Testing Physics of the Early Universe **Observationally**: *Are Primordial Fluctuations Gaussian, or Non-Gaussian?*

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How Do We Test Inflation?

- How can we answer a simple question like this:
  - "*How were primordial fluctuations generated?*"

Power Spectrum

- A very successful explanation (Guth & Pi; Hawking; Starobinsky; Bardeen, Steinhardt & Turner) is:
  - *Primordial fluctuations were generated by quantum fluctuations of the scalar field that drove inflation.*
  - The prediction: a nearly scale-invariant power spectrum in the curvature perturbation, ζ:
    - **$P_\zeta(k) = A/k^{4-n_s}$**
    - where $n_s \sim 1$ and A is a normalization.

Komatsu et al. (2009)

n_s<1 Observed

- The latest results from the WMAP 5-year data:
  - **n_s=0.960 ± 0.013** (68%CL; for tensor modes = zero)
  - **n_s=0.970 ± 0.015** (68%CL; for tensor modes ≠ zero)
    - tensor-to-scalar ratio **< 0.22** (95%CL)
- Another evidence for inflation
- Detection of non-zero tensor modes is a next important step

Anything Else?

- One can also look for other signatures of inflation. For example:
  - **Isocurvature perturbations**
    - Proof of the existence of multiple fields
  - **Non-zero spatial curvature**
    - Evidence for the Land Scape, if curvature is negative. Rules it out if positive.
  - **Scale-dependent n_s (running index)**
    - Complex dynamics of inflation

Komatsu et al. (2009)

Anything Else?

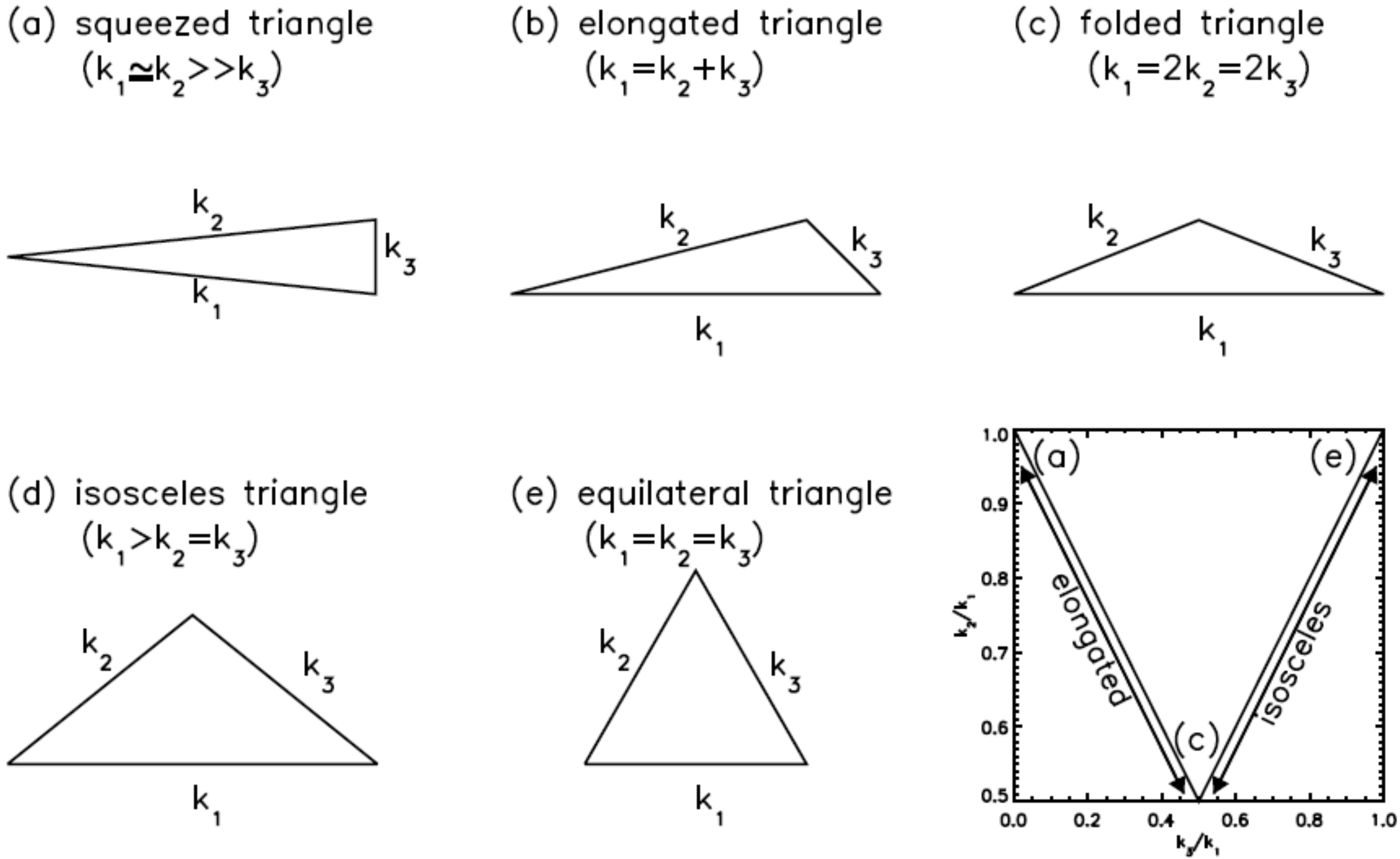
- One can also look for other signatures of inflation. For example:
  - 95%CL limits on **Isocurvature perturbations**
    - **<8.9%** (axion CDM); **<2.1%** (curvaton CDM)
  - 95%CL limits on **Non-zero spatial curvature**
    - **<1.8%** (positive curvature); **<0.8%** (negative curvature)
  - 95%CL limits on **Scale-dependent n_s**
    - $-0.068 < dn_s/d\ln k < 0.012$

Beyond Power Spectrum

- All of these are based upon fitting the observed power spectrum.
- Is there any information one can obtain beyond the power spectrum?

Bispectrum

- Three-point function!
- $B_\zeta(\mathbf{k}_1,\mathbf{k}_2,\mathbf{k}_3)$ = (amplitude) x $(2\pi)^3\delta(\mathbf{k}_1+\mathbf{k}_2+\mathbf{k}_3)b(k_1,k_2,k_3)$



(a) squeezed triangle
(k1≃k2>>k3)
(b) elongated triangle
(k1=k2+k3)
(c) folded triangle
(k1=2k2=2k3)
(d) isosceles triangle
(k1>k2=k3)
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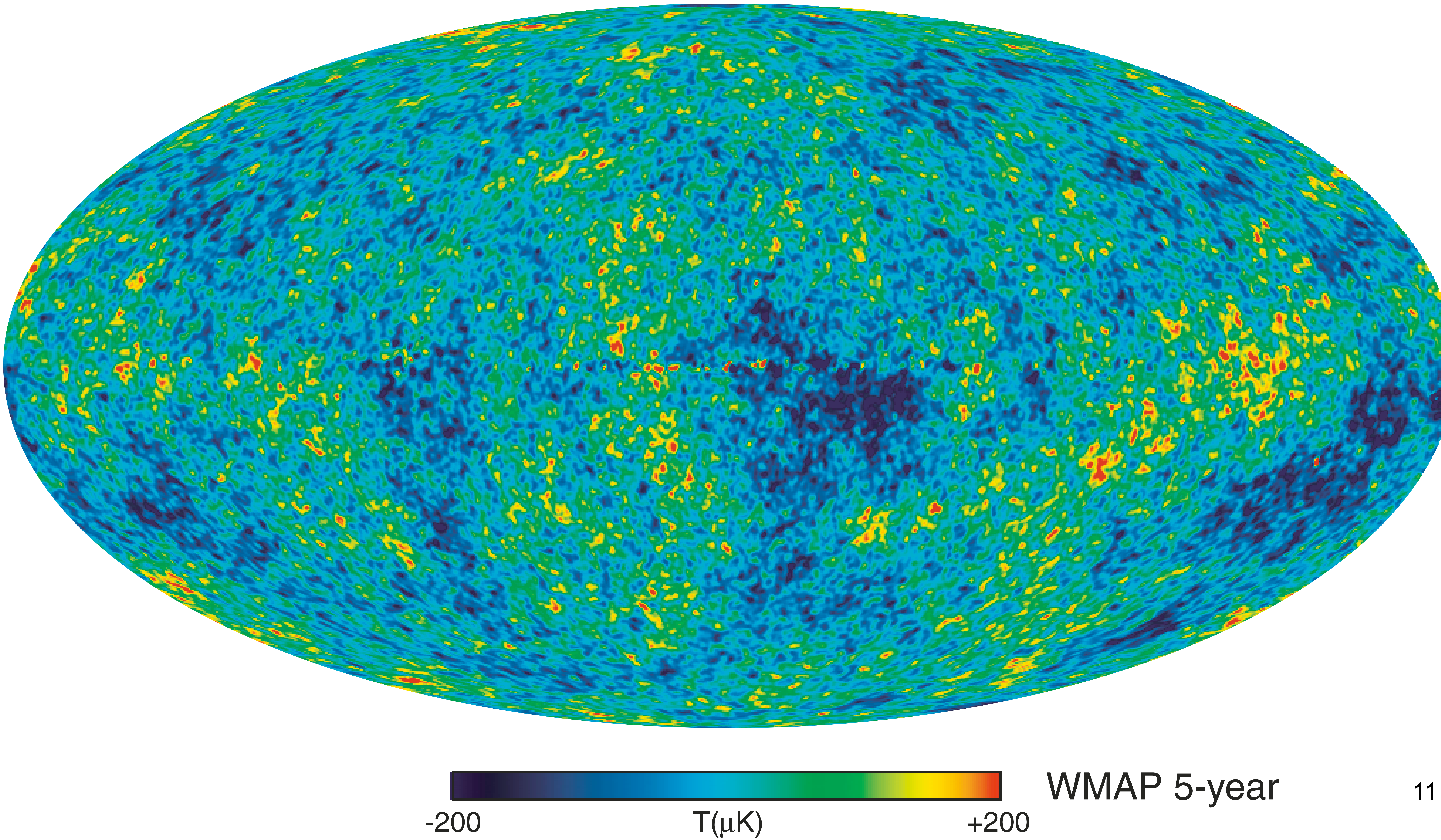
k1
k2
k3
k2/k1
k3/k1
1.0
0.9
0.8
0.7
0.6
0.5
0.0
0.2
0.4
0.6
0.8
1.0
(a)
(e)
(c)
elongated
isosceles

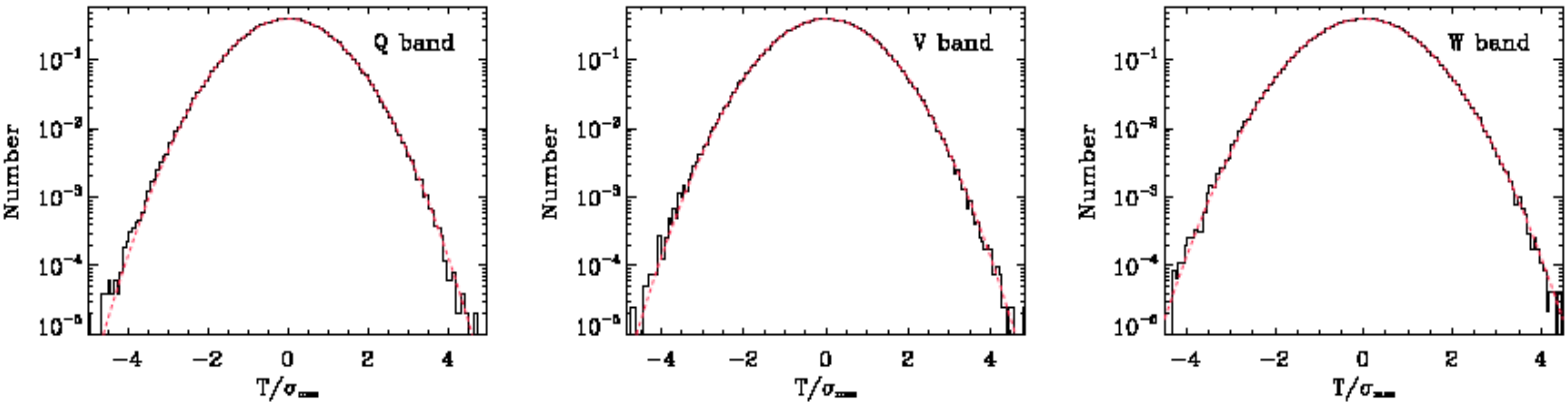
Why Study Bispectrum?

- It probes the interactions of fields - new piece of information that cannot be probed by the power spectrum
- But, above all, it provides us with a **critical test** of the simplest models of inflation: "***are primordial fluctuations Gaussian, or non-Gaussian?***"
- Bispectrum vanishes for Gaussian fluctuations.
- Detection of the bispectrum = detection of non-Gaussian fluctuations

Gaussian?



Take One-point Distribution Function



- The one-point distribution of WMAP map looks pretty Gaussian.
  - –Left to right: Q (41GHz), V (61GHz), W (94GHz).
- Deviation from Gaussianity is small, if any.

Inflation Likes This Result

- According to inflation (Guth & Yi; Hawking; Starobinsky; Bardeen, Steinhardt & Turner), CMB anisotropy was created from **quantum fluctuations of a scalar field in Bunch-Davies vacuum** during inflation
- Successful inflation (with the expansion factor more than e^{60}) *demands* the scalar field be almost interaction-free
- The wave function of free fields in the ground state is a Gaussian!

But, Not Exactly Gaussian

- Of course, there are always corrections to the simplest statement like this
- For one, inflaton field **does** have interactions. They are simply weak – of order the so-called slow-roll parameters, ε and η, which are O(0.01)

Non-Gaussianity from Inflation

- You need cubic interaction terms (or higher order) of fields.
  - $V(\phi)\sim\phi^3$: *Falk, Rangarajan & Srendnicki (1993)* [gravity not included yet]
  - Full expansion of the action, including gravity action, to cubic order was done a decade later by *Maldacena (2003)*

$$\phi = \phi(t) + \varphi(t,x)$$

$$\partial^2\chi = \frac{\dot\phi^2}{2\dot\rho^2}\frac{d}{dt}\left(-\frac{\dot\rho}{\dot\phi}\varphi\right)$$

$$h_{ij} = e^{2\rho}\hat{h}_{ij}$$

$$S_3 = \int e^{3\rho}\left(-\frac{\dot\phi}{4\dot\rho}\varphi\dot\varphi^2 - e^{-2\rho}\frac{\dot\phi}{4\dot\rho}\varphi(\partial\varphi)^2 - \dot\varphi\partial_i\chi\partial_i\varphi + \right.$$
$$+\frac{3\dot\phi^3}{8\dot\rho}\varphi^3 - \frac{\dot\phi^5}{16\dot\rho^3}\varphi^3 - \frac{\dot\phi V''}{4\dot\rho}\varphi^3 - \frac{V'''}{6}\varphi^3 + \frac{\dot\phi^3}{4\dot\rho^2}\varphi^2\dot\varphi + \frac{\dot\phi^2}{4\dot\rho}\varphi^2\partial^2\chi$$
$$\left. + \frac{\dot\phi}{4\dot\rho}(-\varphi\partial_i\partial_j\chi\partial_i\partial_j\chi + \varphi\partial^2\chi\partial^2\chi)\right)$$

Computing Primordial Bispectrum

- Three-point function, using in-in formalism (*Maldacena 2003; Weinberg 2005*)

