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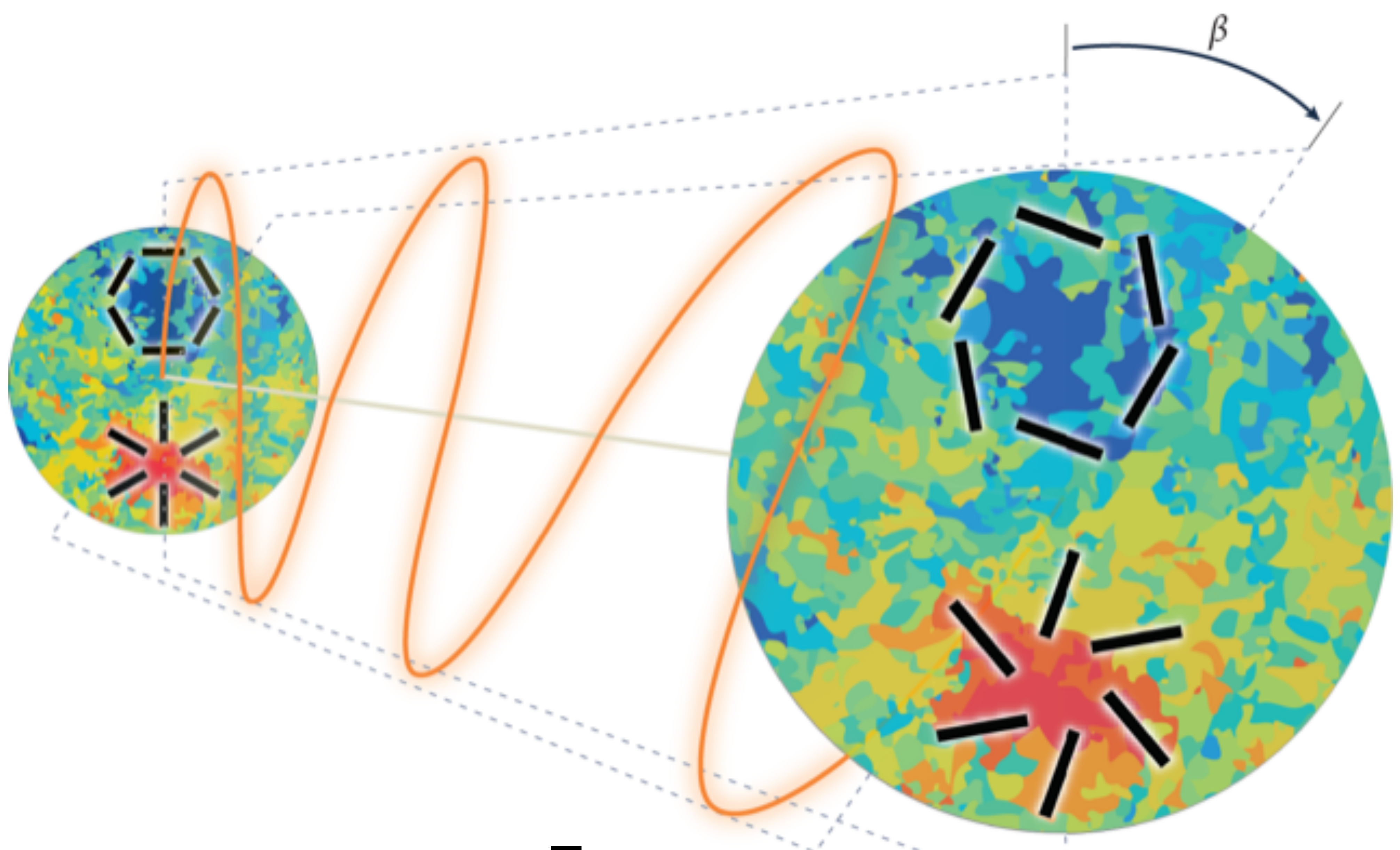


Parity Violation in Cosmology

In search of new physics for the Universe

The lecture slides are available at
https://wwwmpa.mpa-garching.mpg.de/~komatsu/lectures--reviews.html

Day 2

Eiichiro Komatsu (Max Planck Institute for Astrophysics)
Nagoya University, June 6–30, 2023

Topics

From the syllabus

1. What is parity symmetry?

2. Chern-Simons interaction

$$I_{CS} = \int d^4x \sqrt{-g} \left(-\frac{\alpha}{4f} \chi F \tilde{F} \right)$$

