Observing Primordial Fluctuations From the Early Universe: *Gaussian*, or *non-Gaussian*?

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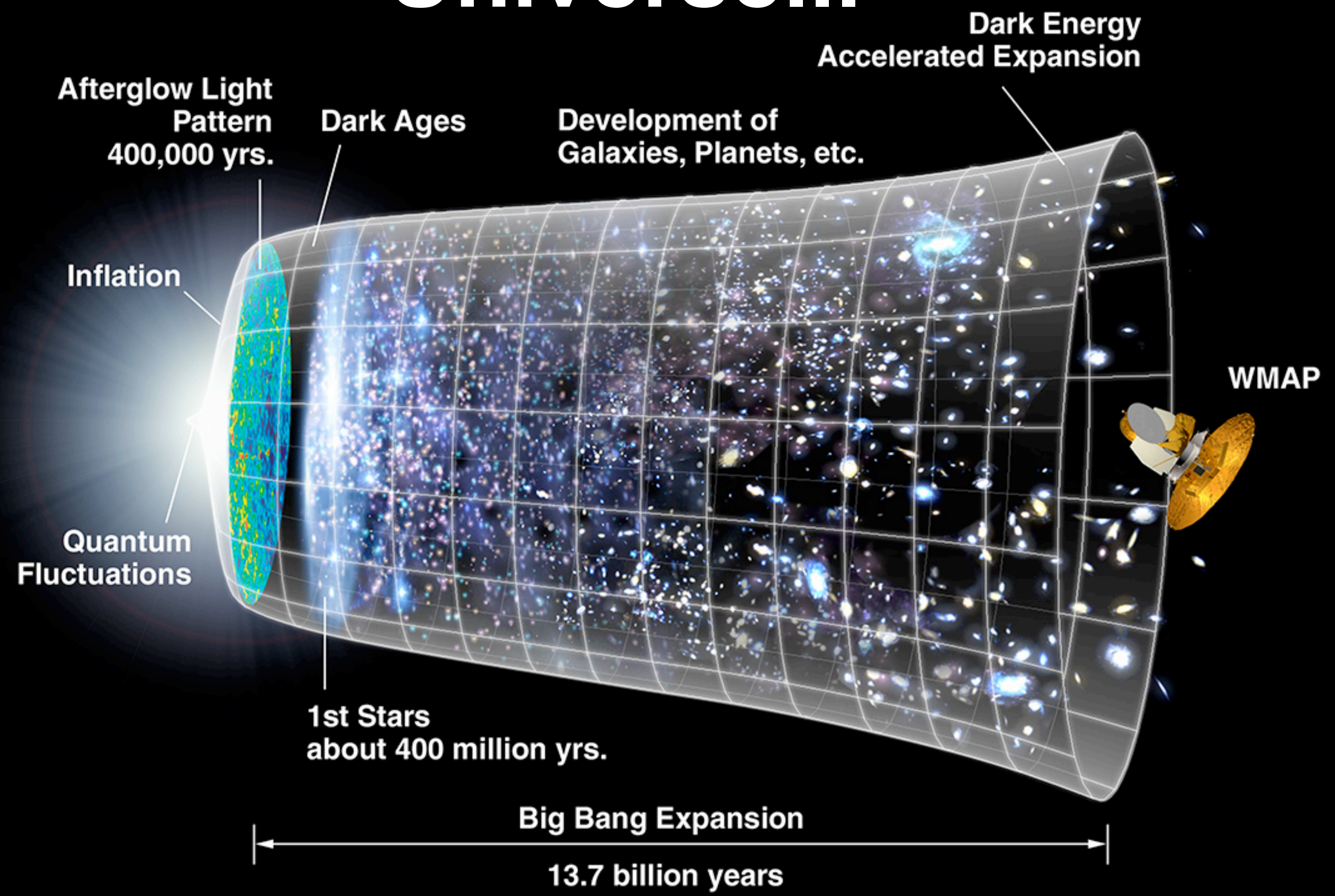
Colloquium at the University of Oklahoma, February 21, 2008

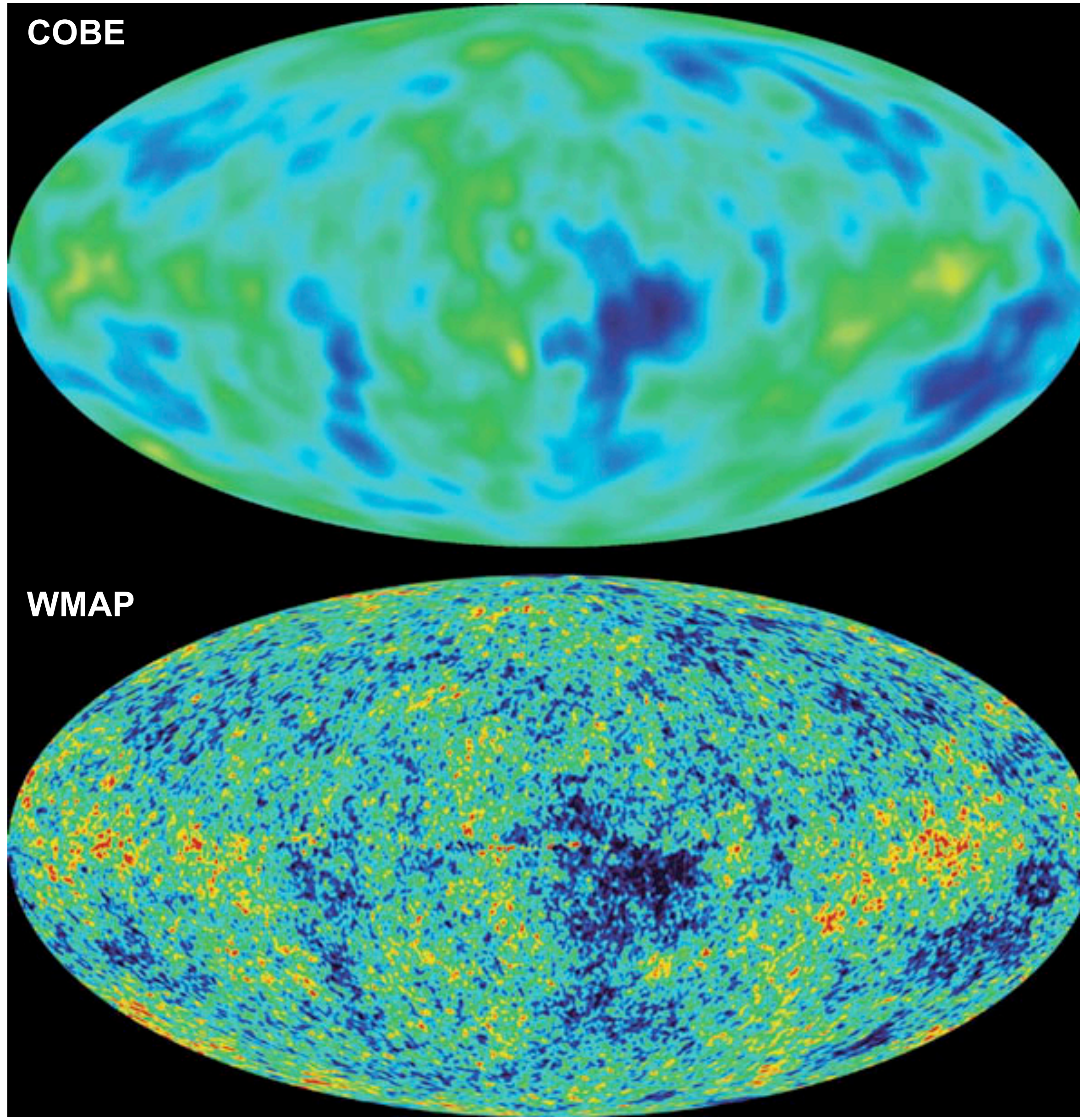
Messages From the Primordial Universe...



NASA/WMAP Science Team

The Cosmic Microwave Background

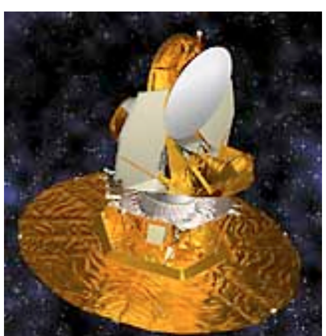
COBE
1989



Press Release from the Nobel Foundation

[COBE's] measurements also marked the inception of cosmology as a precise science. It was not long before **it was followed up,** for instance **by the WMAP satellite, which yielded even clearer images of the background radiation**.

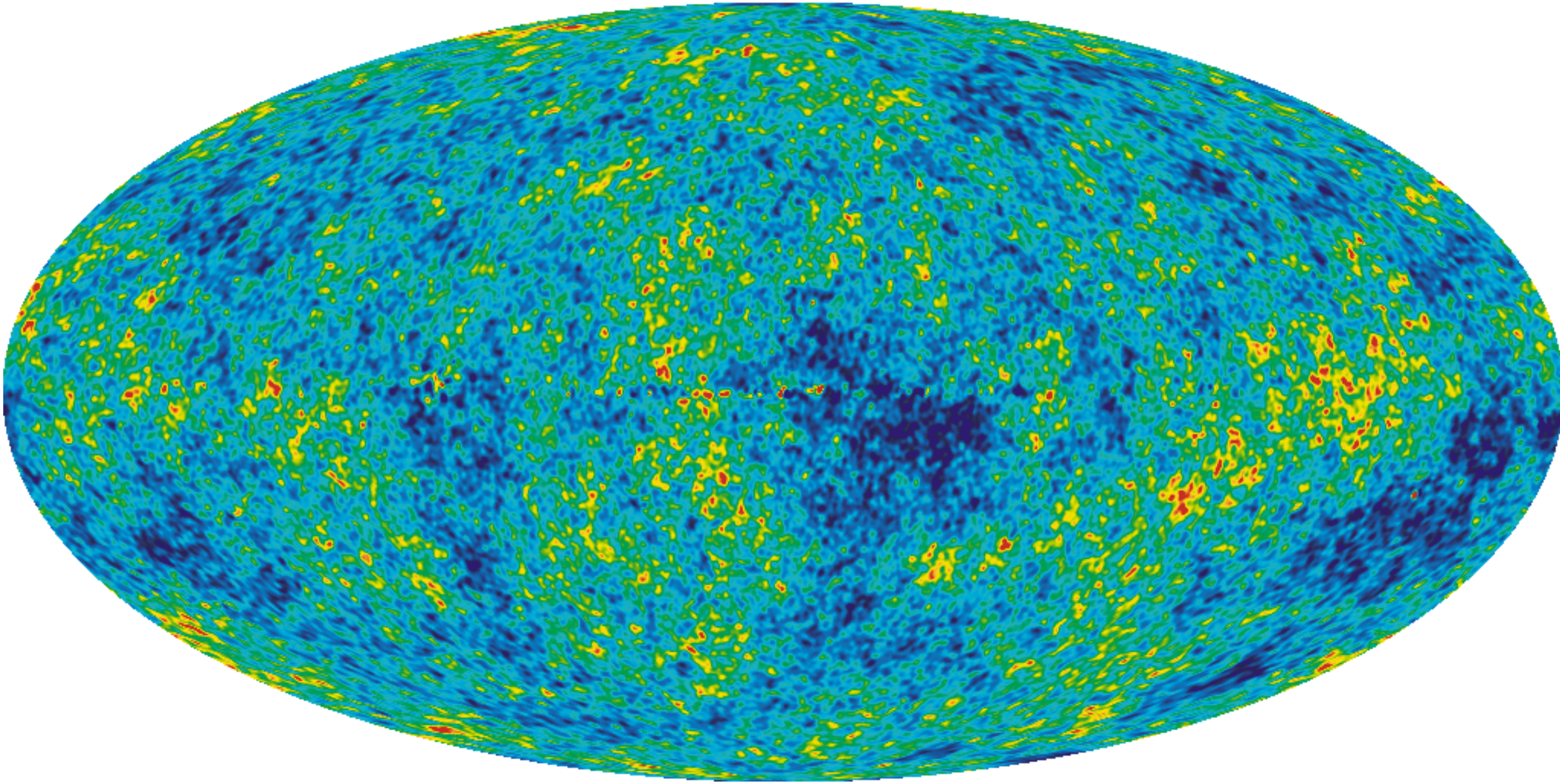
WMAP
2001



A Little Advertisement...

- We have released the 1-year WMAP results in February 2003, and 3-year results in March 2006.
- Well, it has been two years since the last release...
- It’s time to release the 5-year results.
- **The 5-year results coming near you very soon** --- in a week or two!

Microwave Sky (minus the mean temperature) as seen by WMAP



What is shown here?

