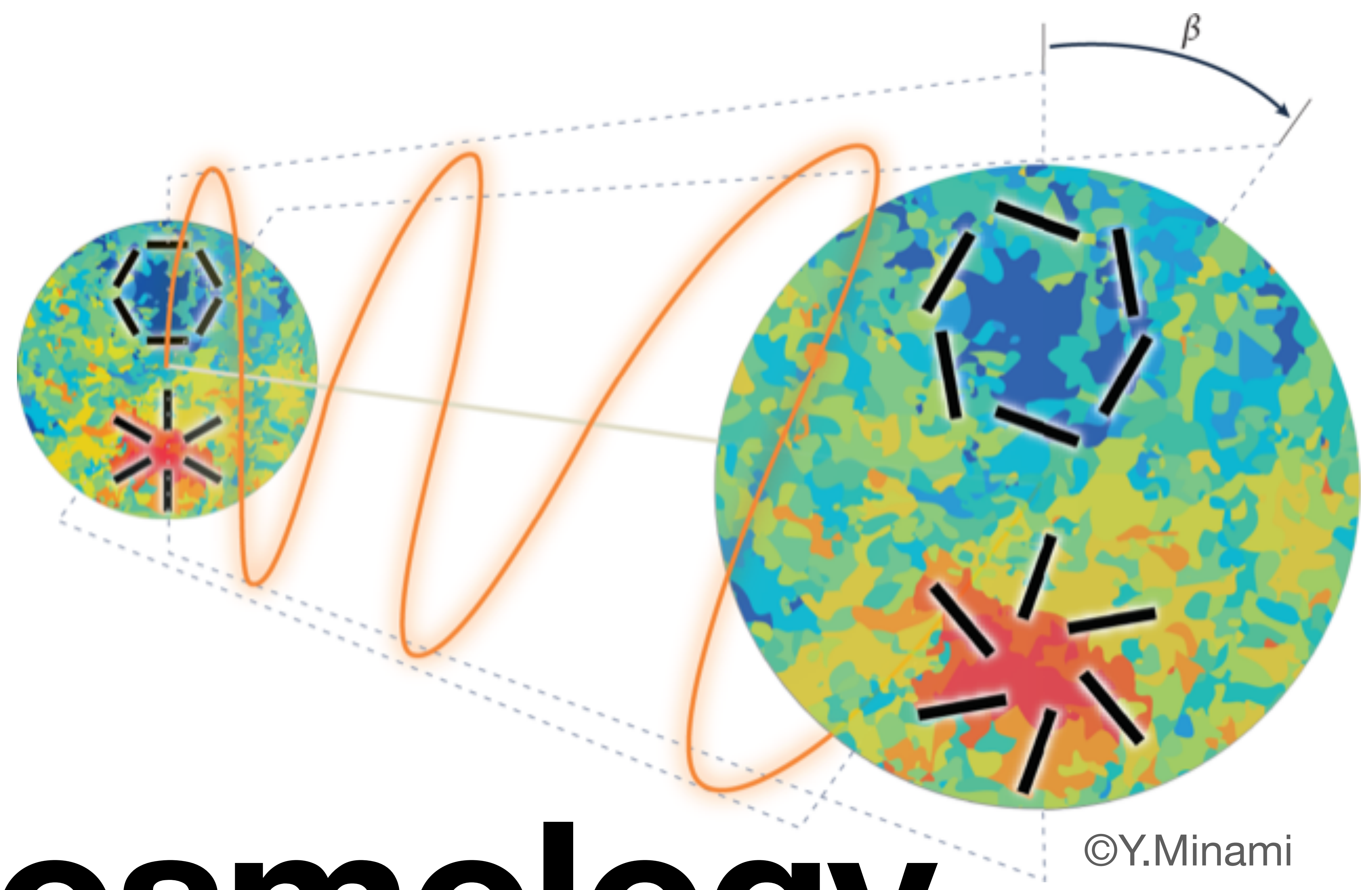
$$I_{\rm CS} = \int d^4x \sqrt{-g}\left(-\frac{\alpha}{4f}\chi F\tilde{F}\right)$$



Parity Violation in Cosmology

In search of new physics for the Universe

The lecture slides are available at
https://wwwmpa.mpa-garching.mpg.de/~komatsu/lectures--reviews.html

Day 5

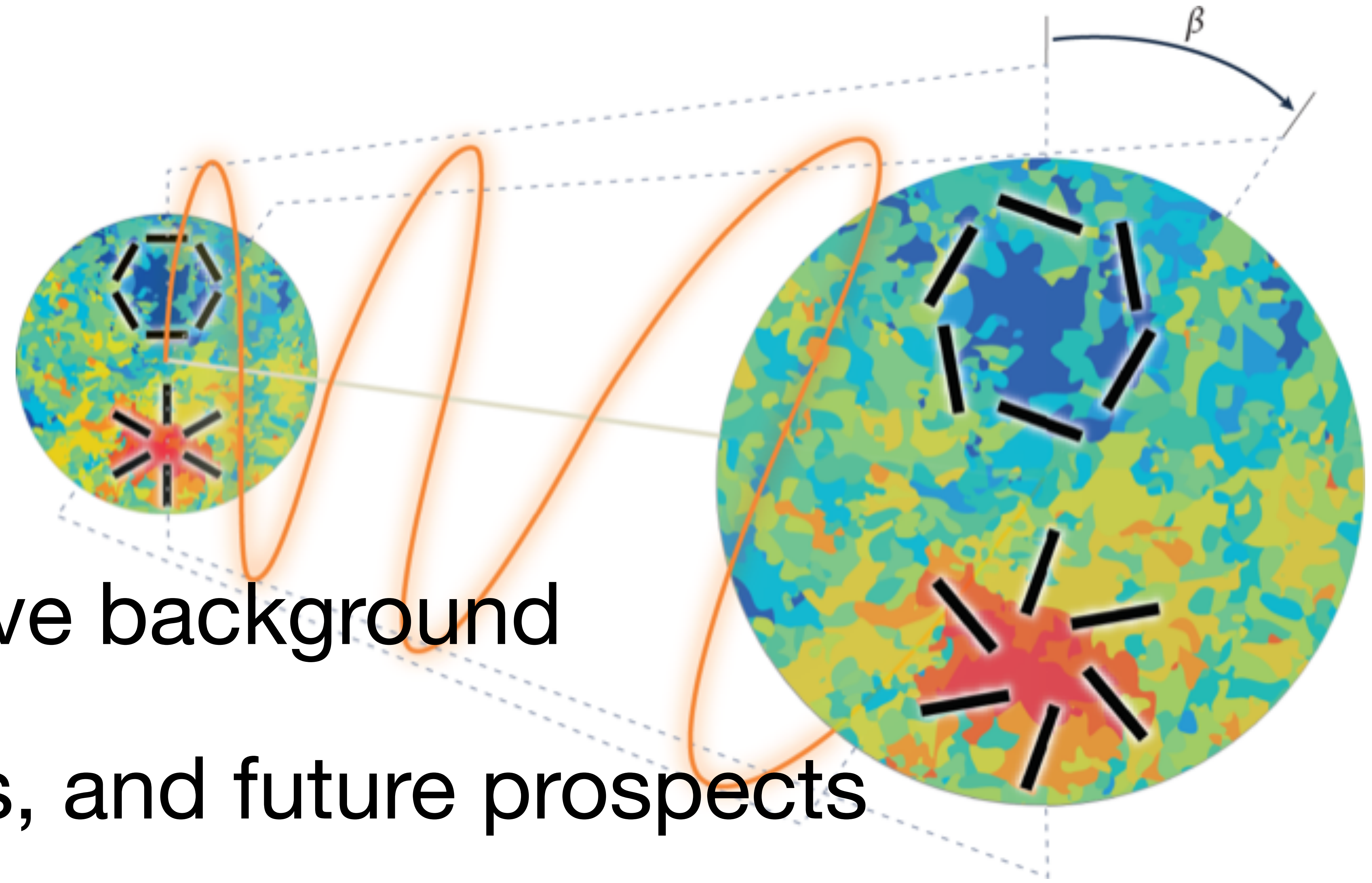
Eiichiro Komatsu (Max Planck Institute for Astrophysics)
Nagoya University, June 6–30, 2023

Topics

From the syllabus

1. What is parity symmetry?
2. Chern-Simons interaction
3. Parity violation 1: Cosmic inflation
4. **Parity violation 2: Dark matter**
5. Parity violation 3: Dark energy
6. Light propagation: birefringence
7. Physics of polarization of the cosmic microwave background
8. Recent observational results, their implications, and future prospects

$$I_{\mathrm{CS}} = \int \mathrm{d}^4x \sqrt{-g}\left(-\frac{\alpha}{4f}\chi F\widetilde{F}\right)$$



4.1 Scalar Field Dark Matter

What is dark matter?

No one knows!

- There can be different types of dark matter (just like in the visible sector).
  - Dark matter can be elementary particles or composite particles (like a pion).
  - Dark matter can be fermions or bosons with arbitrary spins.
  - Dark matter may or may not be coupled to Standard Model particles.
  - …
  - Dark matter may or may not violate parity symmetry.

Scalar field dark matter coupled to the CS term

$$I = \int d^4x\sqrt{-g}\left[-\frac{1}{2}(\partial\chi)^2 - V(\chi) - \frac{1}{4}F^2 - \frac{\alpha}{4f}\chi F\tilde{F}\right]$$

- χ is a neutral pseudoscalar field (spin 0).
- Why consider χ as a good dark matter candidate?
  - *Why not*? We have an example in the Standard Model: a neutral pion.
  - We expect $\alpha \simeq \alpha_{\mathrm{EM}} \simeq 10^{-2}$ and $f < M_{\mathrm{Pl}} \simeq 2.4 \times 10^{18}$ GeV.
- χ can be composed of fermions like a pion, or a fundamental pseudoscalar like an “axion” field.

Cold Dark Matter (CDM)

Is χ pressureless?

- Current observations suggest that dark matter is “cold” (low velocity), which implies that it is practically pressureless, $P \approx 0$.
  - P is given by the velocity dispersion of particles, $P/\rho = \langle v^2 \rangle /3 \ll 1$, where ρ is the mass energy density with $c = 1$.
- What is P/ρ of χ? **It depends on the potential, *V*(χ)!**

Equation of motion in non-expanding space

***c*=1**

$$\Box = \eta^{\mu\nu}\partial_\mu\partial_\nu = -\frac{\partial^2}{\partial t^2} + \nabla^2$$
$$\eta^{\mu\nu} = \mathrm{diag}(-1, \mathbf{1})$$

$$\Box\chi - \frac{\partial V}{\partial \chi} = -\ddot{\chi} + \nabla^2\chi - \frac{\partial V}{\partial \chi} = -\frac{\alpha}{f}\mathbf{E}\cdot\mathbf{B}$$

- The right hand side is the second-order fluctuation (Day 4).
  - **E** and **B** cannot have a uniform background, if we impose spatial isotropy (no preferred direction in space).
- The uniform distribution of dark matter is described by the average value of χ. We decompose $$\chi(t, \mathbf{x}) = \bar{\chi}(t) + \delta\chi(t, \mathbf{x})$$

Equation of motion in non-expanding space

For the homogeneous mode

$$\Box\chi - \frac{\partial V}{\partial \chi} \Rightarrow -\ddot{\bar{\chi}} + \cancel{\nabla^2 \chi} - \frac{\partial V}{\partial \bar{\chi}} = -\frac{\alpha}{f}\cancel{\mathbf{E} \cdot \mathbf{B}}$$

- The energy density and pressure of a homogeneous scalar field are

$$P = \frac{1}{2}\langle\dot{\bar{\chi}}^2\rangle - \langle V(\bar{\chi})\rangle$$

$$\rho = \frac{1}{2}\langle\dot{\bar{\chi}}^2\rangle + \langle V(\bar{\chi})\rangle$$