3. Parity violation 1: Cosmic inflation

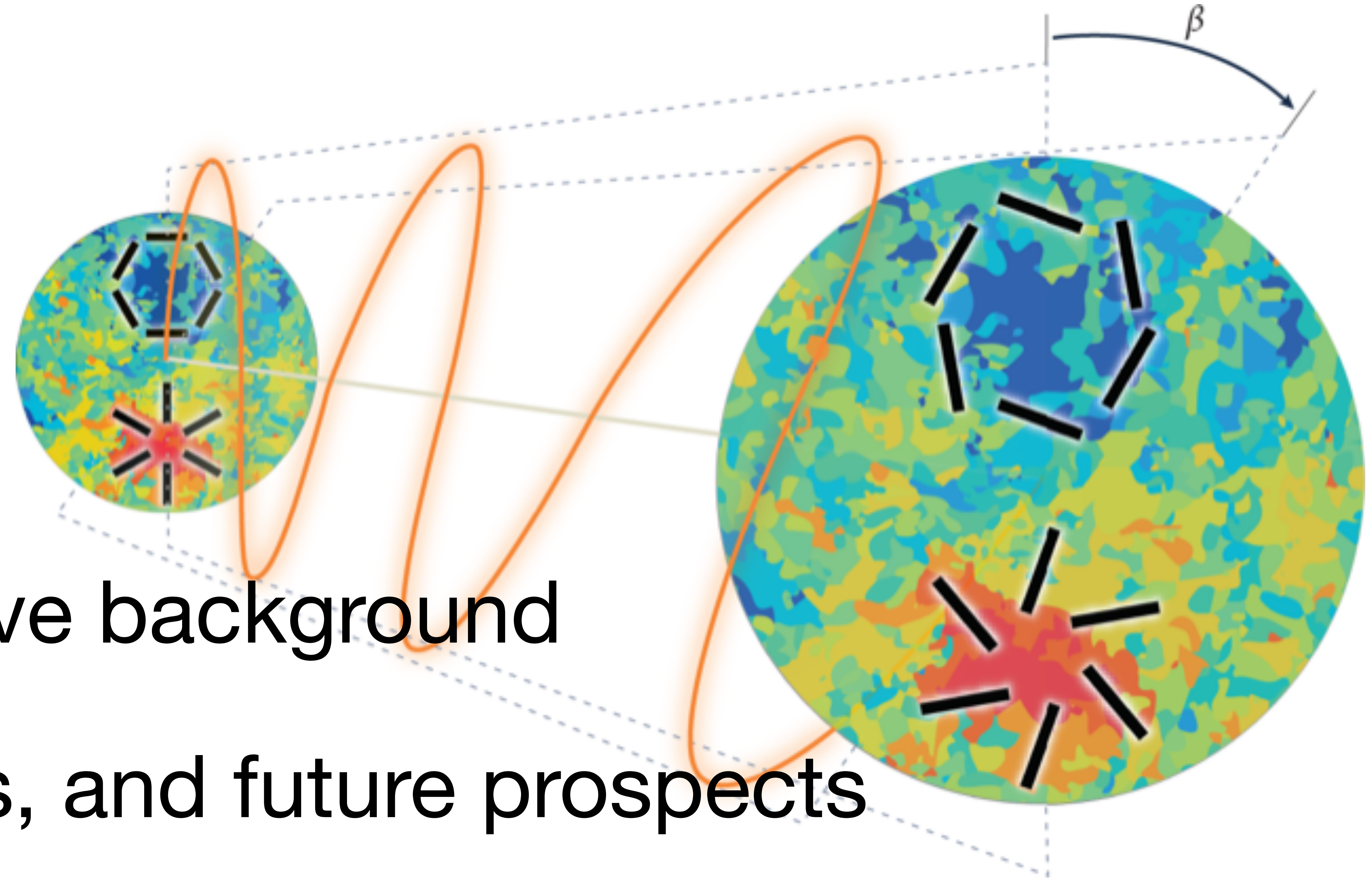
4. Parity violation 2: Dark matter

5. Parity violation 3: Dark energy

6. Light propagation: birefringence

7. Physics of polarization of the cosmic microwave background

8. Recent observational results, their implications, and future prospects



2.1 Parity Symmetry in Electromagnetism (EM)

Maxwell's Equations

In Heaviside units and *c*=1

$$\nabla \cdot \mathbf{E} = \rho\,, \qquad -\dot{\mathbf{E}} + \nabla \times \mathbf{B} = \mathbf{j}$$

$$\nabla \cdot \mathbf{B} = 0\,, \qquad \dot{\mathbf{B}} + \nabla \times \mathbf{E} = 0$$

- These equations are invariant under both spatial translation and rotation.
- They are also invariant under parity transformation, if **E** and **j** are vectors, ρ is a scalar, and **B** is a pseudovector.

Parity-flipping Maxwell's Equations

In Heaviside units and *c*=1

$$(-\nabla)\cdot(-\mathbf{E}) = \rho\,, \qquad -(-\dot{\mathbf{E}}) + (-\nabla)\times\mathbf{B} = (-\mathbf{j})$$

$$(-\nabla)\cdot\mathbf{B} = 0\,, \qquad \dot{\mathbf{B}} + (-\nabla)\times(-\mathbf{E}) = 0$$

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$$(-\nabla)\cdot\mathbf{B} = 0\,, \qquad \dot{\mathbf{B}} + (-\nabla)\times(-\mathbf{E}) = 0$$

If there is a magnetic monopole,
it must be a pseudoscalar!

- They are also invariant under parity transformation, if **E** and **j** are vectors, ρ is a scalar, and **B** is a pseudovector.

Simplifying Maxwell's Equations

Let's go 4D.

$$\nabla \cdot \mathbf{E} = \rho \,, \qquad -\dot{\mathbf{E}} + \nabla \times \mathbf{B} = \mathbf{j}$$

$$\nabla \cdot \mathbf{B} = 0 \,, \qquad \dot{\mathbf{B}} + \nabla \times \mathbf{E} = 0$$

- These equations can be written in a compact form as

$$\partial_\nu F^{\mu\nu} = j^\mu \qquad \partial_\nu \tilde{F}^{\mu\nu} = 0$$

$$\mu = 0, 1, 2, 3 \,, \quad j^\mu = (\rho, \mathbf{j}) \,, \quad \partial_\mu = \partial / \partial x^\mu \,, \quad x^\mu = (t, \mathbf{x})$$

Antisymmetric Field Strength Tensor, $F^{\mu\nu}$

$F^{\mu\nu} = -F^{\nu\mu}$

$$F^{\mu\nu} = \begin{pmatrix} 0 & E_x & E_y & E_z \\ -E_x & 0 & B_z & -B_y \\ -E_y & -B_z & 0 & B_x \\ -E_z & B_y & -B_x & 0 \end{pmatrix}$$

- Equivalently,

$$F^{0i} = E_i$$

$$F^{ij} = \epsilon^{ijk} B_k$$

$F^{12} = B_z \,, F^{23} = B_x \,, F^{31} = B_y$

$F^{21} = -B_z \,, F^{32} = -B_x \,, F^{13} = -B_y$

$$\epsilon^{ijk} = \begin{cases} +1 & \text{if (i,j,k) is even permutation of (1,2,3)} \\ -1 & \text{if (i,j,k) is odd permutation of (1,2,3)} \\ 0 & \text{otherwise} \end{cases}$$

Levi-Civita symbol

$\epsilon^{123} = 1 \,, \epsilon^{132} = -1 \,, \epsilon^{312} = 1 \,, \ldots$

Antisymmetric Field Strength Tensor, $F_{\mu\nu}$

$F_{\mu\nu} = -F_{\nu\mu}$

$$F_{\mu\nu} = \eta_{\mu\alpha}\eta_{\nu\beta}F^{\alpha\beta}$$ where $\eta_{\mu\alpha} = \mathrm{diag}(-1, 1, 1, 1)$

$$F_{\mu\nu} = \begin{pmatrix} 0 & -E_x & -E_y & -E_z \\ E_x & 0 & B_z & -B_y \\ E_y & -B_z & 0 & B_x \\ E_z & B_y & -B_x & 0 \end{pmatrix}$$

- Therefore,

$$F^2 \equiv F_{\mu\nu}F^{\mu\nu} = 2(\mathbf{B}\cdot\mathbf{B} - \mathbf{E}\cdot\mathbf{E})$$

This is a *scalar* and is invariant under parity transformation.