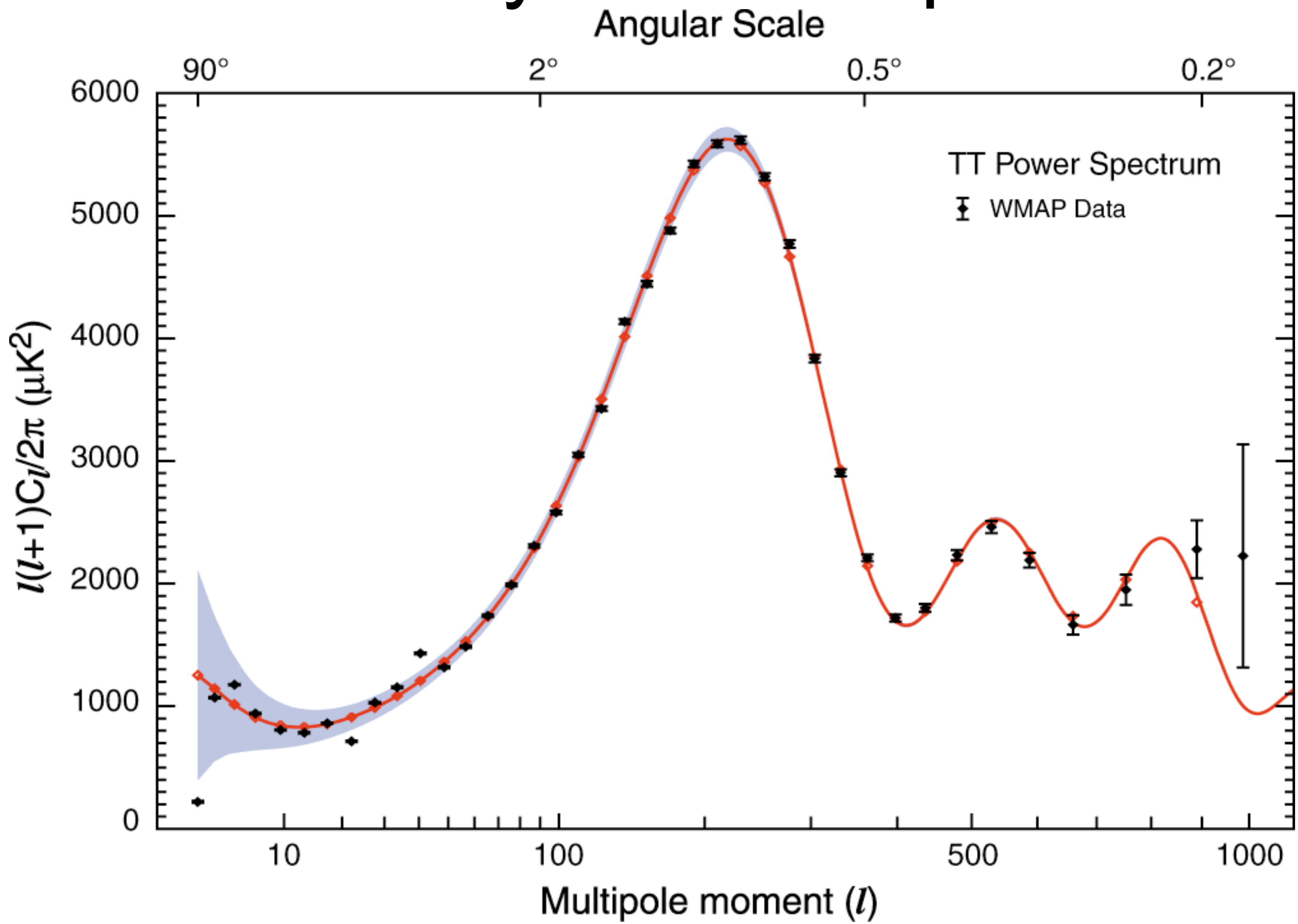
The Angular Power Spectrum

- CMB temperature anisotropy is very close to Gaussian (**but I have a lot to say about this later!**); thus, its spherical harmonic transform, a_{lm}, is also very close to Gaussian.
- If a_{lm} is Gaussian, the power spectrum:

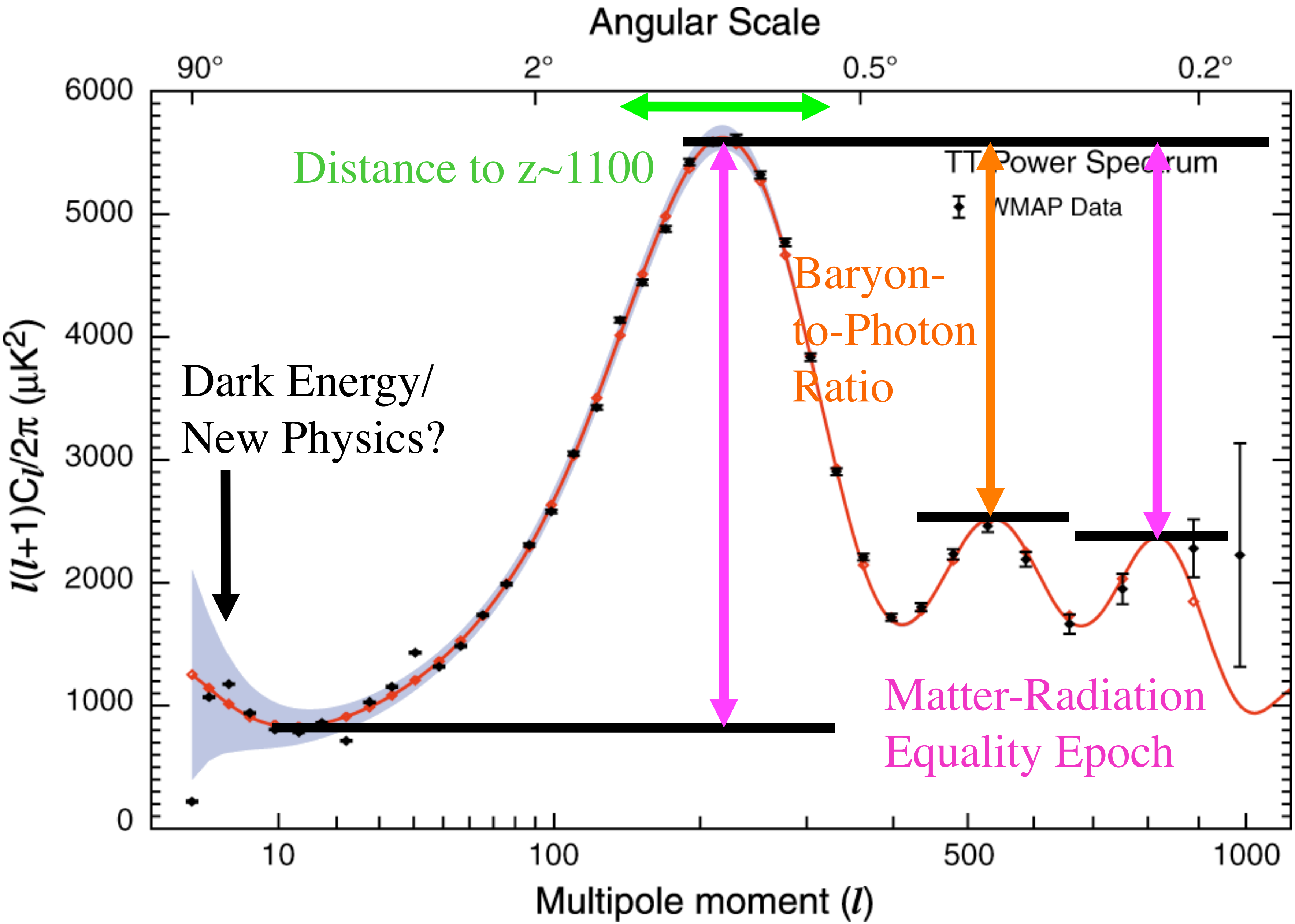
$$C_l = \left\langle a_{lm} a_{lm}^* \right\rangle$$

completely specifies statistical properties of CMB.

WMAP 3-yr Power Spectrum

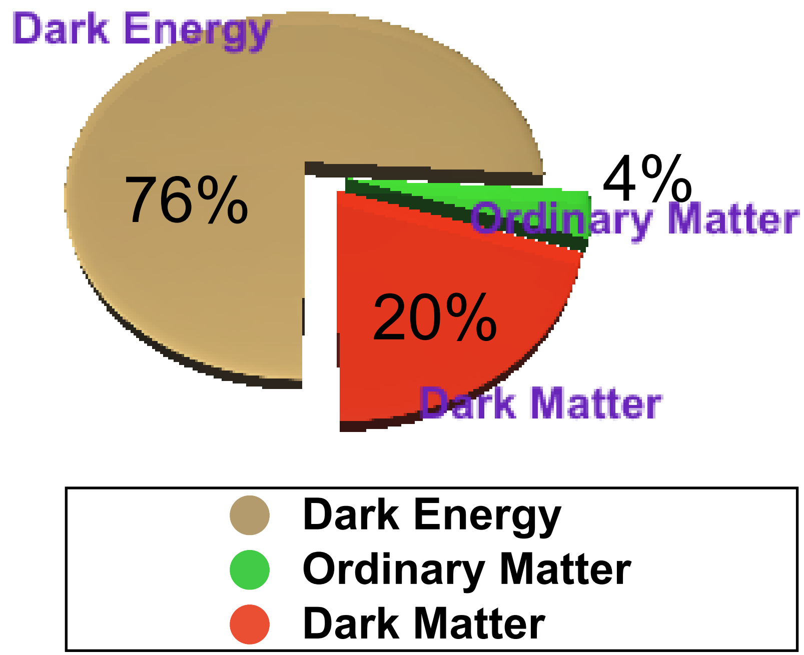


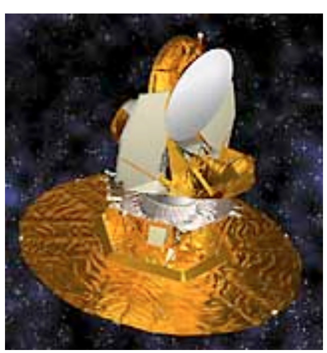
What Temperature Tells Us



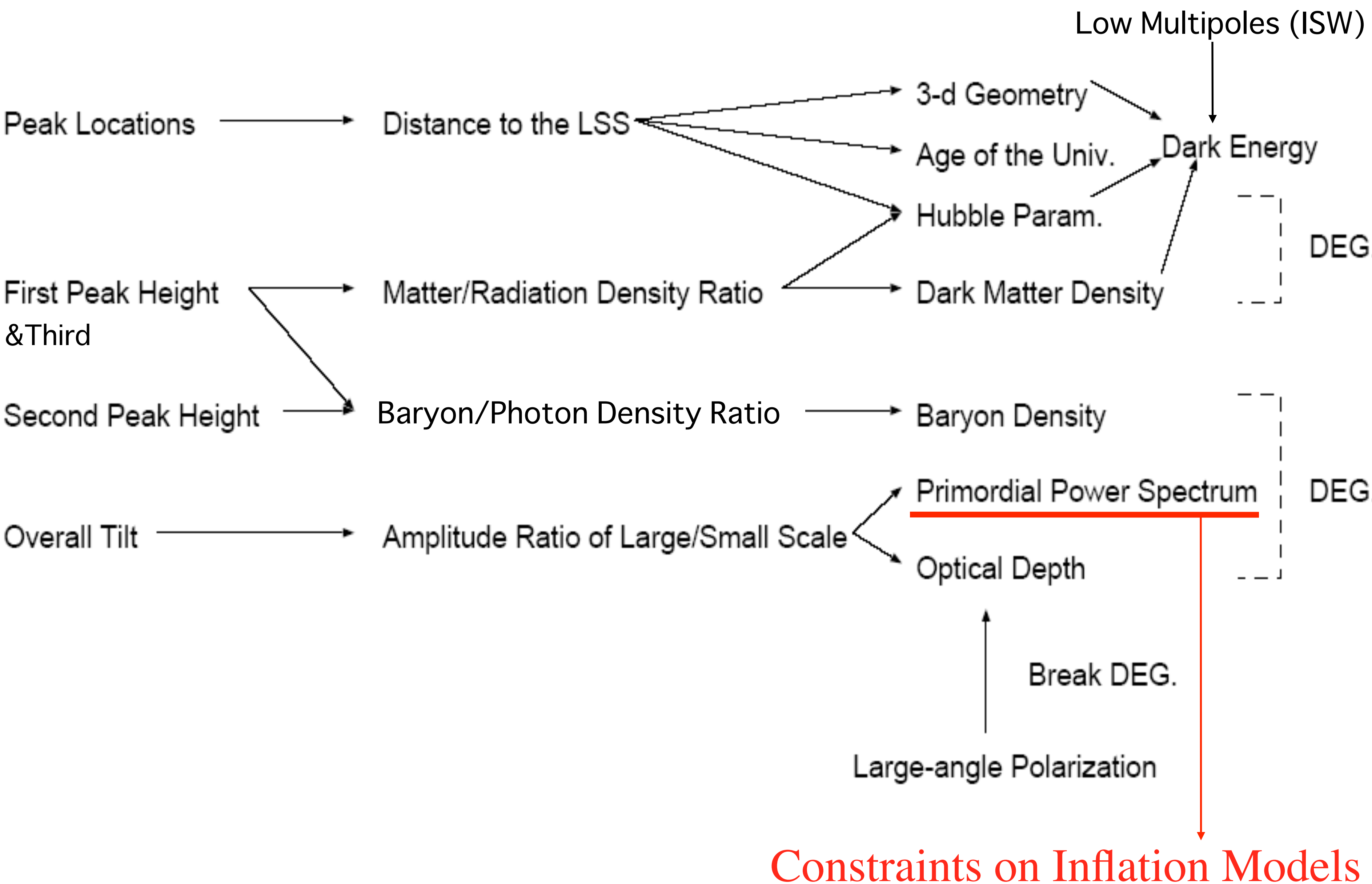
Composition of Our Universe Determined by WMAP 3yr

Mysterious “Dark Energy” occupies **75.9±3.4%** of the total energy of the Universe.