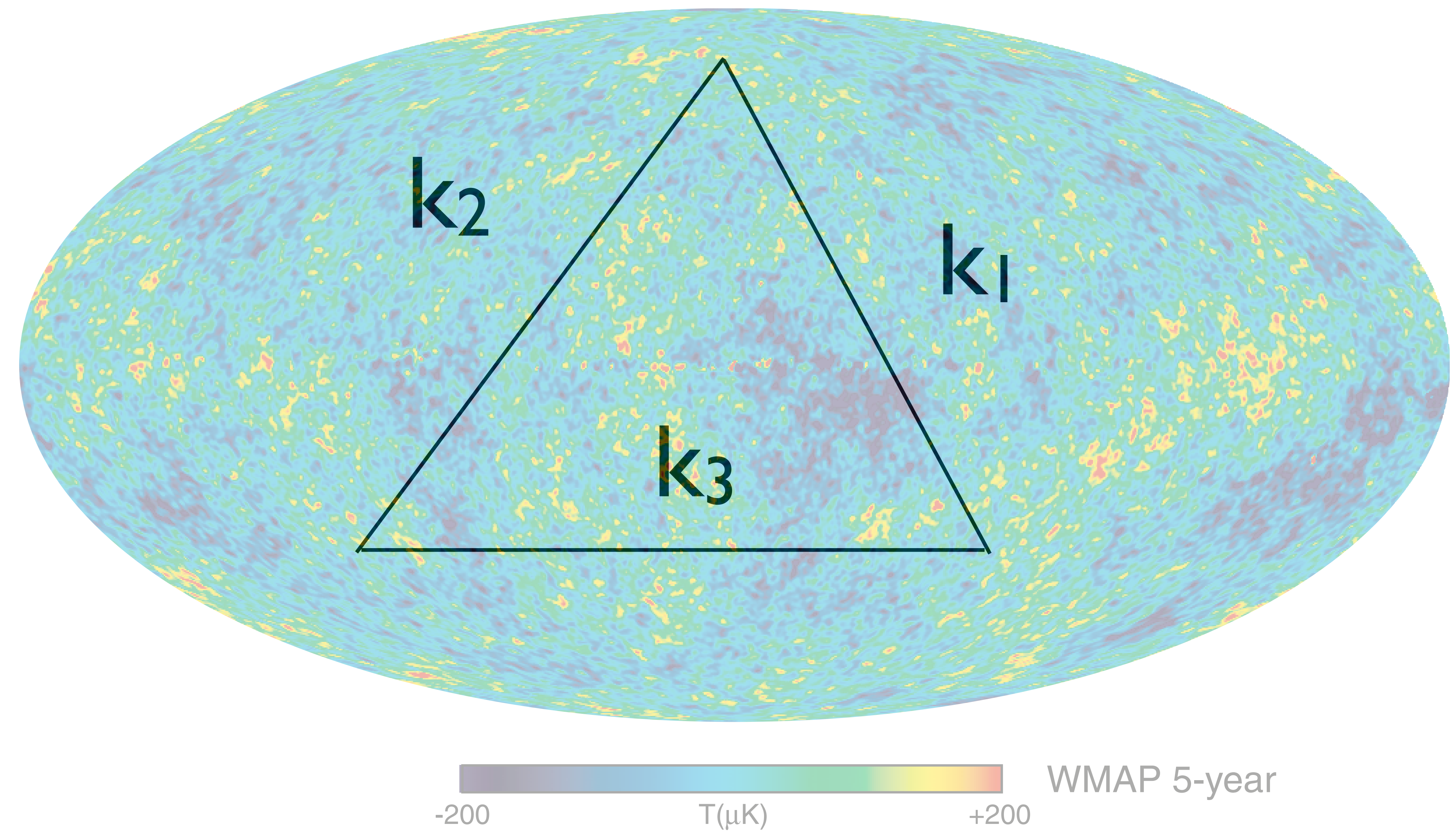
$$\text{3-point function}(\mathbf{x}_1, \mathbf{x}_2, \mathbf{x}_3) = \langle \text{in} \left| \tilde{T} e^{i \int_{-\infty}^{t} H_I(t')dt'} \Phi(\mathbf{x}_1)\Phi(\mathbf{x}_2)\Phi(\mathbf{x}_3) T e^{-i \int_{-\infty}^{t} H_I(t')dt'} \right| \text{in} \rangle$$

- $H_I(t)$: Hamiltonian in interaction picture
  - Model-dependent: this determines which triangle shapes will dominate the signal
- $\Phi(x)$: operator representing curvature perturbations in interaction picture

Why Study Bispectrum?

- Because a detection of the bispectrum has a best chance of **ruling out the largest class of inflation models**.
- Namely, it will rule out inflation models based upon
    - a single scalar field <u>with</u>
    - the canonical kinetic term <u>that</u>
    - rolled down a smooth scalar potential slowly, <u>and</u>
    - was initially in the Bunch-Davies vacuum.
- ***Detection of the bispectrum would be a major breakthrough in cosmology.***

“f_{NL}”



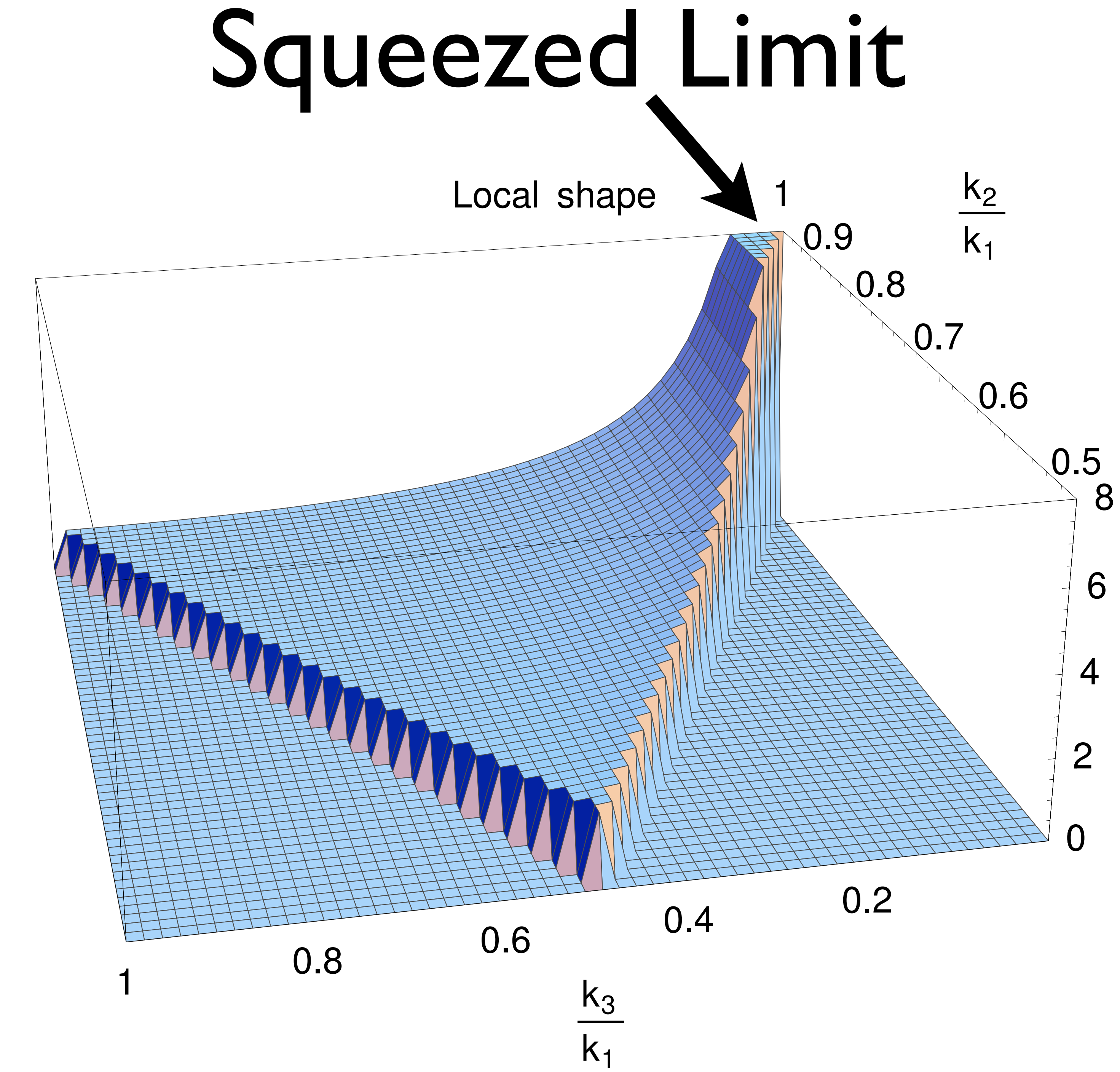
- **f_{NL} = the amplitude of bispectrum**, which is
  - $B_\zeta(\mathbf{k}_1,\mathbf{k}_2,\mathbf{k}_3)=\langle\zeta(\mathbf{k}_1)\zeta(\mathbf{k}_2)\zeta(\mathbf{k}_3)\rangle$

    $=f_{NL}(2\pi)^3\delta(\mathbf{k}_1+\mathbf{k}_2+\mathbf{k}_3)b(k_1,k_2,k_3)$
  - $b(k_1,k_2,k_3)$ is a model-dependent function that defines the shape of triangles predicted by various models.

Forms of $b(k_1,k_2,k_3)$

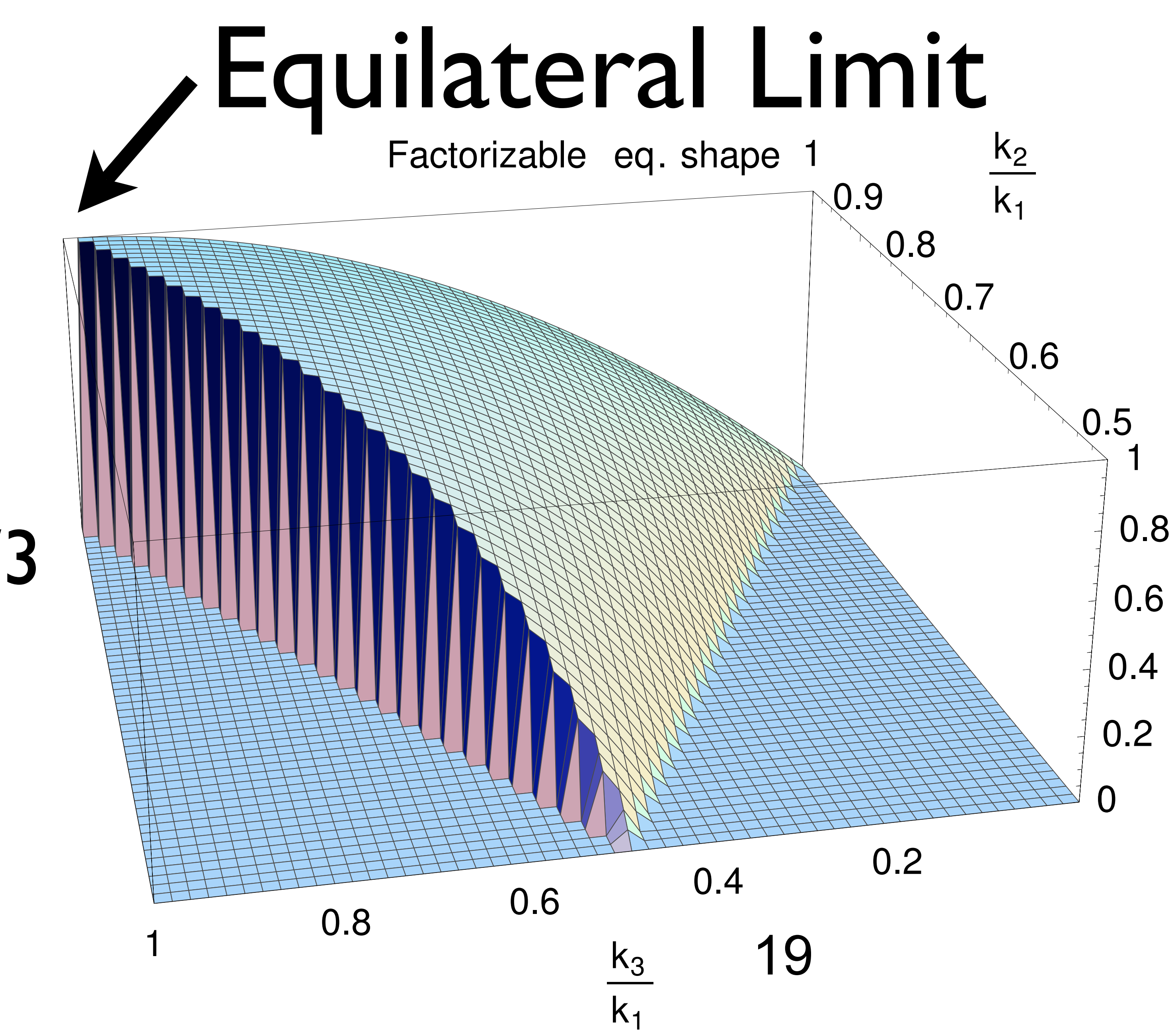
- Local form [*can be generated by multi-field models*]
  - $b^{local}(k_1,k_2,k_3) = (6/5)[P_\zeta(k_1)P_\zeta(k_2)+\text{cyc.}]$

Komatsu & Spergel (2001)



- Equilateral form [*can be generated by non-canonical kinetic terms, e.g., DBI*]
  - $b^{equilateral}(k_1,k_2,k_3) = (18/5)\{-[P_\zeta(k_1)P_\zeta(k_2)+\text{cyc.}] - 2[P_\zeta(k_1)P_\zeta(k_2)P_\zeta(k_3)]^{2/3} + [P_\zeta(k_1)^{1/3}P_\zeta(k_2)^{2/3}P_\zeta(k_3)+\text{cyc.}]\}$

Babich, Creminelli & Zaldarriaga (2004)



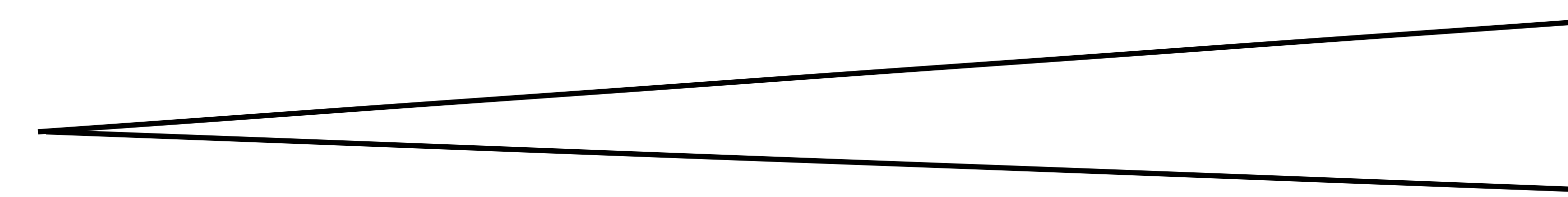
Local Form Non-Gaussianity

- The local form bispectrum,
  $B_\zeta(\mathbf{k}_1,\mathbf{k}_2,\mathbf{k}_3)=(2\pi)^3\delta(\mathbf{k}_1+\mathbf{k}_2+\mathbf{k}_3)f_{NL}^{local}[(6/5)P_\zeta(k_1)P_\zeta(k_2)+cyc.]$
- is equivalent to having the curvature perturbation in position space, in the form of:
  - $\zeta(\mathbf{x})=\zeta_{gaussian}(\mathbf{x}) + (3/5)f_{NL}[\zeta_{gaussian}(\mathbf{x})]^2$
    - This provides a useful model to parametrize non-Gaussianity, and generate initial conditions for, e.g., N-body simulations.
- This can be extended to higher-order:
  - $\zeta(\mathbf{x})=\zeta_{gaussian}(\mathbf{x}) + (3/5)f_{NL}[\zeta_{gaussian}(\mathbf{x})]^2 + (9/25)g_{NL}[\zeta_{gaussian}(\mathbf{x})]^3$

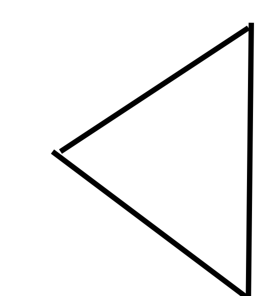
What if f_{NL} is detected?

- A single field, canonical kinetic term, slow-roll, and/or Bunch-Davies vacuum, must be modified.