where $\langle(\ldots)\rangle = \frac{1}{T}\int_0^T dt\ (\ldots)$

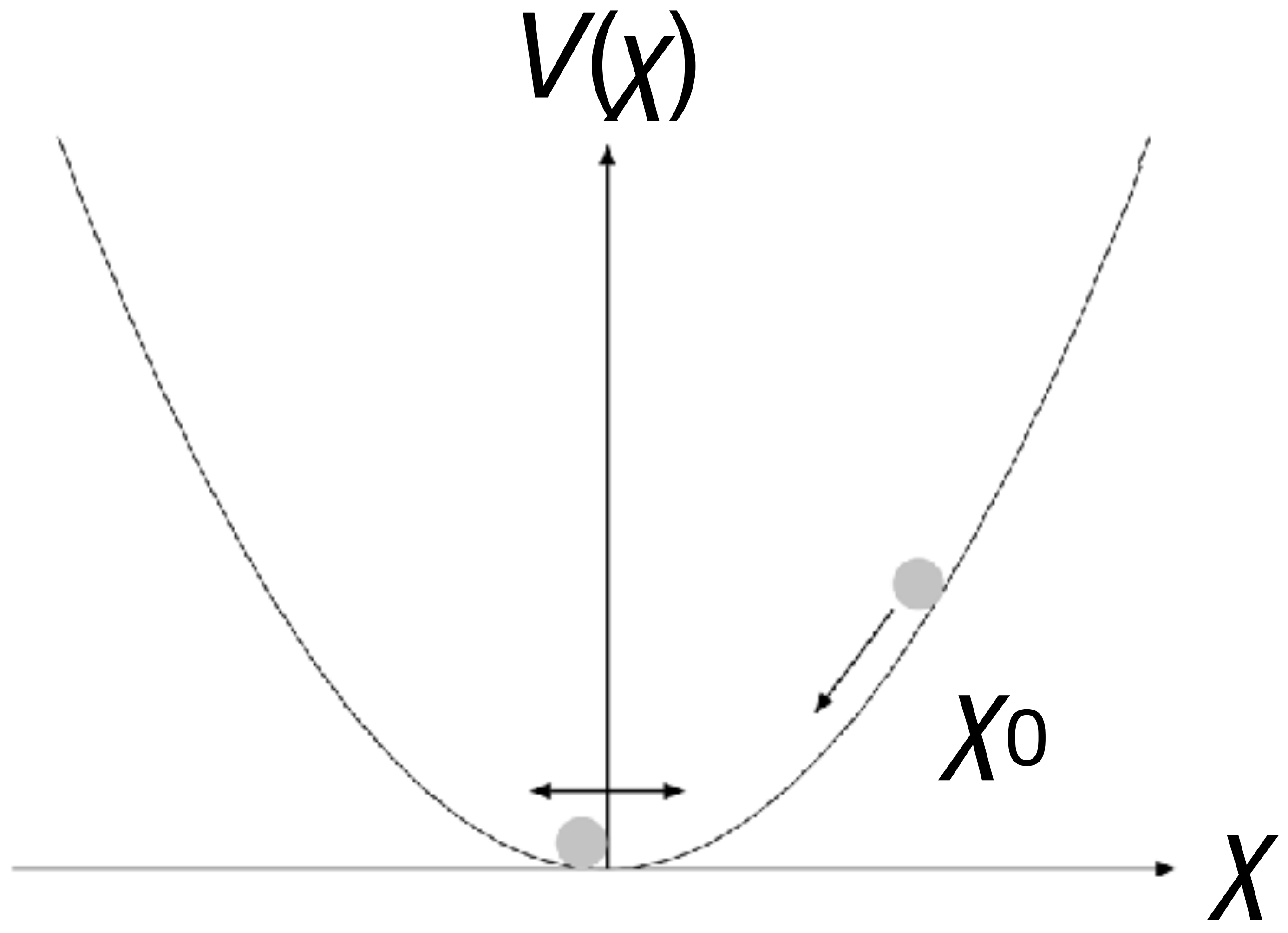
is the average over time. *T* is some characteristic timescale for χ, like the period of oscillations.

Pressure of a massive free scalar field

$V(\chi) = m^2\chi^2/2$ (c=1 and ℏ=1)

- To simplify notation, we will omit the overline, $\bar{\chi}(t) \rightarrow \chi(t)$.
- The equation of motion for a massive free scalar field is

$$\ddot{\chi} + m^2\chi = 0$$



- The solution with $\chi(0) = \chi_0$ and $\dot{\chi}(0) = 0$ is

$$\chi(t) = \chi_0 \cos(mt)$$

Oscillations with the period $T = 2\pi/m$.

$$\Rightarrow \quad \rho = \frac{1}{2}m^2\chi_0^2\,, \quad P = 0$$

Pressureless –> CDM!

Obata, Fujita, Michimura (2018); Fedderke, Graham, Rajendran (2019)

CDM-induced parity violation in EM waves

"Time-domain cosmology"

- The Chern-Simons interaction between photons and CDM gives

$$\ddot{A}_\pm + \left(k^2 \mp \frac{k\alpha\dot{\chi}}{f}\right) A_\pm = 0$$

$$\ddot{A}_\pm + \left[k^2 \pm \frac{k\alpha\chi_0 m}{f} \underline{\sin(mt)}\right] A_\pm = 0$$

Periodic change for parity violation in EM waves

$$T = \frac{2\pi}{m} \simeq 1\ \mathrm{day} \left(\frac{5\times 10^{-20}\ \mathrm{eV}}{m}\right)$$

This is a human timescale!!

Turner (1983)

Problem Set 5

P/ρ for a power-law potential

- In non-expanding space, show that

$$\langle \dot{\chi}^2 \rangle = \langle \chi \partial V / \partial \chi \rangle = 2n \langle V \rangle$$

for a power-law potential, $V(\chi) \propto \chi^{2n}$.

Hint:

$$\dot{\chi}^2 = (\chi\dot{\chi})^{\cdot} - \chi\ddot{\chi}$$

- Show that

$$\frac{P}{\rho} = \frac{n-1}{n+1} = \begin{cases} 0 & \text{for } V(\chi) = \chi^2 \\ 1/3 & \text{for } V(\chi) = \chi^4 \\ 1/2 & \text{for } V(\chi) = \chi^6 \end{cases}$$

4.2 Evolution of χ in Expanding Space

Equation of motion in expanding space

With the physical time *t*, instead of the conformal time *τ*

$$\Box\chi - \frac{\partial V}{\partial \chi} = -\ddot{\chi} - 3H\dot{\chi} - m^2\chi = 0$$

$$\Box = \frac{1}{\sqrt{-g}}\partial_\mu(\sqrt{-g}g^{\mu\nu}\partial_\nu)$$
$$= -\frac{\partial^2}{\partial t^2} - 3\frac{\dot{a}}{a}\frac{\partial}{\partial t} + \frac{1}{a^2}\nabla^2$$

where $g^{\mu\nu} = \mathrm{diag}(-1, \mathbf{1}/a^2)$

$\sqrt{-g} = a^3$

- During the matter-dominated era, $a(t) \propto t^{2/3}$ and $H(t) = 2/(3t)$.
- The solution with $\chi(0) = \chi_0$ and $\dot{\chi}(0) = 0$ is

$$\chi(t) = \chi_0 \frac{\sin(mt)}{mt}$$

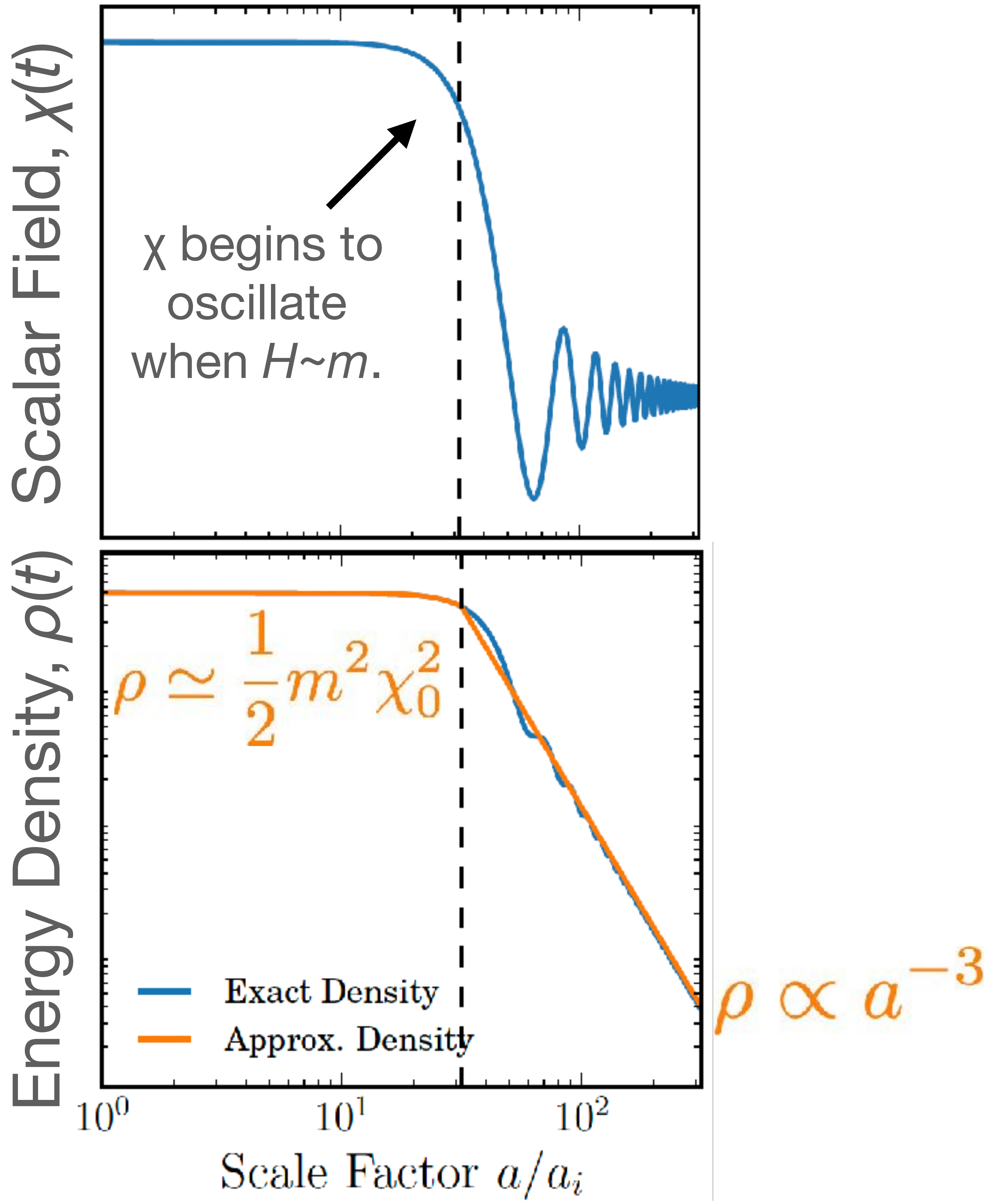
(Non-expanding case ⟷ $\chi(t) = \chi_0 \cos(mt)$)

Evolution of χ in expanding space

***m* < *H* or *m* > *H*?**

$$\chi(t) = \chi_0 \frac{\sin(mt)}{mt}$$

- For $mt \ll 1$ (or $m \ll H$), $\chi(t) \simeq \chi_0$.
  - The energy density is a constant.
- For $mt \gg 1$ (or $m \gg H$), $\chi(t) \propto t^{-1} \propto a^{-3/2}$.
  - The energy density dilutes away as $\rho \propto a^{-3}$, in agreement with pressureless matter.



Evolution of *P/ρ* in expanding space

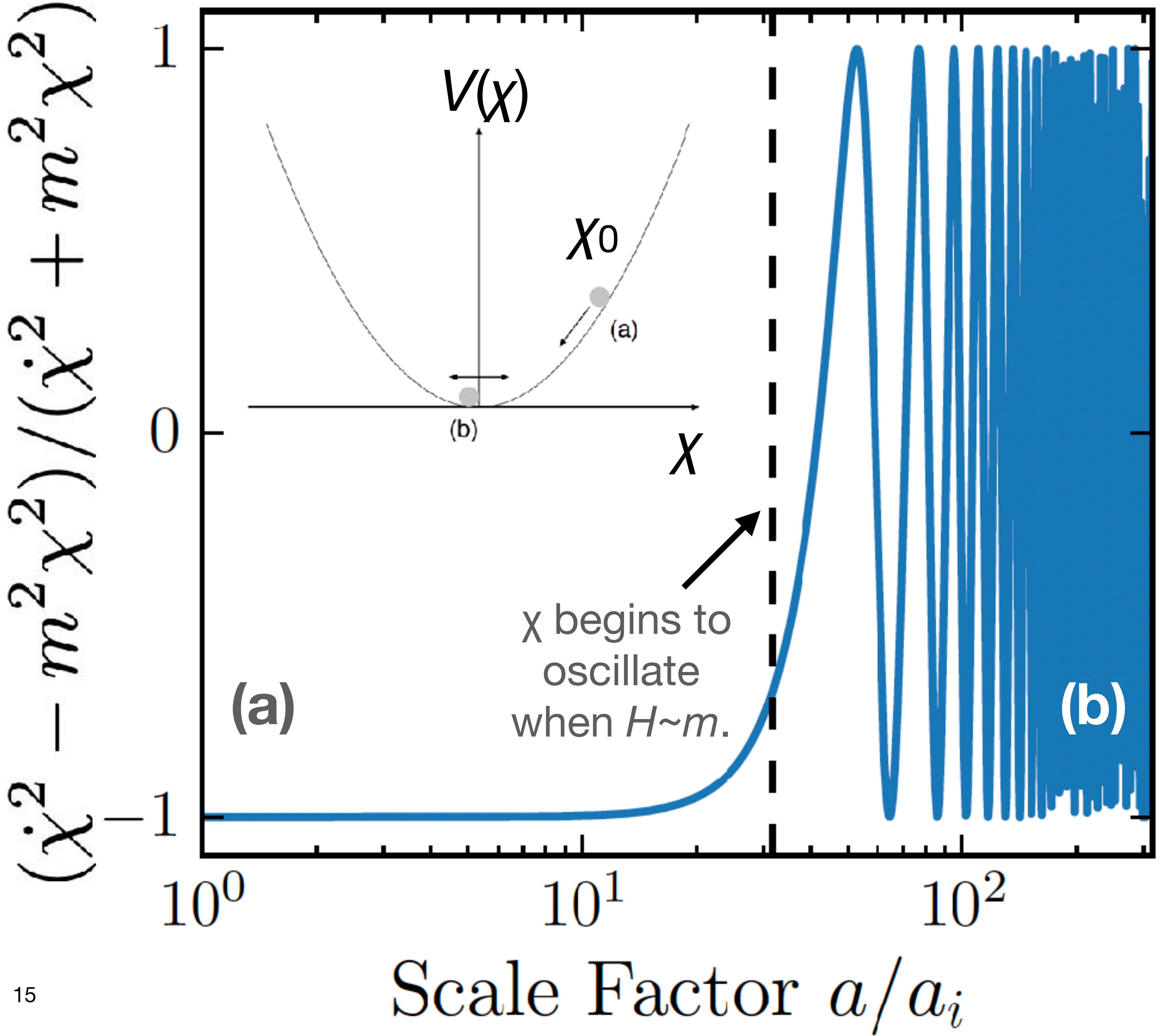
m < H or *m > H*?