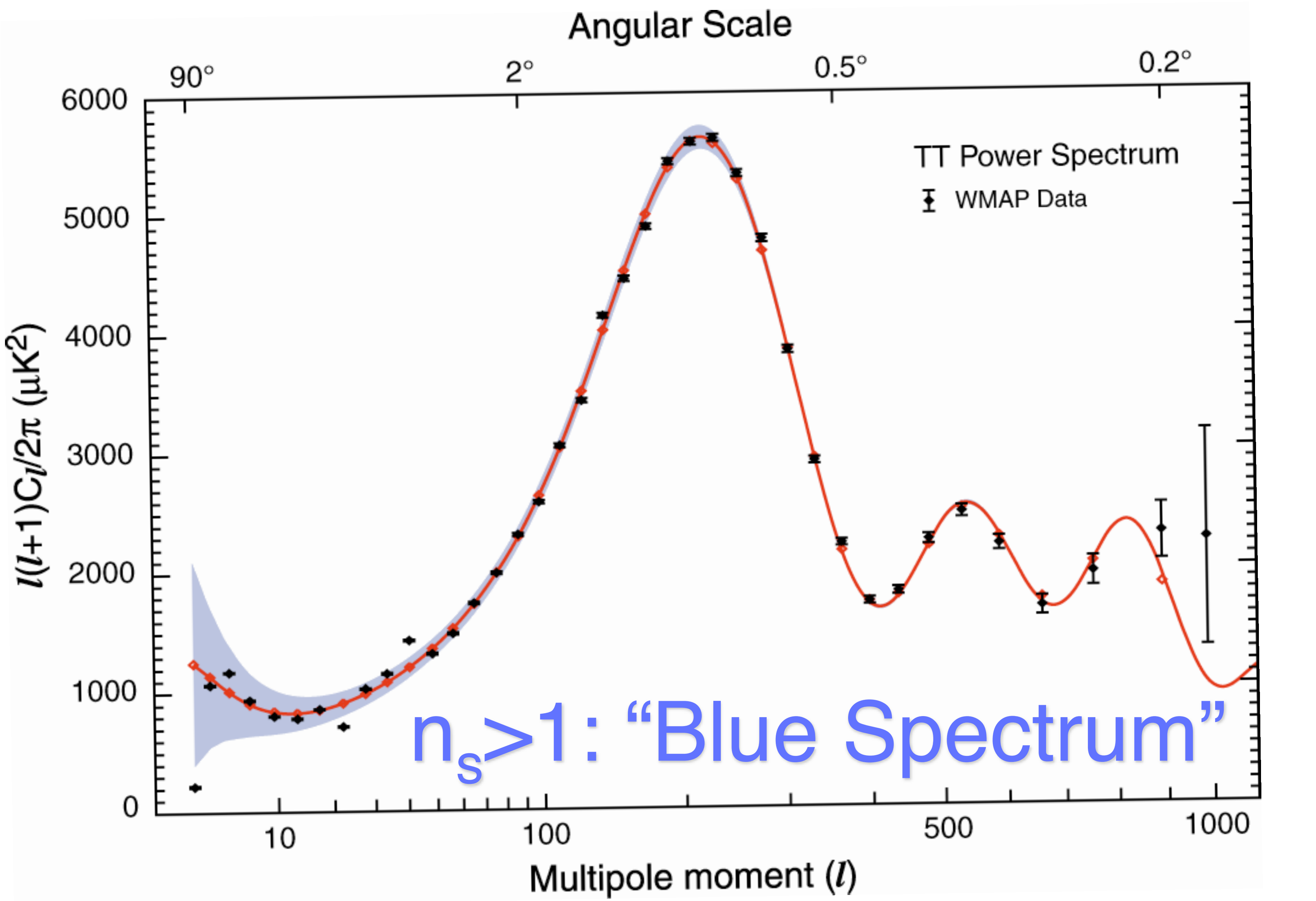




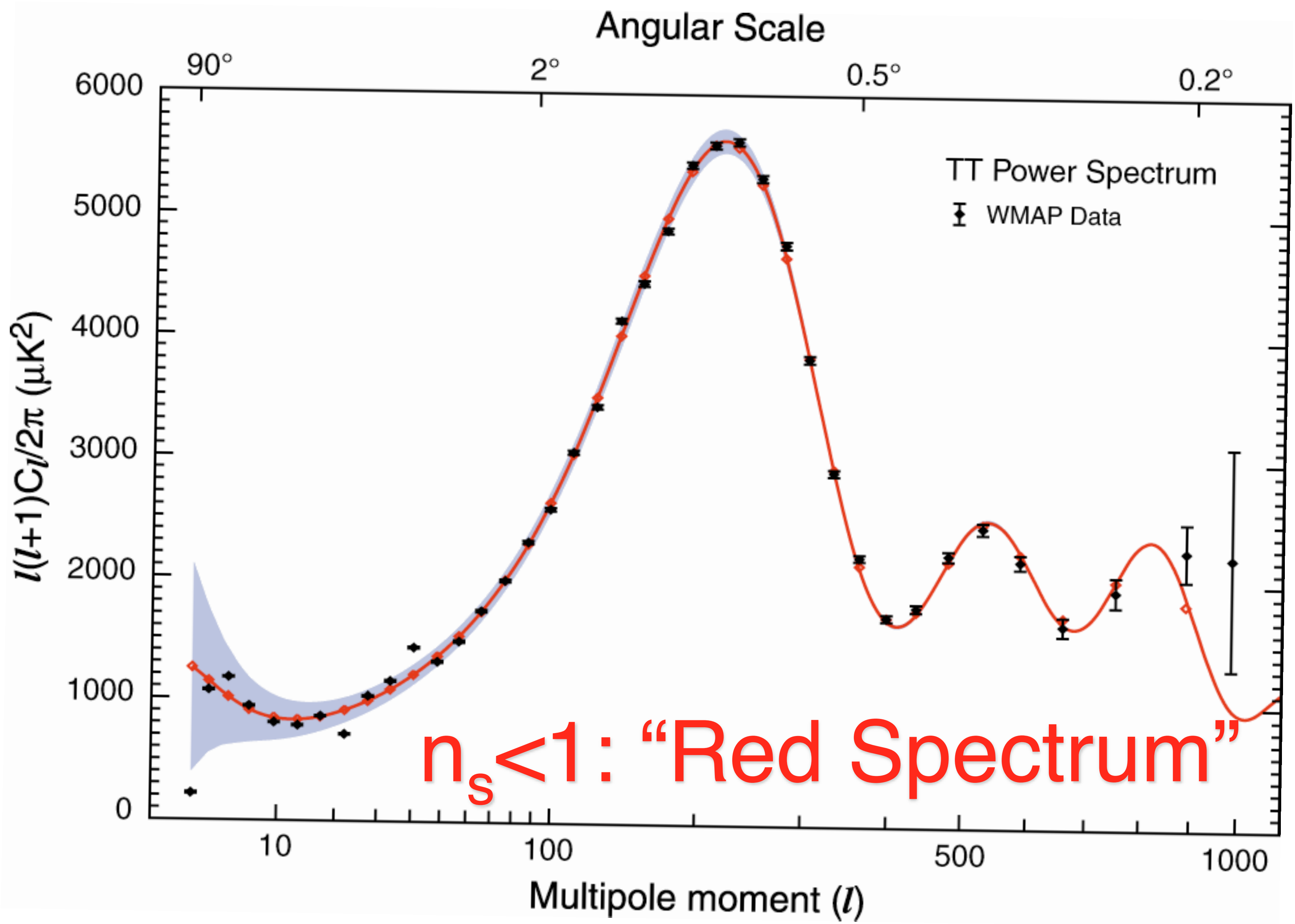
CMB to Cosmology



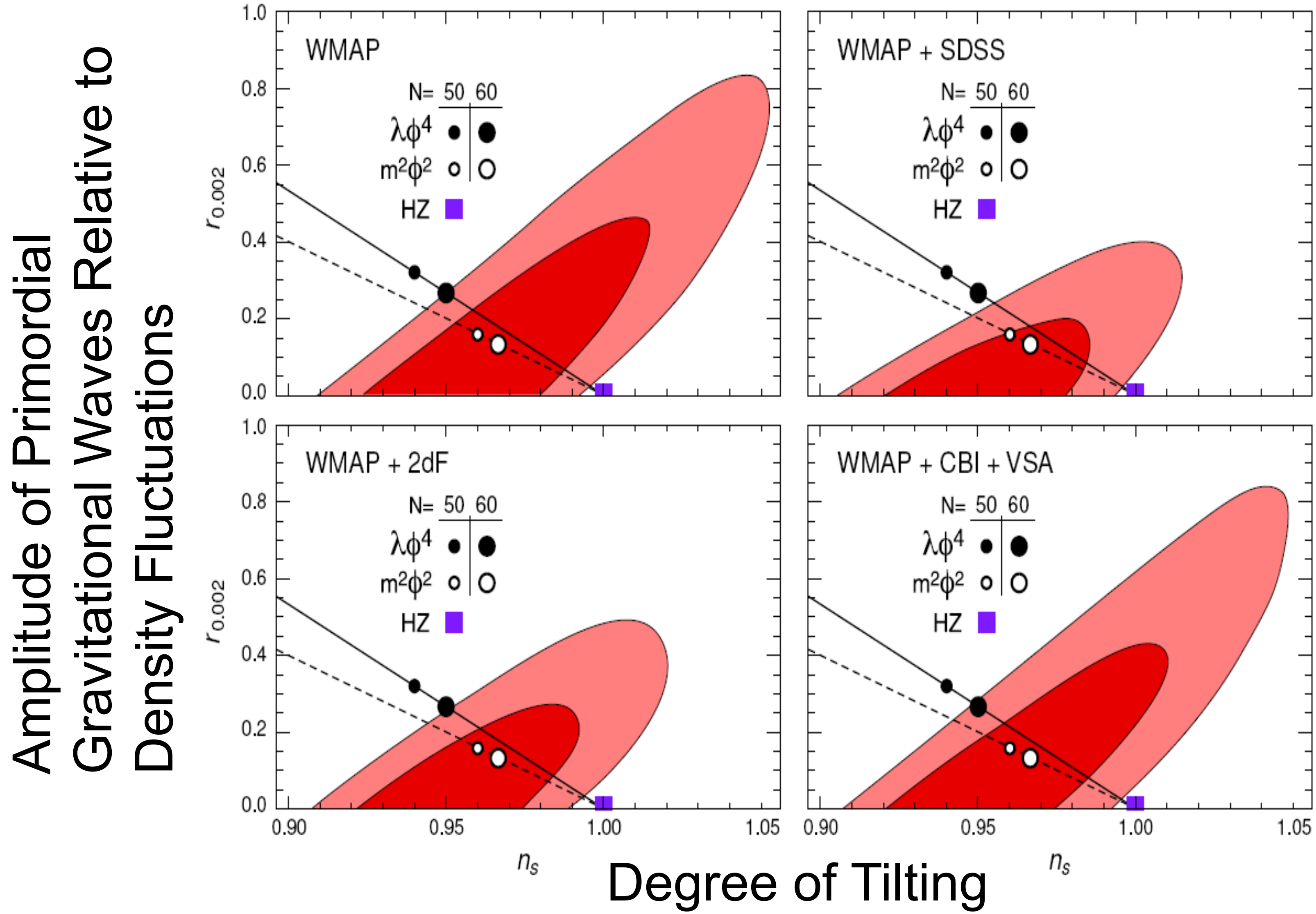
n_s: Tilting Spectrum



n_s: Tilting Spectrum



Seeing the shape and amplitude of the primordial fluctuations



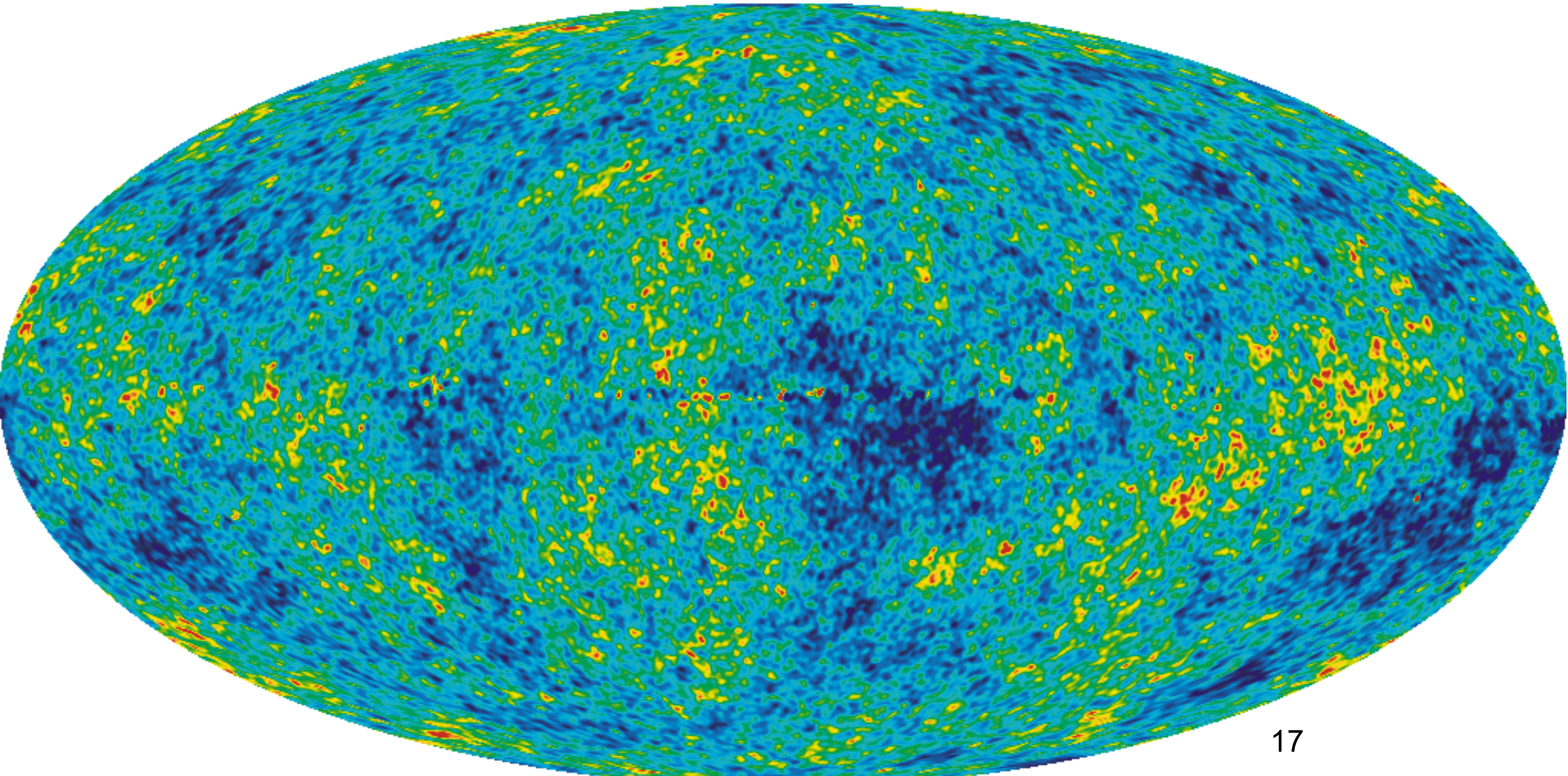
And, WMAP Talk Usually Ends Here. But...

- These results are exciting, but is this all we can learn from the WMAP data?
- In particular, is this all we can learn about the primordial universe from WMAP?

Why Study Non-Gaussianity?

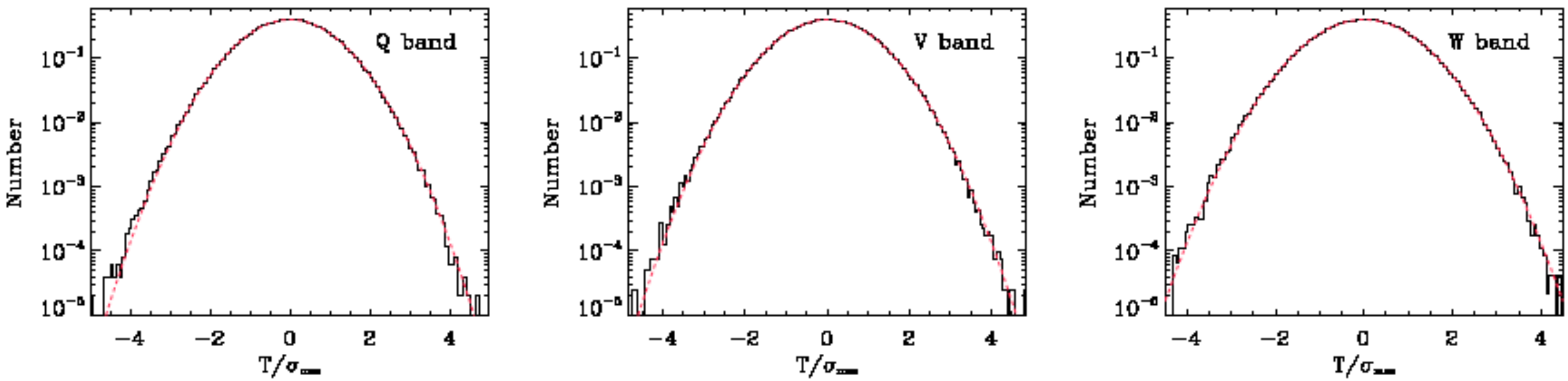
- **Who said that CMB must be Gaussian?**
  - **Don't let people take it for granted.**
  - It is rather remarkable that the distribution of the observed temperatures is so close to a Gaussian distribution.
  - The WMAP map, when smoothed to 1 degree, is entirely dominated by the CMB signal.
    - If it were still noise dominated, no one would be surprised that the map is Gaussian.
  - The WMAP data are telling us that primordial fluctuations are pretty close to a Gaussian distribution.
    - How common is it to have something so close to a Gaussian distribution in astronomy?
  - It is not so easy to explain why CMB is Gaussian, unless we have a compelling early universe model that predicts Gaussian primordial fluctuations: e.g., ***Inflation***.

How Do We Test *Gaussianity of CMB*?



Spergel et al. (2007)

One-point PDF from WMAP



- The one-point distribution of CMB temperature anisotropy looks pretty Gaussian.
  - Left to right: Q (41GHz), V (61GHz), W (94GHz).
- We are therefore talking about quite a subtle effect.

Two Approaches to Testing Non-Gaussianity

- **I. Blind Tests / "Discovery" Mode**
  - This approach has been most widely used in the literature.
  - One may apply one's favorite statistical tools (higher-order correlations, topology, isotropy, etc) to the data, and show that the data are (*in*)*consistent* with Gaussianity at xx% CL.
  - PROS: Model-independent. Very generic.
  - CONS: We don't know how to interpret the results.
    - "**The data are consistent with Gaussianity**" --- what physics do we learn from that? It is not clear what could be ruled out using this kind of test

Two Approaches to Testing Non-Gaussianity

- **II. "Model-testing" Mode**
  - Somewhat more recent approaches.
  - Try to constrain "Non-gaussian parameter(s)" (e.g., f_{NL})
  - PROS: We know what we are testing, we can quantify our constraints, and we can compare different data sets.