Local • Multi-field (curvaton)

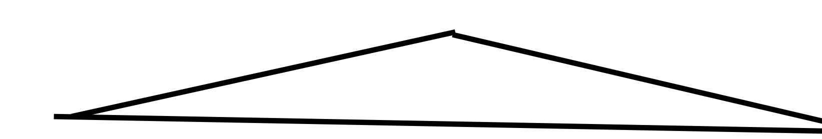


Equil. • Non-canonical kinetic term (k-inflation, DBI)



Bump +Osci. • Temporary fast roll (features in potential)

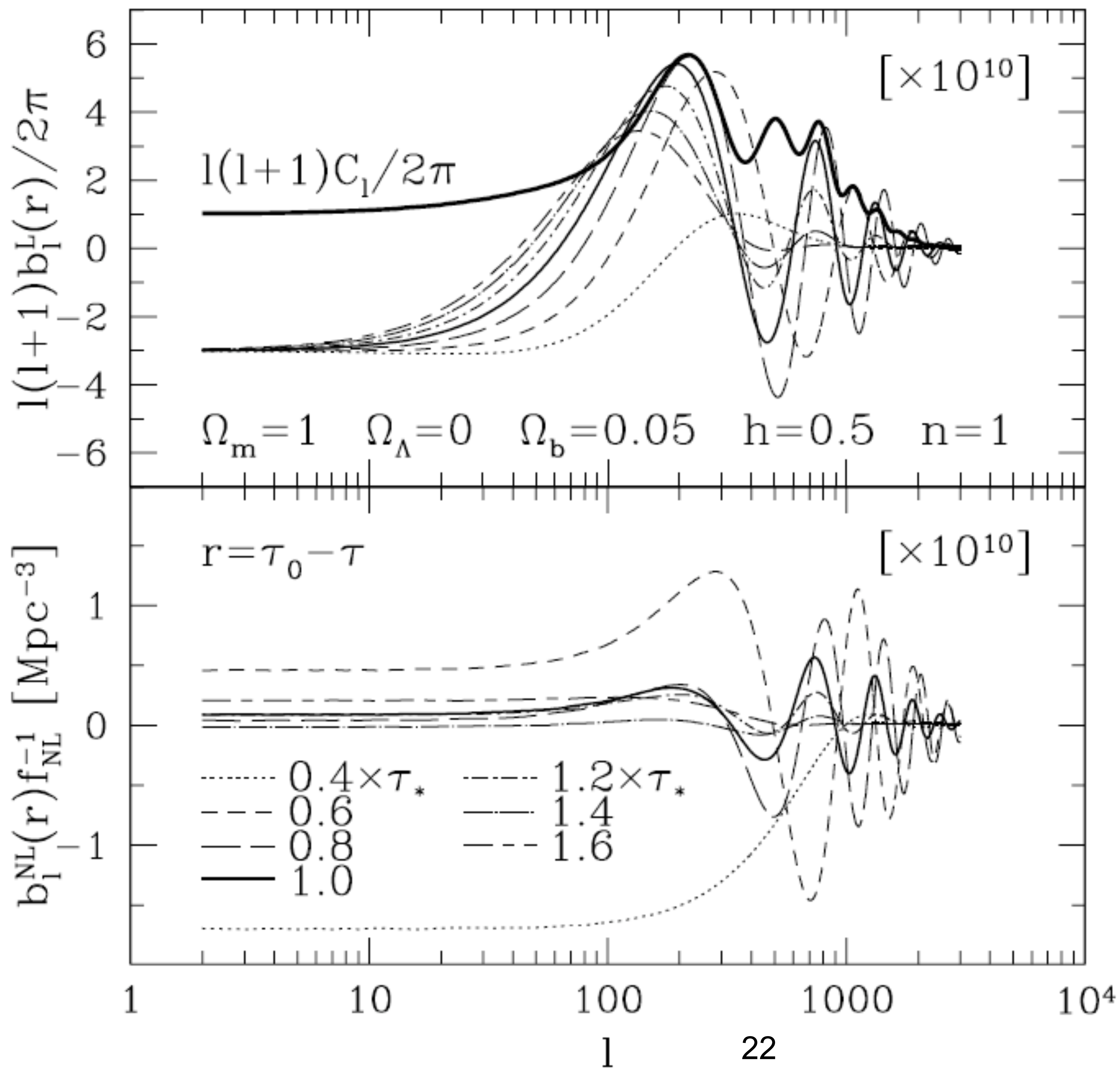
Folded • Departures from the Bunch-Davies vacuum



- *It will give us a lot of clues as to what the correct early universe models should look like.*

Decoding Bispectrum

- Hydrodynamics at z=1090 generates acoustic oscillations in the bispectrum
- Well understood at the linear level (*Komatsu & Spergel 2001*)
- Non-linear extension?
  - *Nitta, Komatsu, Bartolo, Matarrese & Riotto*, arXiv:0903.0894
  - f_{NL}^{local}~0.5



Measurement

- Use everybody's favorite: χ^2 minimization.
  - Minimize:

$$\chi^2 \equiv \sum_{2 \le l_1 \le l_2 \le l_3} \frac{\left(B^{obs}_{l_1 l_2 l_3} - \sum_i A_i B^{(i)}_{l_1 l_2 l_3} \right)^2}{\sigma^2_{l_1 l_2 l_3}}$$

  - with respect to $A_i=(f_{NL}^{local}, f_{NL}^{equilateral}, b_{src})$
  - B^{obs} is the observed bispectrum
  - $B^{(i)}$ is the theoretical template from various predictions

Journal on f_{NL} (95%CL)

- Local
  - $-3500 < f_{NL}^{local} < 2000$ [COBE 4yr, l_{max}=20] Komatsu et al. (2002)
  - $-58 < f_{NL}^{local} < 134$ [WMAP 1yr, l_{max}=265] Komatsu et al. (2003)
  - $-54 < f_{NL}^{local} < 114$ [WMAP 3yr, l_{max}=350] Spergel et al. (2007)
  - **$-9 < f_{NL}^{local} < 111$ [WMAP 5yr, l_{max}=500]** Komatsu et al. (2008)
- Equilateral
  - $-366 < f_{NL}^{equil} < 238$ [WMAP 1yr, l_{max}=405] Creminelli et al. (2006)
  - $-256 < f_{NL}^{equil} < 332$ [WMAP 3yr, l_{max}=475] Creminelli et al. (2007)
  - **$-151 < f_{NL}^{equil} < 253$ [WMAP 5yr, l_{max}=700]** Komatsu et al. (2008)

Latest on f_{NL}^{local}

(Fast-moving field!)

- CMB (WMAP5 + most optimal bispectrum estimator)
  - **$-4 < f_{NL}^{local} < 80$ (95%CL)** Smith et al. (2009)
  - $f_{NL}^{local} = 38 \pm 21$ (68%CL)
- Large-scale Structure (Using the SDSS power spectra)
  - **$-29 < f_{NL}^{local} < 70$ (95%CL)**
  - $f_{NL}^{local} = 31^{+16}_{-27}$ (68%CL) Slosar et al. (2009)

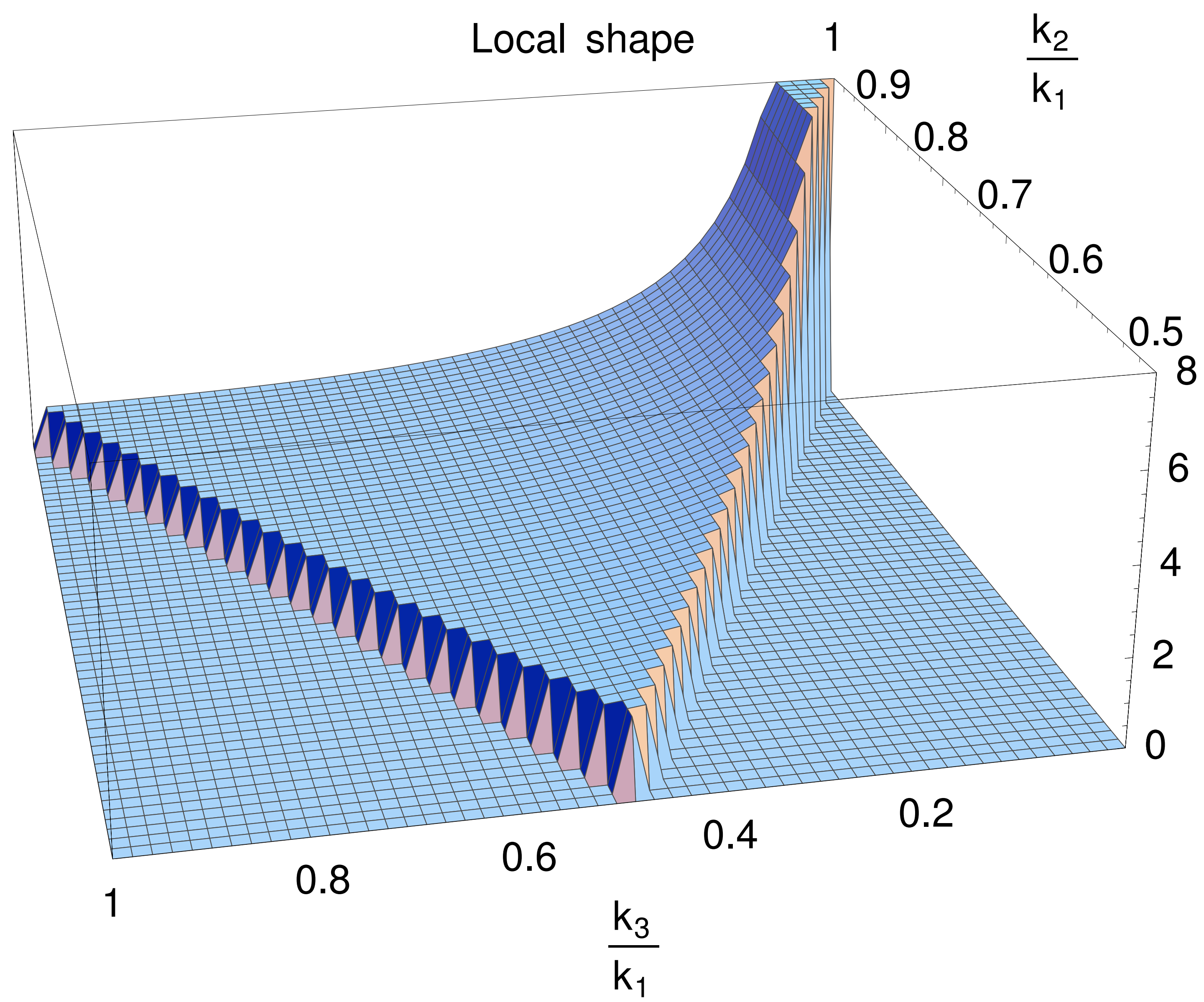
Exciting Future Prospects

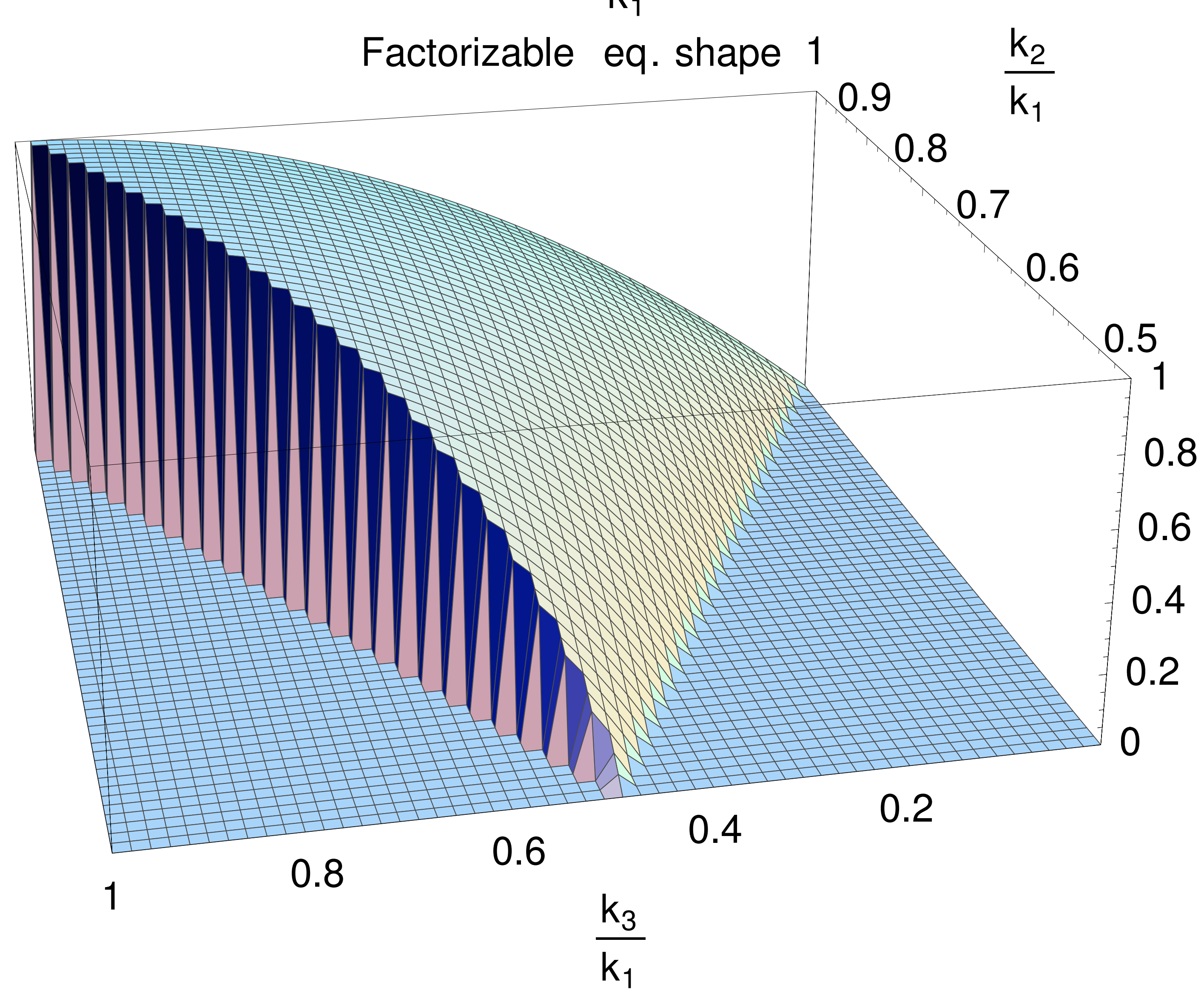
- Planck satellite (to be launched on May 6, 2009)
  - will see f_{NL}^{local} at **8σ**, IF (big if) $f_{NL}^{local}=40$

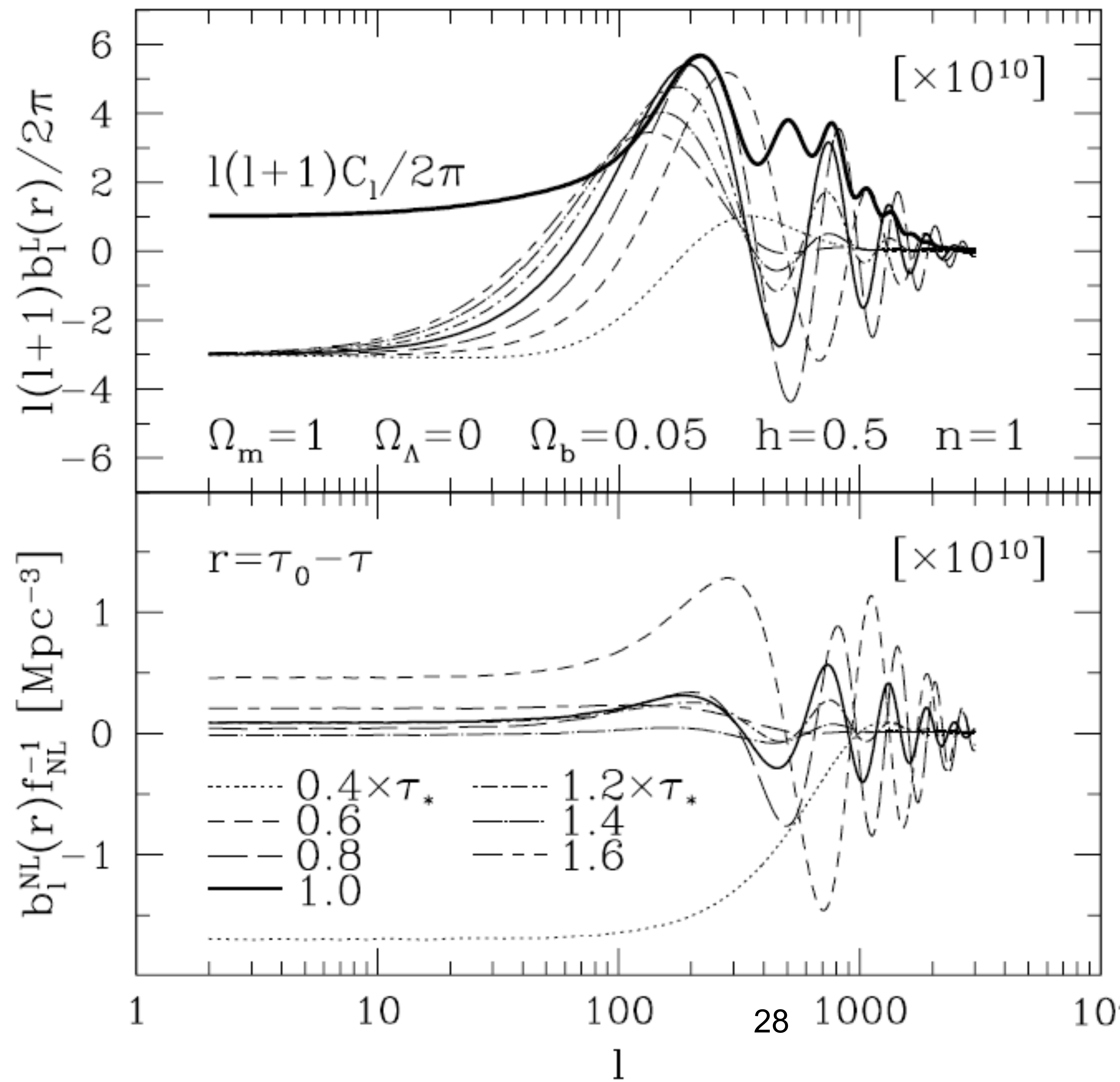
A Big Question

- Suppose that f_{NL} was found in, e.g., WMAP 9-year or Planck. That would be a profound discovery. ***However***:
  - **Q**: How can we convince ourselves and other people that primordial non-Gaussianity was found, rather than some junk?
  - **A**: (i) shape dependence of the signal, (ii) different statistical tools, and (iii) different tracers

(i) Remember These Plots?





