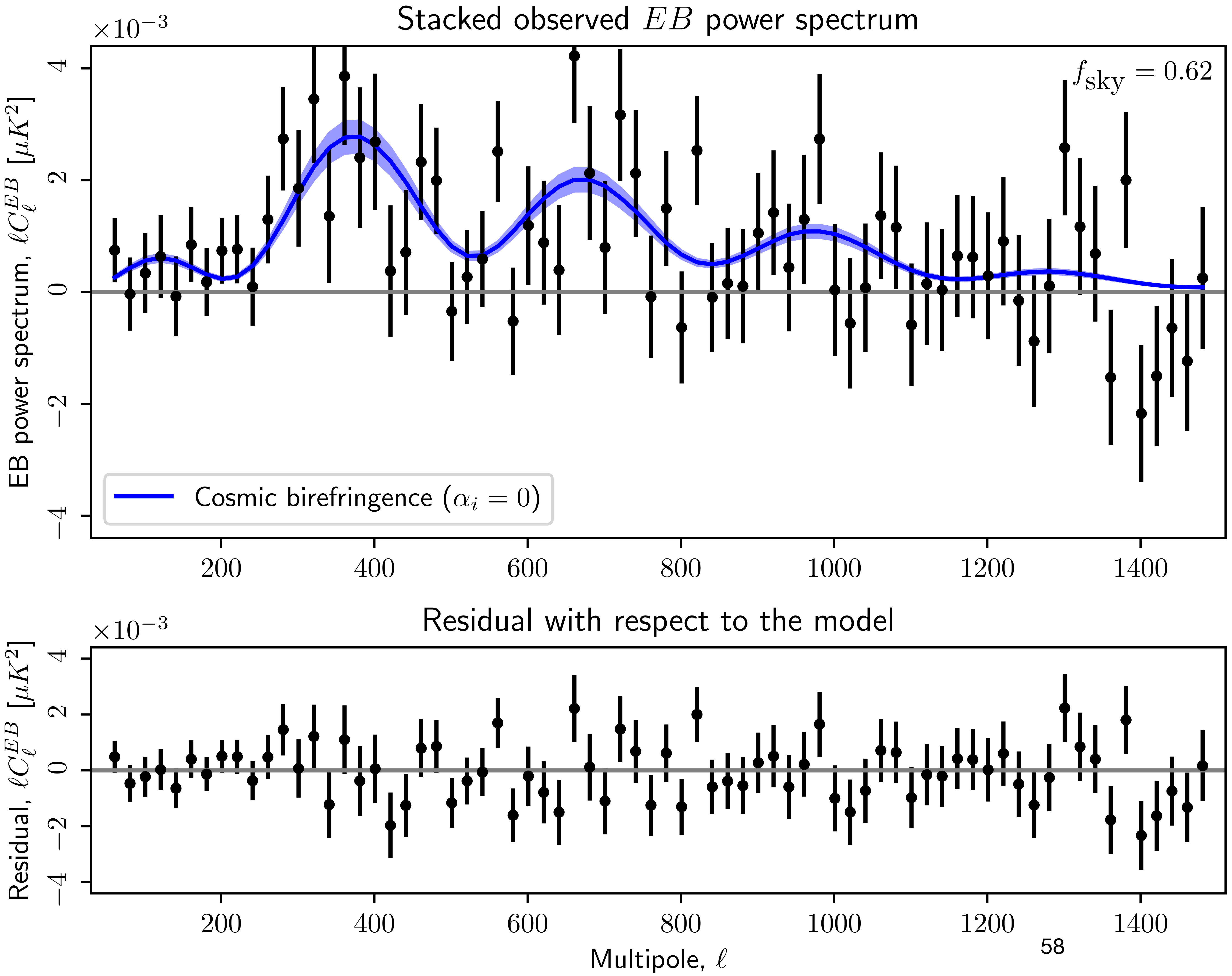

- $\beta$ = 0.288 ± 0.032 deg
- $\chi^2$ = 66.1 for DOF=71
  - Good fit! 9σ detection?

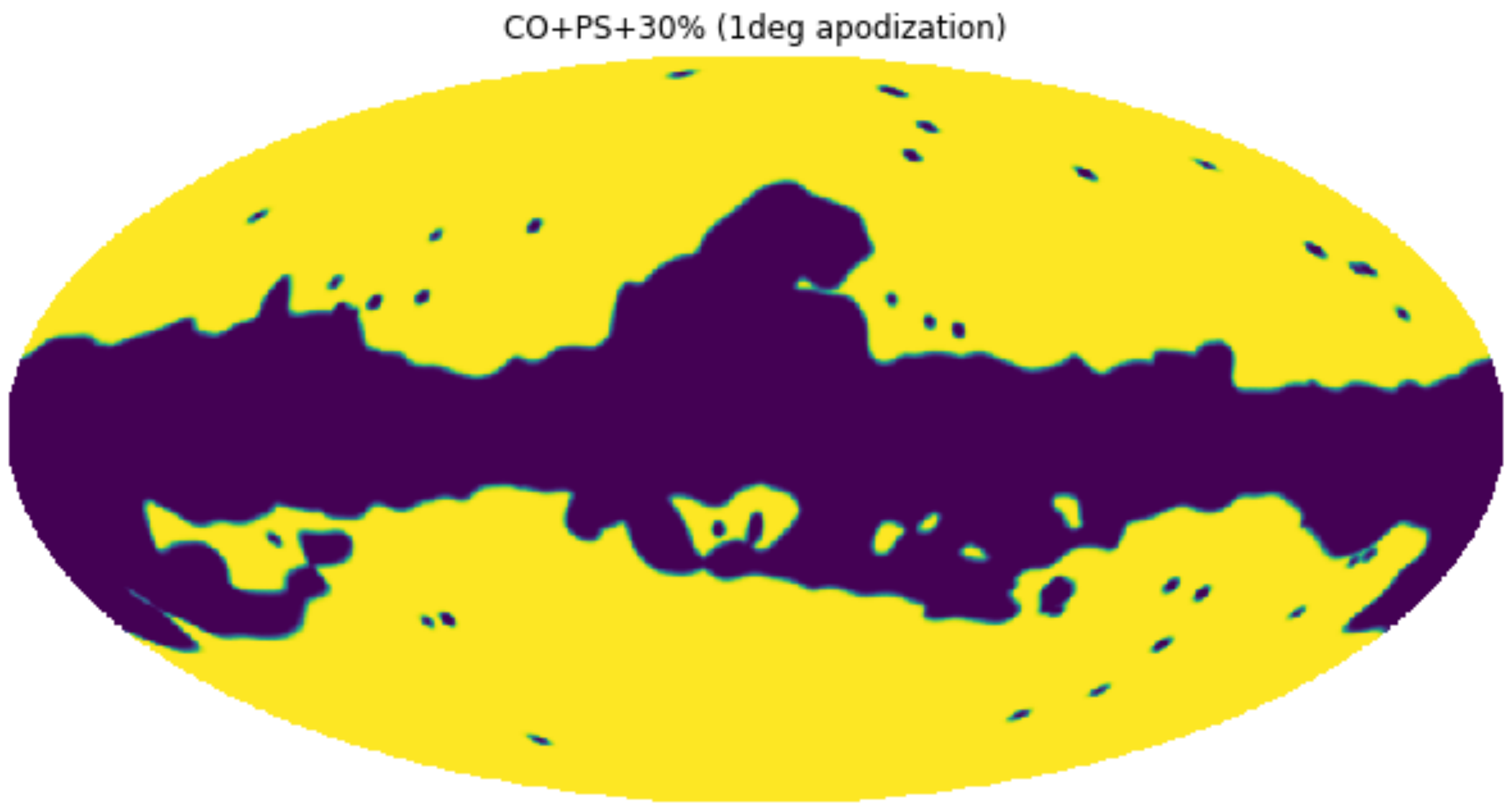
Eskilt, EK (2022)