  - CONS: Highly model-dependent. We may well be missing other important non-Gaussian signatures.

Cosmology and Fundamental Physics: 6 Numbers

- Successful early-universe models **must** satisfy the following observational constraints:
  - The observable universe is nearly flat, **$|\Omega_K|<O(0.02)$**
  - The primordial fluctuations are
    - Nearly Gaussian, **$|f_{NL}|<O(100)$**
    - Nearly scale invariant, **$|n_s-1|<O(0.05)$**, **$|dn_s/d\ln k|<O(0.05)$**
    - Nearly adiabatic, **(non-adi)/(adi)<O(0.2)**

Cosmology and Fundamental Physics: 6 Numbers

- A “generous” theory would make cosmologists very happy by producing detectable primordial gravitational waves ($r>0.01$)…
  - But, this is not a requirement yet.
  - Currently, **$r<O(0.5)$**

Gaussianity vs Flatness

- We are generally happy that geometry of our observable Universe is flat.
  - Geometry of our Universe is consistent with a flat geometry to **~2%** accuracy at 95% CL. (Spergel et al., WMAP 3yr)

- What do we know about Gaussianity?
  - Parameterize non-Gaussianity: $\Phi = \Phi_L + f_{NL} \Phi_L^2$
    - $\Phi_L \sim 10^{-5}$ is a Gaussian, linear curvature perturbation in the matter era
  - Therefore, $f_{NL} < 100$ means that the distribution of Φ is consistent with a Gaussian distribution to $\sim 100 \times (10^{-5})^2 / (10^{-5}) =$ **0.1%** accuracy at 95% CL.

- **"Inflation is supported more by Gaussianity than by flatness."**

What is Φ?

- By Φ I mean the "curvature perturbation," which is minus of the usual Newtonian gravitational potential.
- E.g., in the Schwarzschild spacetime,
  - $\Phi = GM/R$
  - Newtonian potential = $-GM/R$

Why Φ?

- The curvature perturbation generates temperature anisotropy that we observe.
- On very large angular scales (>10 degrees), we have a simple relationship from the cosmological perturbation theory:
  - $dT/T = (-1/3)\Phi$
  - This is called the "Sachs-Wolfe effect" (Sachs & Wolfe 1967)

Why is Φ (so close to) Gaussian?

- Inflation explains this as follows.
- The CMB fluctuations that we observe today in WMAP were created from **quantum fluctuations of a scalar field in vacuum** during the epoch of inflation.
- Inflation demands the scalar field be almost interaction-free.
- Now, quantum mechanics: **the wave function of a non-interacting field in the ground state is a Gaussian!**

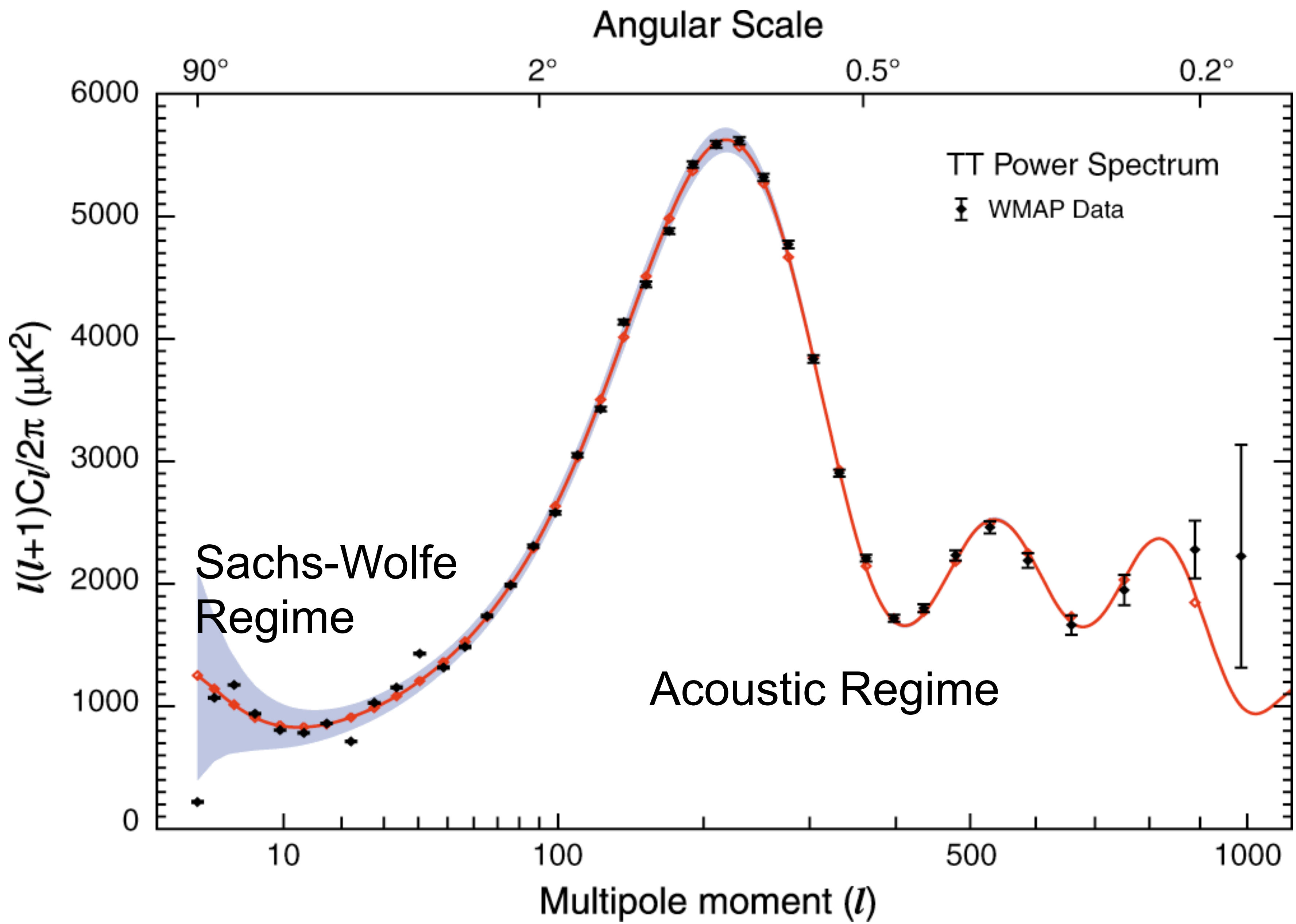
But, not precisely Gaussian...