- If we do not average over time,

$$\frac{\dot{\chi}^2 - m^2\chi^2}{\dot{\chi}^2 + m^2\chi^2}$$

  oscillates rapidly around 0 for *m>H*.

- The ratio is -1 for *m<H.*
  - This means *P*=-*ρ*. *Dark energy!*



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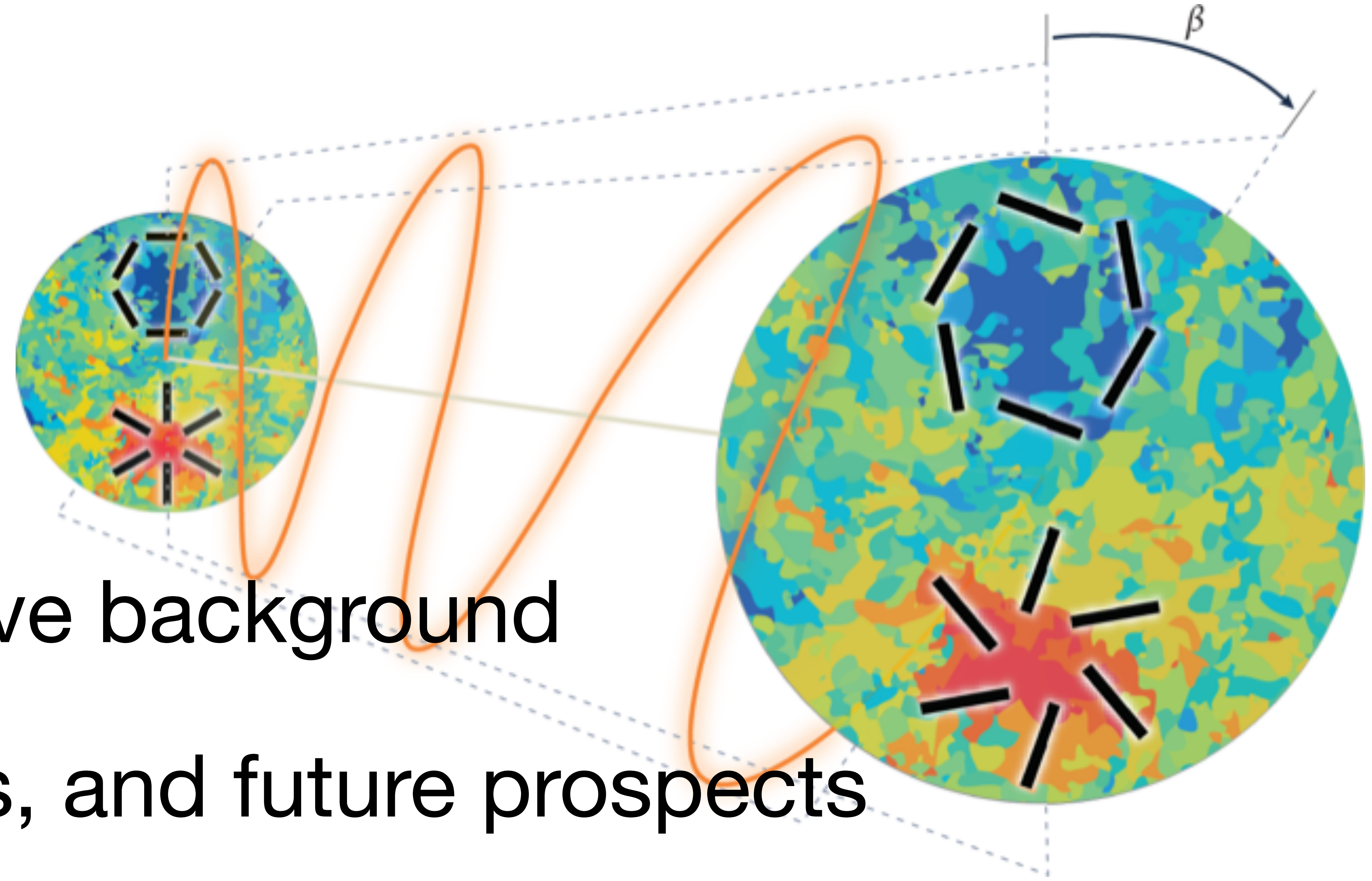


5.1 Scalar Field Dark Energy

What is dark energy (DE)?

No one knows!

- We really have no idea.
  - Most people assume, for practical purposes, that dark energy is Einstein's cosmological constant (Λ). However, recent advances in quantum gravity research suggest that Λ is an unlikely explanation…
  - My approach: Only experiments will tell us the answer!
  - Searching for parity violation might help?

Equation of state parameter of DE

Astronomers have been measuring this parameter for 25 years.

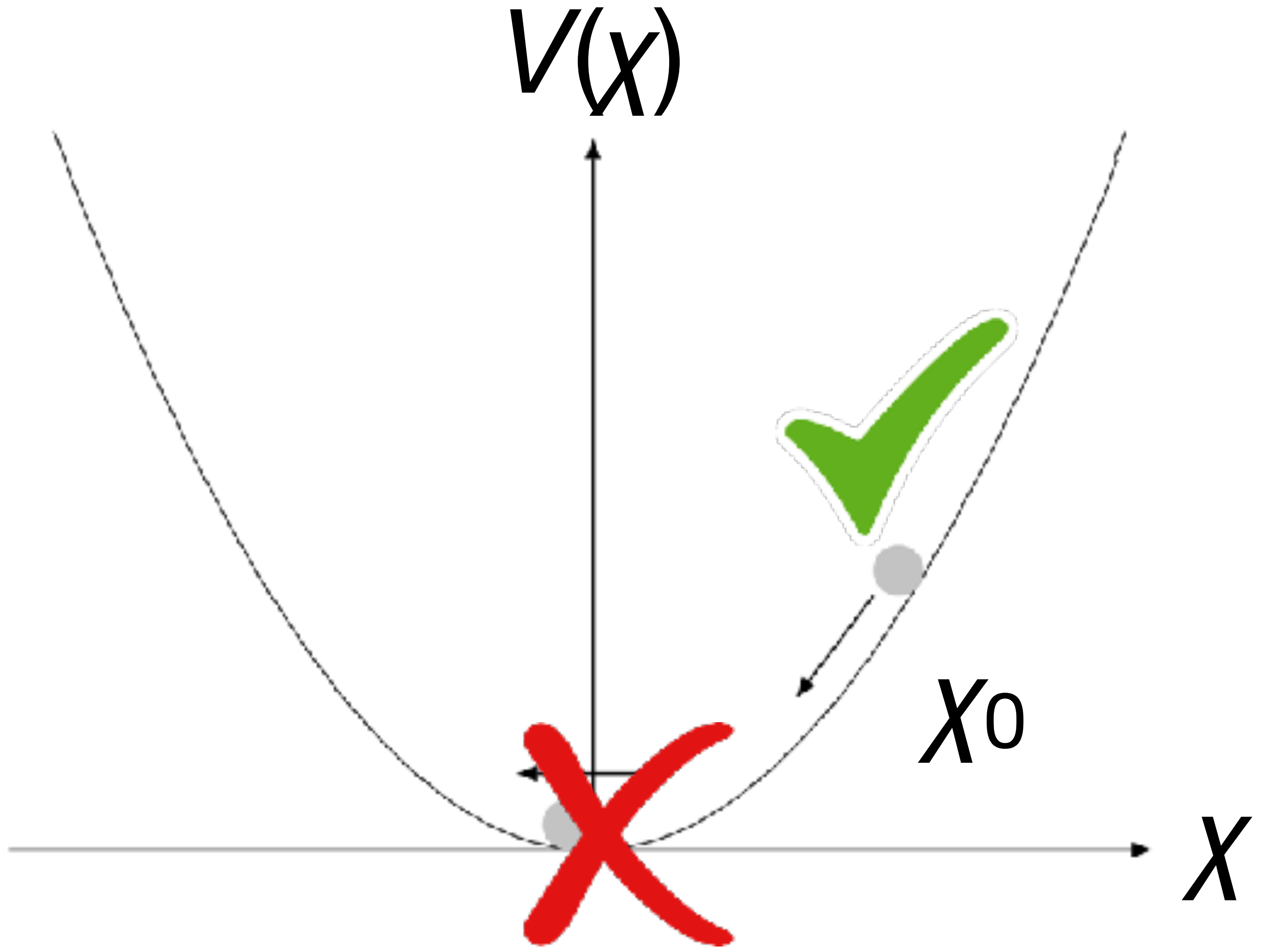
$$w = \frac{P_{\mathrm{DE}}}{\rho_{\mathrm{DE}}} = -0.978^{+0.024}_{-0.031}$$

(68% CL; Brout et al. 2022)

- If DE is a scalar field,

$$w = \frac{\frac{1}{2}\langle\dot{\chi}^2\rangle - \langle V(\chi)\rangle}{\frac{1}{2}\langle\dot{\chi}^2\rangle + \langle V(\chi)\rangle}$$

- Therefore, current observations **require** that $\dot{\chi}^2 \ll V(\chi)$.

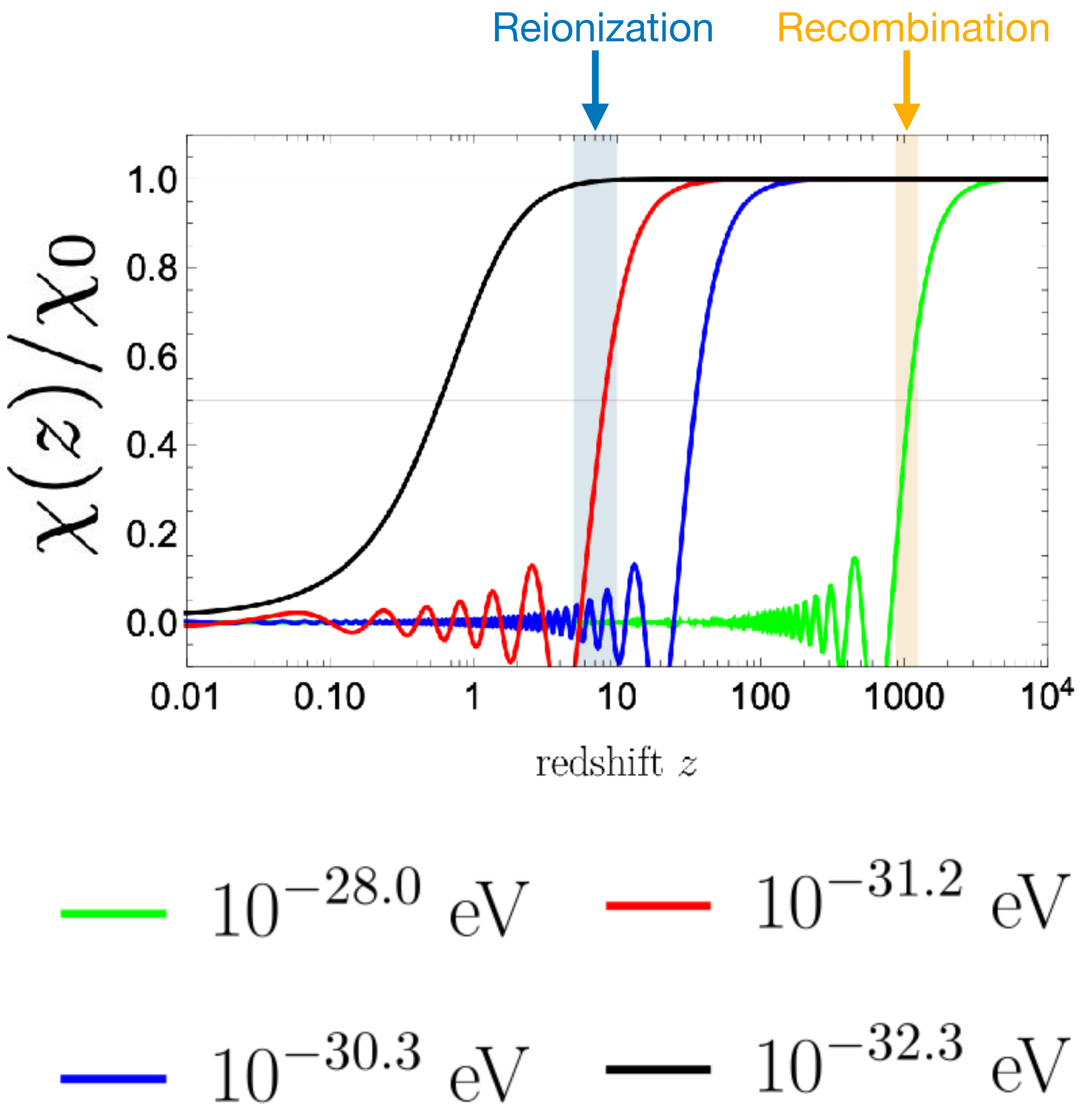


Nakatsuka, Namikawa, EK (2022)

Scalar field dark energy

A ridiculously small "mass"

- The (effective) mass of a scalar field DE must be smaller than the current expansion rate of the Universe (the Hubble constant).
  - $m < H_0 \simeq 10^{-33}$ eV
  - A ridiculously small "mass"!
  - This simply means that the scalar field potential must be nearly flat, and that the scalar field is still slowly rolling down the potential today.