# Cosmic Birefringence fits well(?)

$$C_\ell^{EB,o} = \frac{\tan(4\beta)}{2}\left(C_\ell^{EE,o} - C_\ell^{BB,o}\right)$$

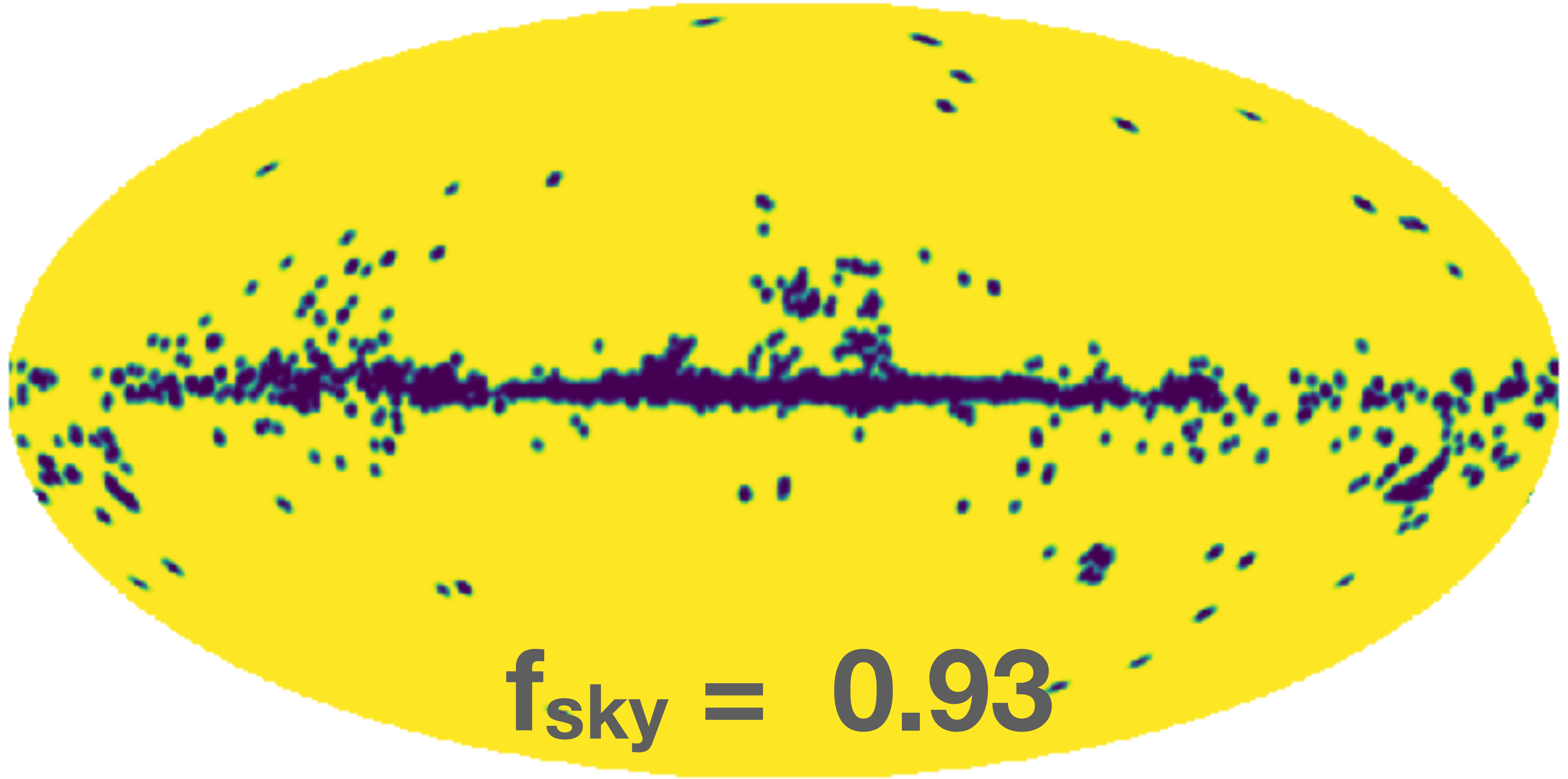
## Galactic plane removed (62% of the sky)




- β = 0.330 ± 0.035 deg
- $\chi^2$ = 64.5
- Signal is robust with respect to the Galactic mask.