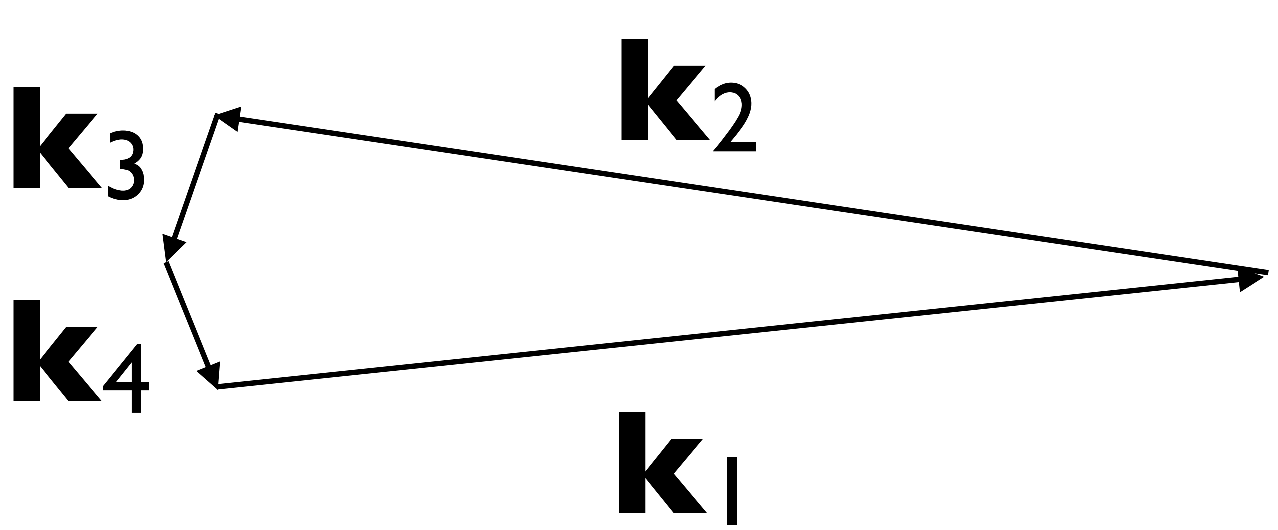


(ii) Different Tools

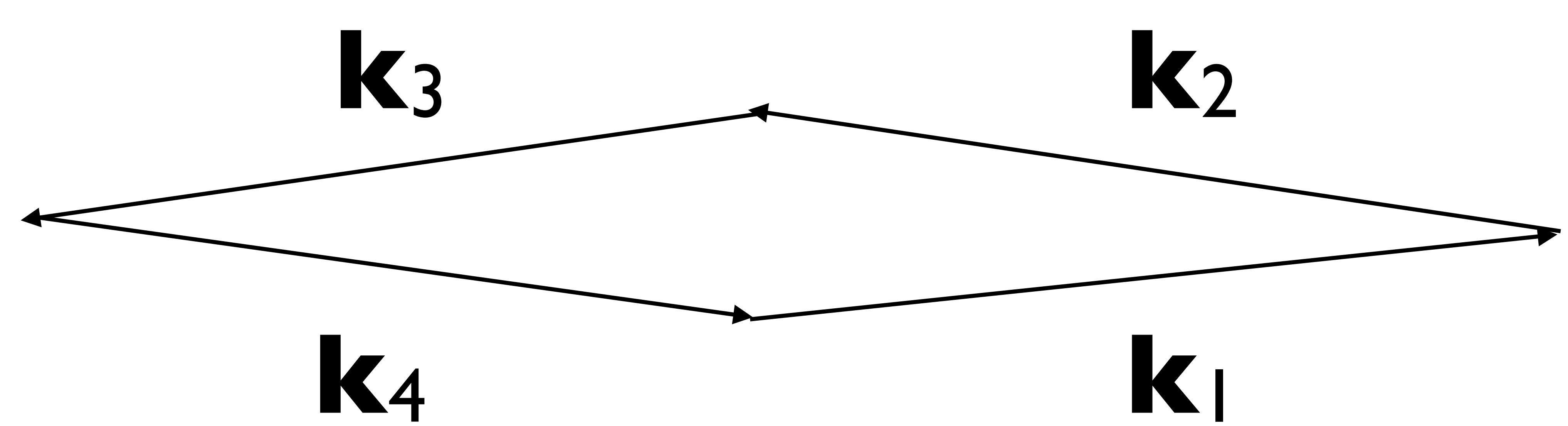
- How about 4-point function (trispectrum)?

Local Form Trispectrum

- For $\zeta(\mathbf{x})=\zeta_{gaussian}(\mathbf{x}) + (3/5)f_{NL}[\zeta_{gaussian}(\mathbf{x})]^2 + (9/25)g_{NL}[\zeta_{gaussian}(\mathbf{x})]^3$, we obtain the trispectrum:
  - $T_\zeta(\mathbf{k}_1,\mathbf{k}_2,\mathbf{k}_3,\mathbf{k}_4)=(2\pi)^3\delta(\mathbf{k}_1+\mathbf{k}_2+\mathbf{k}_3+\mathbf{k}_4)$ $\{\mathbf{g_{NL}}[(54/25)P_\zeta(k_1)P_\zeta(k_2)P_\zeta(k_3)+\text{cyc.}] +$ $\mathbf{(f_{NL})^2}[(18/25)P_\zeta(k_1)P_\zeta(k_2)(P_\zeta(|\mathbf{k}_1+\mathbf{k}_3|)+P_\zeta(|\mathbf{k}_1+\mathbf{k}_4|))+\text{cyc.}]\}$



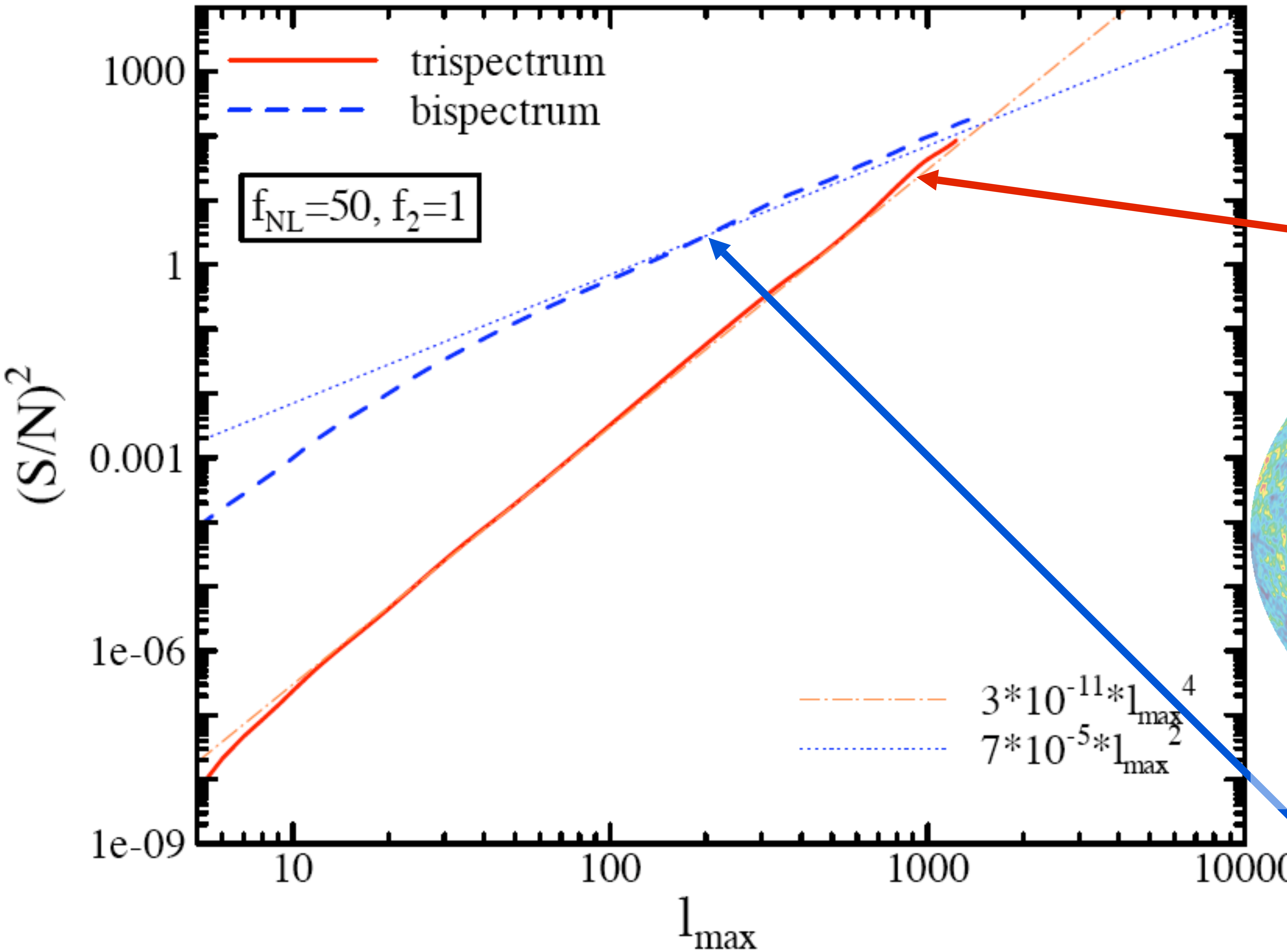
g_{NL}



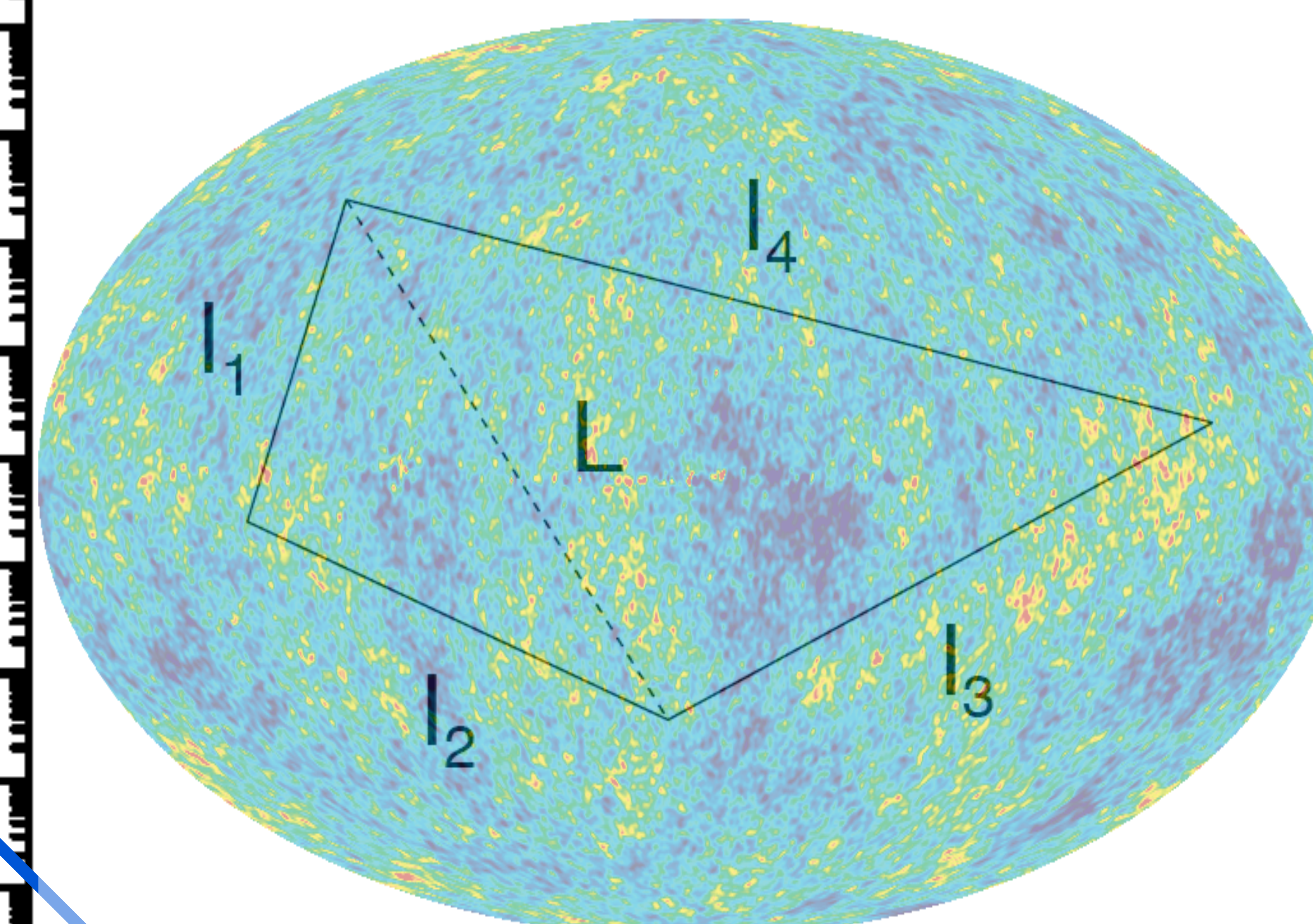
$f_{NL}{}^2$

Kogo & Komatsu (2006)

Trispectrum: if f_{NL} is ~50, excellent cross-check for Planck



Trispectrum (~f_{NL}^2)

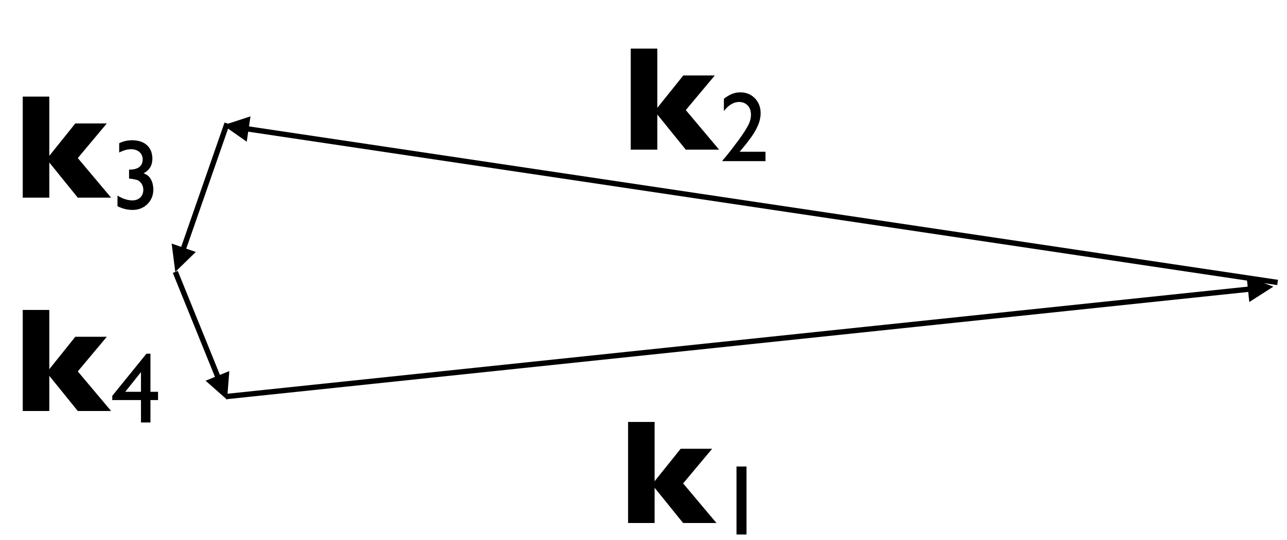


Bispectrum (~ f_{NL})