Dual Field Strength Tensor, $\widetilde{F}^{\mu\nu}$

$\widetilde{F}^{\mu\nu} = -\widetilde{F}^{\nu\mu}$

$$\tilde{F}^{\mu\nu} = \frac{1}{2}\epsilon^{\mu\nu\alpha\beta}F_{\alpha\beta}$$ where $$\epsilon^{\mu\nu\alpha\beta} = \begin{cases} +1 & \text{if } (\mu,\nu,\alpha,\beta) \text{ is even perm. of } (0,1,2,3) \\ -1 & \text{if } (\mu,\nu,\alpha,\beta) \text{ is odd perm. of } (0,1,2,3) \\ 0 & \text{otherwise} \end{cases}$$

Levi-Civita symbol

$$\tilde{F}^{\mu\nu} = \begin{pmatrix} 0 & B_x & B_y & B_z \\ -B_x & 0 & -E_z & E_y \\ -B_y & E_z & 0 & -E_x \\ -B_z & -E_y & E_x & 0 \end{pmatrix}$$

- Equivalently,

$$\tilde{F}^{0i} = B_i$$
$$\tilde{F}^{ij} = -\epsilon^{ijk}E_k$$

- Therefore,

$$F\tilde{F} \equiv F_{\mu\nu}\tilde{F}^{\mu\nu} = -4\mathbf{B}\cdot\mathbf{E}$$

This is a *pseudoscalar* and changes sign under parity transformation!

2.2 Action Principle for EM

$$\partial_\nu F^{\mu\nu} = j^\mu, \quad \partial_\nu \tilde{F}^{\mu\nu} = 0$$

What is the action that gives Maxwell's equations?

In Heaviside units and *c*=1

- The answer is

$$I = -\frac{1}{4}\int d^4x\ F^2 + \int d^4x\ A_\mu j^\mu \qquad \boxed{d^4x = dt d^3\mathbf{x}}$$

with

Vector potential

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu \quad \text{where} \quad A_\mu = (-\phi, \mathbf{A})$$

Therefore,

$$\begin{cases} F_{i0} = \partial_i A_0 - \dot{A}_i = E_i & \Rightarrow \mathbf{E} = -\nabla\phi - \dot{\mathbf{A}} \\ F_{ij} = \partial_i A_j - \partial_j A_i = \epsilon_{ijk} B_k & \Rightarrow \mathbf{B} = \nabla \times \mathbf{A} \end{cases}$$

$$\partial_\nu F^{\mu\nu} = j^\mu, \checkmark \partial_\nu \tilde{F}^{\mu\nu} = 0$$

What is the action that gives Maxwell's equations?

In Heaviside units and *c*=1

- The answer is

$$I = -\frac{1}{4}\int d^4x\ F^2 + \int d^4x\ A_\mu j^\mu \qquad d^4x = dt d^3\mathbf{x}$$

with

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu \quad \text{where } A_\mu = (-\phi, \mathbf{A})$$

One set of Maxwell's equations is simply given by the definition of $F_{\mu\nu}$:

$$\partial_\nu \tilde{F}^{\mu\nu} = \partial_\nu \frac{1}{2}\epsilon^{\mu\nu\alpha\beta}(\partial_\alpha A_\beta - \partial_\beta A_\alpha) = \epsilon^{\mu\nu\alpha\beta}\partial_\nu\partial_\alpha A_\beta = 0$$

$$\partial_\nu F^{\mu\nu} = j^\mu, \quad \partial_\nu \tilde{F}^{\mu\nu} = 0$$

What is the action that gives Maxwell's equations?

In Heaviside units and *c*=1

$$I = -\frac{1}{4}\int d^4x\ F^2 + \int d^4x\ A_\mu j^\mu$$

- **The idea**: The equation of motion for A_μ is the path that gives a stationary point. For a small change in $A_\mu \rightarrow A_\mu + \delta A_\mu$, the corresponding change in $I \rightarrow I + \delta I$ is also small.

$$\delta I = \int d^4x\ F^{\mu\nu}\partial_\nu(\delta A_\mu) + \int d^4x\ (\delta A_\mu) j^\mu$$

Integration by parts

$$= \int d^4x\ (-\partial_\nu F^{\mu\nu} + j^\mu)\delta A_\mu = 0 \Rightarrow \boxed{\partial_\nu F^{\mu\nu} = j^\mu}$$

Hint:

$$\delta(F^2) = 2F^{\mu\nu}\delta F_{\mu\nu}$$
$$= -4F^{\mu\nu}\partial_\nu(\delta A_\mu)$$

Finding symmetries in the action

It is like a treasure hunt!

$$I = -\frac{1}{4}\int d^4x\ F^2 + \int d^4x\ A_\mu j^\mu$$

- This action is invariant under spatial translation, rotation, and parity transformation.

- **It is also invariant under the following "gauge transformation",**

$$A_\mu \to A_\mu + \partial_\mu f$$

Hint:

Integration by parts

$$\int d^4x\ (\partial_\mu f) j^\mu = -\int d^4x\ f \partial_\mu j^\mu = 0$$

- Here, *f* is an arbitrary scalar function.

due to the charge conservation:

$$\partial_\mu j^\mu = 0 \Rightarrow \dot{\rho} + \nabla \cdot \mathbf{j} = 0$$

Finding symmetries in the action

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- This action is invariant under spatial translation, rotation, and parity transformation.

- **It is also invariant under the following "gauge transformation",**

$$\phi \to \phi - \dot{f}$$
$$\mathbf{A} \to \mathbf{A} + \nabla f$$

Hint:

Integration by parts

$$\int d^4x\ (\partial_\mu f) j^\mu = -\int d^4x\ f \partial_\mu j^\mu = 0$$

due to the charge conservation:

$$\partial_\mu j^\mu = 0 \Rightarrow \dot{\rho} + \nabla \cdot \mathbf{j} = 0$$

- Here, *f* is an arbitrary scalar function.

Problem Set 2

Playing with Maxwell

1. Derive Maxwell's equations from $\partial_\nu F^{\mu\nu} = j^\mu\,,\ \partial_\nu \tilde{F}^{\mu\nu} = 0$.

2. Show that $F\tilde{F}$ is a total derivative and can be written as

$$F_{\mu\nu}\tilde{F}^{\mu\nu} = 2\partial_\mu(A_\nu \tilde{F}^{\mu\nu})$$

2.3 $F\tilde{F}$ in the action

$F\tilde{F}$ in the action?

$$I = -\frac{1}{4}\int d^4x\ F^2 + \int d^4x\ A_\mu j^\mu$$

- This action is sufficient to produce all of Maxwell's equations.