CO+PS (1deg apodization)

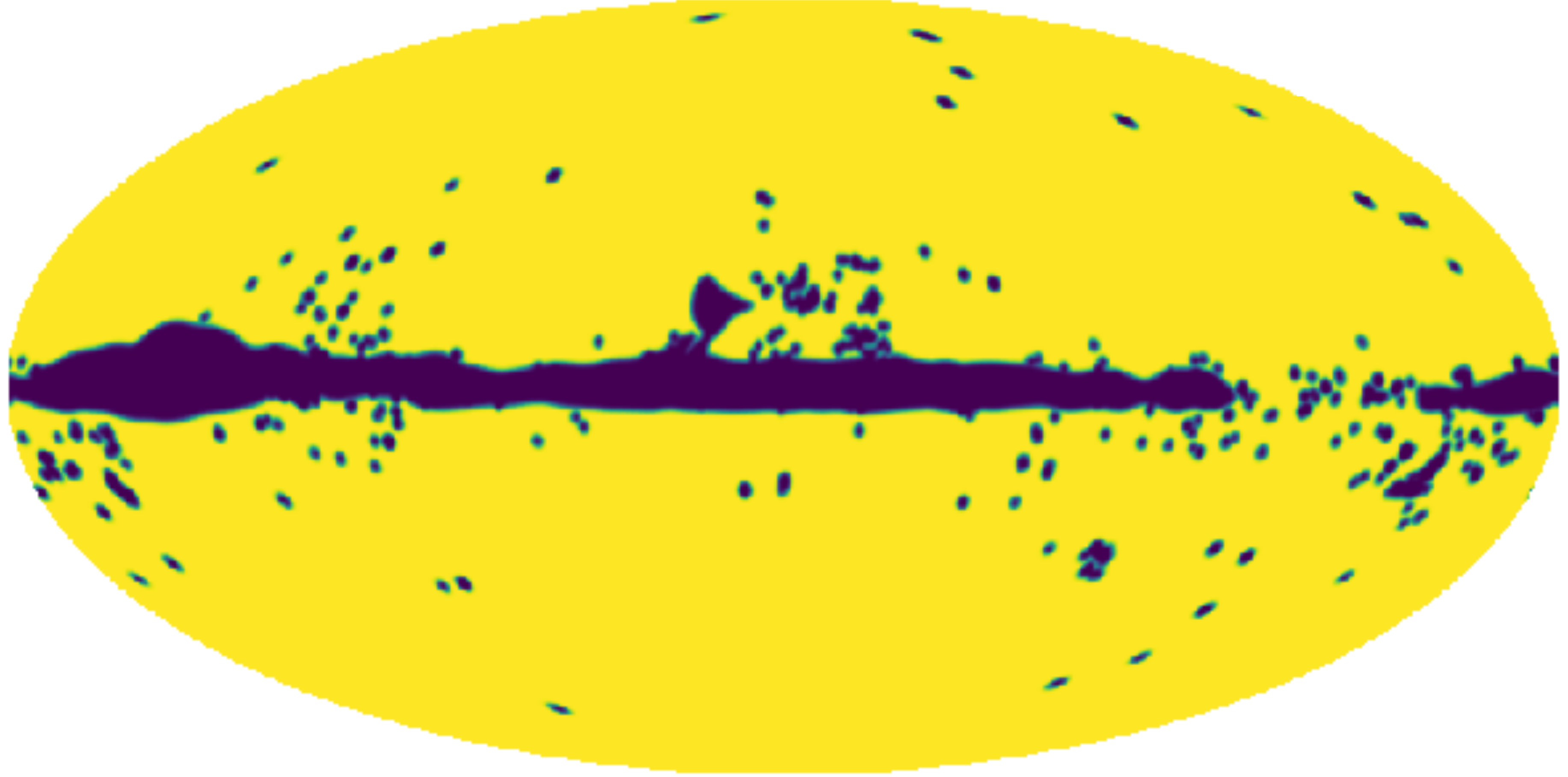


$f_{sky}$ = **0.93**

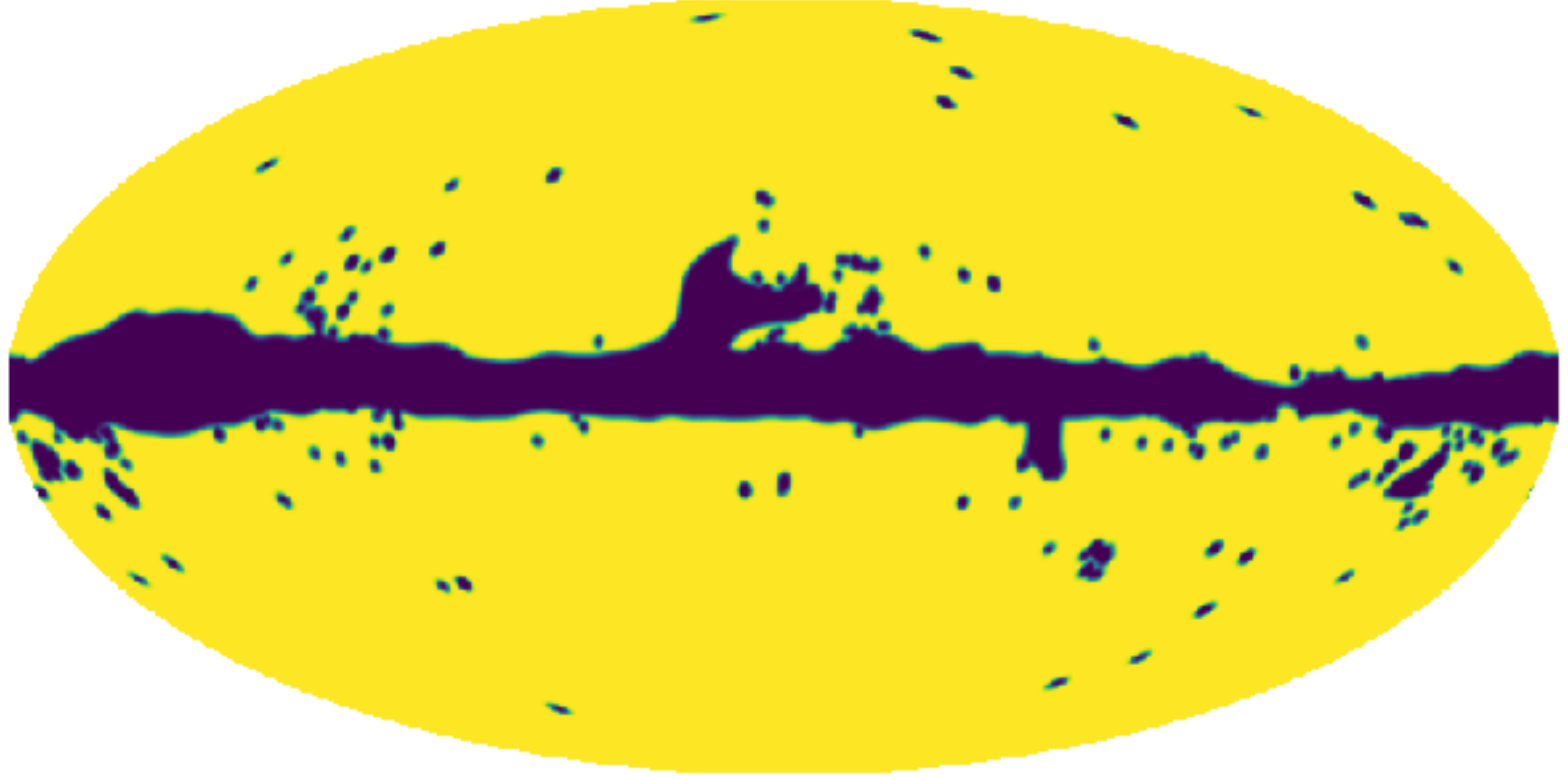
**= nearly full sky**

$f_{sky}$ = 0.90

CO+PS+5% (1deg apodization)



CO+PS+10% (1deg apodization)



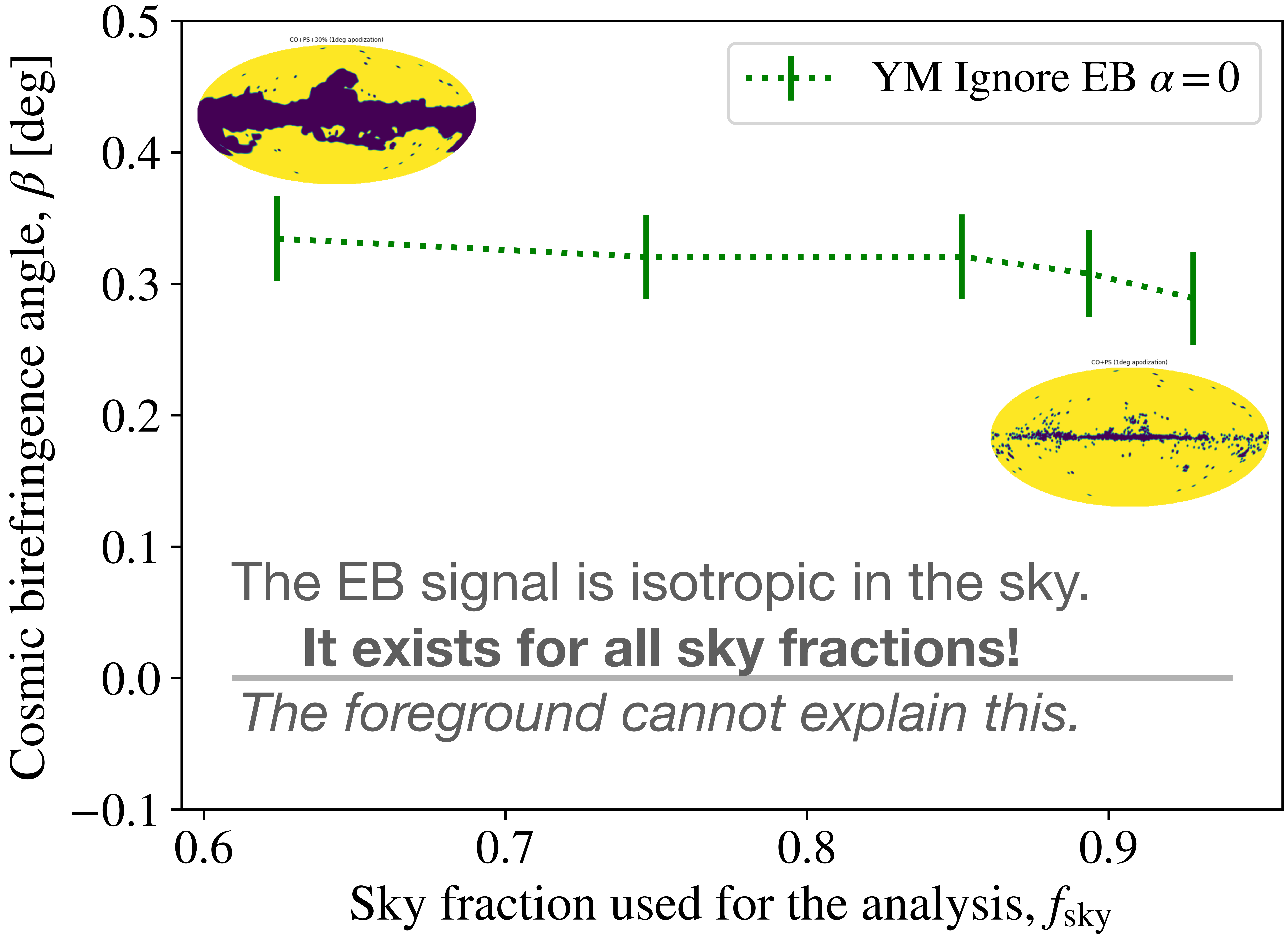
$f_{sky}$ = 0.85

CO+PS+20% (1deg apodization)

$f_{sky}$ = 0.75

CO+PS+30% (1deg apodization)

$f_{sky}$ = 0.63



CO+PS+30% (1deg apodization)
YM Ignore EB $\alpha = 0$
CO+PS (1deg apodization)
The EB signal is isotropic in the sky.
It exists for all sky fractions!
The foreground cannot explain this.
Cosmic birefringence angle, $\beta$ [deg]
0.5
0.4
0.3
0.2
0.1
0.0
−0.1
0.6
0.7
0.8
0.9
Sky fraction used for the analysis, $f_{\rm sky}$