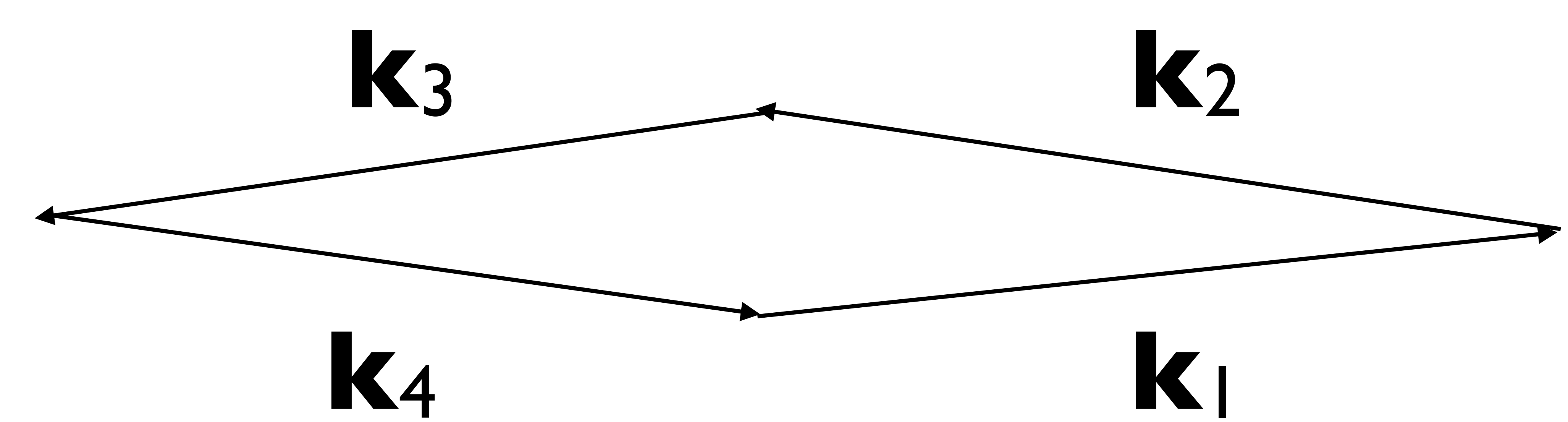
(Slightly) Generalized Trispectrum

- $T_\zeta(\mathbf{k}_1,\mathbf{k}_2,\mathbf{k}_3,\mathbf{k}_4)=(2\pi)^3\delta(\mathbf{k}_1+\mathbf{k}_2+\mathbf{k}_3+\mathbf{k}_4)$ $\{\mathbf{g_{NL}}[(54/25)P_\zeta(k_1)P_\zeta(k_2)P_\zeta(k_3)+\text{cyc.}]$ $+\boldsymbol{\tau_{NL}}[(18/25)P_\zeta(k_1)P_\zeta(k_2)(P_\zeta(|\mathbf{k}_1+\mathbf{k}_3|)+P_\zeta(|\mathbf{k}_1+\mathbf{k}_4|))+\text{cyc.}]\}$

The local form consistency relation, $\tau_{NL}=(f_{NL})^2$, may not be respected – additional test of multi-field inflation!



g_{NL}



$f_{NL}{}^2$

Trispectrum: Next Frontier

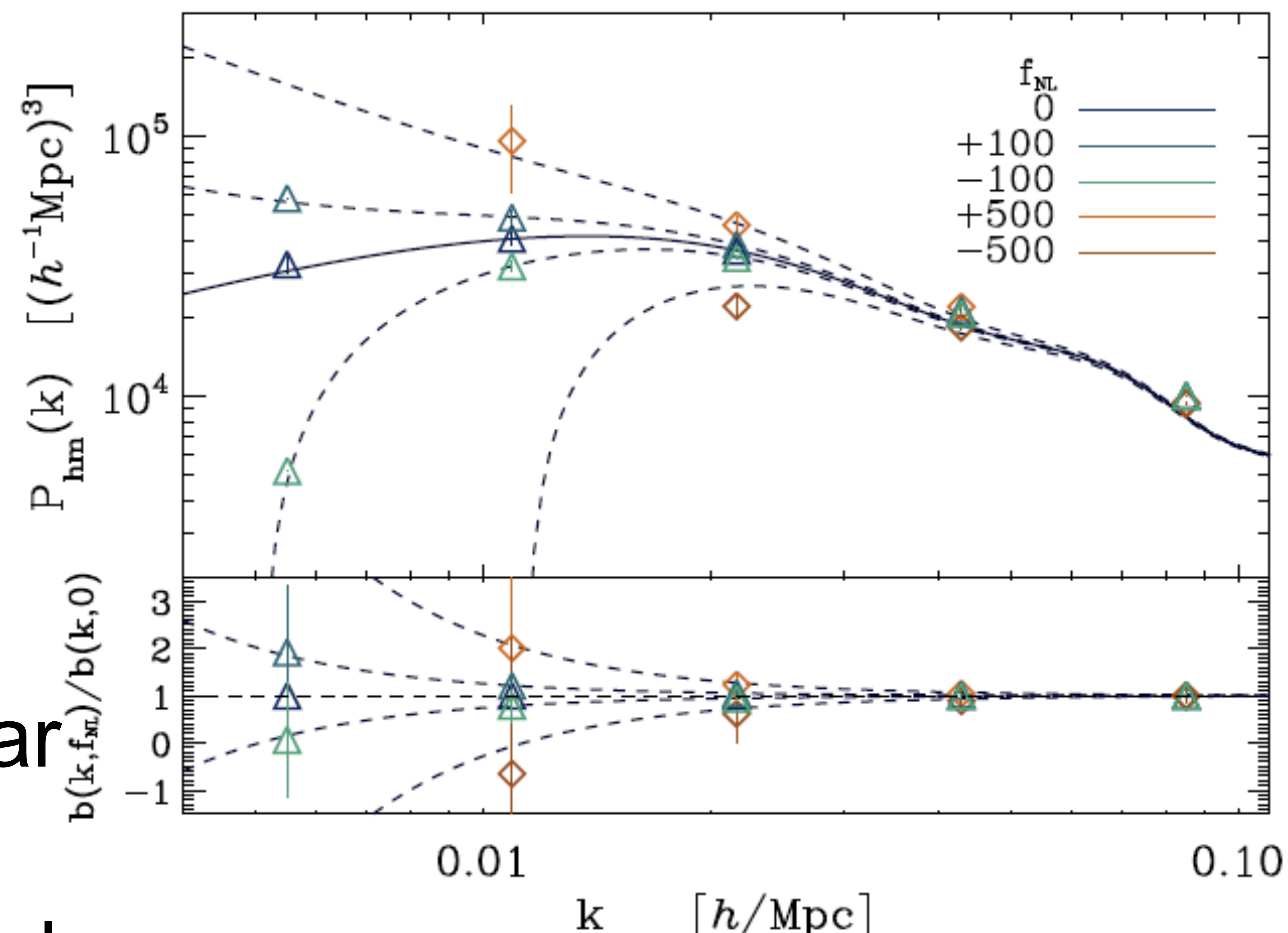
- A new phenomenon: many talks given at the IPMU non-Gaussianity workshop emphasized the importance of the trispectrum as a source of additional information on the physics of inflation.
- $\tau_{NL} \sim f_{NL}^2$; $\tau_{NL} \sim f_{NL}^{4/3}$; $\tau_{NL} \sim (\text{isocurv.})*f_{NL}^2$; $g_{NL} \sim f_{NL}$; $g_{NL} \sim f_{NL}^2$; or they are completely independent
- Shape dependence? (Squares from ghost condensate, diamonds and rectangles from multi-field, etc)

(ii) Different Tracers

- New frontier: large-scale structure of the universe as a probe of primordial non-Gaussianity

New, Powerful Probe of f_{NL}

- f_{NL} modifies the power spectrum of galaxies on very large scales
  - *–Dalal et al.; Matarrese & Verde*
  - *–Mcdonald; Afshordi & Tolley*
- The statistical power of this method is **VERY** promising
  - –SDSS: $-29 < f_{NL} < 70$ (95%CL); Slosar et al.
  - –Comparable to the WMAP 5-year limit already
  - –Expected to beat CMB, and reach a sacred region: **$f_{NL}\sim 1$**



Effects of f_{NL} on the statistics of PEAKS

- The effects of f_{NL} on the power spectrum of peaks (i.e., galaxies) are profound.
- **How about the bispectrum of galaxies?**

Previous Calculation

- Scoccimarro, Sefusatti & Zaldarriaga (2004); Sefusatti & Komatsu (2007)
  - Treated the distribution of galaxies as a *continuous distribution*, biased relative to the matter distribution:
    - $\delta_g = b_1\delta_m + (b_2/2)(\delta_m)^2 + \ldots$
  - Then, the calculation is straightforward. Schematically:
    - $\langle\delta_g^3\rangle = (b_1)^3\langle\delta_m^3\rangle + (b_1^2 b_2)\langle\delta_m^4\rangle + \ldots$

Non-linear Gravity
Primordial NG

Non-linear Bias Bispectrum

Previous Calculation

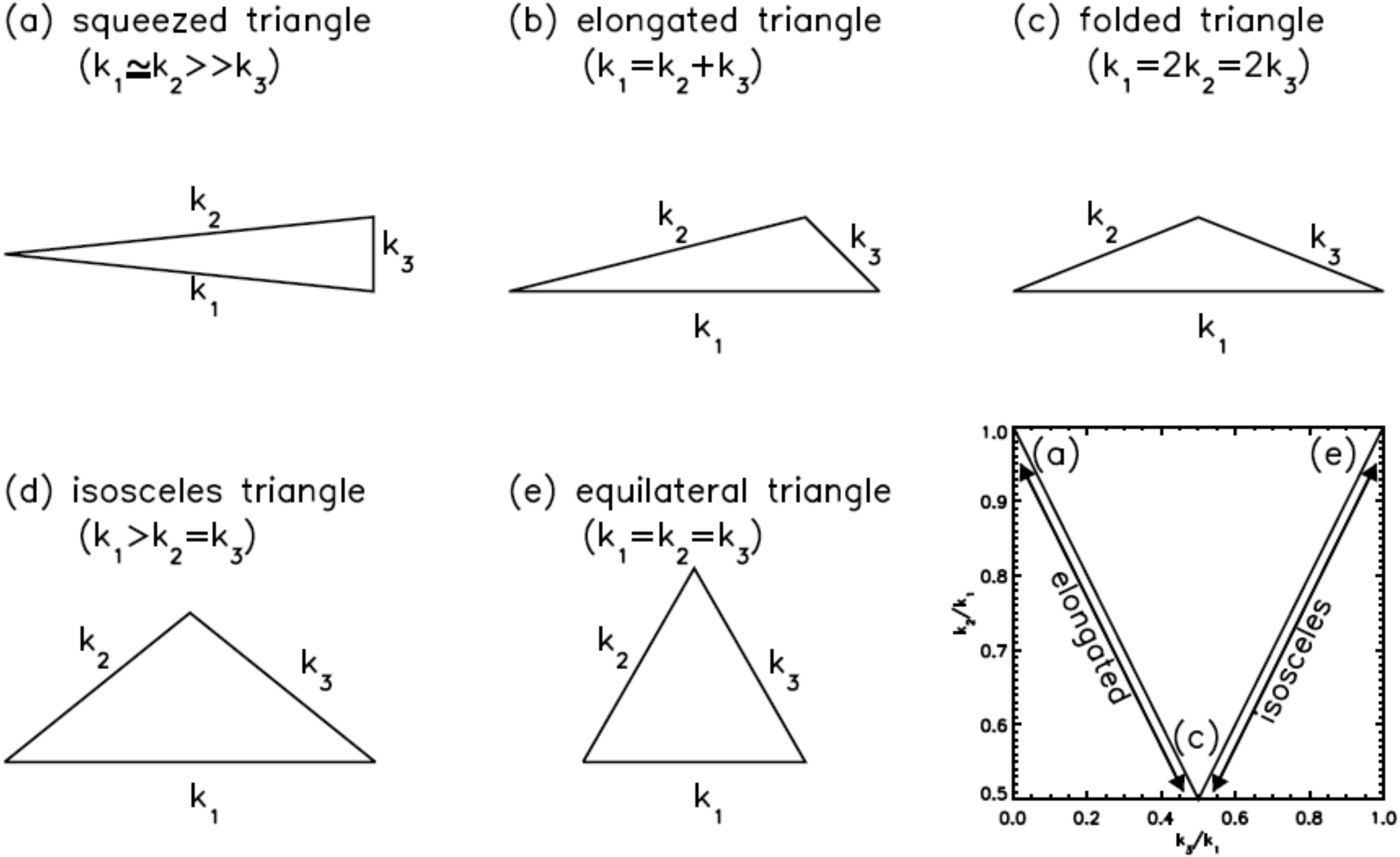
$$
\begin{aligned}
& B_g(k_1, k_2, k_3, z) \\
&= 3b_1^3 f_{\rm NL}\Omega_m H_0^2 \left[\frac{P_m(k_1,z)}{k_1^2 T(k_1)} \frac{P_m(k_2,z)}{k_2^2 T(k_2)} \frac{k_3^2 T(k_3)}{D(z)} + ({\rm cyclic}) \right] \\
&+ 2b_1^3 \left[F_2^{(s)}(\boldsymbol{k}_1, \boldsymbol{k}_2) P_m(k_1,z) P_m(k_2,z) + ({\rm cyclic}) \right] \\
&+ b_1^2 b_2 \left[P_m(k_1,z) P_m(k_2,z) + ({\rm cyclic}) \right]
\end{aligned}
$$

Primordial NG

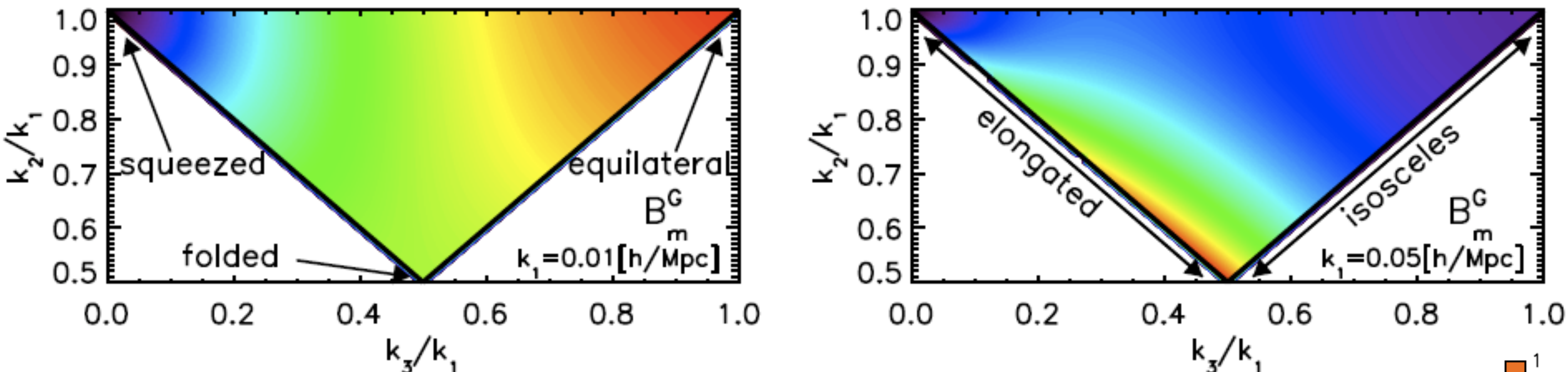
Non-linear Gravity

Non-linear Bias

- We find that this formula captures only a part of the full contributions. In fact, this formula is sub-dominant in the squeezed configuration, and the new terms are dominant.