- However small they are, there are always corrections to such a simple statement.
- Interactions are small, but they are not zero.
- What if the initial state was not in vacuum?
- A simple-minded form of the correction:

$$\Phi = \Phi_L + f_{NL} \Phi_L^2$$

What Non-Gaussianity Does

- In the Sachs-Wolfe limit,
  - $dT/T = (-1/3)[\Phi + f_{NL}\Phi^2]$
    - where Φ is a Gaussian random field.
  - dT/T is no longer Gaussian!
- For small angular scales, the Sachs-Wolfe formula is no longer true, and we must take into account the acoustic physics at the decoupling epoch at z~1090.



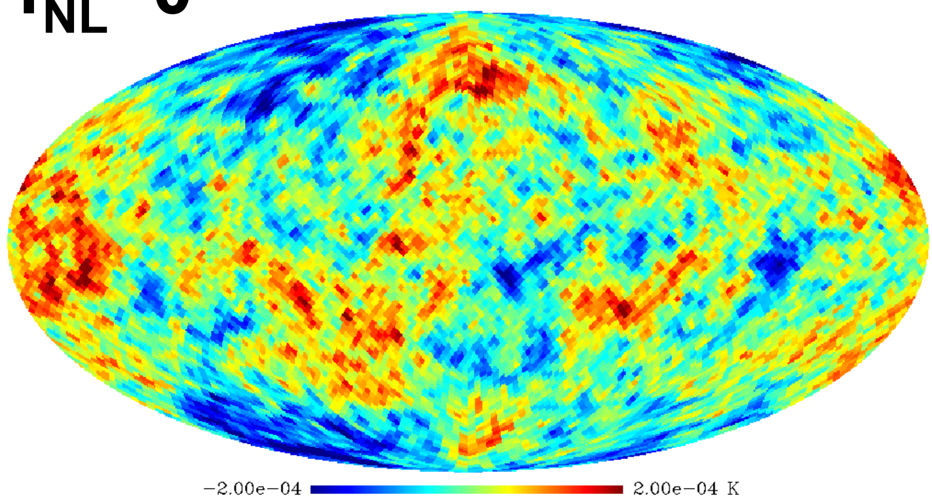
Angular Scale
90°
2°
0.5°
0.2°
TT Power Spectrum
WMAP Data
Sachs-Wolfe
Regime
Acoustic Regime
6000
5000
4000
3000
2000
1000
0
$l(l+1)C_l/2\pi$ (μK^2)
10
100
500
1000
Multipole moment (l)

Positive f_{NL} = More Cold Spots

Simulated temperature maps from $\Phi(x) = \Phi_G(x) + f_{NL}\Phi_G^2(x)$

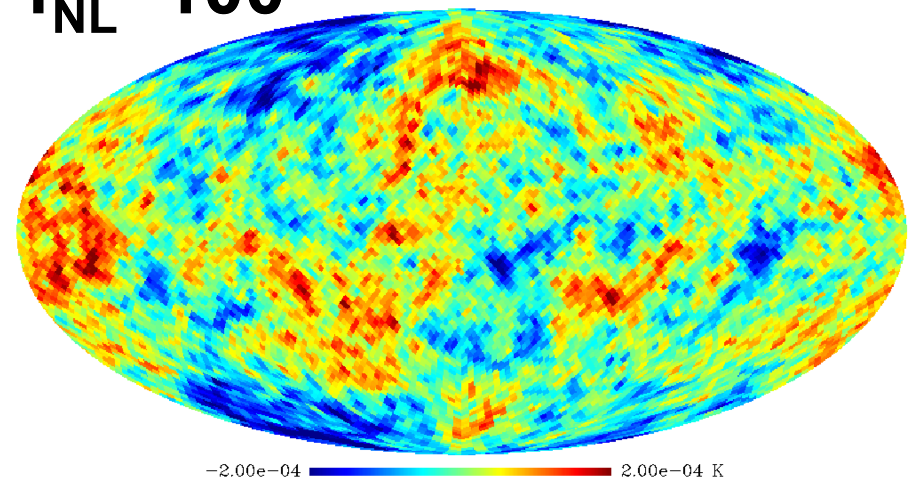
f_{NL}=0

Gaussian simulation, n=1024~3



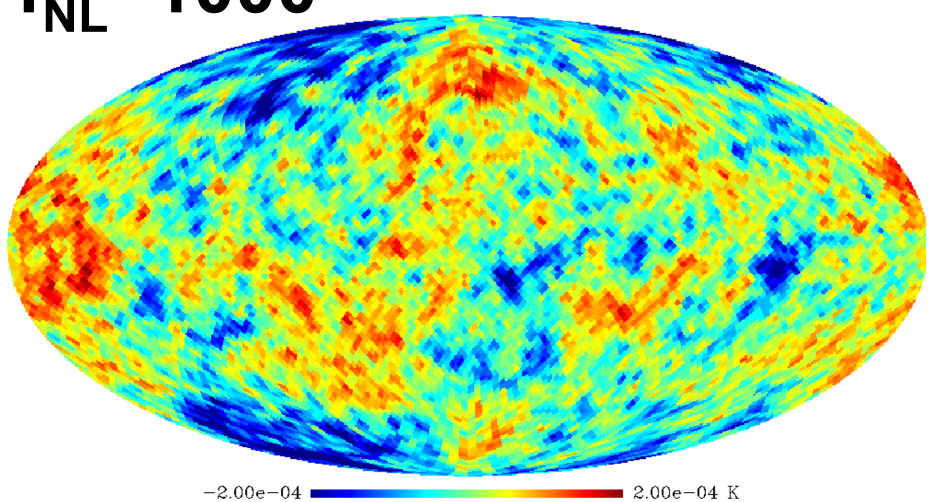
f_{NL}=100

Gaussian simulation, fNL=100, 1024~3



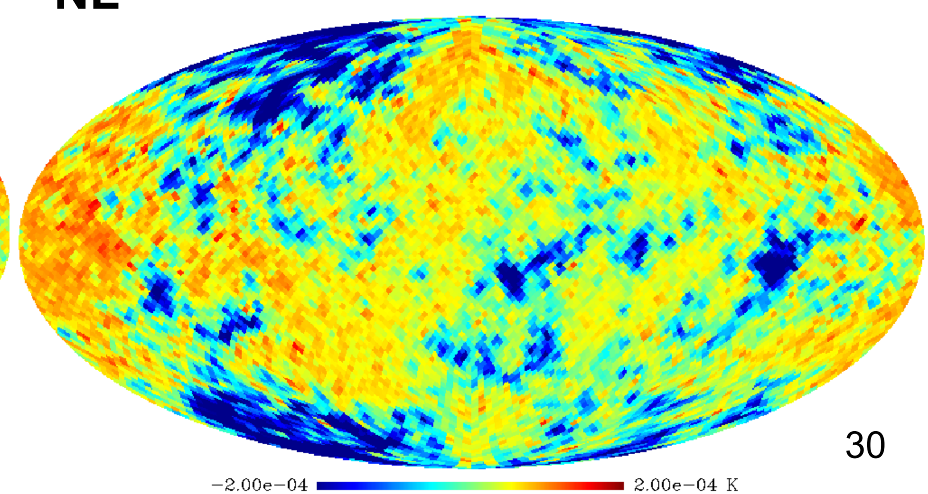
f_{NL}=1000

sian simulation, fNL=1000, n=1024~3

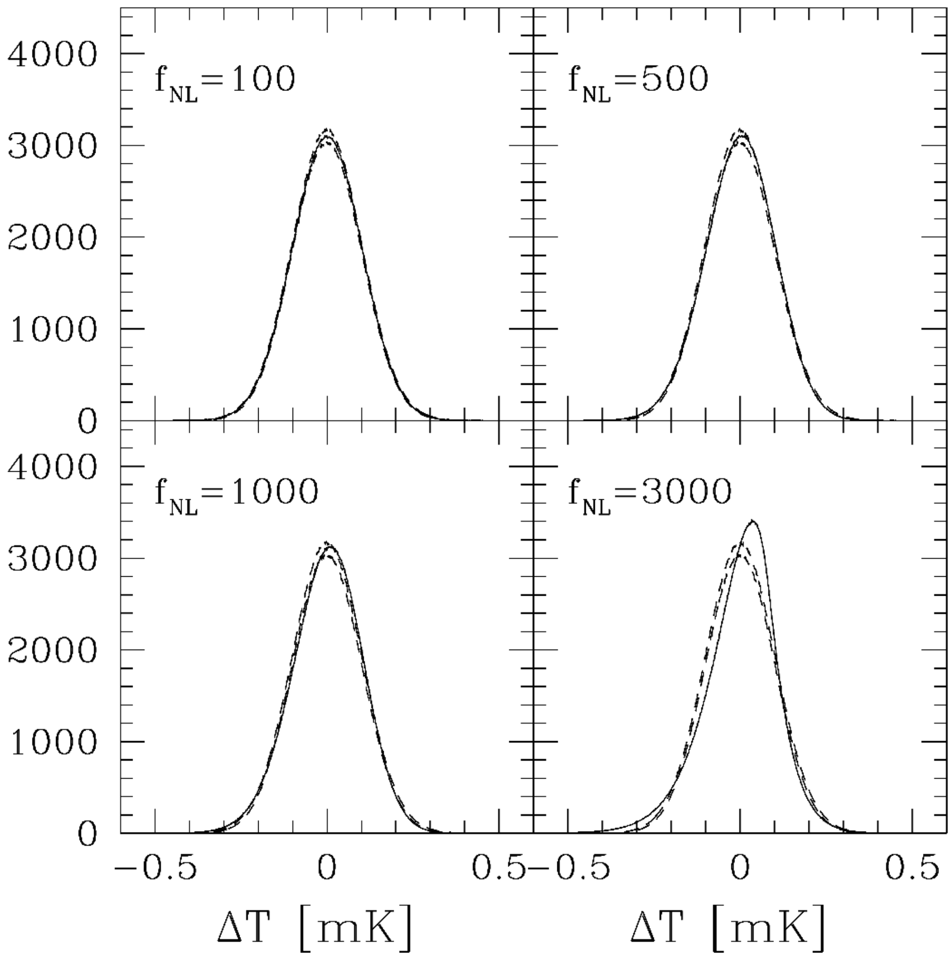


f_{NL}=5000

sian simulation, fNL=5000, n=1024~3



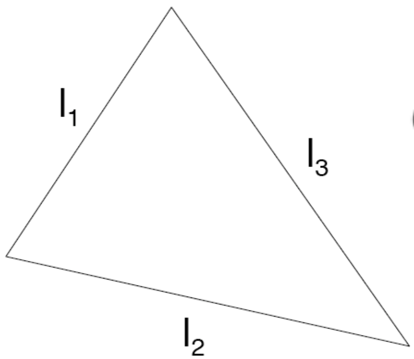
How Would f_{NL} Modify PDF?