- Can we add $\int d^4x\ F\tilde{F}$ to the action?
  - The answer is yes. However, **this is only a surface term**, since $F\tilde{F}$ is a total derivative (as shown in Problem Set 2).

Ni (1977); Turner, Widrow (1987); Carroll, Field, Jackiw (1990)

$F\tilde{F}$ in the action

Chern-Simons term

- Consider $I_{\rm CS} = -\frac{1}{4}\alpha \int d^4x\ \theta F\tilde{F}$ with $F\tilde{F} = 2\partial_\mu(A_\nu \tilde{F}^{\mu\nu})$
  - α: a dimensionless constant
  - *θ*: a dimensionless pseudoscalar field
- **This is not a surface term!** Integration by parts gives

$$I_{\rm CS} = \frac{1}{2}\alpha \int d^4x\ (\partial_\mu \theta) A_\nu \tilde{F}^{\mu\nu}$$

Jim Simons in 2023

https://einstein-chair.github.io/simons2023/

- This is a special case of the so-called *Chern-Simons term*, $p_\mu A_\nu \tilde{F}^{\mu\nu}$ with $p_\mu = \partial_\mu \theta$

Carroll, Field, Jackiw (1990)

Consistency with gauge invariance

p_μ cannot be arbitrary

$$I_{\rm CS} = \frac{1}{2}\alpha \int d^4x \; p_\mu A_\nu \tilde{F}^{\mu\nu}$$

- This action is invariant under the gauge transformation, $A_\nu \to A_\nu + \partial_\nu f$

  if $\partial_\nu p_\mu - \partial_\mu p_\nu = 0$

  Hint: Use integration by parts and the identity $\partial_\nu \tilde{F}^{\mu\nu} = 0$

- For example:
  - p_μ is a constant vector and not dynamical.
  - p_μ is a gradient of a dynamical (pseudo)scalar field, such as $p_\mu = \partial_\mu \theta$.

Carroll, Field, Jackiw (1990)

Consistency with gauge invariance

p_μ cannot be arbitrary

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- This action is invariant under the gauge transformation, $A_\nu \rightarrow A_\nu + \partial_\nu f$

  if $\partial_\nu p_\mu - \partial_\mu p_\nu = 0$

  Hint: Use integration by parts and the identity $\partial_\nu \tilde{F}^{\mu\nu} = 0$

- For example:

  This implies the presence of a preferred direction in spacetime and violation of Lorentz invariance!

  - p_μ is a constant vector and not dynamical, or
  - p_μ is a gradient of a dynamical (pseudo)scalar field, such as $p_\mu = \partial_\mu\theta$.

The main goals of this lecture series

Let's find new physics!

- We will study the cosmological consequence of

$$I_{\mathrm{CS}} = -\frac{1}{4}\alpha \int d^4x \; \theta F\tilde{F}$$

- Specifically, we ask if θ is —
    - active during cosmic inflation,
    - responsible for dark matter, or
    - responsible for dark energy.

The main goals of this lecture series

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- Specifically, we ask if θ is —
  - active during cosmic inflation,
  - responsible for dark matter, or
  - responsible for dark energy.
- We will also study observational signatures in —
  - Cosmic microwave background,
  - Gravitational waves, and
  - Large-scale structure of the Universe.

Is there a known example of this term in particle physics?

Yes, a pion.

Credit: HiggsTan

- A pion is a composite meson composed of a quark and an antiquark.
  - A neutral pion, π^0, is composed of either $u\bar{u}$ or $d\bar{d}$, and **is a pseudoscalar**.
    (Chinowsky & Steinberger, 1954)
  - π^0 is coupled to photons via l_{CS} where
    - $\theta = \pi^0 / f_\pi$ with $f_\pi \sim 184$ MeV (pion decay constant)
    - $\alpha = 2\alpha_{EM} N_c / (3\pi)$ with $N_c = 3$ (the number of quark colors) and $\alpha_{EM} \sim 1/137$ (EM fine structure constant)
- **π^0 decays into 2 photons via this term, which has been observed.** So, what we are going to study in this lecture is not completely crazy!

Correction to Maxwell's equations

In Heaviside units and *c*=1

- We now derive the correction to Maxwell's equations from

$$d^4x = dtd^3\mathbf{x}$$

$$I = -\frac{1}{4}\int d^4x \, \left(F^2 + \alpha\theta F\tilde{F}\right) + \int d^4x \, A_\mu j^\mu$$

Hint: $\delta(F\tilde{F}) = \epsilon^{\mu\nu\alpha\beta}(\delta F_{\mu\nu})F_{\alpha\beta}$
$= -4(\partial_\nu \delta A_\mu)\tilde{F}^{\mu\nu}$

- Finding the path that gives a stationary point,

$$\delta I = \int d^4x \, (F^{\mu\nu} + \alpha\theta\tilde{F}^{\mu\nu})\partial_\nu(\delta A_\mu) + \int d^4x \, (\delta A_\mu)j^\mu$$

Integration by parts

$$= \int d^4x \, \left[-\partial_\nu(F^{\mu\nu} + \alpha\theta\tilde{F}^{\mu\nu}) + j^\mu\right]\delta A_\mu = 0$$

$=0$

Correction to Maxwell's equations

In Heaviside units and *c*=1

- Therefore, the correction to Maxwell's equations is

Hint: $\partial_\nu \tilde{F}^{\mu\nu} = 0$

$$\partial_\nu F^{\mu\nu} + \alpha(\partial_\nu \theta)\tilde{F}^{\mu\nu} = j^\mu$$

- The result is

$$\delta I = \int d^4x \, (F^{\mu\nu} + \alpha\theta\tilde{F}^{\mu\nu})\partial_\nu(\delta A_\mu) + \int d^4x \, (\delta A_\mu) j^\mu$$

Integration by parts

$$= \int d^4x \, \left[-\partial_\nu(F^{\mu\nu} + \alpha\theta\tilde{F}^{\mu\nu}) + j^\mu\right]\delta A_\mu = 0$$

=0

2.4 Parity Violation in EM Waves

Warm-up: The wave equation for *A*µ

Maxwell's equations in vacuum

- Maxwell's equations in vacuum $\partial_\nu F^{\mu\nu} = 0$ gives

$$-\Box A^\mu + \eta^{\mu\alpha}\partial_\alpha(\partial_\nu A^\nu) = 0$$

where

$$\Box = \eta^{\alpha\beta}\partial_\alpha\partial_\beta = -\frac{\partial^2}{\partial t^2} + \nabla^2$$

$$\eta^{\alpha\beta} = \mathrm{diag}(-1, \mathbf{1})$$

$$A^\mu = \eta^{\mu\alpha} A_\alpha = (\phi, \mathbf{A})$$

"Lorenz gauge condition"