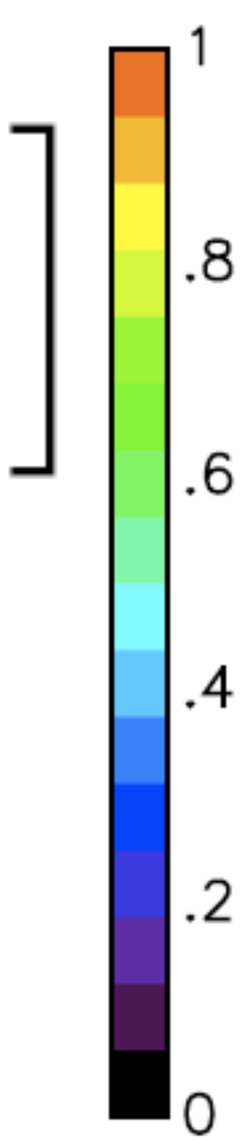

(a) squeezed triangle
(k1≃k2>>k3)
k2
k3
k1
(b) elongated triangle
(k1=k2+k3)
k2
k3
k1
(c) folded triangle
(k1=2k2=2k3)
k2
k3
k1
(d) isosceles triangle
(k1>k2=k3)
k2
k3
k1
(e) equilateral triangle
(k1=k2=k3)
k2
k3
k1
1.0
0.9
0.8
0.7
0.6
0.5
k2/k1
0.0
0.2
0.4
0.6
0.8
1.0
k3/k1
(a)
(e)
(c)
elongated
isosceles

Non-linear Gravity

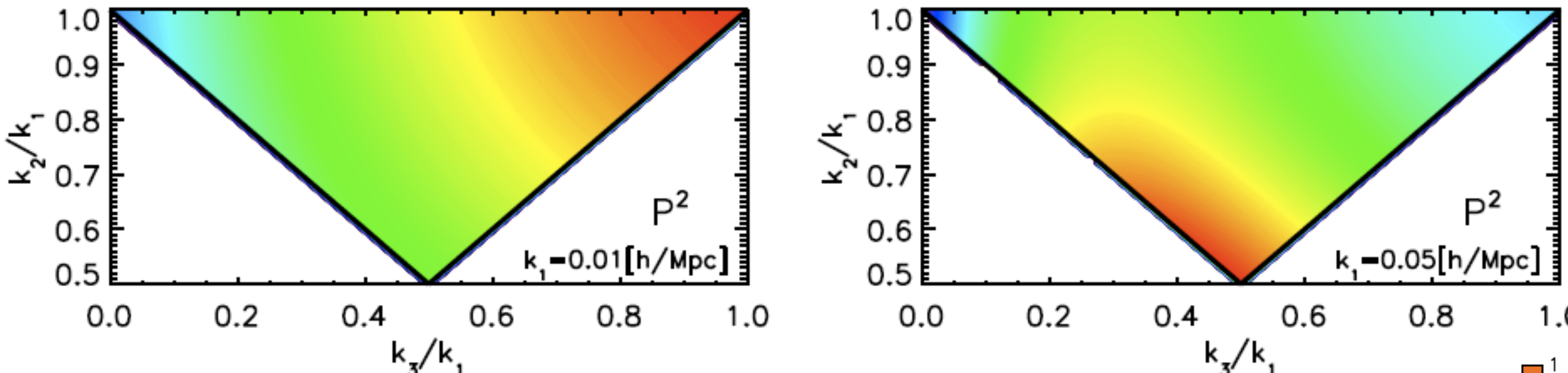


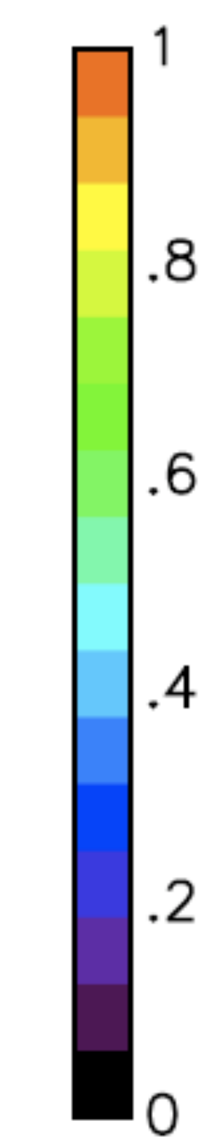
$$2b_1^3\left[F_2^{(s)}(\boldsymbol{k}_1,\boldsymbol{k}_2)P_m(k_1,z)P_m(k_2,z)+(\text{cyclic})\right]$$

- For a given k_1, vary k_2 and k_3, with $k_3 \leq k_2 \leq k_1$
- $F_2(k_2,k_3)$ vanishes in the squeezed limit, and peaks at the elongated triangles.



Non-linear Galaxy Bias

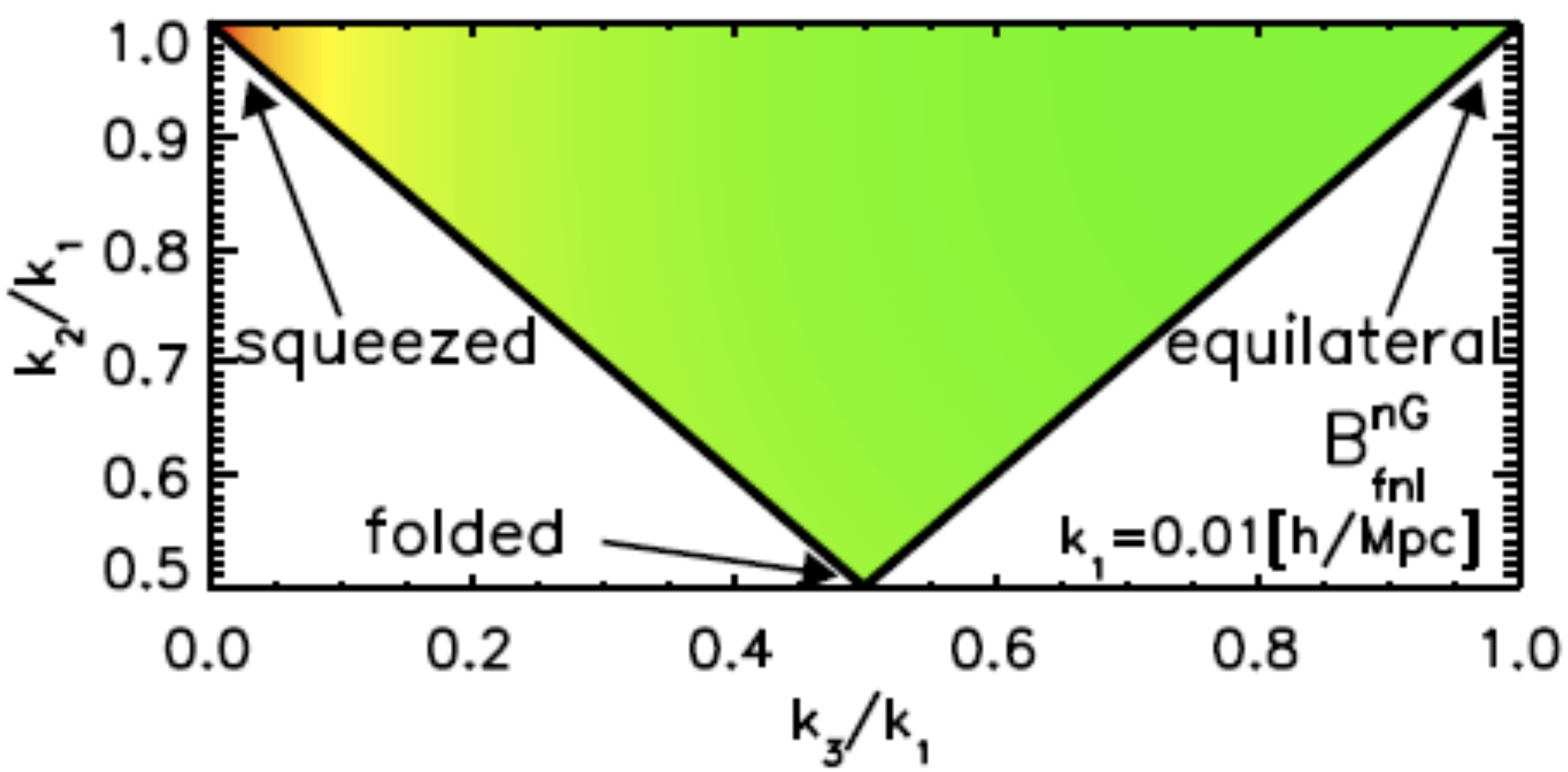


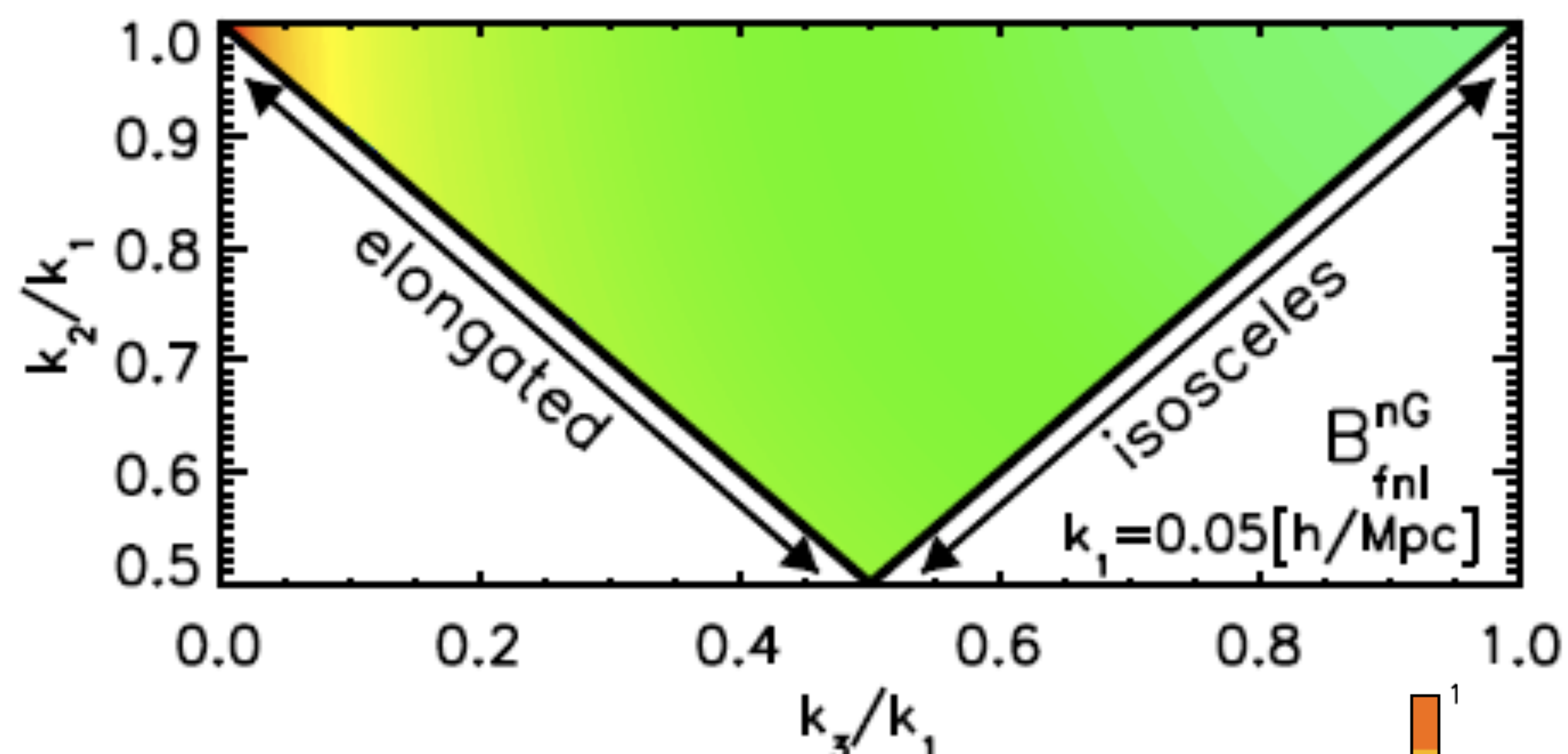


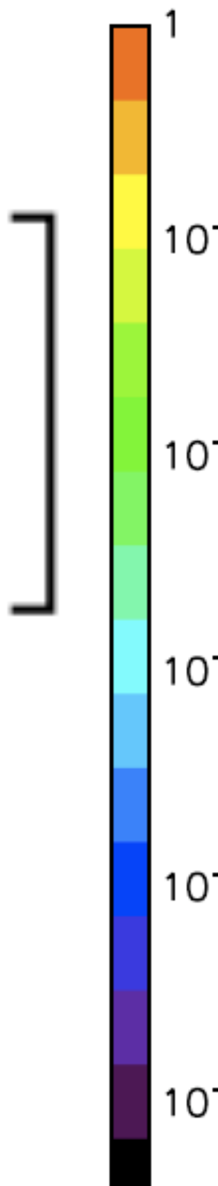
$$b_1^2 b_2 \left[P_m(k_1, z) P_m(k_2, z) + (\text{cyclic}) \right]$$

- There is no F_2: less suppression at the squeezed, and less enhancement along the elongated triangles.
- Still peaks at the equilateral or elongated forms.

Primordial NG (SK07)







$$3b_1^3 f_{\rm NL}\Omega_m H_0^2 \left[\frac{P_m(k_1,z)}{k_1^2 T(k_1)}\frac{P_m(k_2,z)}{k_2^2 T(k_2)}\frac{k_3^2 T(k_3)}{D(z)} + ({\rm cyclic})\right]$$

- Notice the factors of k^2 in the denominator.
- This gives the peaks at the squeezed configurations.

New Terms

- But, it turns our that Sefusatti & Komatsu's calculation, which is valid only for the continuous field, misses the dominant terms that come from the statistics of PEAKS.
- Jeong & Komatsu, arXiv:0904.0497

Donghui Jeong

Matarrese, Lucchin & Bonometto (1986)

MLB Formula

$$1 + \xi_h(x_{12}) + \xi_h(x_{23}) + \xi_h(x_{31}) + \zeta_h(\boldsymbol{x}_1, \boldsymbol{x}_2, \boldsymbol{x}_3)$$

$$= \exp\left[\frac{1}{2}\frac{\nu^2}{\sigma_R^2}\sum_{i\neq j}\xi_R^{(2)}(x_{ij}) + \sum_{n=3}^{\infty}\left\{\sum_{m_1=0}^{n}\sum_{m_2=0}^{n-m_1}\frac{\nu^n\sigma_R^{-n}}{m_1!m_2!m_3!}\right.\right.$$

$$\times\xi_R^{(n)}\begin{pmatrix}\boldsymbol{x}_1,\cdots,\boldsymbol{x}_1, & \boldsymbol{x}_2,\cdots,\boldsymbol{x}_2, & \boldsymbol{x}_3,\cdots,\boldsymbol{x}_3\\ m_1\,\text{times} & m_2\,\text{times} & m_3\,\text{times}\end{pmatrix}$$

$$\left.\left.-3\frac{\nu^n\sigma_R^{-n}}{n!}\xi_R^{(n)}\begin{pmatrix}\boldsymbol{x},\cdots,\boldsymbol{x}\\ n\,\text{times}\end{pmatrix}\right\}\right]$$

- N-point correlation function of peaks is the sum of M-point correlation functions, where M≥N.

Bottom Line

- **The bottom line is:**
- The power spectrum (2-pt function) of peaks is sensitive to the power spectrum of the underlying mass distribution, and the bispectrum, and the trispectrum, etc.
    - Truncate the sum at the bispectrum: sensitivity to f_{NL}
    - Dalal et al.; Matarrese&Verde; Slosar et al.; Afshordi&Tolley

Bottom Line

- **The bottom line is:**
- The bispectrum (3-pt function) of peaks is sensitive to the bispectrum of the underlying mass distribution, and the trispectrum, and the quadspectrum, etc.
  - Truncate the sum at the trispectrum: sensitivity to τ_{NL} ($\sim f_{NL}^2$) and g_{NL}!
  - This is the new effect that was missing in Sefusatti & Komatsu (2007).

Real-space 3pt Function

$$\zeta_h(\boldsymbol{x}_1, \boldsymbol{x}_2, \boldsymbol{x}_3) = \frac{\nu^3}{\sigma_R^3}\xi_R^{(3)}(\boldsymbol{x}_1, \boldsymbol{x}_2, \boldsymbol{x}_3)$$
$$+ \frac{\nu^4}{\sigma_R^4}\left[\xi_R^{(2)}(x_{12})\xi_R^{(2)}(x_{23}) + (\text{cyclic})\right]$$
$$+ \frac{\nu^4}{2\sigma_R^4}\left[\xi_R^{(4)}(\boldsymbol{x}_1, \boldsymbol{x}_1, \boldsymbol{x}_2, \boldsymbol{x}_3) + (\text{cyclic})\right]$$

- Plus 5-pt functions, etc...

New Bispectrum Formula

$$B_h(\boldsymbol{k}_1, \boldsymbol{k}_2, \boldsymbol{k}_3)$$

$$= b_1^3 \Bigg[B_R(\boldsymbol{k}_1, \boldsymbol{k}_2, \boldsymbol{k}_3) + \frac{b_2}{b_1} \{ P_R(k_1) P_R(k_2) + (\text{cyclic}) \}$$

$$+ \frac{\delta_c}{2\sigma_R^2} \int \frac{d^3 q}{(2\pi)^3} T_R(\boldsymbol{q}, \boldsymbol{k}_1 - \boldsymbol{q}, \boldsymbol{k}_2, \boldsymbol{k}_3) + (\text{cyclic}) \Bigg].$$

- First: bispectrum of the underlying mass distribution.
- Second: non-linear bias
- Third: trispectrum of the underlying mass distribution.

Local Form Trispectrum

$\Phi=(3/5)\zeta$

$$\Phi(\boldsymbol{x}) = \phi(\boldsymbol{x}) + f_{\rm NL}\left[\phi^2(\boldsymbol{x}) - \langle\phi^2\rangle\right] + g_{\rm NL}\phi^3(\boldsymbol{x})$$

$$T_\Phi(\boldsymbol{k}_1, \boldsymbol{k}_2, \boldsymbol{k}_3, \boldsymbol{k}_4)$$
$$= 6g_{\rm NL}\left[P_\phi(k_1)P_\phi(k_2)P_\phi(k_3) + ({\rm cyclic})\right] + 2f_{\rm NL}^2$$
$$\times\left[P_\phi(k_1)P_\phi(k_2)\left\{P_\phi(k_{13}) + P_\phi(k_{14})\right\} + ({\rm cyclic})\right]$$

- For general multi-field models, f_{NL}^2 can be more generic: often called τ_{NL}.
- Exciting possibility for testing more about inflation!