One-point PDF is not useful for measuring primordial NG. We need something better:

- Three-point Function
  - Bispectrum
- Four-point Function
  - Trispectrum
- Morphological Test
  - Minkowski Functionals

Bispectrum of CMB



$$a_{lm} \equiv \int d^2\hat{\mathbf{n}} \frac{\Delta T(\hat{\mathbf{n}})}{T} Y^*_{lm}(\hat{\mathbf{n}})$$

$$= 4\pi(-i)^l \int \frac{d^3\mathbf{k}}{(2\pi)^3} \Phi(\mathbf{k}) g_{Tl}(k) Y^*_{lm}(\hat{\mathbf{k}})$$

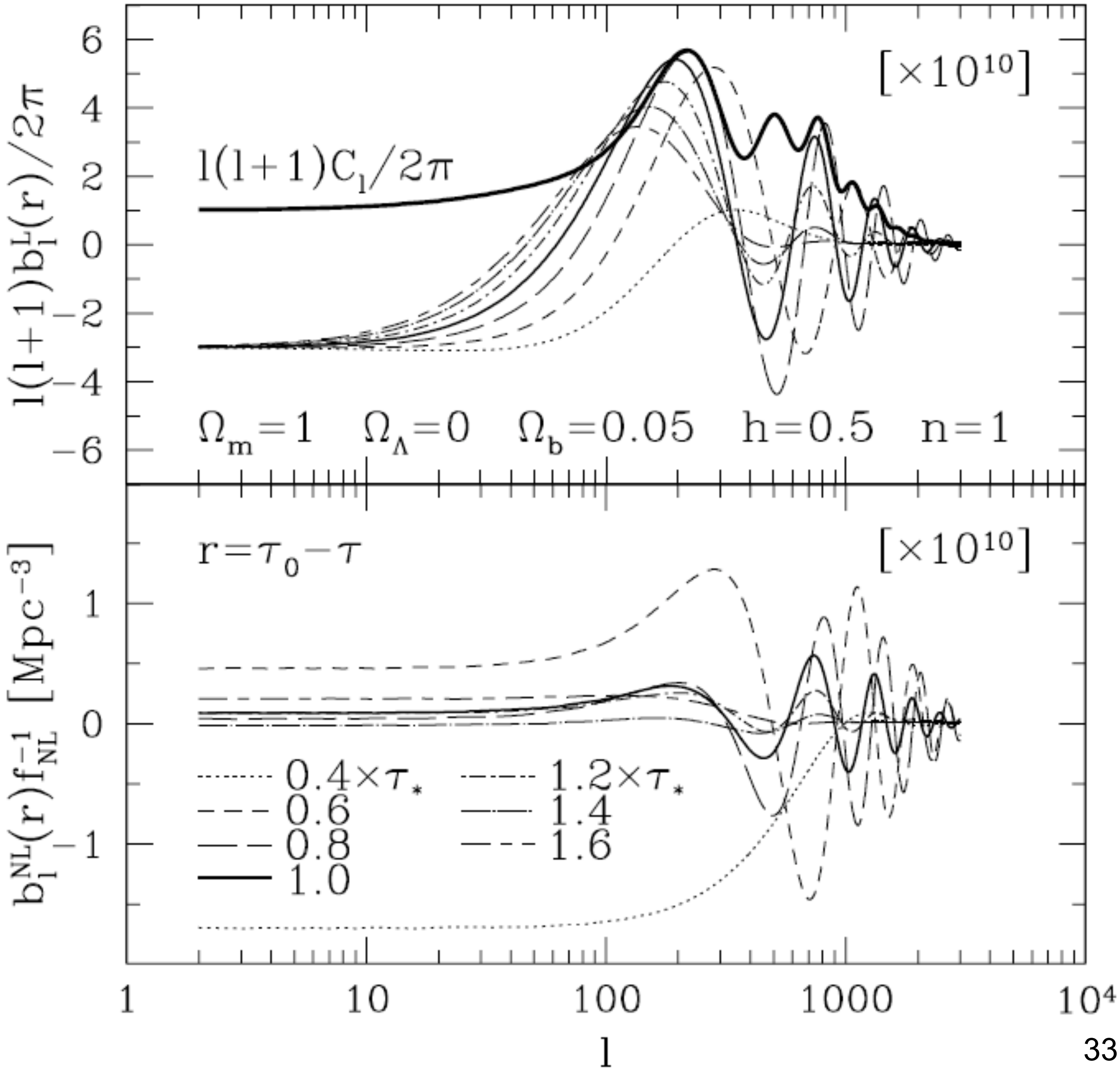
$$B^{m_1 m_2 m_3}_{l_1 l_2 l_3} \equiv \langle a_{l_1 m_1} a_{l_2 m_2} a_{l_3 m_3} \rangle = \mathcal{G}^{m_1 m_2 m_3}_{l_1 l_2 l_3} b_{l_1 l_2 l_3}$$

$$b^{primary}_{l_1 l_2 l_3} = 2 \int_0^\infty r^2 dr \left[b^L_{l_1}(r) b^L_{l_2}(r) b^{NL}_{l_3}(r) + \right.$$ (cyclic)

$$b^L_l(r) \equiv \frac{2}{\pi} \int_0^\infty k^2 dk P_\Phi(k) g_{Tl}(k) j_l(kr),$$

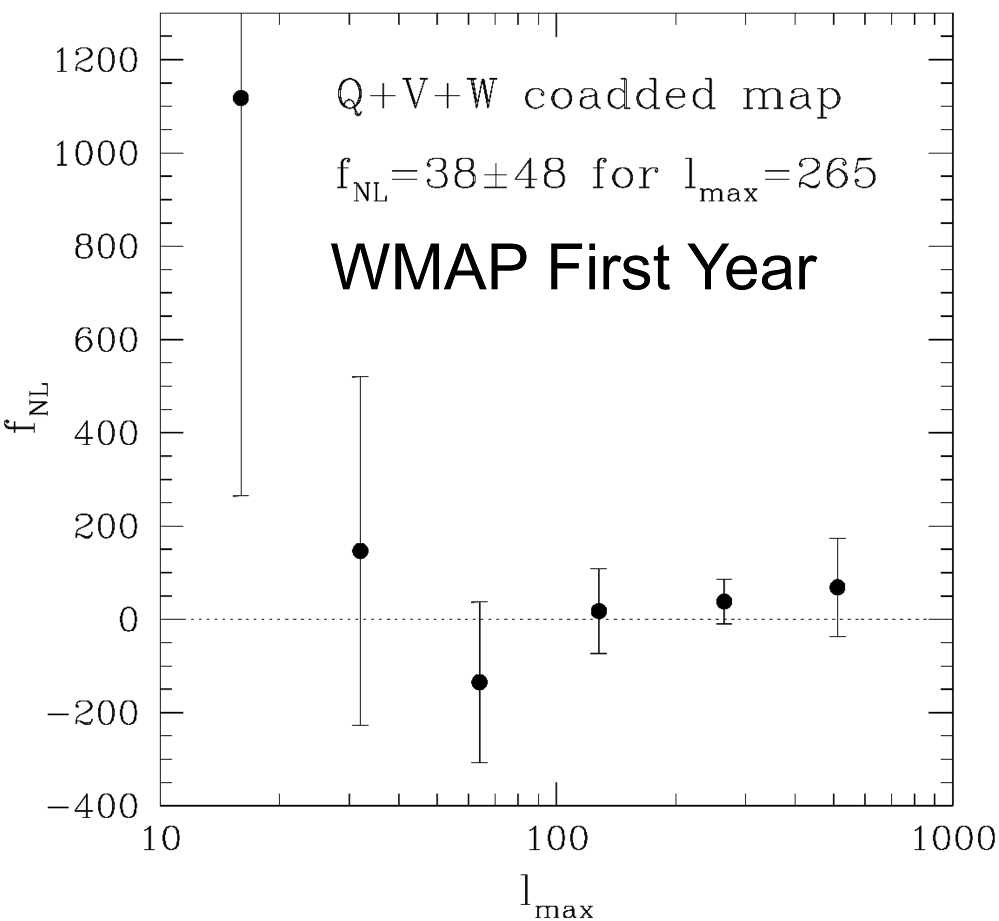
$$b^{NL}_l(r) \equiv \frac{2}{\pi} \int_0^\infty k^2 dk f_{NL} g_{Tl}(k) j_l(kr).$$

Komatsu & Spergel (2001)



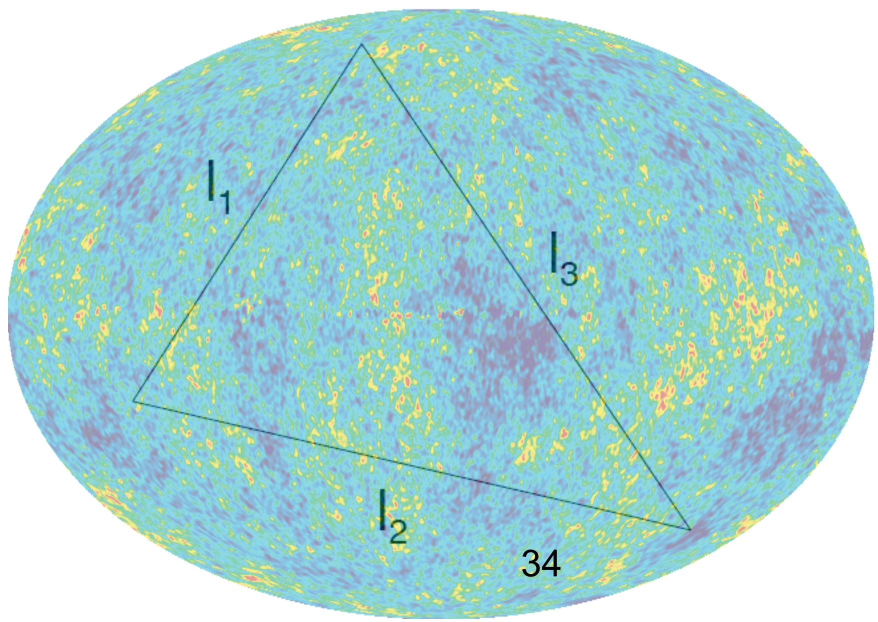
Komatsu et al. (2003); Spergel et al. (2007)

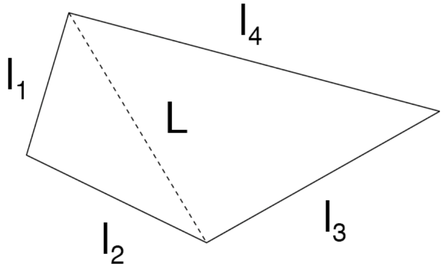
Bispectrum Constraints



$-58 < f_{NL} < +134$ (95% CL) (1yr)

$-54 < f_{NL} < +114$ (95% CL) (3yr)





Okamoto & Hu (2002); Kogo & Komatsu (2006)

Trispectrum of CMB

$$\langle a_{l_1m_1}a_{l_2m_2}a_{l_3m_3}a_{l_4m_4}\rangle = \sum_{LM}(-1)^M \begin{pmatrix} l_1 & l_2 & L \\ m_1 & m_2 & -M \end{pmatrix}\begin{pmatrix} l_3 & l_4 & L \\ m_3 & m_4 & M \end{pmatrix} T^{l_1l_2}_{l_3l_4}(L)$$

$$T_c{}^{l_1l_2}_{l_3l_4}(L) = P^{l_1l_2}_{l_3l_4}(L) + (2L+1)\sum_{L'}\left[(-1)^{l_2+l_3}\begin{Bmatrix} l_1 & l_2 & L \\ l_4 & l_3 & L' \end{Bmatrix} P^{l_1l_3}_{l_2l_4}(L') + (-1)^{L+L'}\begin{Bmatrix} l_1 & l_2 & L \\ l_3 & l_4 & L' \end{Bmatrix} P^{l_1l_4}_{l_3l_2}(L')\right],$$

where

$$P^{l_1l_2}_{l_3l_4}(L) = t^{l_1l_2}_{l_3l_4}(L) + (-1)^{2L+l_1+l_2+l_3+l_4} t^{l_2l_1}_{l_4l_3}(L) + (-1)^{L+l_3+l_4} t^{l_1l_2}_{l_4l_3}(L) + (-1)^{L+l_1+l_2} t^{l_2l_1}_{l_3l_4}(L).$$

$$\begin{aligned} t^{l_1l_2}_{l_3l_4}(L) &= \int r_1^2dr_1r_2^2dr_2\ F_L(r_1,r_2)\alpha_{l_1}(r_1)\beta_{l_2}(r_1)\alpha_{l_3}(r_2)\beta_{l_4}(r_2)h_{l_1Ll_2}h_{l_3Ll_4} \\ &+ \int r^2dr\ \beta_{l_2}(r)\beta_{l_4}(r)\left[\mu_{l_1}(r)\beta_{l_3}(r) + \beta_{l_1}(r)\mu_{l_3}(r)\right]h_{l_1Ll_2}h_{l_3Ll_4}, \end{aligned}$$