- Now, using invariance under the gauge transformation, we can set $\partial_\nu A^\nu = 0$ by choosing $\Box f = -\partial_\nu A^\nu$ in $A_\mu \to A_\mu + \partial_\mu f$. Then…

Warm-up: The wave equation for A^μ

The use case of the gauge invariance

"Lorenz gauge condition"

- Maxwell's equations in vacuum $\partial_\nu F^{\mu\nu} = 0$ and $\partial_\nu A^\nu = 0$ gives

$$\Box A^\mu = 0$$

→ The equation for a wave traveling at the speed of light!

where

$$\Box = \eta^{\alpha\beta}\partial_\alpha\partial_\beta = -\frac{\partial^2}{\partial t^2} + \nabla^2$$

$$A^\mu = \eta^{\mu\alpha}A_\alpha = (\phi, \mathbf{A})$$ with $\dot{\phi} + \nabla \cdot \mathbf{A} = 0$

- The number of degrees of freedom for A^μ is **3** due to the Lorenz gauge condition.

Physical degrees of freedom of EM waves

3? 2?

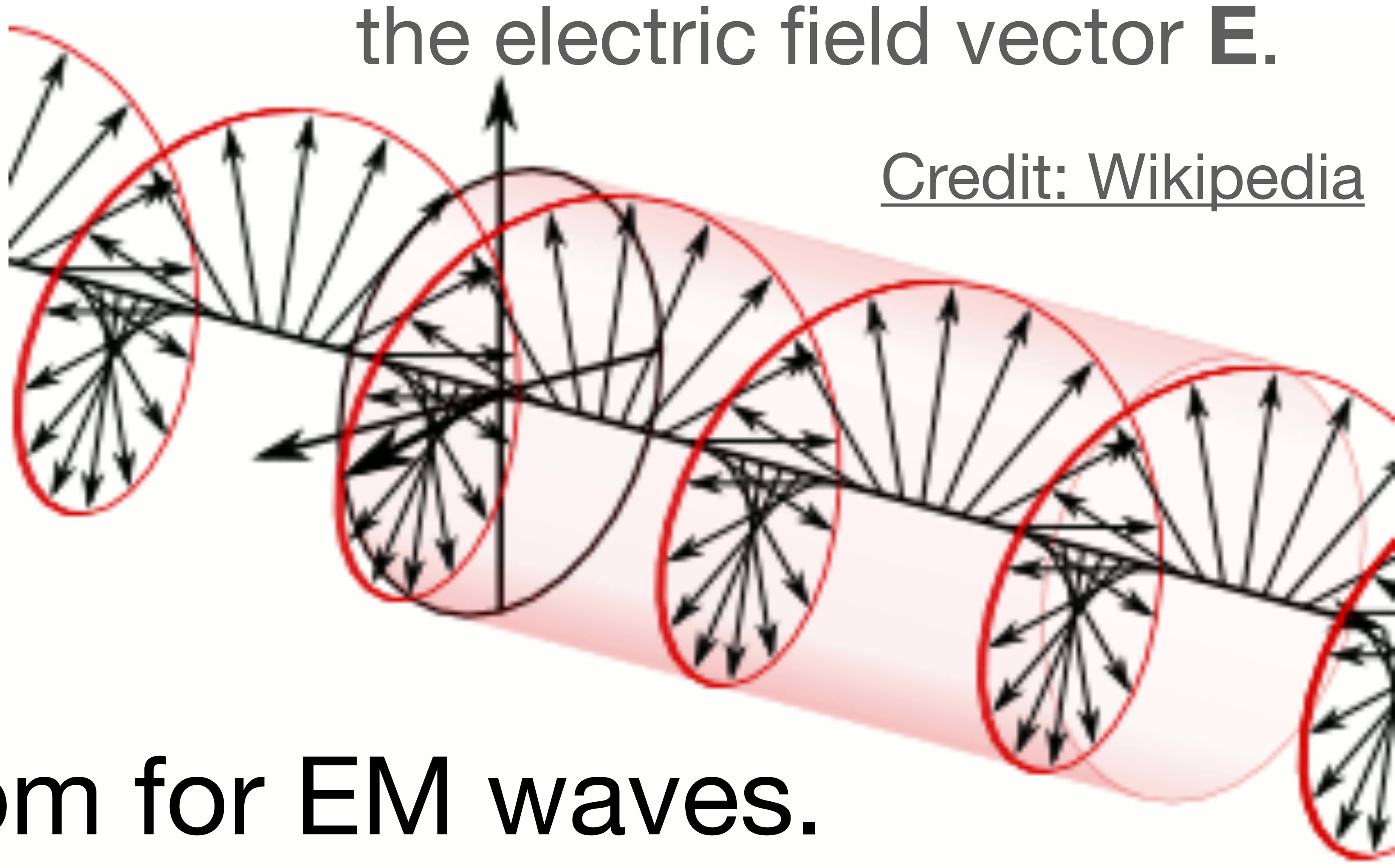
- We know that photons must have only 2 helicity states, $\lambda=\pm1$ (two circular polarization states).
  - Shouldn't the number of physical degrees of freedom be **2**, instead of 3? The answer is yes.
- The Lorenz gauge does not fully specify A^μ. We can still add

  $$A_\mu \to A_\mu + \partial_\mu f_2$$

  which satisfies $\Box f_2 = 0$
- Choosing f_2 will fully specify A^μ. This leaves **2** degrees of freedom.

EM waves must be transverse

The arrows show directions of the electric field vector **E**.



Credit: Wikipedia

- It is common to choose *f* and f_2 such that

$$\phi = 0, \qquad \nabla \cdot \mathbf{A} = 0$$

in vacuum — "Coulomb gauge condition"

- We have 2 conditions. That leaves **2** degrees of freedom for EM waves.
- This choice is consistent with the Lorenz gauge condition $\dot{\phi} + \nabla \cdot \mathbf{A} = 0$
- $\nabla \cdot \mathbf{A} = 0$ requires that the EM wave be *transverse*, i.e., the change in **A** is perpendicular to the direction of propagation of the EM wave.
- **We will use this condition throughout the lecture.**

Correction to the EM wave equation

With the Chern-Simons term

$$\partial_\nu F^{\mu\nu} + \alpha(\partial_\nu\theta)\tilde{F}^{\mu\nu} = 0$$

- With $A^0=\phi=0$ in the Lorenz gauge, we find

$$-\Box A^i + \alpha(\partial_\nu\theta)\tilde{F}^{i\nu} = 0$$

$$\Box = \eta^{\alpha\beta}\partial_\alpha\partial_\beta = -\frac{\partial^2}{\partial t^2} + \nabla^2$$

$$A^\mu = \eta^{\mu\alpha}A_\alpha = (\phi, \mathbf{A})$$

$$\Rightarrow \quad \ddot{\mathbf{A}} - \nabla^2\mathbf{A} + \alpha\left[-\dot{\theta}(\nabla\times\mathbf{A}) + (\nabla\theta)\times\dot{\mathbf{A}}\right] = 0$$

Correction to the EM wave equation!