Local Form Trispectrum

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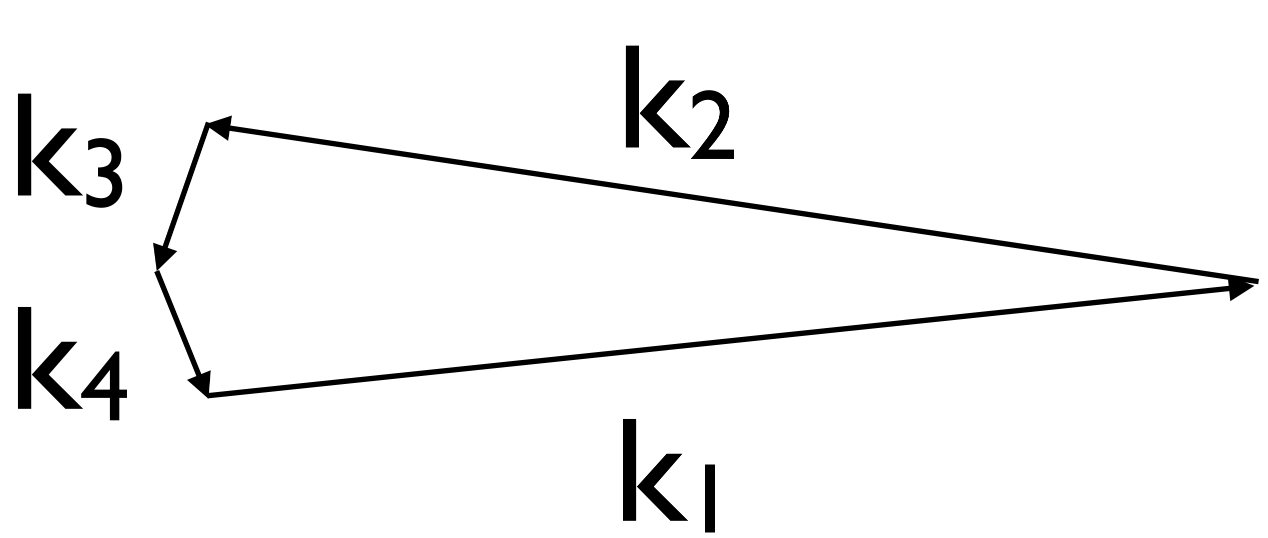
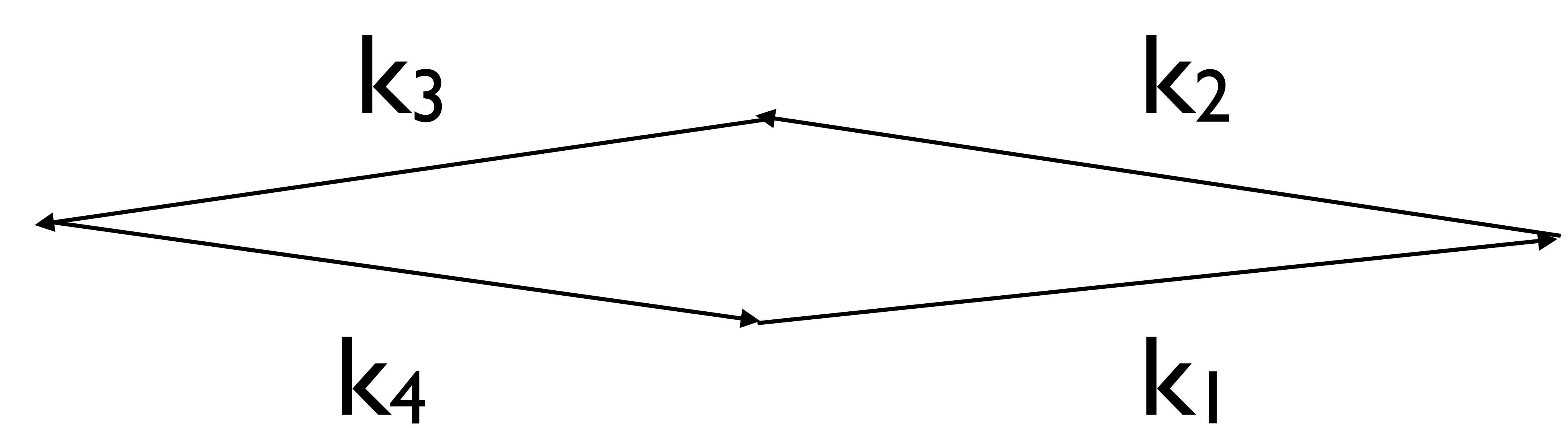
$$T_\Phi(\boldsymbol{k}_1,\boldsymbol{k}_2,\boldsymbol{k}_3,\boldsymbol{k}_4)$$
$$=6g_{\rm NL}\left[P_\phi(k_1)P_\phi(k_2)P_\phi(k_3)+({\rm cyclic})\right]+2f_{\rm NL}^2$$
$$\times\left[P_\phi(k_1)P_\phi(k_2)\left\{P_\phi(k_{13})+P_\phi(k_{14})\right\}+({\rm cyclic})\right]$$



g_{NL}



f_{NL}^2 (or τ_{NL})

Trispectrum Term

$$\frac{\delta_c}{2\sigma_R^2}\int\frac{d^3q}{(2\pi)^3}\left[T_R(\boldsymbol{q},\boldsymbol{k}_1-\boldsymbol{q},\boldsymbol{k}_2,\boldsymbol{k}_3)+(\mathrm{cyclic})\right]$$

$$=g_{\mathrm{NL}}B^{nG}_{g_{\mathrm{NL}}}(k_1,k_2,k_3)+f_{\mathrm{NL}}^2B^{nG}_{f^2_{\mathrm{NL}}}(k_1,k_2,k_3),$$

$$B^{nG}_{g_{\mathrm{NL}}}(k_1,k_2,k_3)\equiv\frac{\delta_c}{2\sigma_R^2}\Big[6\mathcal{M}_R(k_2)\mathcal{M}_R(k_3)\left[P_\phi(k_2)+P_\phi(k_3)\right]\int\frac{d^3q}{(2\pi)^3}\mathcal{M}_R(q)\mathcal{M}_R(|\boldsymbol{k}_1-\boldsymbol{q}|)P_\phi(q)P_\phi(|\boldsymbol{k}_1-\boldsymbol{q}|)+(\mathrm{cyclic})$$
$$+12\mathcal{M}_R(k_2)\mathcal{M}_R(k_3)P_\phi(k_2)P_\phi(k_3)\int\frac{d^3q}{(2\pi)^3}\mathcal{M}_R(q)\mathcal{M}_R(|\boldsymbol{k}_1-\boldsymbol{q}|)P_\phi(q)+(\mathrm{cyclic})\Big].\qquad(20)$$

$$B^{nG}_{f^2_{\mathrm{NL}}}(k_1,k_2,k_3)\approx\frac{\delta_c}{2\sigma_R^2}\Big[8\mathcal{M}_R(k_2)\mathcal{M}_R(k_3)P_\phi(k_1)\left[P_\phi(k_2)+P_\phi(k_3)\right]\int\frac{d^3q}{(2\pi)^3}\mathcal{M}_R(q)\mathcal{M}_R(|\boldsymbol{k}_1-\boldsymbol{q}|)P_\phi(q)+(\mathrm{cyclic})$$
$$+4\mathcal{M}_R(k_2)\mathcal{M}_R(k_3)P_\phi(k_2)P_\phi(k_3)\int\frac{d^3q}{(2\pi)^3}\mathcal{M}_R(q)\mathcal{M}_R(|\boldsymbol{k}_1-\boldsymbol{q}|)$$
$$\times\left[P_\phi(|\boldsymbol{k}_2+\boldsymbol{q}|)+P_\phi(|\boldsymbol{k}_3+\boldsymbol{q}|)\right]+(\mathrm{cyclic})\Big].\qquad(21)$$

Trispectrum Term

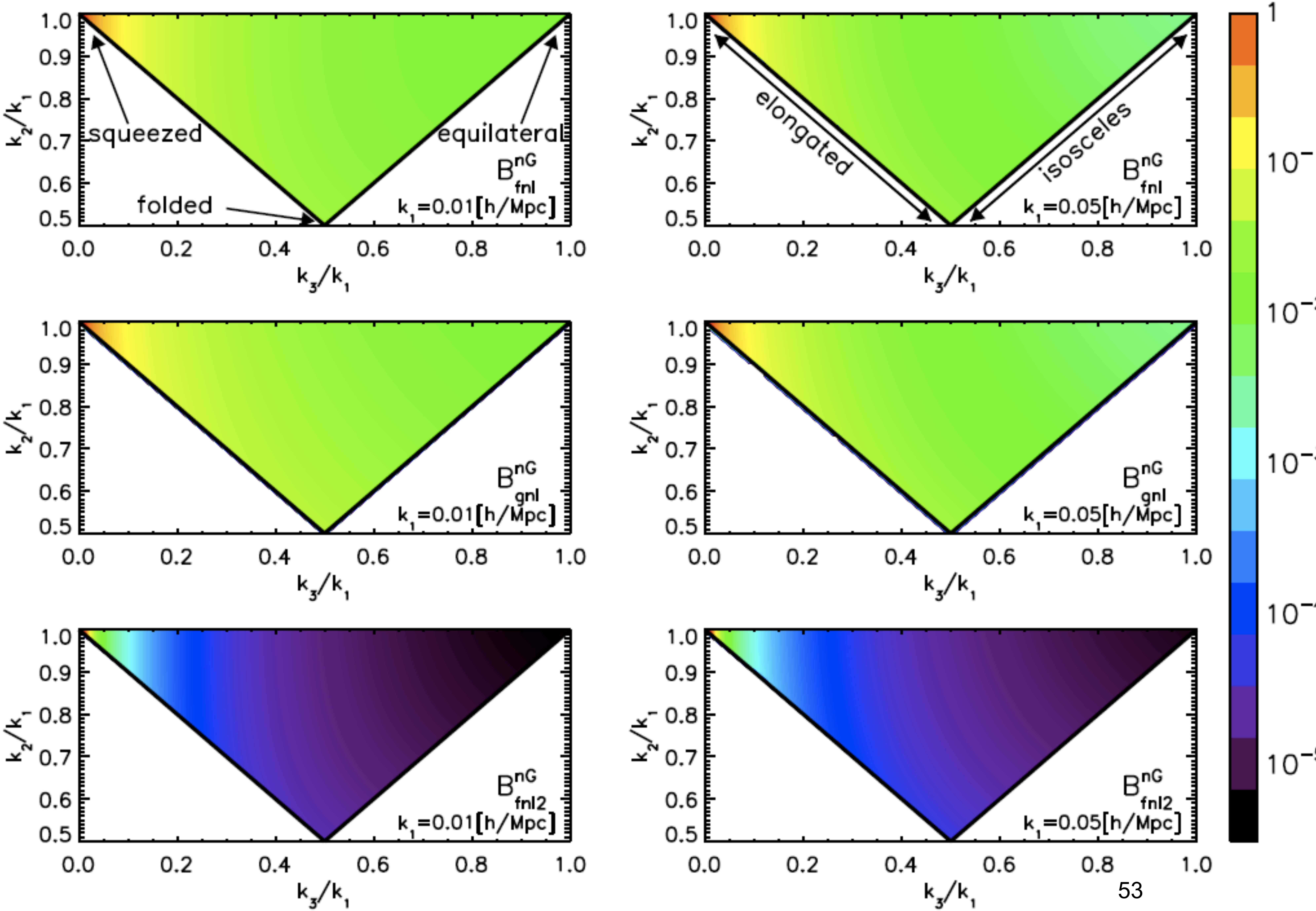
$$\frac{\delta_c}{2\sigma_R^2}\int\frac{d^3q}{(2\pi)^3}\left[T_R(\boldsymbol{q},\boldsymbol{k}_1-\boldsymbol{q},\boldsymbol{k}_2,\boldsymbol{k}_3)+(\text{cyclic})\right]$$

$$=g_{\rm NL}B^{nG}_{g_{\rm NL}}(k_1,k_2,k_3)+f_{\rm NL}^2B^{nG}_{f_{\rm NL}^2}(k_1,k_2,k_3),$$

$$B^{nG}_{g_{\rm NL}}(k_1,k_2,k_3)\equiv\frac{\delta_c}{2\sigma_R^2}\Bigg[6\mathcal{M}_R(k_2)\mathcal{M}_R(k_3)\left[P_\phi(k_2)+P_\phi(k_3)\right]\int\frac{d^3q}{(2\pi)^3}\mathcal{M}_R(q)\mathcal{M}_R(|\boldsymbol{k}_1-\boldsymbol{q}|)P_\phi(q)P_\phi(|\boldsymbol{k}_1-\boldsymbol{q}|)+(\text{cyclic})$$
$$+12\mathcal{M}_R(k_2)\mathcal{M}_R(k_3)P_\phi(k_2)P_\phi(k_3)\int\frac{d^3q}{(2\pi)^3}\mathcal{M}_R(q)\mathcal{M}_R(|\boldsymbol{k}_1-\boldsymbol{q}|)P_\phi(q)+(\text{cyclic})\Bigg].\qquad(20)$$

$$B^{nG}_{f_{\rm NL}^2}(k_1,k_2,k_3)\approx\frac{\delta_c}{2\sigma_R^2}\Bigg[\underline{8\mathcal{M}_R(k_2)\mathcal{M}_R(k_3)P_\phi(k_1)\left[P_\phi(k_2)+P_\phi(k_3)\right]\int\frac{d^3q}{(2\pi)^3}\mathcal{M}_R(q)\mathcal{M}_R(|\boldsymbol{k}_1-\boldsymbol{q}|)P_\phi(q)+(\text{cyclic})}$$
$$+4\mathcal{M}_R(k_2)\mathcal{M}_R(k_3)P_\phi(k_2)P_\phi(k_3)\int\frac{d^3q}{(2\pi)^3}\mathcal{M}_R(q)\mathcal{M}_R(|\boldsymbol{k}_1-\boldsymbol{q}|)$$
$$\times\left[P_\phi(|\boldsymbol{k}_2+\boldsymbol{q}|)+P_\phi(|\boldsymbol{k}_3+\boldsymbol{q}|)\right]+(\text{cyclic})\Bigg].\qquad(21)$$

Most Dominant in the Squeezed Limit

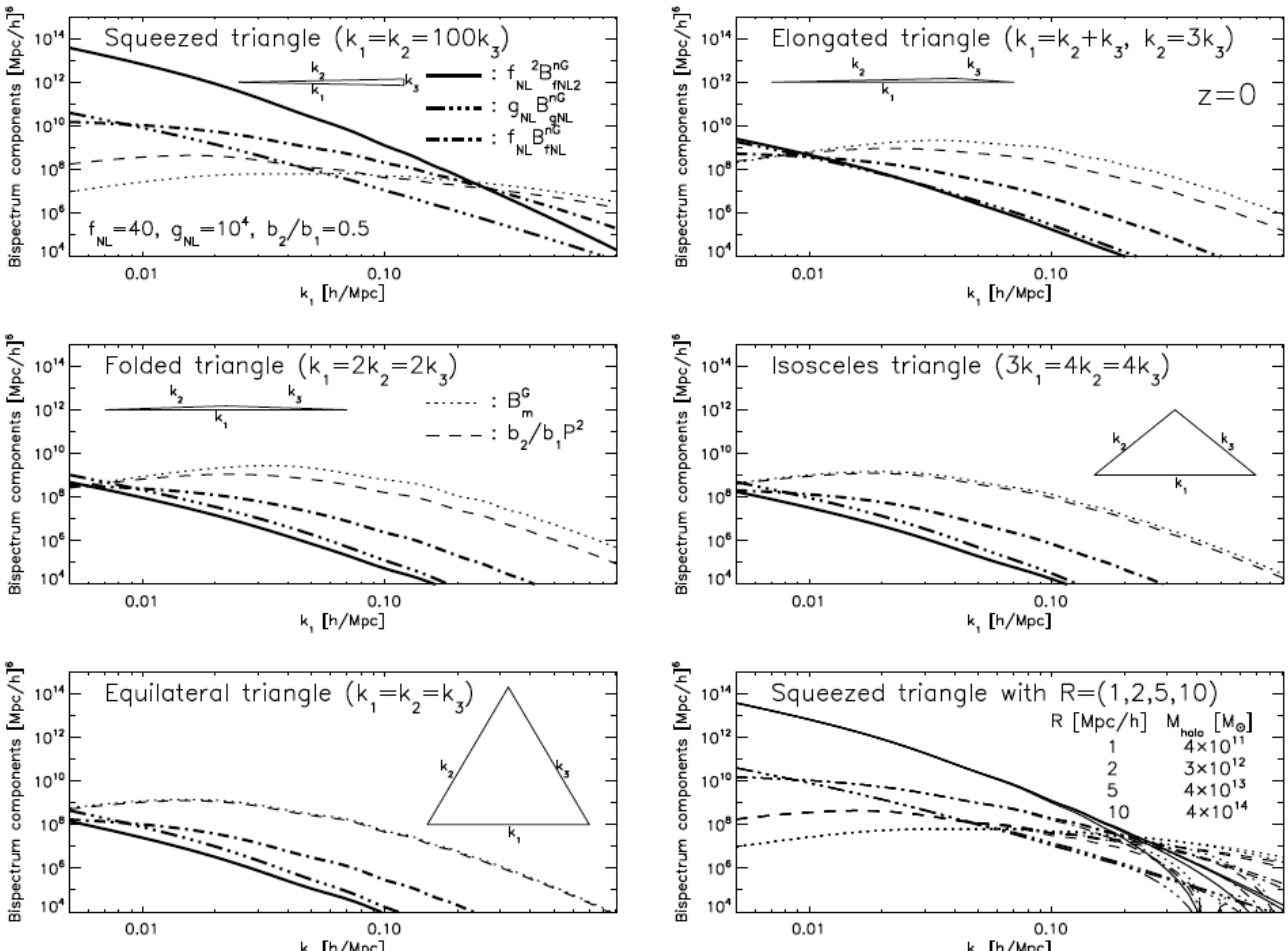

squeezed
equilateral
folded
elongated
isosceles
B^{nG}_{fnl}
k_1=0.01[h/Mpc]
k_1=0.05[h/Mpc]
B^{nG}_{gnl}
B^{nG}_{fnl2}
k_2/k_1
k_3/k_1