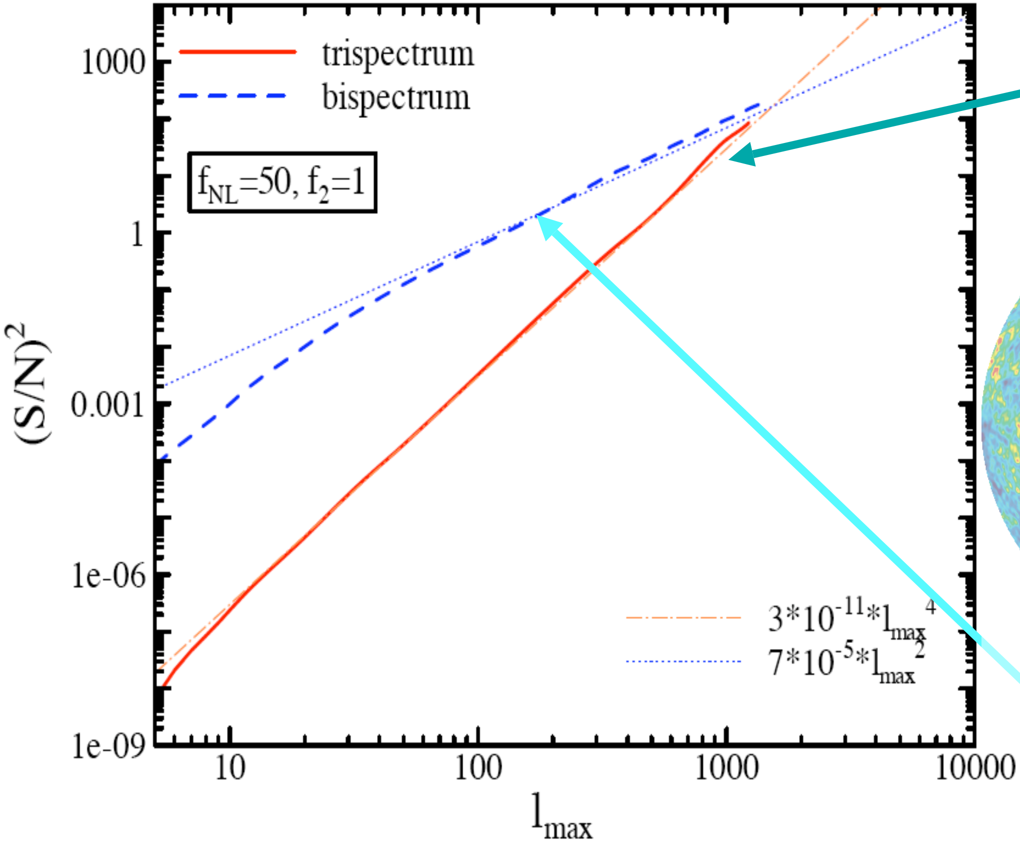
$\text{alpha}_l(r)=2b_l^{NL}(r)$; $\text{beta}_l(r)=b_l^{L}(r)$; $\mu_l(r) \equiv \frac{2}{\pi}\int k^2dk\ f_2 g_{Tl}(k) j_l(kr)$

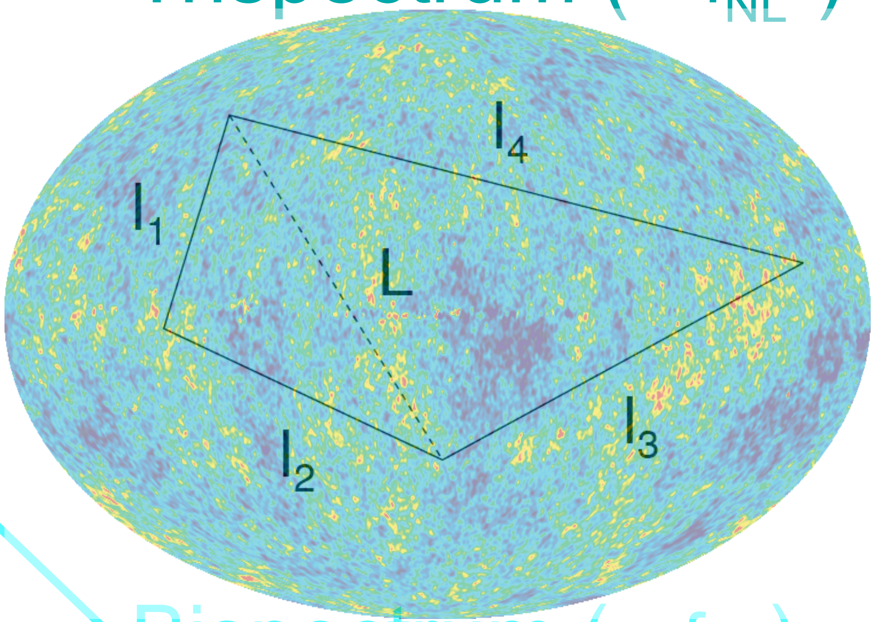
Measuring Trispectrum

- It's pretty painful to measure all the quadrilateral configurations.
  - Measurements from the COBE 4-year data (Komatsu 2001; Kunz et al. 2001)
- Only limited configurations measured from the WMAP 3-year data
  - Spergel et al. (2007)
- No evidence for non-Gaussianity, but f_{NL} has not been constrained by the trispectrum yet. (Work to do.)

Kogo & Komatsu (2006)

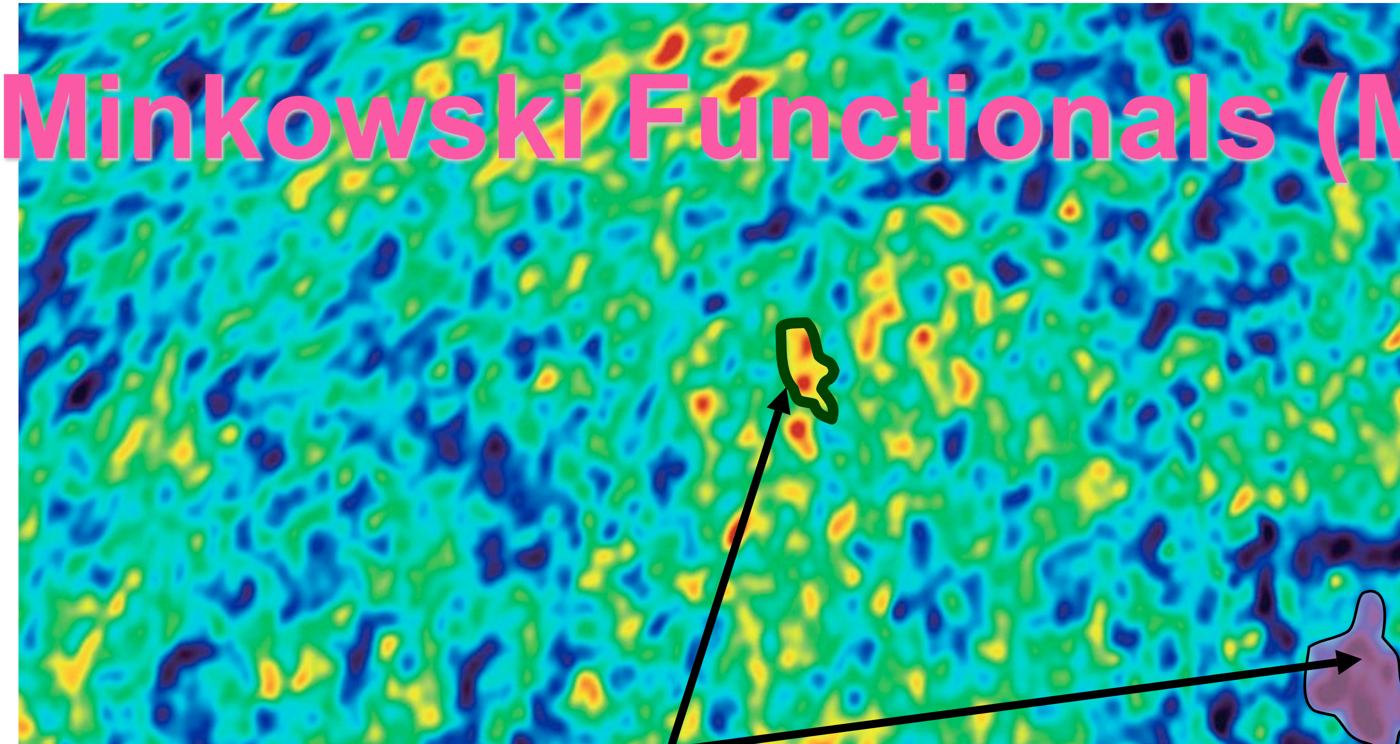
Trispectrum: Not useful for WMAP, but maybe useful for Planck, if f_{NL} is greater than ~50





- Trispectrum (~ f_{NL}^2)
- Bispectrum (~ f_{NL})

Minkowski Functionals (MFs)

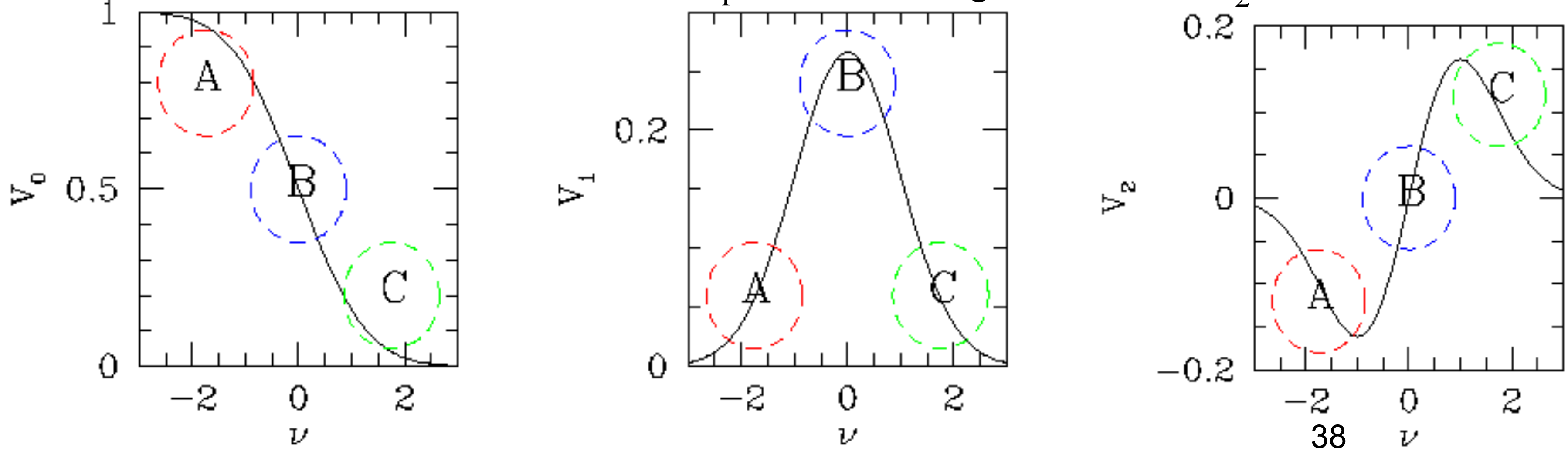


The number of hot spots minus cold spots.

V_0:surface area

V_1: Contour Length

V_2: Euler Characteristic



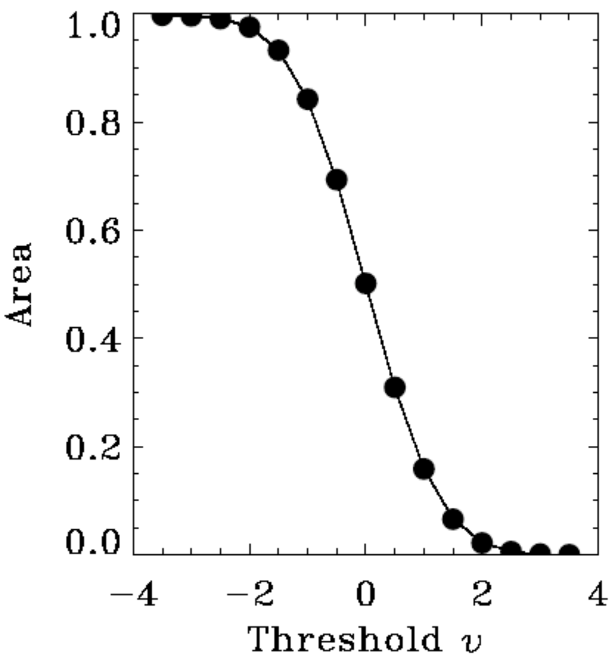
Komatsu et al. (2003); Spergel et al. (2007); Hikage et al. (2008)

MFs from *WMAP*

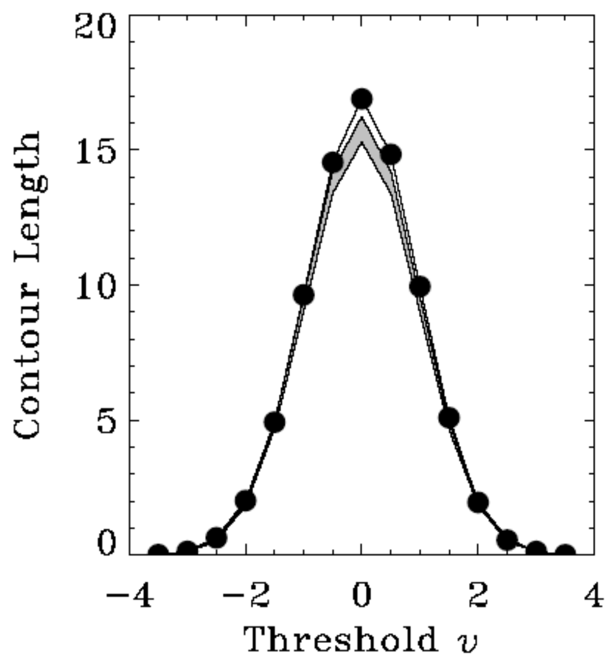
(1yr)

(3yr)