Note: **A** is a vector and θ is a pseudoscalar.

Helicity Basis

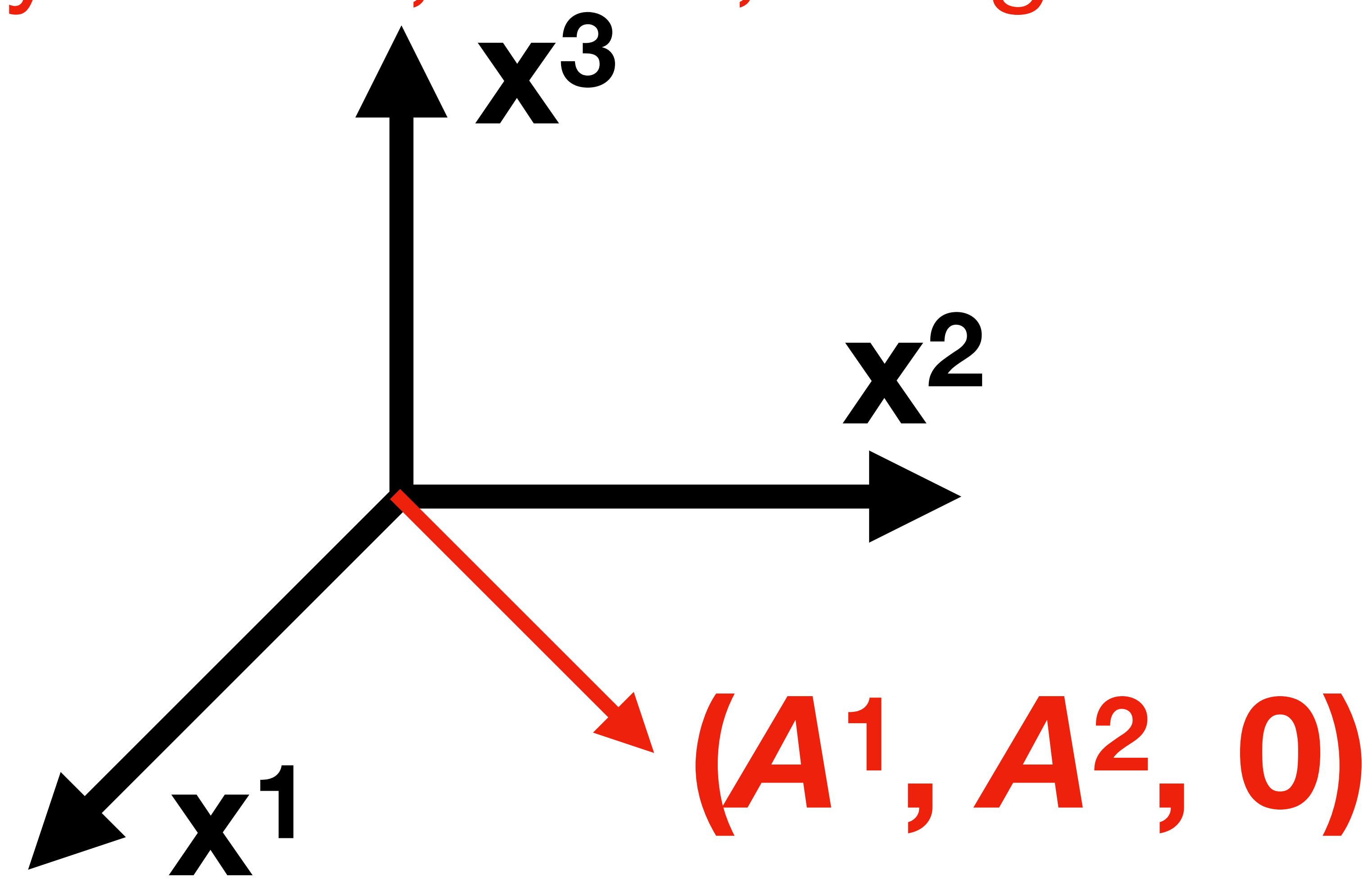
Going to Fourier space

- Fourier transform of **A**(t,**x**) is $\mathbf{A}(t,\mathbf{x}) = (2\pi)^{-3/2} \int d^3\mathbf{k}\ \mathbf{A}_\mathbf{k}(t) e^{i\mathbf{k}\cdot\mathbf{x}}$
  - The EM wave propagates in the direction of **k**. The change in $\mathbf{A}_\mathbf{k}$ is perpendicular to **k**.

"Coulomb gauge" $\nabla \cdot \mathbf{A}(t,\mathbf{x}) = 0 \to \mathbf{k} \cdot \mathbf{A}_\mathbf{k}(t) = 0$

- Choose **k** to be on the z(=x^3) axis. The helicity states, λ=±1, are given for each Fourier mode by