Carroll (1998)

DE-induced parity violation in EM waves

- The Chern-Simons interaction between photons and DE gives

$$A''_{\pm} + \left(k^2 \mp \frac{k\alpha\chi'}{f}\right) A_{\pm} = 0$$

- The slow-roll of χ implies

$$\ddot{\chi} + 3H\dot{\chi} = -\frac{\partial V}{\partial \chi} \quad \Rightarrow \quad \dot{\chi} = \frac{1}{a}\chi' \simeq -\frac{1}{3H}\frac{\partial V}{\partial \chi}$$

negligible

Gasparotto, Obata (2022)

DE-induced parity violation in EM waves

- The Chern-Simons interaction between photons and DE gives

$$A''_\pm + \left(k^2 \pm \frac{k\alpha a}{3Hf}\boxed{\frac{\partial V}{\partial \chi}}\right) A_\pm = 0$$

It is the **slope** of the potential, rather than the mass (second derivative), that determines the magnitude of the parity violation in EM waves due to DE.

- The slow-roll of χ implies

$$\cancel{\ddot{\chi}} + 3H\dot{\chi} = -\frac{\partial V}{\partial \chi} \quad \Rightarrow \quad \dot{\chi} = \frac{1}{a}\chi' \simeq -\frac{1}{3H}\frac{\partial V}{\partial \chi}$$

negligible

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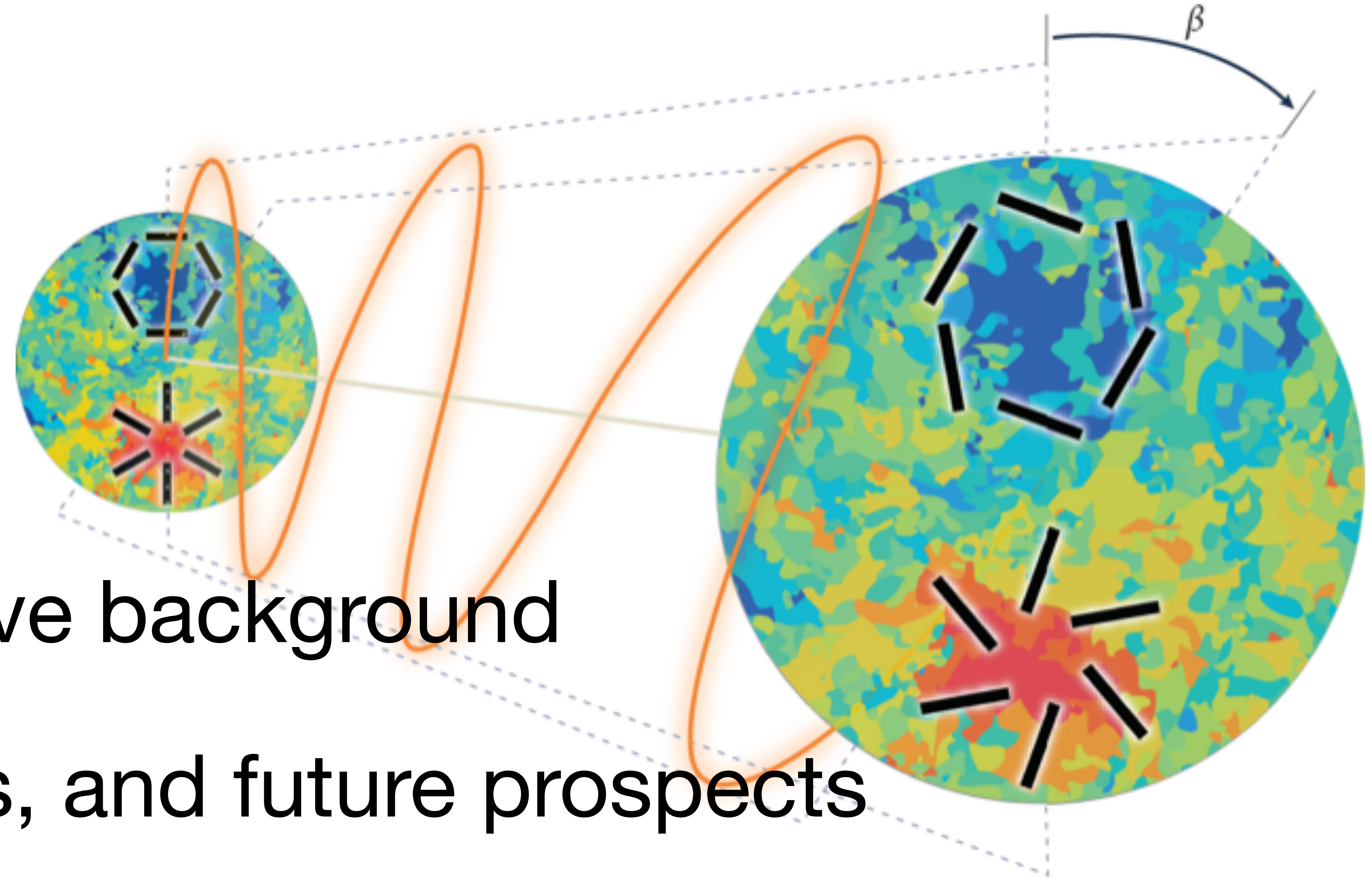


6.1 Polarization of Light

Phase velocity of circular polarization states

***c*=1**

- We write

$$A''_\pm + \omega_\pm^2 A_\pm = 0, \quad \omega_\pm^2 = k^2 \mp \frac{k\alpha\chi'}{f}$$

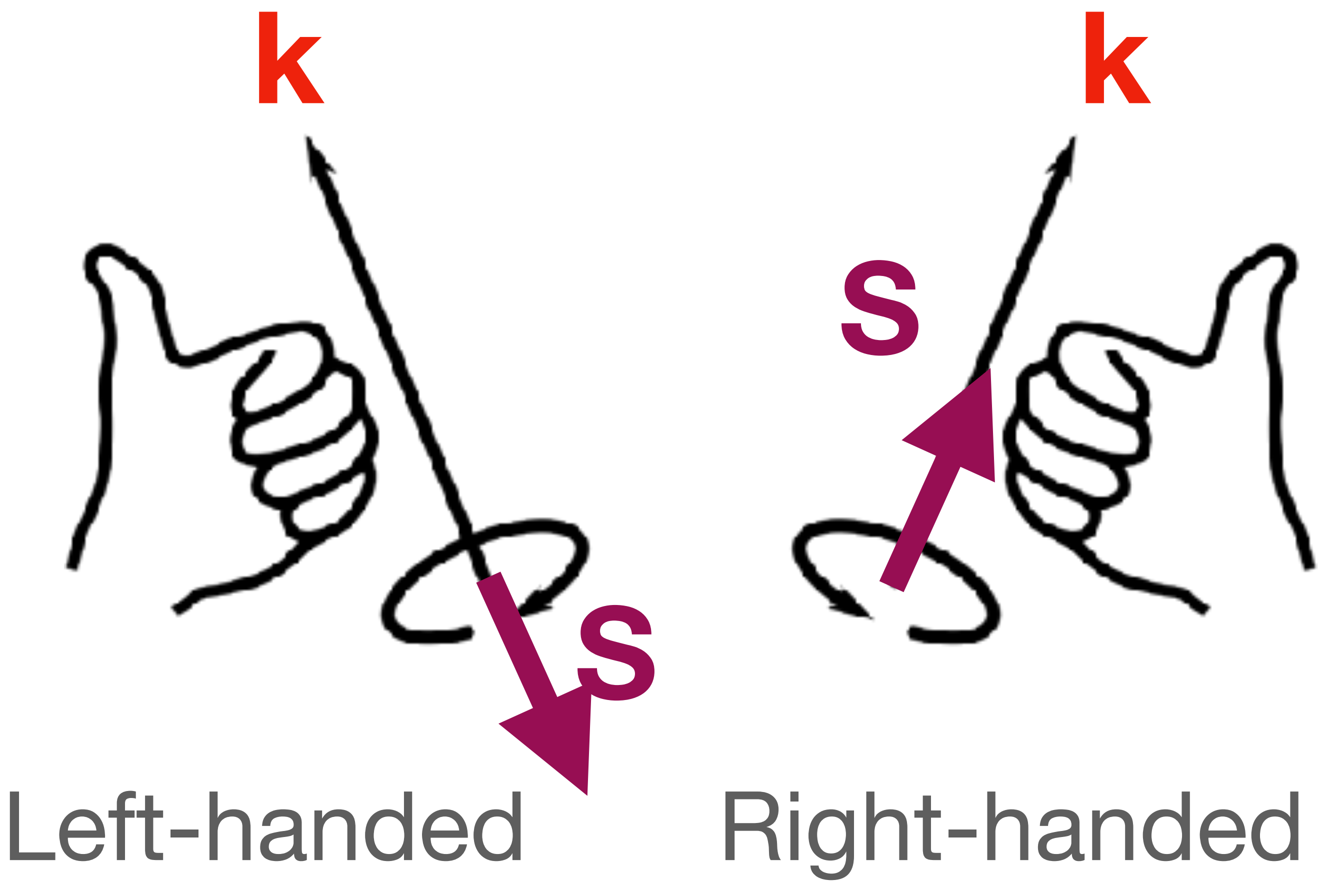
- In contrast to inflation, where $\omega_\pm^2$ can become negative (Day 3), we will work in the limit of $k^2 \gg k\alpha\chi'/f$. This approximation is accurate for the photons we observe today.

- The phase velocity of circular polarization states, $\omega_\pm/k$, is

$$\frac{\omega_\pm}{k} \simeq 1 \mp \frac{\alpha\chi'}{2kf}$$

+: Right-handed state

–: Left-handed state



Plane-wave Solution

c=1

$$A''_\pm + \omega_\pm^2 A_\pm = 0, \quad \omega_\pm \simeq k \mp \frac{\alpha\chi'}{2f}$$

- For $|\omega'_\pm| \ll \omega_\pm^2$, which is satisfied here, an accurate solution is given by

$$A_\pm \simeq C_\pm \frac{\exp\left(-i\int d\tau\ \omega_\pm + i\delta_\pm\right)}{\boxed{\sqrt{2\omega_\pm}} \simeq \sqrt{2k}}$$

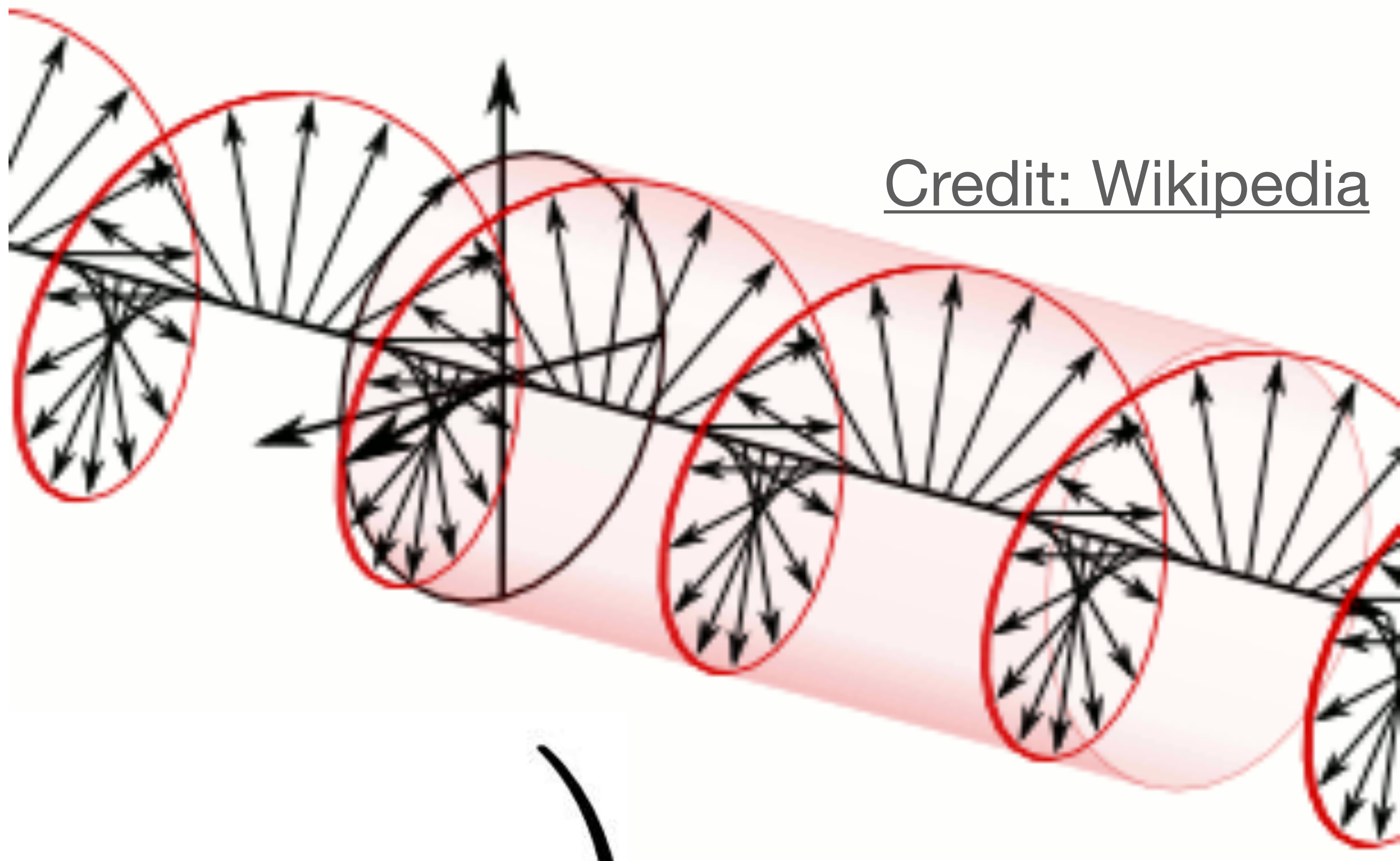
We can replace $\omega_\pm$ in amplitude (but not in phase) with k.

where $C_\pm$ is the initial amplitude and $\delta_\pm$ is the initial phase.