# What is the origin of this signal? See Patricia's talk.

# Parity Violation during Cosmic Inflation

$$I_{\rm CS} = \int d^4x \sqrt{-g} \left( -\frac{\alpha}{4f} \chi F \widetilde{F} \right) \rightarrow$$

**Scalar fluctuations**

$$\Box\chi - \frac{\partial V}{\partial \chi} = -\frac{\alpha}{f} \mathbf{E} \cdot \mathbf{B}$$

**Gravitational waves**

$$\Box h_{ij} = 16\pi G (E_i E_j + B_i B_j)^{\rm TT}$$

Dark Energy
Accelerated Expansion
Afterglow Light
Pattern
375,000 yrs.
Dark Ages
Development of
Galaxies, Planets, etc.
Inflation
WMAP
Quantum
Fluctuations
1st Stars
about 400 million yrs.
Big Bang Expansion
13.77 billion years
NASA/WMAP Science Team

# *Cosmic Inflation*: Key Features

## More than 40 years of research in a single slide

- Inflation is the period of **accelerated** expansion in the very early Universe.
  - If the distance between two points increases as $a(t)$, **$d^2a/dt^2 > 0$**.

This is the definition of inflation.

- *Primordial fluctuations* are generated **quantum mechanically**.
  - Scalar modes: Density fluctuations –> The origin of all cosmic structure.
  - Tensor modes: Gravitational waves –> Yet to be discovered.
  - Vector modes: ?
- **A New Paradigm**: Sourced contributions (this talk)

Anber, Sorbo (2010); Barnaby, Peloso (2011); Sorbo (2011); Barnaby, Namba, Peloso (2011)

# The full action

## Observational consequences

Similar phenomenology for non-Abelian gauge fields (Maleknejad et al.)

$$F\tilde{F} = F^a_{\mu\nu}F^{\mu\nu a}$$

$$I = I_{\text{inflation}}$$ [no one understands this]

$$+ \int d\tau d^3\mathbf{x}\sqrt{-g}\left[\frac{R}{16\pi G}\right.$$

→ **Gravitational waves**

$$\Box h_{ij} = 16\pi G(E_iE_j + B_iB_j)^{\mathrm{TT}}$$

$$-\frac{1}{2}(\partial\chi)^2 - V(\chi)$$

→ **Scalar fluctuations**

$$\Box\chi - \frac{\partial V}{\partial\chi} = -\frac{\alpha}{f}\mathbf{E}\cdot\mathbf{B}$$

$$\left.-\frac{1}{4}F^2 - \frac{\alpha}{4f}\chi F\tilde{F}\right]$$

→ **Parity violation in $A_\mu$**

$$A''_\pm + \omega^2_\pm A_\pm = 0, \quad \omega^2_\pm = k^2 \mp \frac{k\alpha\chi'}{f}$$

# A note on terminology

## "Photons" = Massless spin-1 particles

- Since inflation occurred long before the electroweak symmetry breaking, "photons" as we know them did not exist during inflation.
- We should think of them more generally as "**massless spin-1 particles**".

**Gravitational waves**

$$\Box h_{ij} = 16\pi G(\underline{E_i E_j + B_i B_j})^{\mathrm{TT}}$$

**Scalar fluctuations**

$$\Box\chi - \frac{\partial V}{\partial\chi} = -\frac{\alpha}{f}\underline{\mathbf{E}\cdot\mathbf{B}}$$

Spin-1 sources, which violate parity symmetry due to the Chern-Simons term.

→ **Non-Gaussian and parity-violating gravitational waves and scalar fluctuations!**

Anber, Sorbo (2010)

# Particle production due to $\chi F\tilde{F}$ during inflation

## Kinetic energy of $\chi$ is used to produce massless spin-1 particles

$$A''_\pm + \omega_\pm^2 A_\pm = 0 \quad \text{where} \quad \begin{cases} \omega_\pm^2 = k^2 \mp \dfrac{2k\xi}{-\tau} \\ \xi = \dfrac{\alpha\dot{\chi}}{2fH} \end{cases} \quad (-\infty < \tau < 0)$$