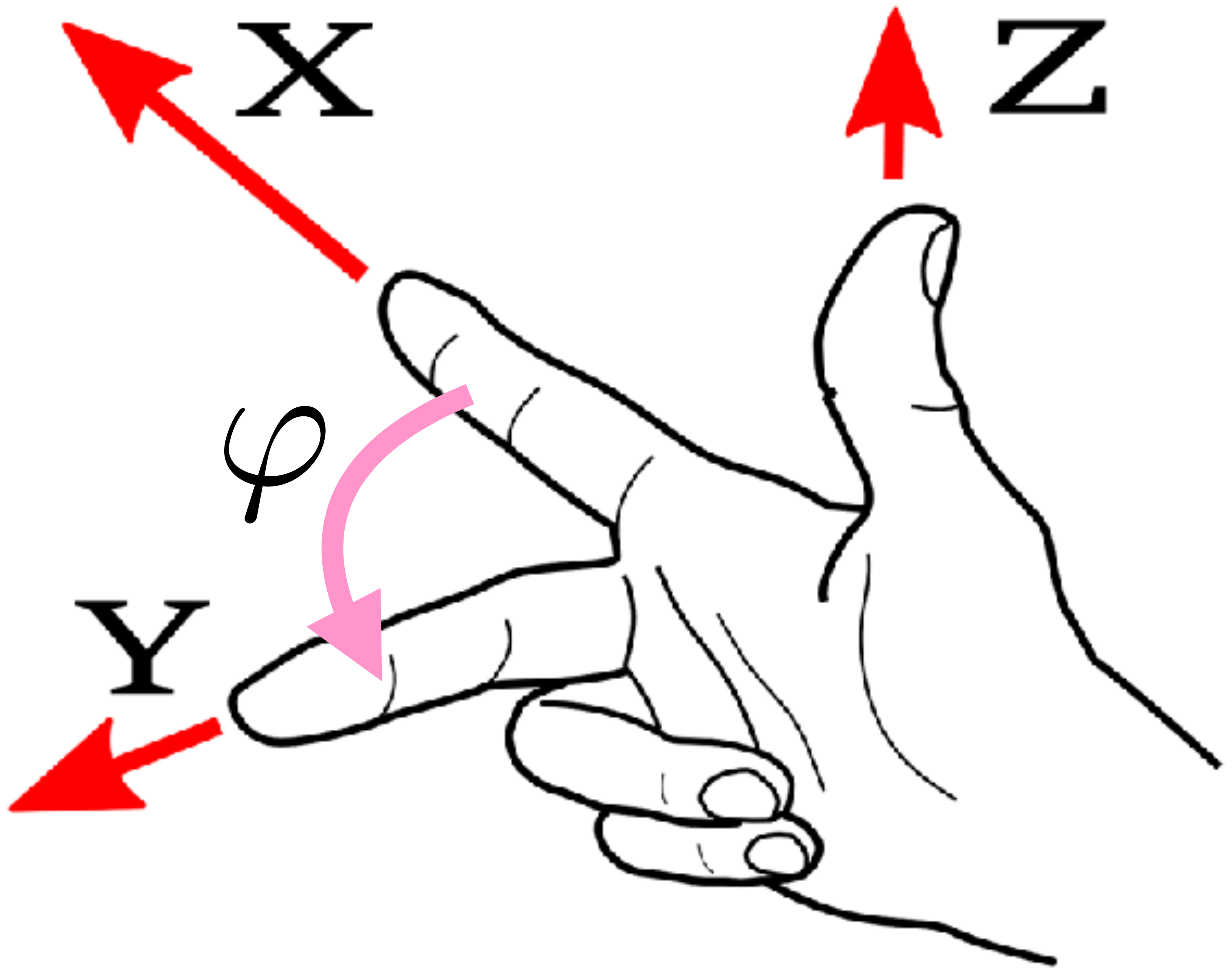
$$A_\pm = \frac{A^1_\mathbf{k} \mp i A^2_\mathbf{k}}{\sqrt{2}}$$



Helicity Basis

Transformation property under rotation

- To show that $A_\pm$ represents the helicity states, rotate the spatial coordinates around the z axis in the right-handed system by an angle φ.