Electric Field

In the circular polarization basis

The arrows show directions of the electric field vector **E**.



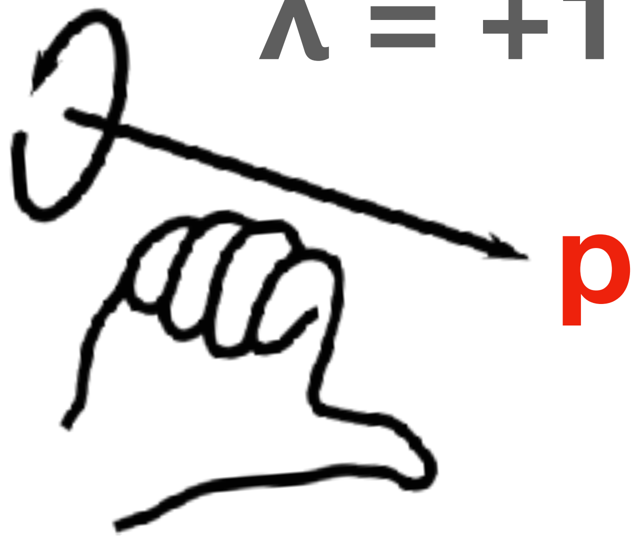
Credit: Wikipedia

- As $\mathbf{E} = -a^{-2}\mathbf{A}'$,

$$E_\pm \simeq i\sqrt{\frac{k}{2}}\frac{C_\pm}{a^2(\tau)}\exp\left(-i\int d\tau\ \omega_\pm + i\delta_\pm\right)$$

where $a(\tau_{\rm ini}) = 1$ at the initial conformal time, $\tau_{\rm ini}$.

Right-handed
λ = +1

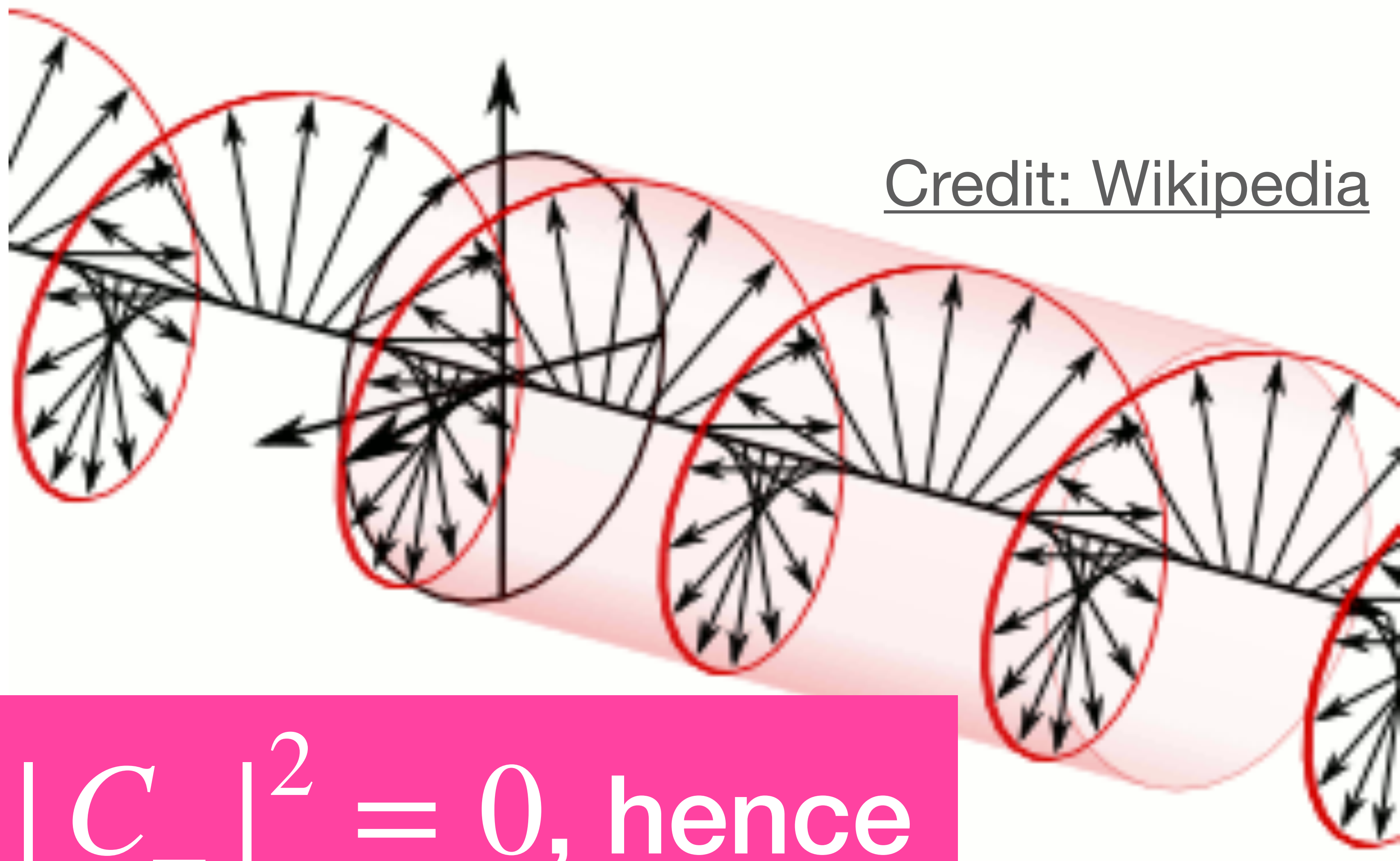


- The circular polarization is given by $V = |E_+|^2 - |E_-|^2 \propto |C_+|^2 - |C_-|^2$. Therefore, the **Chern-Simons term with $|\omega_\pm'| \ll \omega_\pm^2$ does not create new circular polarization**, if there was no circular polarization to begin with.

Electric Field

In the circular polarization basis

The arrows show directions of the electric field vector **E**.



- As $\mathbf{E} = -a^{-2}\mathbf{A}'$,

$$E_\pm \simeq i\sqrt{\frac{k}{2}}\frac{C_\pm}{a^2(\tau)}$$

We will assume $|C_+|^2 - |C_-|^2 = 0$, hence no circular polarization. But, it can be *linearly polarized*.

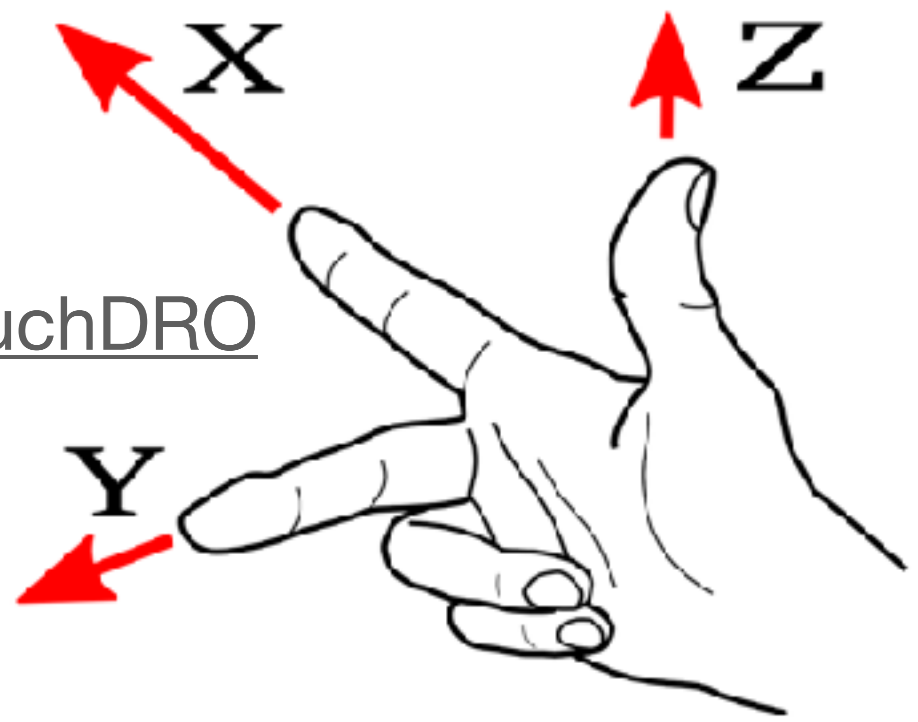
where $a(\tau_{\text{ini}}) = 1$ at the initial conformal time, τ_{ini}.

- The circular polarization is given by $V = |E_+|^2 - |E_-|^2 \propto |C_+|^2 - |C_-|^2$. Therefore, the **Chern-Simons term with $|\omega'_\pm| \ll \omega^2_\pm$ does not create new circular polarization**, if there was no circular polarization to begin with.

Linear Polarization: Stokes Parameters

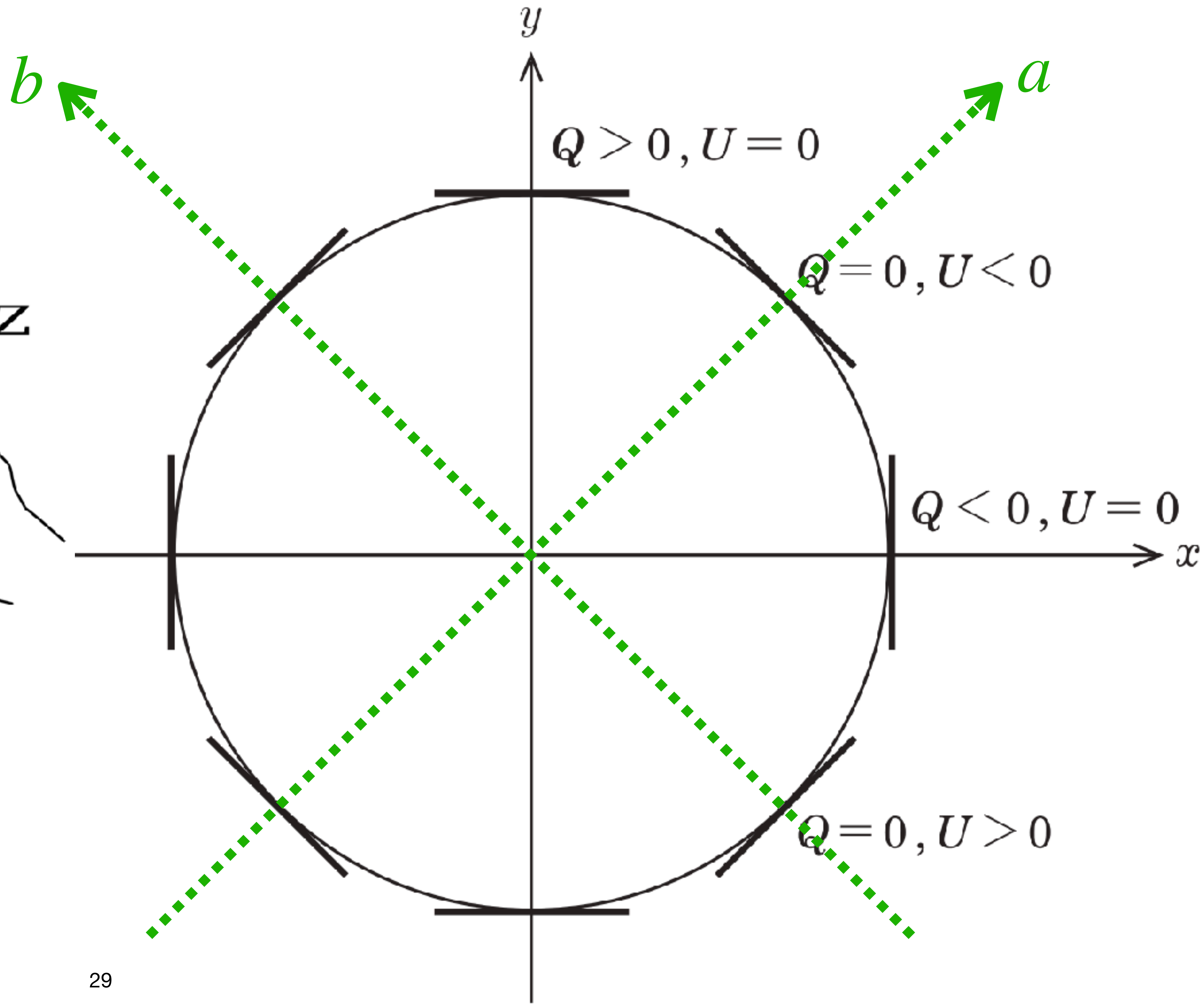
Q and *U*

- In the right-handed coordinate system, the light is coming towards us in the z-direction.