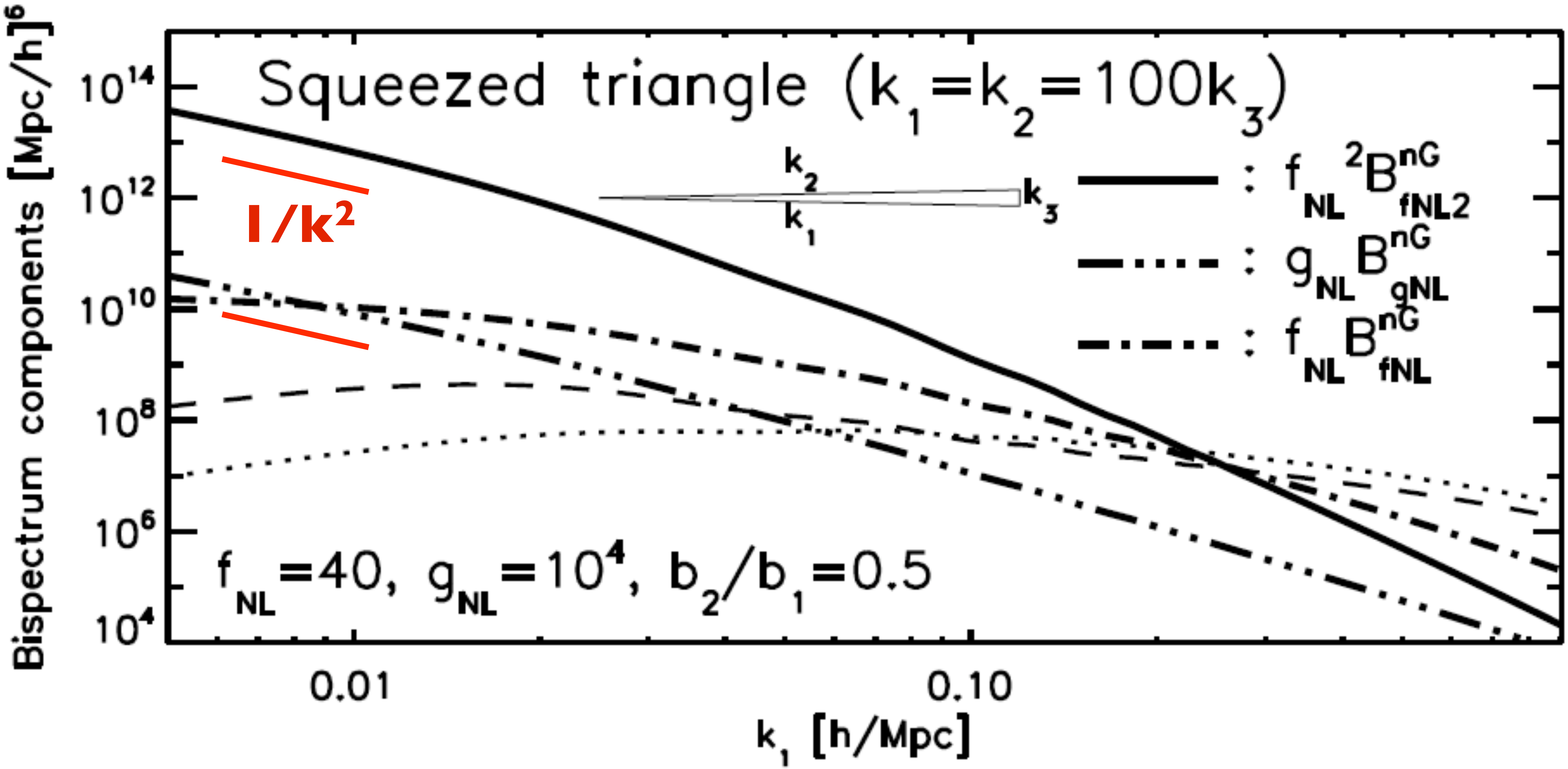
Shape Results

- The primordial non-Gaussianity terms peak at the squeezed triangle.
- f_{NL} and g_{NL} terms have the same shape dependence:
  - For $k_1=k_2=\alpha k_3$, (f_{NL} term)~α and (g_{NL} term)~α
- f_{NL}^2 (τ_{NL}) is more sharply peaked at the squeezed:
  - (f_{NL}^2 term)~α^3

Key Question

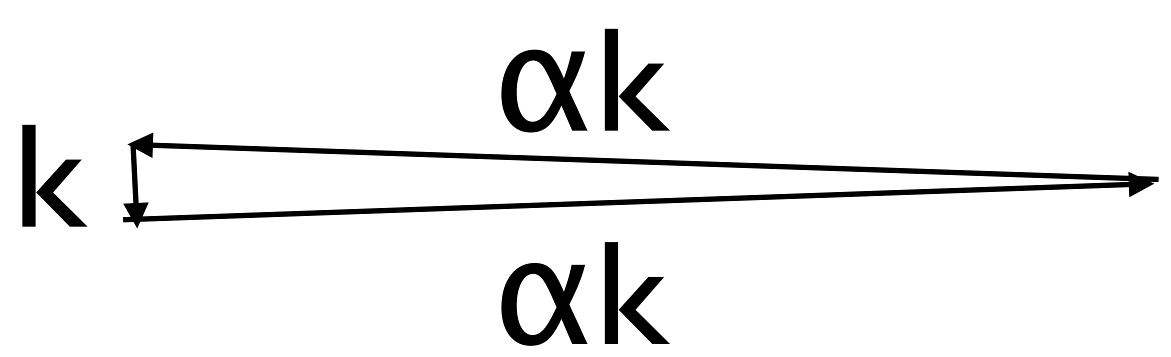
- Are g_{NL} or τ_{NL} terms important?


Squeezed triangle (k1=k2=100k3)
: fNL^2 B^nG_fNL2
: gNL B^nG_gNL
: fNL B^nG_fNL
fNL=40, gNL=10^4, b2/b1=0.5
Elongated triangle (k1=k2+k3, k2=3k3)
z=0
Folded triangle (k1=2k2=2k3)
: B^G_m
: b2/b1 P^2
Isosceles triangle (3k1=4k2=4k3)
Equilateral triangle (k1=k2=k3)
Squeezed triangle with R=(1,2,5,10)
R [Mpc/h] M_halo [M_⊙]
1 4×10^11
2 3×10^12
5 4×10^13
10 4×10^14
Bispectrum components [Mpc/h]^6
k1 [h/Mpc]
0.01
0.10
10^4
10^6
10^8
10^10
10^12
10^14
k1
k2
k3



Squeezed triangle (k1=k2=100k3)
k2
k1
k3
1/k²
: fNL²B^nG_fNL2
: gNL B^nG_gNL
: fNL B^nG_fNL
fNL=40, gNL=10⁴, b2/b1=0.5
Bispectrum components [Mpc/h]⁶
10¹⁴
10¹²
10¹⁰
10⁸
10⁶
10⁴
0.01
0.10
k1 [h/Mpc]

Importance Ratios



$$\frac{f_{\rm NL} B^{nG}_{f_{\rm NL}}}{f^2_{\rm NL} B^{nG}_{f^2_{\rm NL}}} \simeq \frac{1}{f_{\rm NL}\alpha^2} \frac{\mathcal{M}_0 k^2}{2\mathcal{I}_0 \delta_c/\sigma^2_R}$$

$$\simeq 0.0016 \left(\frac{100}{\alpha}\right)^2 \frac{40}{f_{\rm NL}} \left(\frac{k}{0.01\ h\ {\rm Mpc}^{-1}}\right)^2, \qquad (29)$$

$$\frac{g_{\rm NL} B^{nG}_{g_{\rm NL}}}{f^2_{\rm NL} B^{nG}_{f^2_{\rm NL}}} \simeq \frac{4}{3\alpha^2} \frac{g_{\rm NL}}{f^2_{\rm NL}}$$

$$\simeq 0.0008 \left(\frac{100}{\alpha}\right)^2 \left(\frac{40}{f_{\rm NL}}\right)^2 \frac{g_{\rm NL}}{10^4}, \qquad (30)$$

$$\frac{f_{\rm NL} B^{nG}_{f_{\rm NL}}}{g_{\rm NL} B^{nG}_{g_{\rm NL}}} \simeq \frac{f_{\rm NL}}{g_{\rm NL}} \frac{\mathcal{M}_0 k^2}{3\mathcal{I}_0 \delta_c/\sigma^2_R}$$

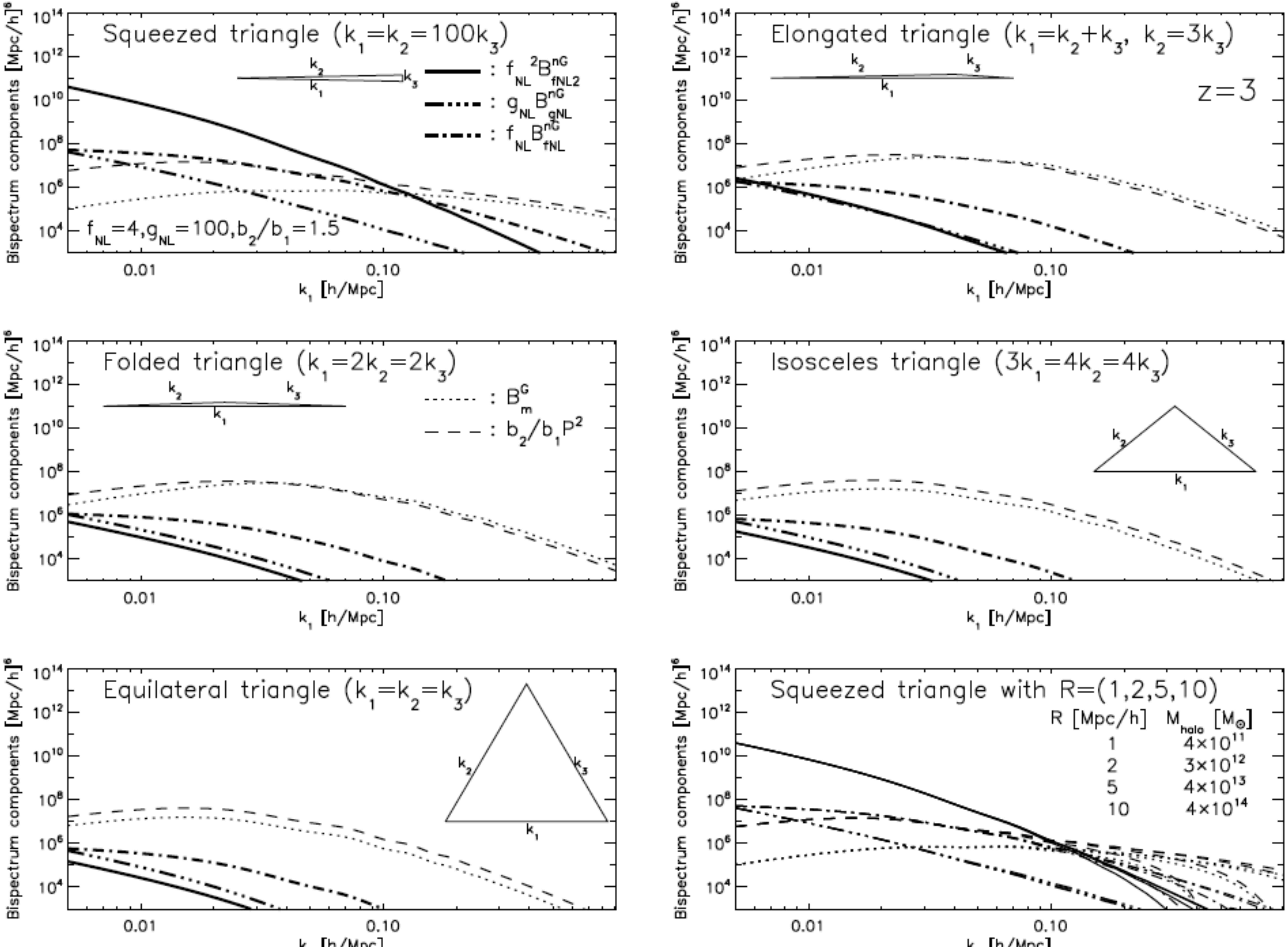
$$\simeq 1.7 \frac{f_{\rm NL}}{40} \frac{10^4}{g_{\rm NL}} \left(\frac{k}{0.01\ h\ {\rm Mpc}^{-1}}\right)^2. \qquad (31)$$

- **$f_{\rm NL}^2$ dominates over $f_{\rm NL}$ term easily for $f_{\rm NL}>1$!**

Redshift Dependence

$$B_h(k_1,k_2,k_3,z) = b_1^3(z)D^4(z)\Bigg[B_m^G(k_1,k_2,k_3) + \frac{b_2(z)}{b_1(z)}\{P_R(k_1)P_R(k_2) + (\text{cyclic})\} + f_{\rm NL}\frac{B_{f_{\rm NL}}^{nG}(k_1,k_2,k_3)}{D(z)}$$
$$+ f_{\rm NL}^2\frac{B_{f_{\rm NL}^2}^{nG}(k_1,k_2,k_3)}{D^2(z)} + g_{\rm NL}\frac{B_{g_{\rm NL}}^{nG}(k_1,k_2,k_3)}{D^2(z)}\Bigg],$$

- Primordial non-Gaussianity terms are more important at higher redshifts.
- The new trispectrum terms are even more important.


Squeezed triangle (k1=k2=100k3)
k2
k1
k3
: fNL 2BnGfNL2
: gNL BnGgNL
: fNL BnGfNL
fNL=4,gNL=100,b2/b1=1.5
Elongated triangle (k1=k2+k3, k2=3k3)
z=3
Folded triangle (k1=2k2=2k3)
: BGm
: b2/b1P2
Isosceles triangle (3k1=4k2=4k3)
Equilateral triangle (k1=k2=k3)
Squeezed triangle with R=(1,2,5,10)
R [Mpc/h]
Mhalo [M⊙]
1
4×10^11
2
3×10^12
5
4×10^13
10
4×10^14
Bispectrum components [Mpc/h]^6
k1 [h/Mpc]
0.01
0.10
10^4
10^6
10^8
10^10
10^12
10^14

Summary

- **Non-Gaussianity is a *new, powerful* probe of physics of the early universe**
  - It has a best chance of ruling out the largest class of inflation models
- Various forms of f_{NL} available today — 1.8σ at the moment, wait for WMAP 9-year (2011) and Planck (2012) for more σ's (if it's there!)
- To convince ourselves of detection, we need to see the acoustic oscillations, and the same signal in the bispectrum and trispectrum, of both CMB and the large-scale structure of the universe.

Additional Remarks

- Unusually healthy interactions between observers and theorists: astronomers, cosmologists, phenomenologists, high-energy theorists
  - The list of participants in workshops on non-Gaussianity speaks for its diversity
  - Interdisciplinary efforts
- Lots of important contributions from young people
- **New “industry” – active field, something new every month**

Now, let's pray:

- May Planck succeed!

Now, let's pray:

- May the signal be there!