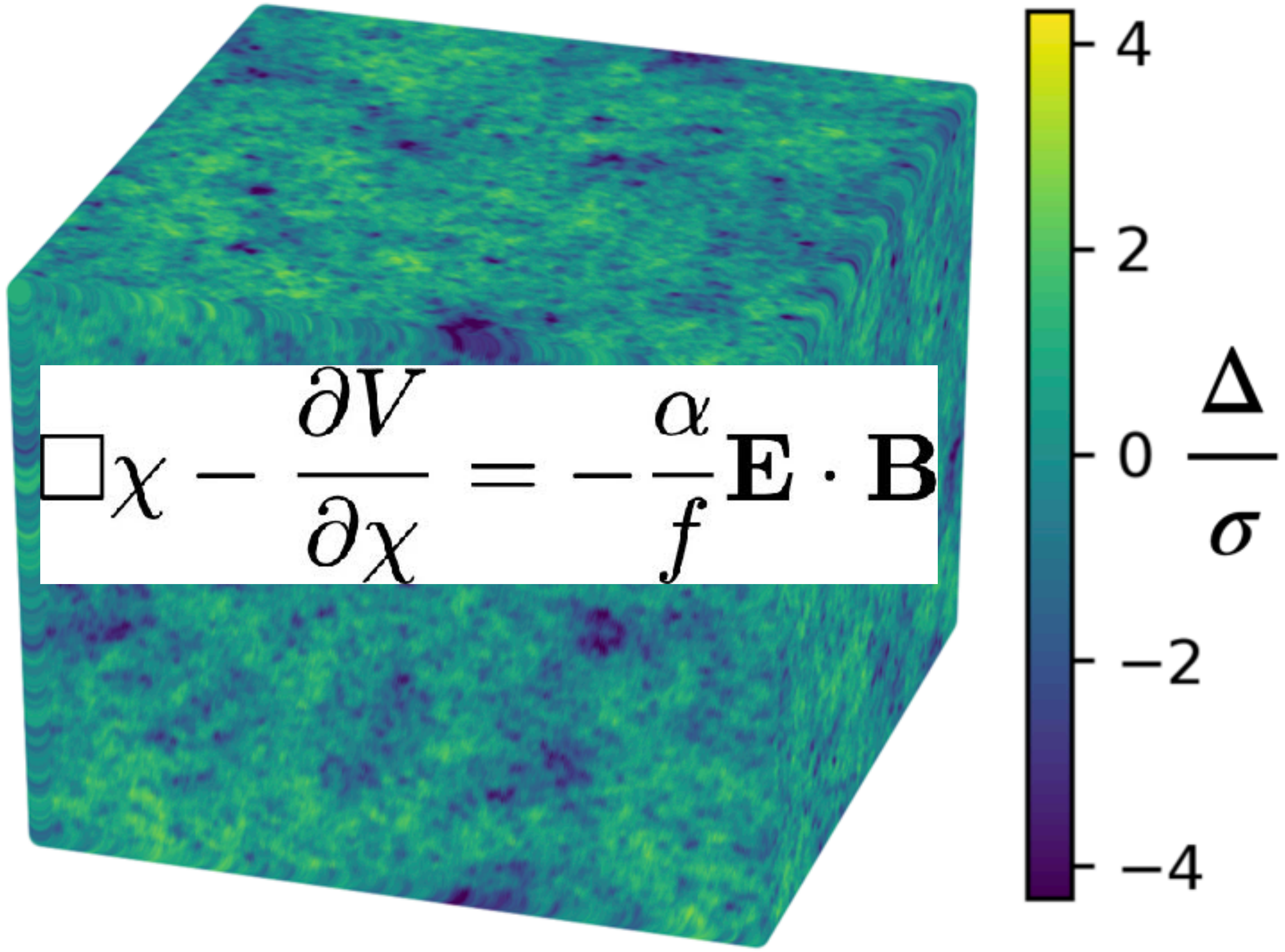
- Instability occurs when $\omega_+^2 < 0$ or $\omega_-^2 < 0$. In other words, $-k\tau < 2|\xi|$.
  - The mode function for *one of the helicity states* is **amplified** on large scales (small $-k\tau$) **relative to the vacuum solution, $e^{-ik\tau}/\sqrt{2k}$.**
  - The right-handed (+ helicity) state is amplified for $\xi>0$, whereas the left-handed (- helicity) state remains close to the vacuum solution.
  - **Parity violation!**

Caravano, EK, Lozanov, Weller (2023)

# Truly *ab initio* simulation!

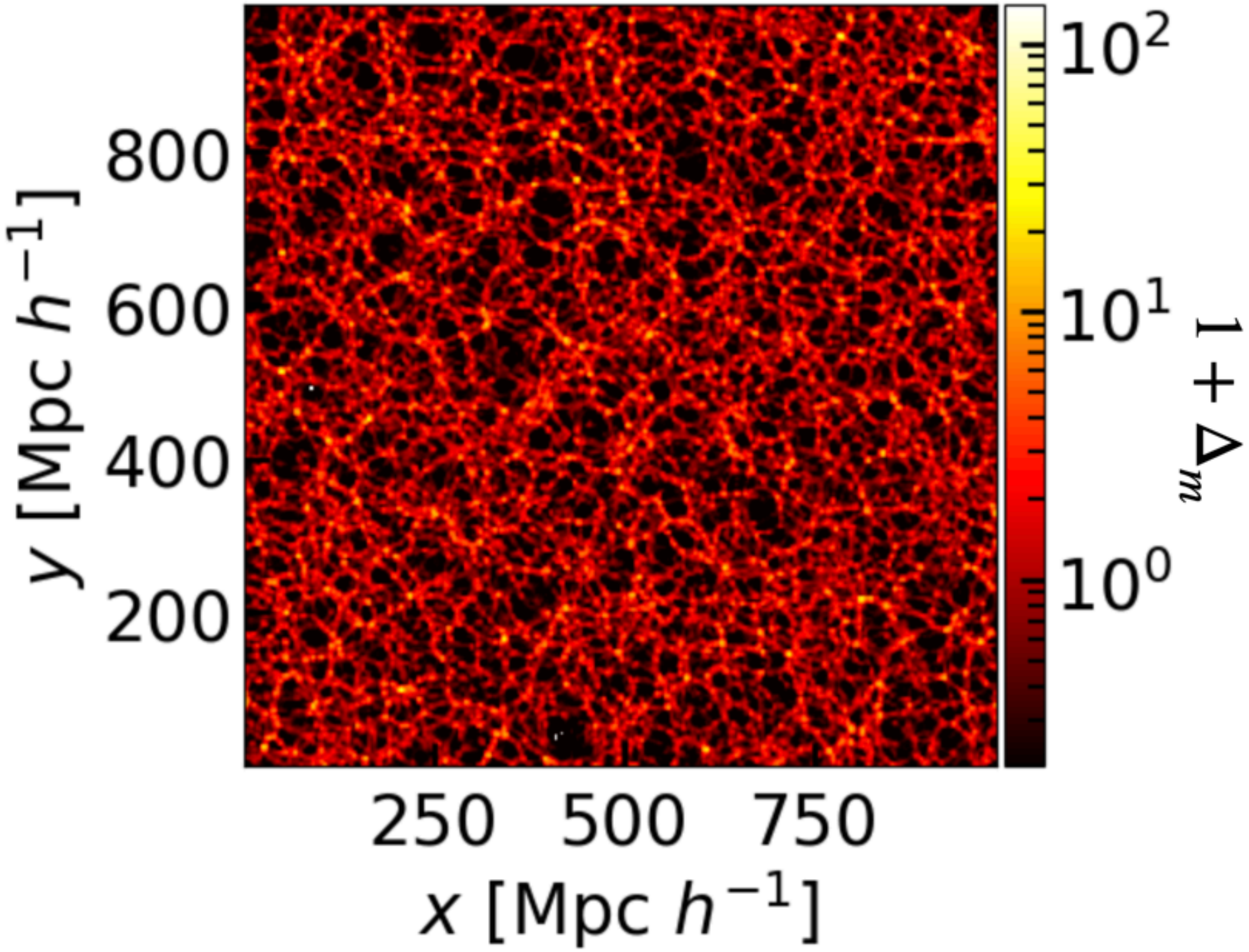
## World's first lattice simulation of inflation

Angelo Caravano




$$\Box\chi - \frac{\partial V}{\partial \chi} = -\frac{\alpha}{f}\mathbf{E}\cdot\mathbf{B}$$

Drew Jamieson




- (Left) Parity-violating and non-Gaussian density fluctuation during inflation.
- (Right) Outcome of N-body simulation at $z$=0, using the left panel as the initial condition.