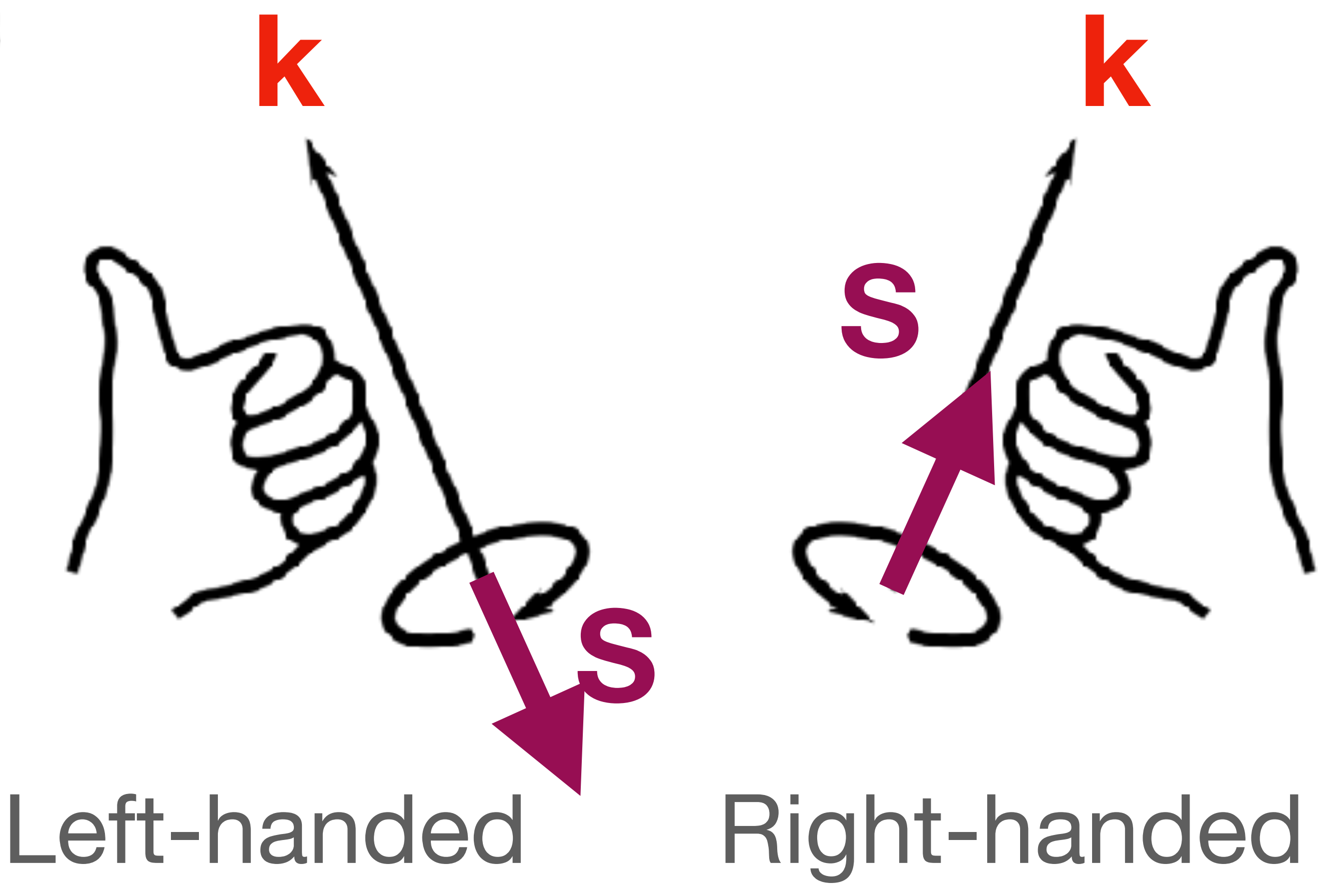


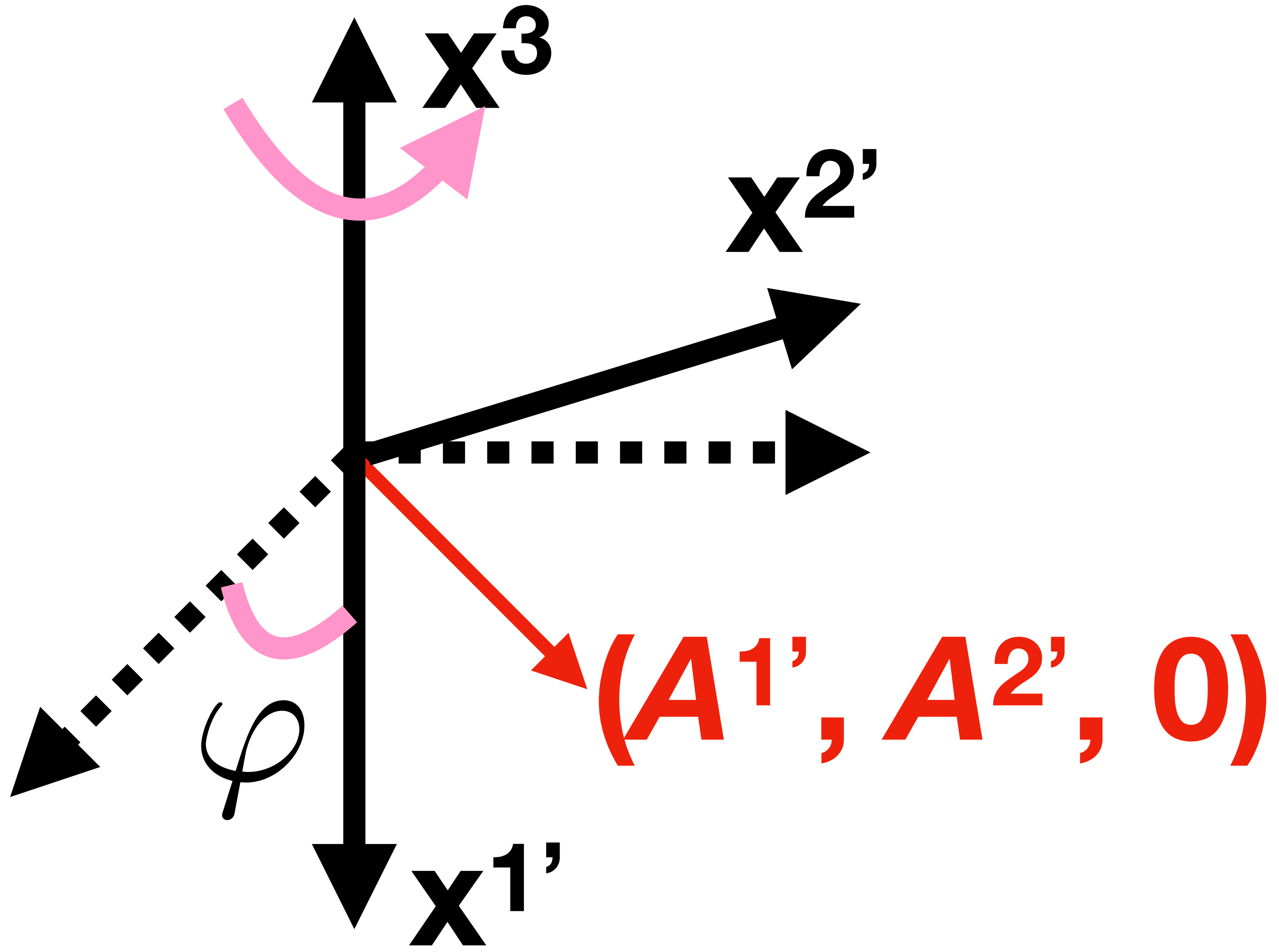
- The helicity states, λ=±1, transform as

$$A_\lambda \to A'_\lambda = e^{i\lambda\varphi} A_\lambda$$

Helicity

A_+: Right-handed state
A_-: Left-handed state





Correction to the EM wave equation

In the helicity basis

Note: **A** is a vector and θ is a pseudoscalar.

$$\ddot{\mathbf{A}} - \nabla^2 \mathbf{A} + \alpha \left[-\dot{\theta}(\nabla \times \mathbf{A}) + (\nabla\theta) \times \dot{\mathbf{A}} \right] = 0$$

Correction to the EM wave equation!

- To simplify the analysis, assume that θ is homogeneous and depends only on time, θ(t,**x**) –> $\bar{\theta}$(t). Then in Fourier space

$$\ddot{\mathbf{A}}_{\mathbf{k}} + k^2 \mathbf{A}_{\mathbf{k}} - i\alpha\dot{\bar{\theta}}(\mathbf{k} \times \mathbf{A}) = 0$$

$$\Rightarrow \quad \ddot{A}_{\pm} + \left(k^2 \mp k\alpha\dot{\bar{\theta}}\right) A_{\pm} = 0$$

Parity violation

The equation of motion depends on handedness!

Recap: Day 2

- There are 2 scalars composed of $F_{\mu\nu}$ and $\tilde{F}_{\mu\nu}$.
  - F^2 is a scalar, whereas $F\tilde{F}$ is a pseudoscalar.
  - The action $\int dt d^3\mathbf{x}\, F^2$ gives Maxwell's equations, whereas $\int dt d^3\mathbf{x}\, F\tilde{F}$ is a surface term.
- A Chern-Simons interaction between a pseudoscalar field θ and photons, $\int dt d^3\mathbf{x}\, \theta F\tilde{F}$, modifies Maxwell's equations.
  - The equation of motion for EM waves depends explicitly on helicity states. This is the signature of violation of parity symmetry! $\ddot{A}_\pm + \left(k^2 \mp k\alpha\dot{\bar{\theta}}\right) A_\pm = 0$
  - ***What is the cosmological implication of this term? Let's find out!***