Credit: TouchDRO

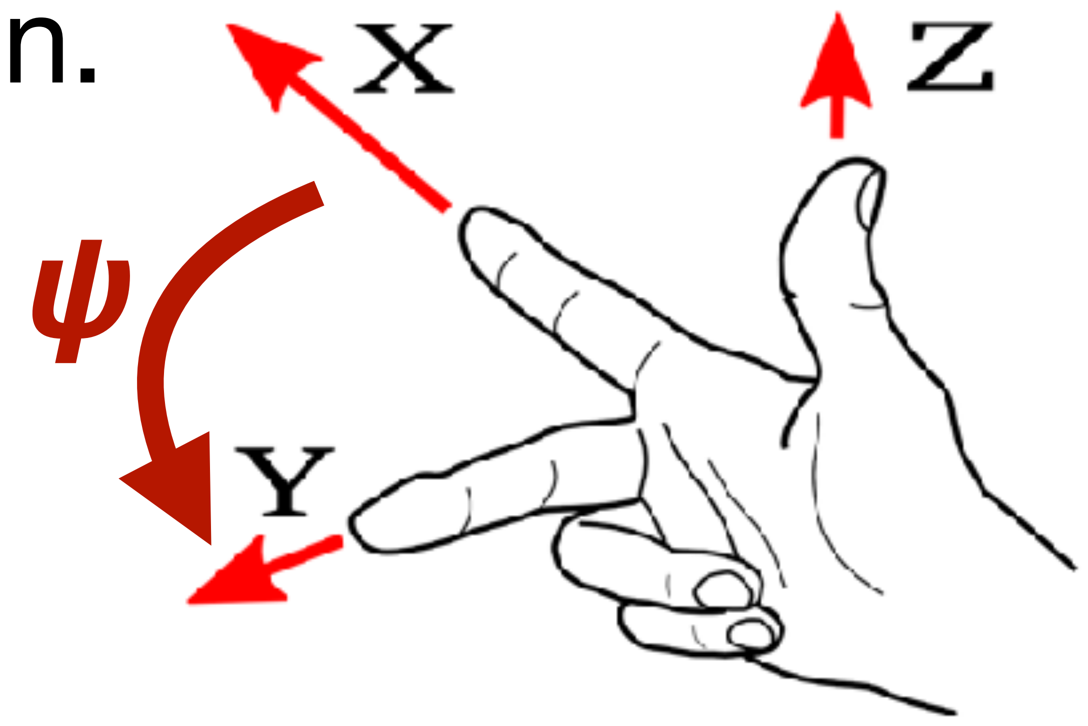
- Each thick black line shows the direction of linear polarization.
- $Q \propto |E_x|^2 - |E_y|^2$
- $U \propto |E_a|^2 - |E_b|^2 = 2\text{Re}(E_x^* E_y)$



Linear Polarization: Stokes Parameters

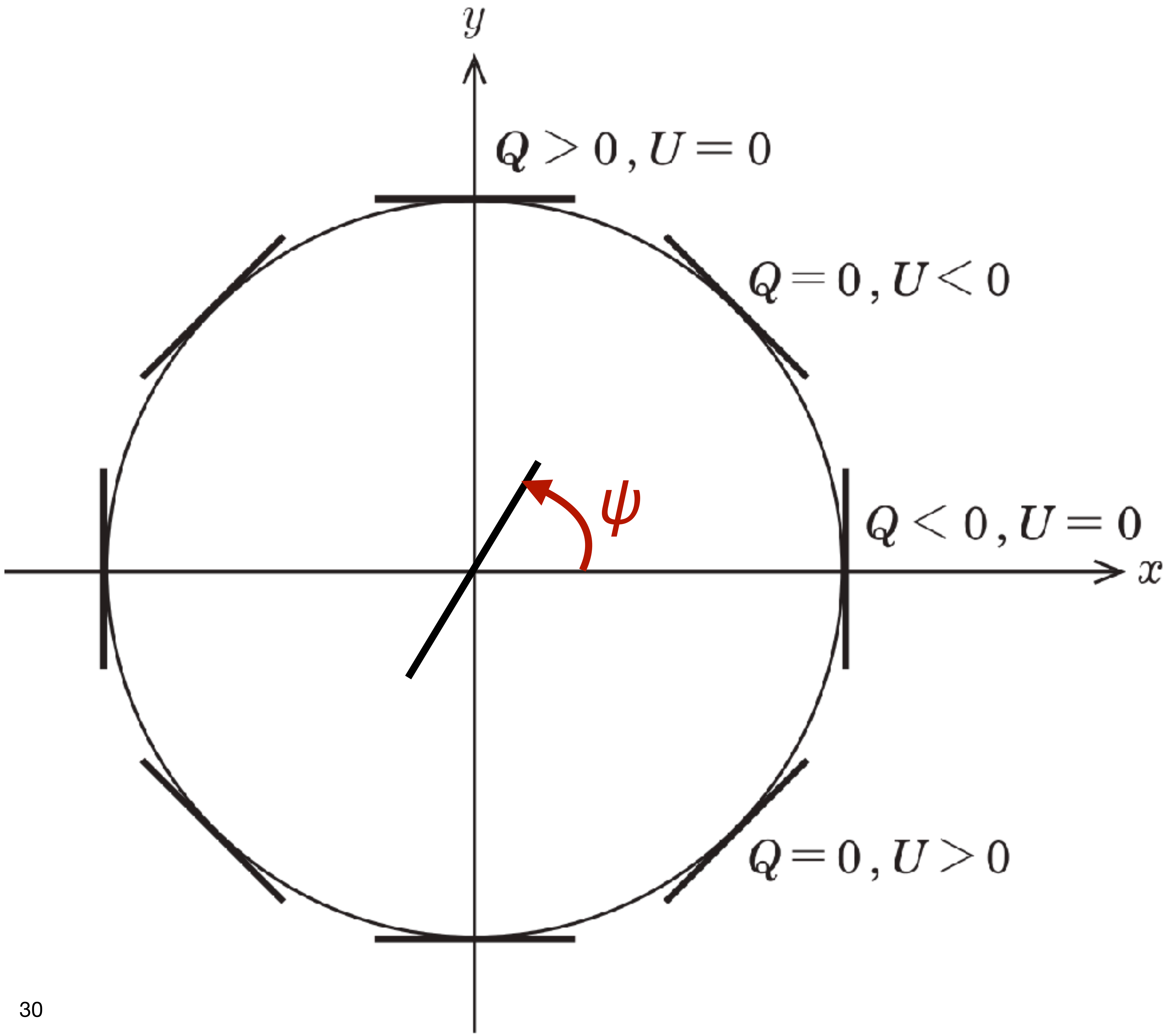
ψ: Position Angle (PA)

- In the right-handed coordinate system, the light is coming towards us in the z-direction.



- The position angle (PA) the plane of linear polarization is given by

$$\frac{U}{Q} = \tan(2\psi)$$



Linear Polarization: Stokes Parameters

Q±i*U*: Spin-2 Field

- The complex combination,

$$Q \pm iU = Pe^{\pm 2i\psi}$$

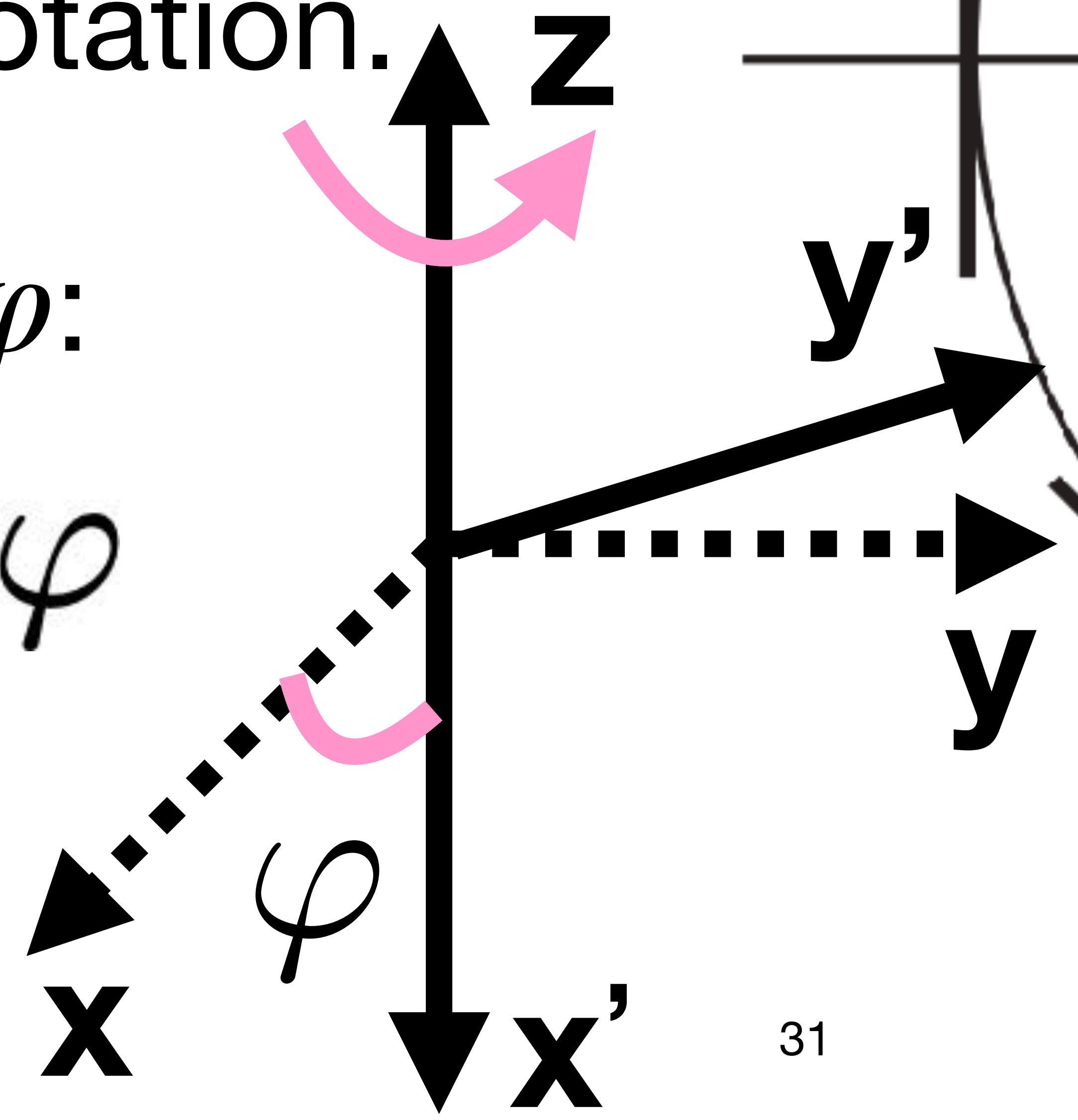
  where *P* is the "polarization intensity", transforms as a spin-2 field under coordinate rotation.

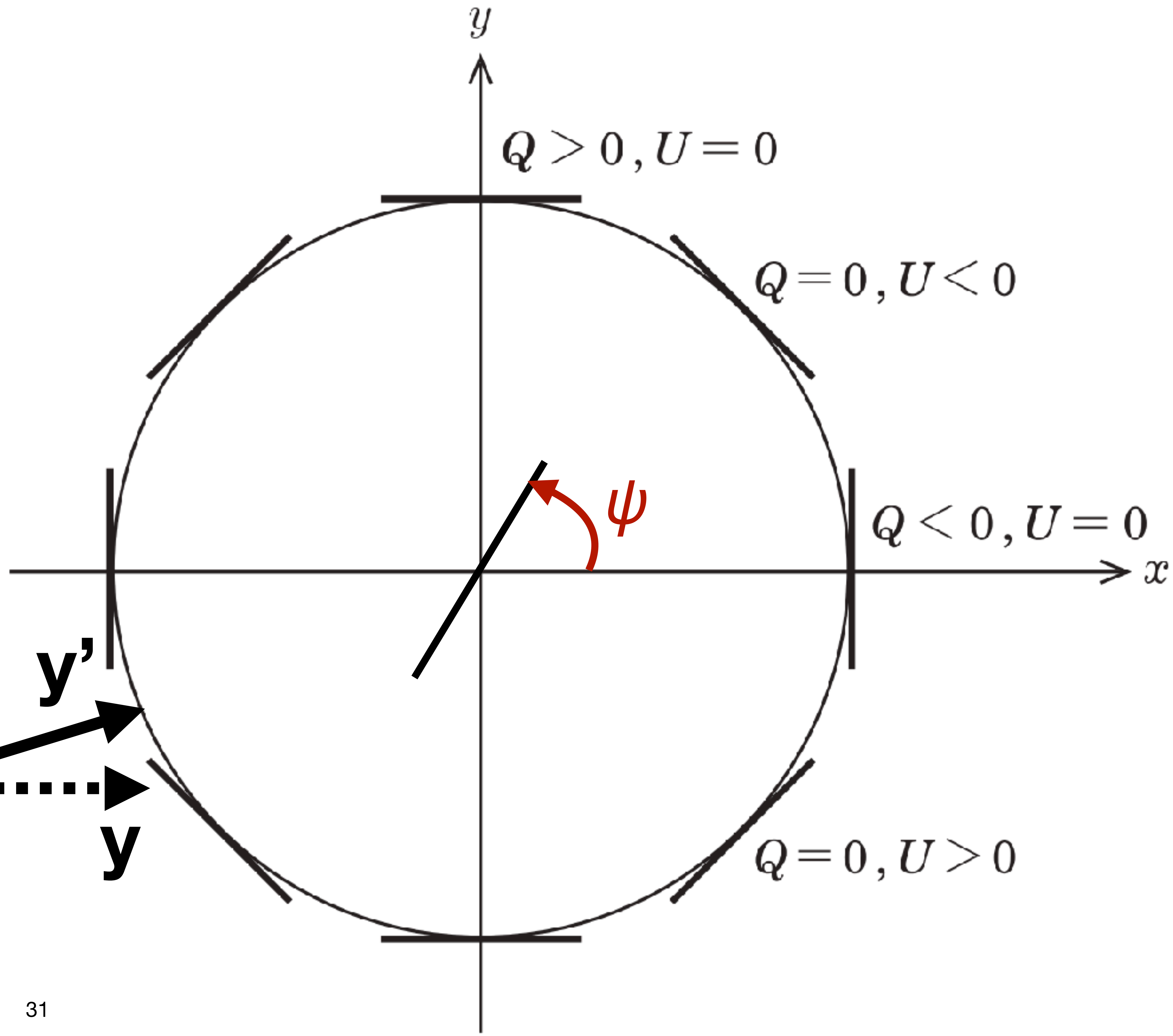
- Coordinate rotation by φ:

$$\psi \to \psi' = \psi - \varphi$$

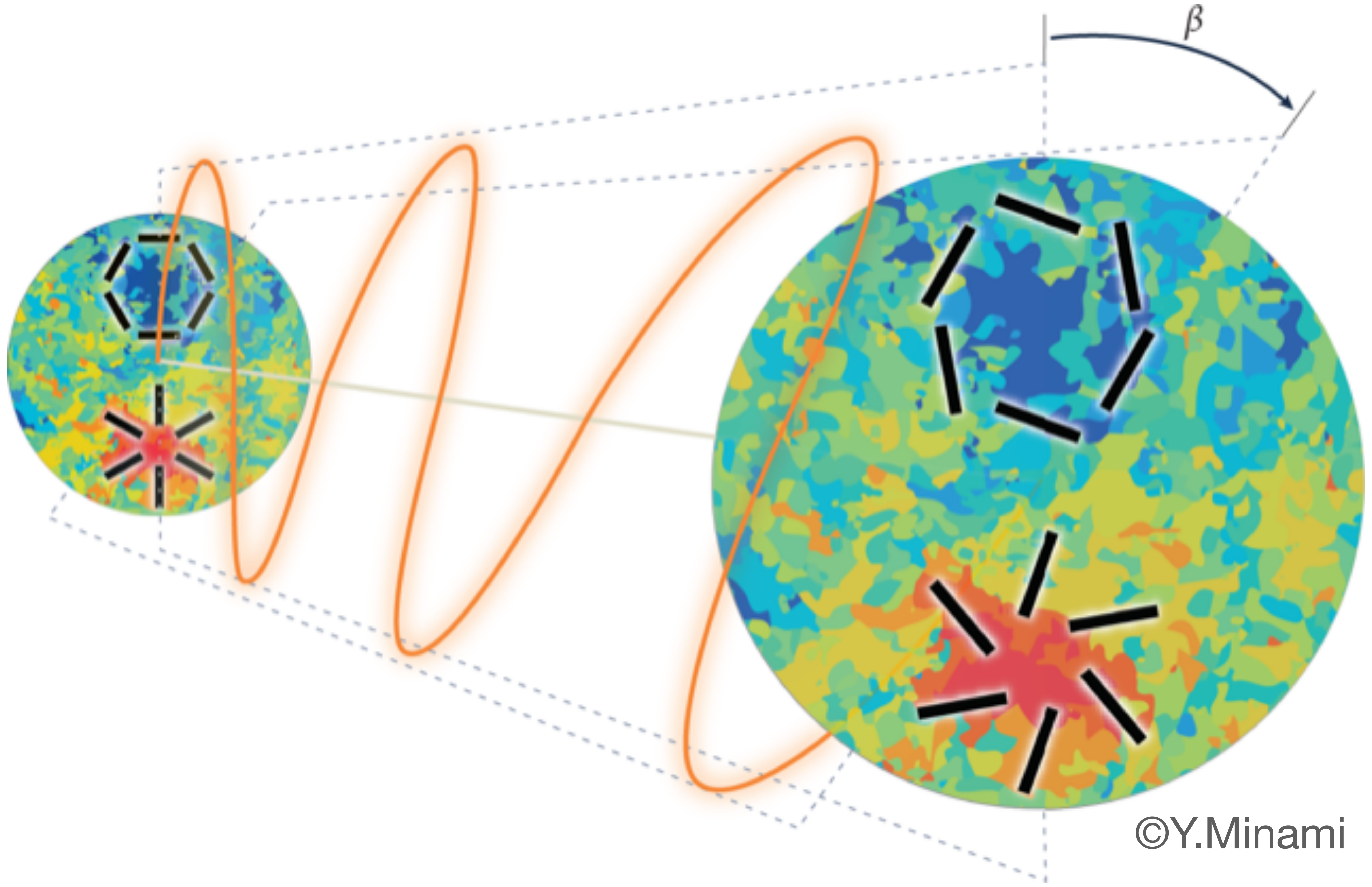
- Thus,

$$Q' \pm iU' = e^{\mp 2i\varphi}(Q \pm iU)$$





6.2 Cosmic Birefringence



Let’s calculate the linear polarization

From $E_\pm$ to E_x, E_y

- $E_\pm = (E_x \mp iE_y)/\sqrt{2}$ (Day 2)
  - $E_x = (E_+ + E_y)/\sqrt{2}$
  - $E_y = i(E_+ - E_-)/\sqrt{2}$
- $Q \propto |E_x|^2 - |E_y|^2 = 2\text{Re}(E_+^* E_-)$
- $U \propto 2\text{Re}(E_x^* E_y) = 2\text{Im}(E_+^* E_-)$

Carroll, Field, Jackiw (1990); Carroll, Field (1991); Harari, Sikivie (1992)

Cosmic Birefringence due to the CS term

Rotation of the plane of linear polarization

$$A''_\pm + \omega_\pm^2 A_\pm = 0, \quad \omega_\pm \simeq k \mp \frac{\alpha\chi'}{2f}$$

- $E_\pm = (E_x \mp iE_y)/\sqrt{2}$ (Day 2)
  - $E_x = (E_+ + E_y)/\sqrt{2}$
  - $E_y = i(E_+ - E_-)/\sqrt{2}$
- $Q \propto |E_x|^2 - |E_y|^2 = 2\mathrm{Re}(E_+^* E_-)$
- $U \propto 2\mathrm{Re}(E_x^* E_y) = 2\mathrm{Im}(E_+^* E_-)$

$$E_\pm \propto \exp\left(-i\int d\tau\ \omega_\pm + i\delta_\pm\right)$$

↓

$$Q \propto \cos\left[\int d\tau(\omega_+ - \omega_-) - (\delta_+ - \delta_-)\right]$$

$$U \propto \sin\left[\int d\tau(\omega_+ - \omega_-) - (\delta_+ - \delta_-)\right]$$

Carroll, Field, Jackiw (1990); Carroll, Field (1991); Harari, Sikivie (1992)

Cosmic Birefringence due to the CS term

Rotation of the plane of linear polarization

$$A''_\pm + \omega_\pm^2 A_\pm = 0, \quad \omega_\pm \simeq k \mp \frac{\alpha\chi'}{2f}$$

- $E_\pm = (E_x \mp iE_y)/\sqrt{2}$ (Day 2)
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- $U \propto 2\mathrm{Re}(E_x^* E_y) = 2\mathrm{Im}(E_+^* E_-)$

$$E_\pm \propto \exp\left(-i\int d\tau\, \omega_\pm + i\delta_\pm\right)$$

$$Q \propto \cos\left[\int d\tau(\omega_+ - \omega_-) - (\delta_+ - \delta_-)\right]$$

$$\frac{U}{Q} = \tan(2\psi)$$

$$U \propto \sin\left[\int d\tau(\omega_+ - \omega_-) - (\delta_+ - \delta_-)\right]$$

Rotation of PA! Initial PA

$$\psi = \frac{1}{2}\int d\tau(\omega_+ - \omega_-) - \frac{1}{2}(\delta_+ - \delta_-)$$

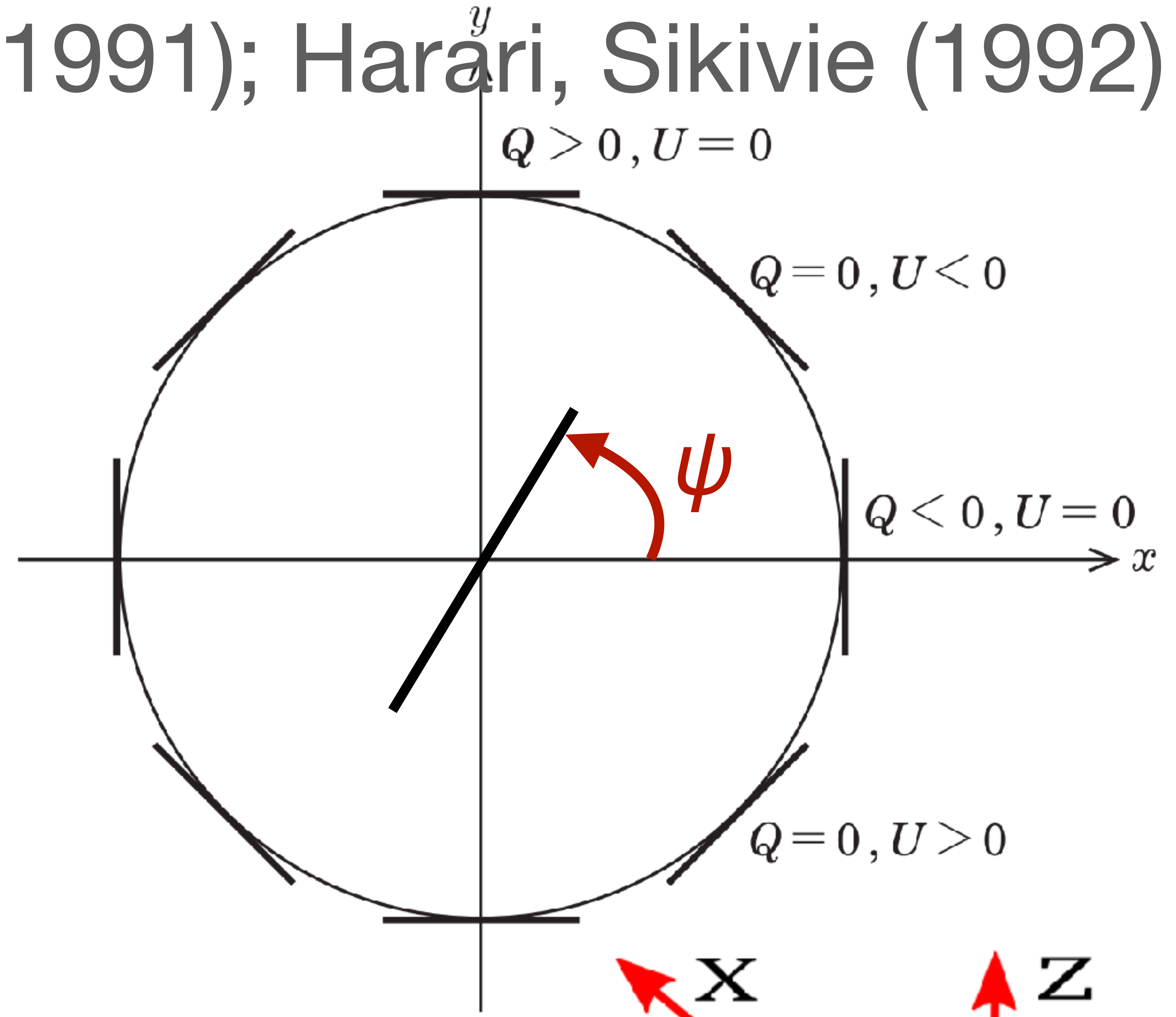
Carroll, Field, Jackiw (1990); Carroll, Field (1991); Harari, Sikivie (1992)

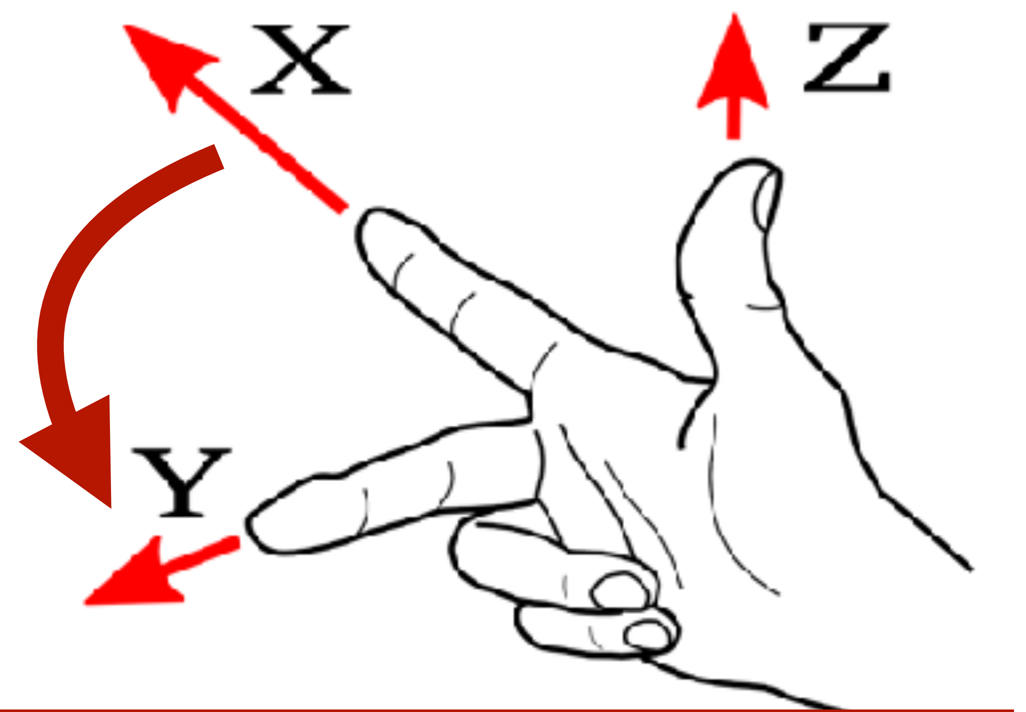
Cosmic Birefringence

$$\omega_\pm \simeq k \mp \frac{\alpha\chi'}{2f}$$

- Using $\omega_+ - \omega_- = -\alpha\chi'/f$, we find

$$\psi_{\rm obs} - \psi_{\rm em} = -\frac{\alpha}{2f}\int_{\tau_{\rm em}}^{\tau_{\rm obs}} d\tau\ \chi'$$
$$= -\frac{\alpha}{2f}\left[\chi(\tau_{\rm obs}) - \chi(\tau_{\rm em})\right]$$