# GR + Maxwell (+ Chern-Simons)

$$\Box = \frac{1}{\sqrt{-g}}\partial_\mu(\sqrt{-g}g^{\mu\nu}\partial_\nu)$$
$$= \frac{1}{a^2}\left(-\frac{\partial^2}{\partial\tau^2} - 2\frac{a'}{a}\frac{\partial}{\partial\tau} + \nabla^2\right)$$

where $g^{\mu\nu} = a^{-2}\mathrm{diag}(-1, \mathbf{1})$

$\sqrt{-g} = a^4$

$$I = \int d\tau d^3\mathbf{x}\sqrt{-g}\left(\frac{R}{16\pi G} - \frac{1}{4}F^2 - \frac{\alpha}{4f}\chi F\tilde{F}\right)$$

- The $F^2$ term contributes to the equation of motion for the GW via the stress-energy tensor (this is the second-order fluctuation).

$$\Box h_{ij} = 16\pi G(E_iE_j + B_iB_j)^{\mathrm{TT}}$$ "Transverse and Traceless"

- The $F\tilde{F}$ term does **not** contribute directly to the equation of motion for the GW.
    - But, it creates a parity violation in **E** and **B**, which also creates a parity violation in the GW.

Sorbo (2011); Barnaby, Namba, Peloso (2011)

# Parity Violation in GW

**For a slowly varying ξ>0**

$$\xi = \frac{\alpha\dot{\bar{\theta}}}{2H} = \frac{\alpha\dot{\chi}}{2Hf}$$

$$\frac{k^3 P_{+2}(k)}{2\pi^2} \simeq \frac{2}{M_{\rm Pl}^2}\left(\frac{H}{2\pi}\right)^2 \left[1 + 8.6\times 10^{-7}\frac{H^2}{M_{\rm Pl}^2}\frac{e^{4\pi\xi}}{\xi^6}\right]$$

$$\frac{k^3 P_{-2}(k)}{2\pi^2} \simeq \frac{2}{M_{\rm Pl}^2}\left(\frac{H}{2\pi}\right)^2 \left[1 + 1.8\times 10^{-9}\frac{H^2}{M_{\rm Pl}^2}\frac{e^{4\pi\xi}}{\xi^6}\right]$$

Vacuum contribution

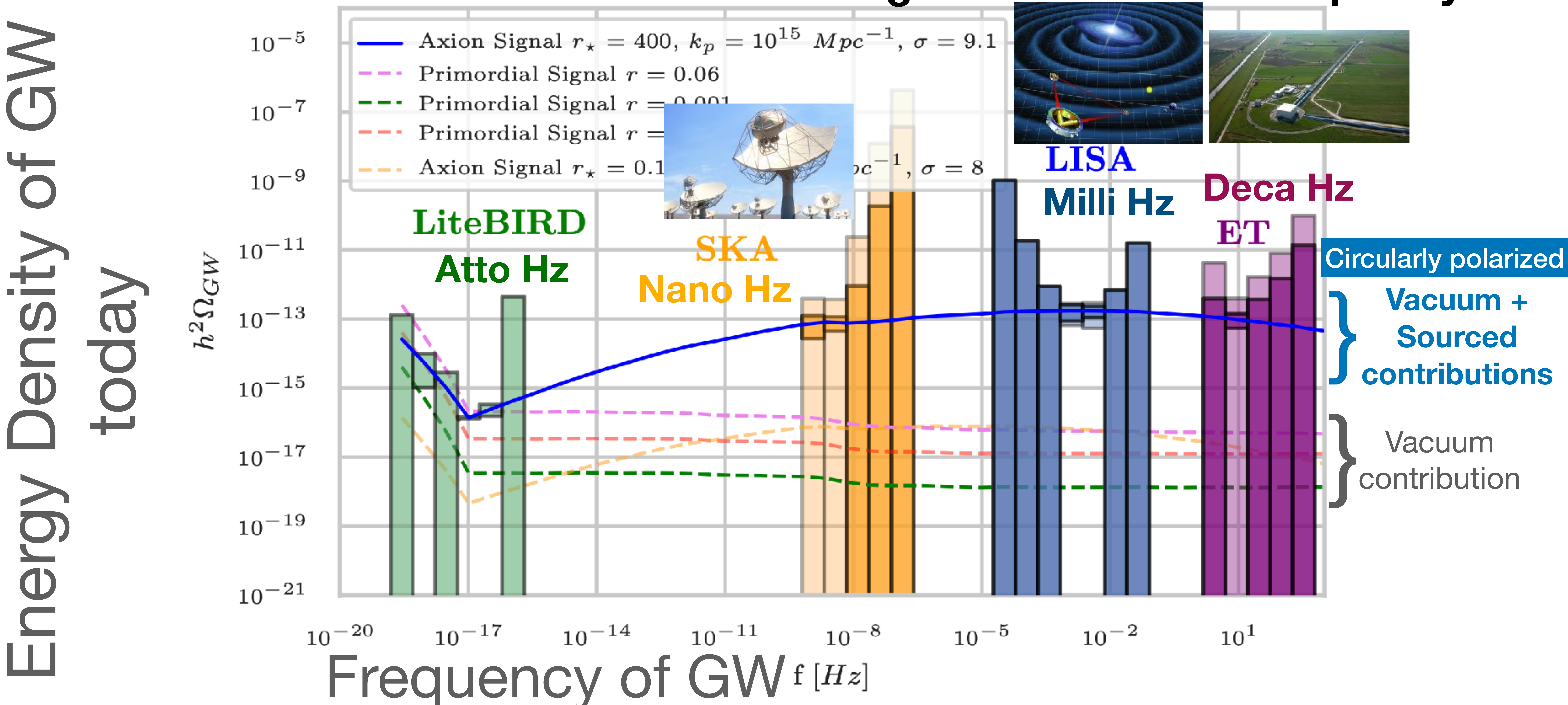
**Parity Violation!**

- The sourced contributions are almost perfectly circularly polarized.
- The sum of the vacuum and sourced contributions is partially circularly polarized. **This can be observationally tested!** (Seto 2006; Seto, Taruya 2007)