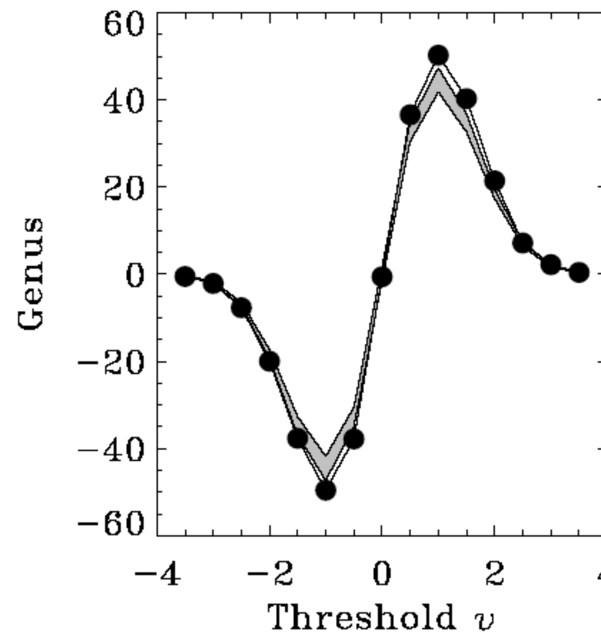
f_{NL} < +117 (95% CL) → **-70 < f_{NL} < +90** (95% CL)



Area



Contour Length



Euler Characteristic

Gaussianity vs Flatness: Future

- **Flatness will never beat Gaussianity.**
  - In 5-10 years, we will know **flatness** to **0.1%** level.
  - In 5-10 years, we will know **Gaussianity** to **0.01%** level (f_{NL}~10), or even to **0.005%** level (f_{NL}~5), at 95% CL.
- However, a real potential of Gaussianity test is that **we might detect something at this level** (multi-field, curvaton, DBI, ghost cond., new ekpyrotic…)

More On Future Prospects

- CMB: Planck (temperature + polarization): **f_{NL}(local)<6 (95%)**
  - Yadav, Komatsu & Wandelt (2007)
- Large-scale Structure: e.g., ADEPT, CIP: **f_{NL}(local)<7 (95%); f_{NL}(equilateral)<90 (95%)**
  - Sefusatti & Komatsu (2007)
- CMB and LSS are independent. By combining these two constraints, we get **f_{NL}(local)<4.5**. This is currently the best constraint that we can possibly achieve in the foreseeable future (~10 years)

If f_{NL} is found, what are the implications?

Three Sources of Non-Gaussianity

- It is important to remember that f_{NL} receives **<u>three contributions</u>**:

1. Non-linearity in inflaton fluctuations, $\delta\phi$
   - Falk, Rangarajan & Srendnicki (1993)
   - Maldacena (2003)
2. Non-linearity in Φ-$\delta\phi$ relation
   - Salopek & Bond (1990; 1991)
   - Matarrese et al. (2nd order PT papers)
   - δN papers; gradient-expansion papers
3. Non-linearity in $\Delta T/T$-Φ relation
   - Pyne & Carroll (1996)
   - Mollerach & Matarrese (1997)

1. Generating Non-Gaussian $\delta\phi$

- You need cubic interaction terms (or higher order) of fields.
  - $V(\phi)\sim\phi^3$: Falk, Rangarajan & Srendnicki (1993) [gravity not included yet]
  - Full expansion of the action, including gravity action, to cubic order was done a decade later by Maldacena (2003)

$$\phi = \phi(t) + \varphi(t,x)$$

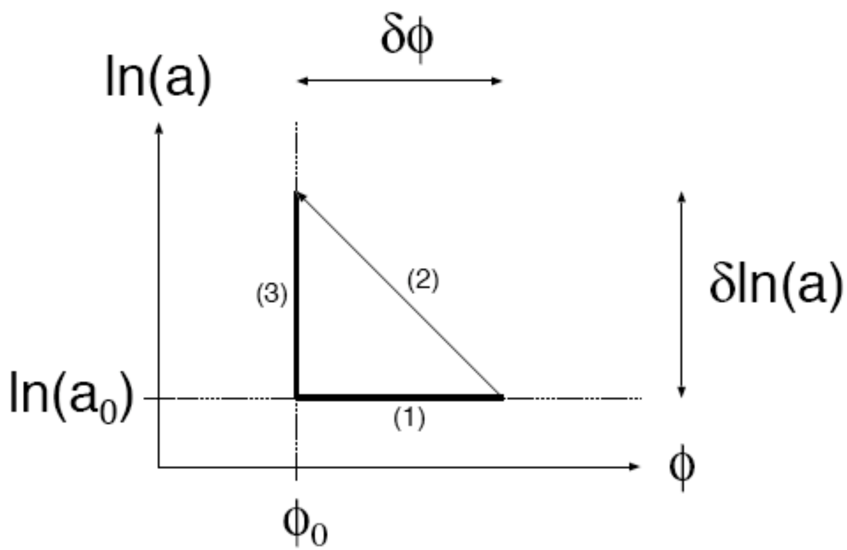
$$\partial^2\chi = \frac{\dot\phi^2}{2\dot\rho^2}\frac{d}{dt}\left(-\frac{\dot\rho}{\dot\phi}\varphi\right)$$

$$h_{ij} = e^{2\rho}\hat h_{ij}$$

$$S_3 = \int e^{3\rho}\left(-\frac{\dot\phi}{4\dot\rho}\varphi\dot\varphi^2 - e^{-2\rho}\frac{\dot\phi}{4\dot\rho}\varphi(\partial\varphi)^2 - \dot\varphi\partial_i\chi\partial_i\varphi + \right.$$
$$+\frac{3\dot\phi^3}{8\dot\rho}\varphi^3 - \frac{\dot\phi^5}{16\dot\rho^3}\varphi^3 - \frac{\dot\phi V''}{4\dot\rho}\varphi^3 - \frac{V'''}{6}\varphi^3 + \frac{\dot\phi^3}{4\dot\rho^2}\varphi^2\dot\varphi + \frac{\dot\phi^2}{4\dot\rho}\varphi^2\partial^2\chi$$
$$\left. + \frac{\dot\phi}{4\dot\rho}(-\varphi\partial_i\partial_j\chi\partial_i\partial_j\chi + \varphi\partial^2\chi\partial^2\chi)\right)$$

2. Non-linear Mapping

- The observable is the curvature perturbation, R. How do we relate R to the scalar field perturbation $\delta\phi$?
- Hypersurface transformation (Salopek & Bond 1990); a.k.a. δN formalism.



(1)Scalar field perturbation
(2)Evolve the scale factor, a, until ϕ matches ϕ_0
(3)$R = \ln(a) - \ln(a_0)$

Komatsu, astro-ph/0206039

Result of Non-linear Mapping

$$N = -\frac{4\pi G}{\partial H/\partial \phi}$$ [N is the Lapse function.]

$$\mathcal{R}_{\rm com} = -\int_{\phi_0}^{\phi_0+\delta\phi_{\rm flat}} d\phi\, \frac{N(\phi)H(\phi)}{\dot{\phi}} = 4\pi G \int_{\phi_0}^{\phi_0+\delta\phi_{\rm flat}} d\phi \left[\frac{\partial \ln H}{\partial \phi}\right]^{-1}$$

Expand R to the quadratic order in $\delta\phi$:

$$\mathcal{R}_{\rm com} = \mathcal{R}^{\rm L}_{\rm com} + \mathcal{R}^{\rm NL}_{\rm com}$$

$$\mathcal{R}^{\rm L}_{\rm com} = 4\pi G \left(\frac{\partial \ln H}{\partial \phi}\right)^{-1} \delta\phi_{\rm flat},$$

$$\mathcal{R}^{\rm NL}_{\rm com} = -\frac{1}{8\pi G}\left(\frac{\partial^2 \ln H}{\partial \phi^2}\right)\left(\mathcal{R}^{\rm L}_{\rm com}\right)^2.$$

$$f_{\rm NL} = -\frac{5}{24\pi G}\left(\frac{\partial^2 \ln H}{\partial \phi^2}\right) \approx -\frac{5}{48\pi G}\left(\frac{\partial^2 \ln V}{\partial \phi^2}\right).$$ [For Gaussian $\delta\phi$]

For standard slow-roll inflation models, this is of order the slow-roll parameters, O(0.01).

Multi-field Generalization

$$\mathcal{R}_{\rm com} = -\int_{\phi_{0A}}^{\phi_{0A}+\delta\phi_{{\rm flat}A}} d\phi_A \frac{N(\phi_A)H(\phi_A)}{\dot{\phi}_A}$$

A=1,..., # of fields in the system

Then, again by expanding R to the quadratic order in $\delta\phi_A$, one can find f_{NL} for the multi-field case.

Example: the curvaton scenario, in which the second derivative of the integrand with respect to ϕ_2, the "curvaton field," divided by the square of the first derivative is much larger than slow-roll param.

3. Curvature Perturbation to CMB

- The linear Sachs-Wolfe effect is given by $dT/T = -(1/3)\Phi_H = +(1/3)\Phi_A$
- The non-linear SW effect is

$$\frac{\Delta T}{T} = \frac{1}{3}\Phi_A + \frac{1}{18}\Phi_A^2 - \nabla^{-4}\partial_i\partial^j(\partial^i\Phi_H\partial_j\Phi_H) - \frac{1}{3}\nabla^{-2}(\partial^i\Phi_H\partial_i\Phi_H)$$

where time-dependent terms (called the integrated SW effect) are not shown. (Bartolo et al. 2004)

- These terms generate f_{NL} of order unity.

Implications of a detection of f_{NL}, if it is found

- f_{NL} never exceeds 10 in the conventional picture of inflation in which
  - All fields are **slowly rolling**, and
  - All fields have the **canonical kinetic term**.
- Therefore, an unambiguous detection of $f_{NL} > 10$ rules out most (>99%) of the existing inflation models.
- Who would the "survivors" be?

3 Ways to Get Larger Non-Gaussianity from Early Universe

1. **Break slow-roll**
   – Features (steps, bumps…) in $V(\phi)$
     - Kofman, Blumenthal, Hodges & Primack (1991); Wang & Kamionkowski (2000); Komatsu et al. (2003); Chen, Easther & Lim (2007)

   – Ekpyrotic model, old and new
     - Buchbinder, Khoury & Ovrut (2007); Koyama, Mizuno, Vernizzi & Wands (2007)

3 Ways to Get Larger Non-Gaussianity from Early Universe

2. Amplify field interactions

– Often done by **non-canonical kinetic terms**

$$S=\int d^4x\,\frac{1}{2}\dot{\pi}^2-\frac{\alpha^2}{2M^2}(\nabla^2\pi)^2-\frac{\beta}{2M^2}\dot{\pi}(\nabla\pi)^2+\cdots$$

- Ghost inflation
  - Arkani-Hamed, Creminelli, Mukohyama & Zaldarriaga (2004)

$$\mathcal{L}_{\rm eff}=-\frac{1}{g_s}\sqrt{-g}\left(f(\phi)^{-1}\sqrt{1+f(\phi)g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi}+V(\phi)\right)$$

- DBI Inflation
  - Alishahiha, Silverstein & Tong (2004)

– Any other models with a low effective sound speed of scalar field: they yield $f_{NL}\sim -1/(c_s)^2$

- Chen, Huang, Kachru & Shiu (2004); Cheung, Creminelli, Fitzpatrick, Kaplan & Senatore (2007)

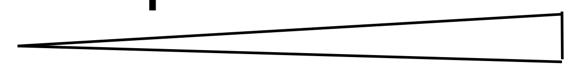
3 Ways to Get Larger Non-Gaussianity from Early Universe

3. Use multi-field:

- A class of multi-models called "curvaton models" can generate large non-Gaussianity
  - Linde & Mukhanov (1997); Lyth & Wands (2002); Lyth, Ungarelli & Wands (2002)

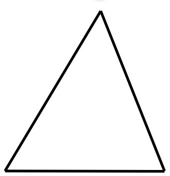
Subtlety: Triangle Dependence

- There are actually two f_{NL}
  - "Local," which has the largest amplitude in the squeezed configuration



Local

Eq.



  - "Equilateral," which has the largest amplitude in the equilateral configuration

- So the question is, "which model gives f_{NL}(local), and which f_{NL}(equilateral)?"

Classifying Non-Gaussianities in the Literature

- Local Form
  - Ekpyrotic models
  - Curvaton models
- Equilateral Form
  - Ghost condensation, DBI, low speed of sound models
- Other Forms
  - Features in potential, which produce large non-Gaussianity within narrow region in l

•Is any of these a winner?
•Non-Gaussianity may tell us soon. We will find out!

Summary

- Since the introduction of f_{NL}, the research on non-Gaussianity as a probe of the physics of early universe has evolved tremendously.
- I hope I convinced you that f_{NL} is as important a tool as $\Omega_{\mathbf{K}}$, n_s, $dn_s/d\ln k$, and r, for constraining inflation models.
- In fact, it has the best chance of ruling out the largest population of models...

Concluding Remarks

- Stay tuned: WMAP continues to observe, and Planck will soon be launched (Oct 31, this year)
- Non-Gaussianity has provided cosmologists, and physicists who work on fundamental physics, with a unique opportunity to work together.
- This is probably the most important contribution that non-Gaussianity has made to the community.