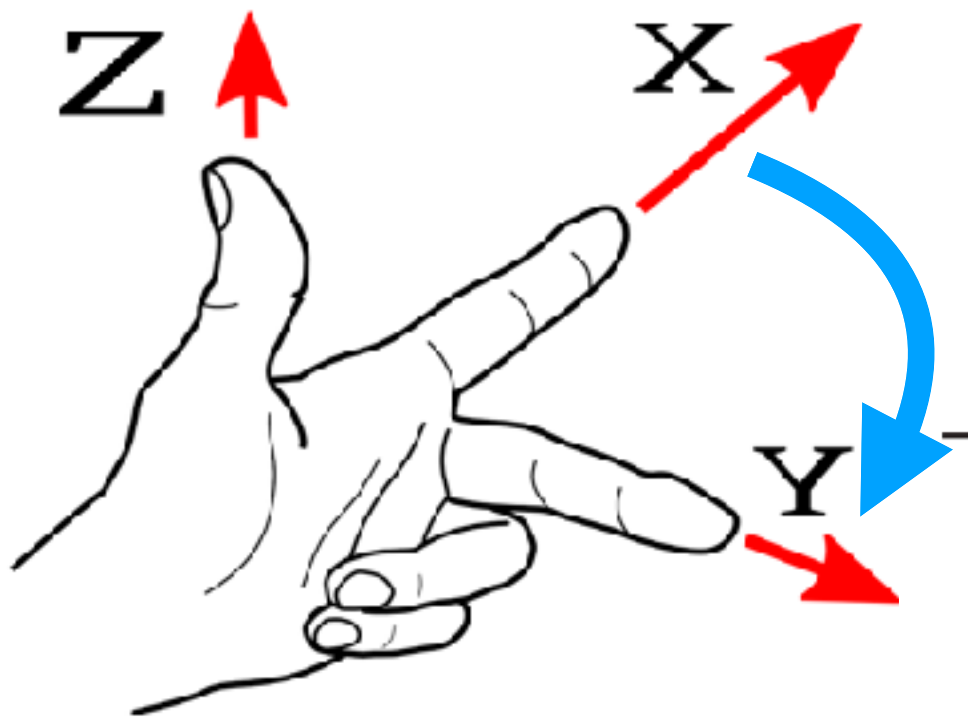
The rotation angle is given by the difference between scalar field values at the emission and observation times and is independent of events in-between.

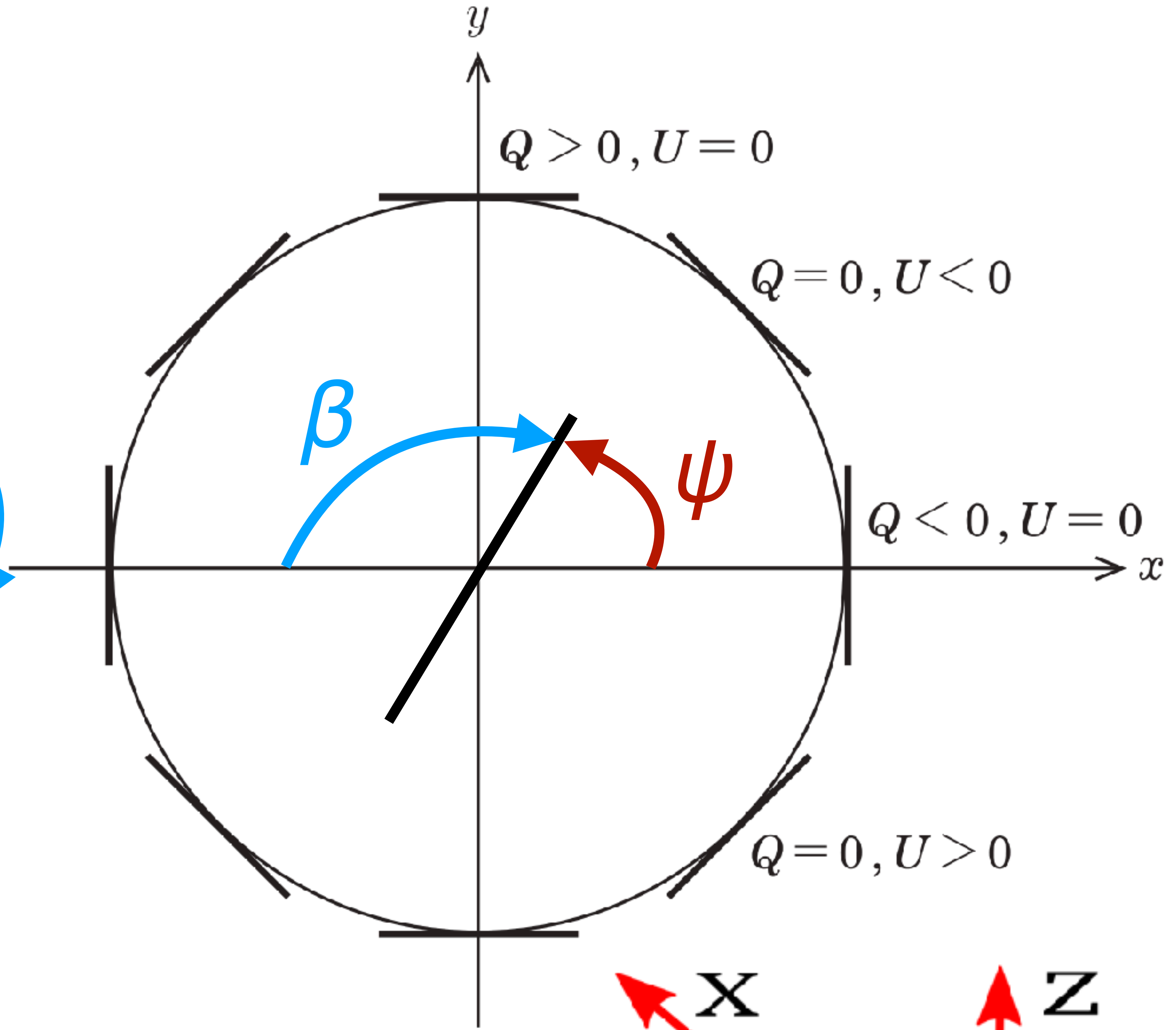
$\psi > 0$ is a *counter-clockwise* rotation on the sky.

di Serego Alighieri (2017)

Cosmic Birefringence

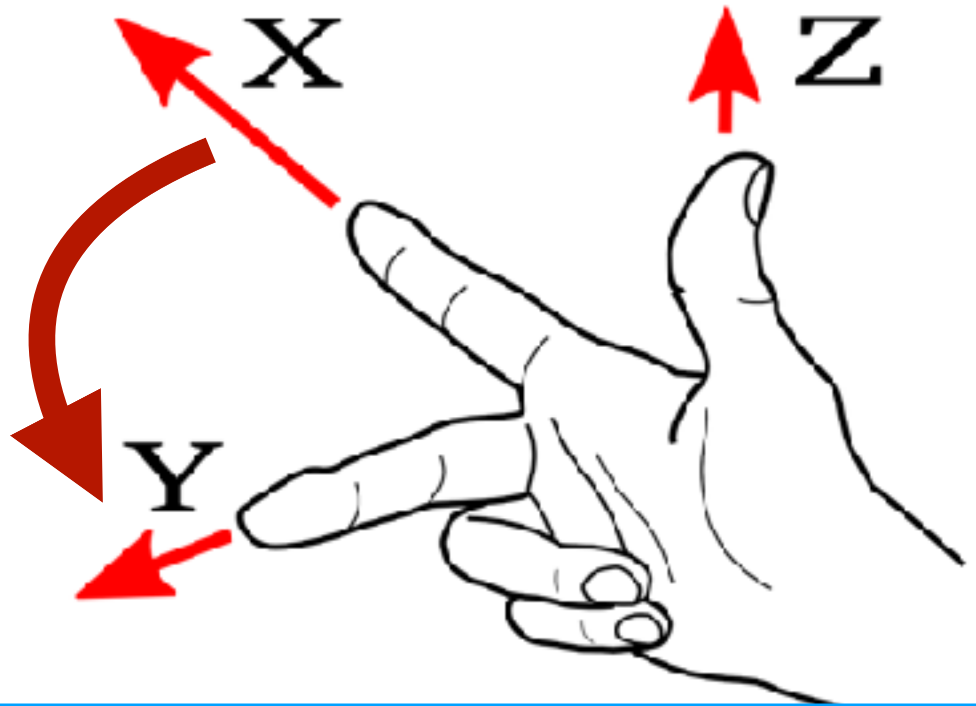
In “CMB Convention”

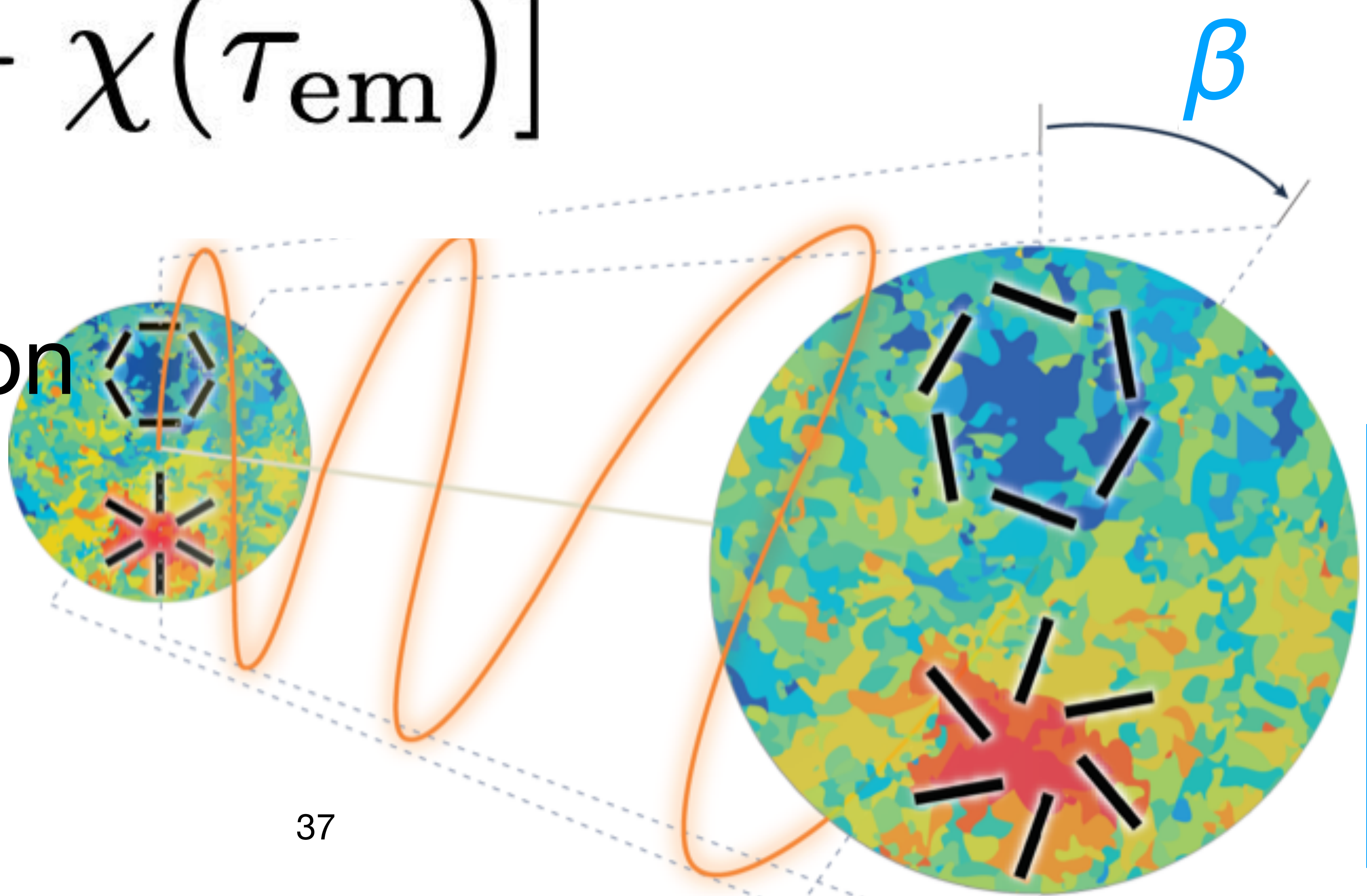




- People working on the cosmic microwave background (CMB) use the **opposite sign** for the angle, called “CMB convention”.

$$\beta = +\frac{\alpha}{2f}\left[\chi(\tau_{\rm obs}) - \chi(\tau_{\rm em})\right]$$





- We will use the CMB convention for the rest of this lecture.

$\beta > 0$ is a *clockwise* rotation on the sky.

Recap: Day 5

- A scalar field is a candidate for dark matter and dark energy.
- For a massive free field with $V(\chi) = m^2\chi^2/2$, the cosmological evolution of χ is very different for $m < H$ (χ~const.) and $m > H$ (oscillation).
- The Chern-Simons interaction between χ and photons rotates the plane of linear polarization of light.
  - **This effect, called "cosmic birefringence", is a signature of parity violation** and a useful probe of the nature of dark matter and dark energy!

$$I_{\rm CS} = \int d^4x \sqrt{-g}\left(-\frac{\alpha}{4f}\chi F\tilde{F}\right) \Rightarrow \beta = +\frac{\alpha}{2f}\left[\chi(\tau_{\rm obs}) - \chi(\tau_{\rm em})\right]$$