Campeti, EK, Poletti, Baccigalupi (2021)

# GWs from the early Universe are everywhere!

## We can measure it across 21 orders of magnitude in the GW frequency



# Experimental Strategy Commonly Assumed So Far

1. Detect CMB polarization in multiple frequencies, to make sure that it is from the CMB (i.e., Planck spectrum)

2. Check for scale invariance: Consistent with a scale invariant spectrum?

   - Yes => Announce discovery of the vacuum fluctuation in spacetime

   - No => WTF?

# New Experimental Strategy: New Standard!

1. Detect CMB polarization in multiple frequencies, to make sure that it is from the CMB (i.e., Planck spectrum)

2. Check for scale invariance: Consistent with a scale invariant spectrum?

3. Parity violating correlations consistent with zero?

4. Consistent with Gaussianity?

- If, and **ONLY IF** Yes to **all** => Announce discovery of the vacuum fluctuation in spacetime

**If not, you may have just discovered new physics during inflation!**

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   ma rum)

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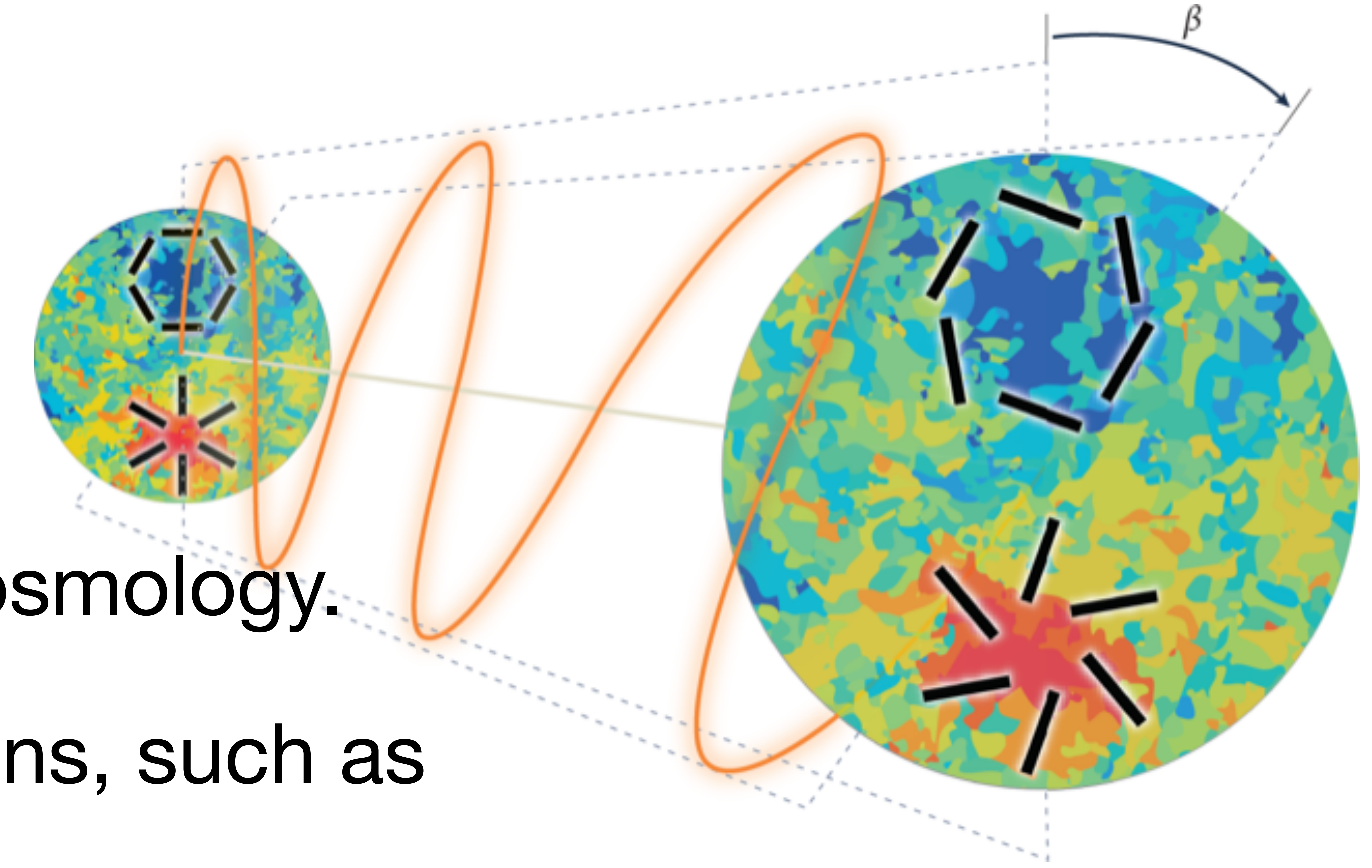
# Summary

## *Let’s find new physics!*




- Violation of parity symmetry is a new topic in cosmology.
- It may hold the answers to fundamental questions, such as
  - *What is Dark Matter and Dark Energy?*
  - *What is the fundamental physics behind cosmic inflation?*
- Rich phenomenology of Chern-Simons term: $I_{CS} = \int d^4x \sqrt{-g} \left( -\frac{\alpha}{4f} \chi F \tilde{F} \right)$
  - Cosmic birefringence **3.6σ hint of the signal**
  - Parity-violating and non-Gaussian gravitational waves and scalar fluctuations

Abelian and non-Abelian gauge fields; Gravitational CS; …

- **What else should we look at? New and great topics of research.**