Cosmology in the Next Decade

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Cosmology: Next Decade?

- Astro2010: Astronomy & Astrophysics Decadal Survey
  - Report from *Cosmology and Fundamental Physics* Panel (Panel Report, Page T-3):

TABLE I Summary of Science Frontiers Panels' Findings

Panel		Science Questions
Cosmology and Fundamental Physics	CFP 1	How Did the Universe Begin?
	CFP 2	Why Is the Universe Accelerating?
	CFP 3	What Is Dark Matter?
	CFP 4	What Are the Properties of Neutrinos?

Cosmology: Next Decade?

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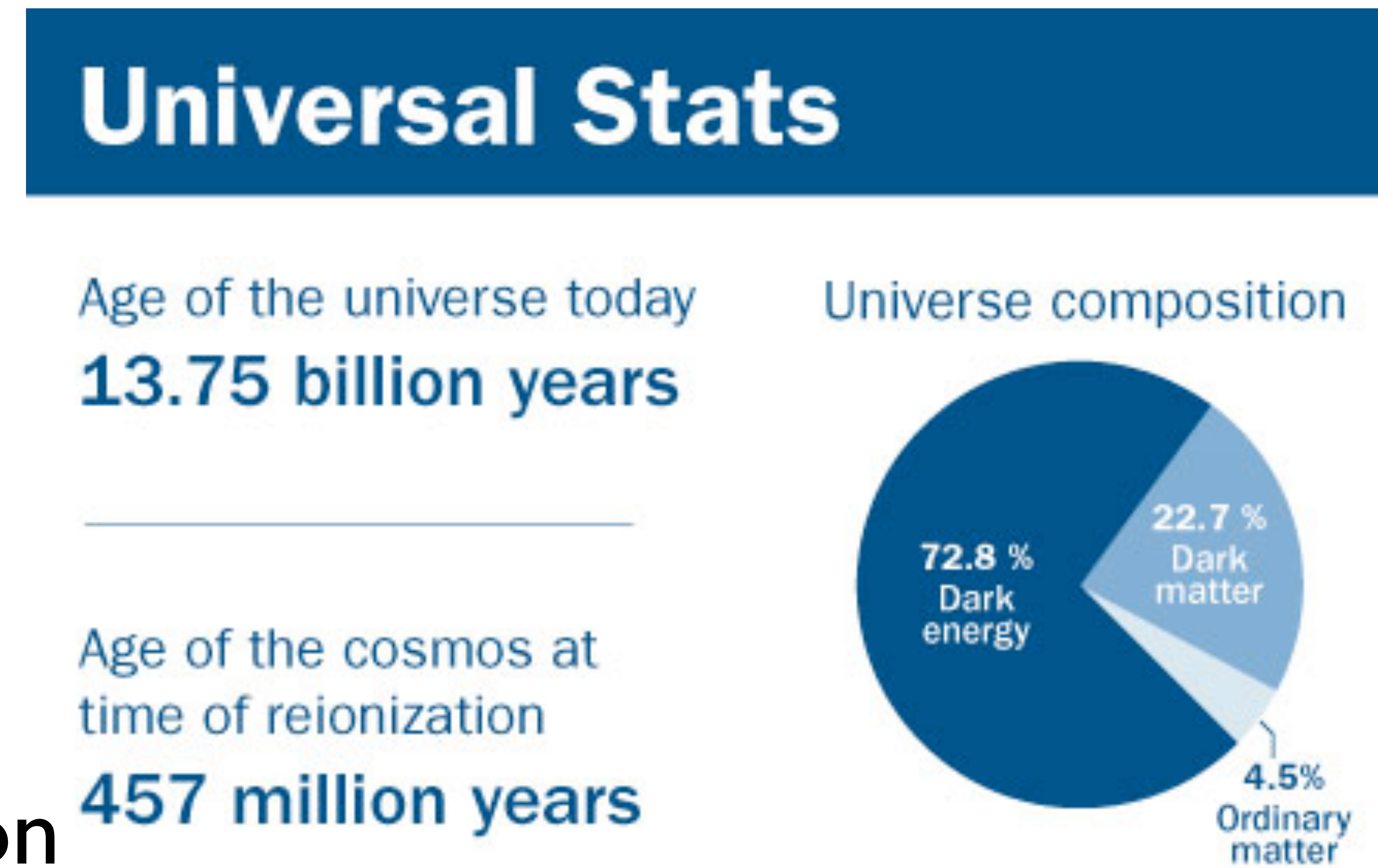
TABLE I Summary of Science Frontiers Panels' Findings

Panel		Science Questions	
Cosmology and Fundamental Physics	CFP 1	How Did the Universe Begin	*Inflation*
	CFP 2	Why Is the Universe Acceler:	*Dark Energy*
	CFP 3	What Is Dark Matter?	*Dark Matter*
	CFP 4	What Are the Properties of N	*Neutrino Mass*

Cosmology Update: WMAP 7-year+

- **Standard Model**
  - H&He = 4.58% (±0.16%)
  - Dark Matter = 22.9% (±1.5%)
  - Dark Energy = 72.5% (±1.6%)
  - H_0=70.2±1.4 km/s/Mpc
  - Age of the Universe = 13.76 billion years (±0.11 billion years)



"ScienceNews" article on the WMAP 7-year results

Can we prove/falsify inflation* ?

* A period of rapidly accelerating phase of the early universe.

What does inflation do?

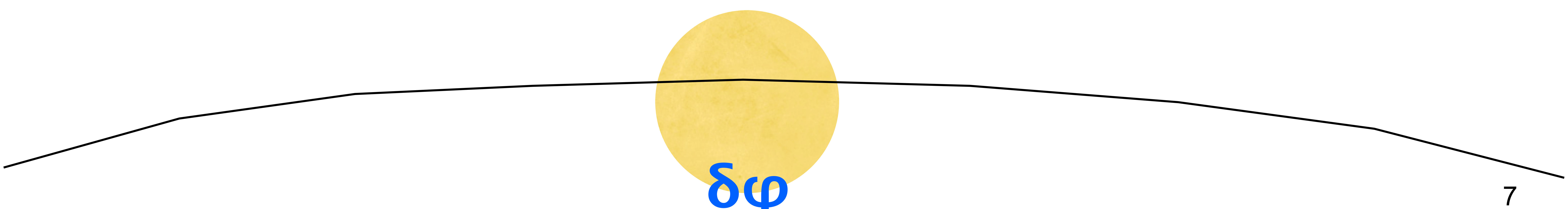
- Inflation can:
  - Make 3d geometry of the observable universe flatter than that imposed by the initial condition
  - Produce scalar quantum fluctuations which can seed the observed structures, with a nearly scale-invariant spatial spectrum
  - Produce tensor quantum fluctuations which can be observed in the form of primordial gravitational waves, with a nearly scale-invariant spectrum

Stretching Micro to Macro

H^{-1} = Hubble Size

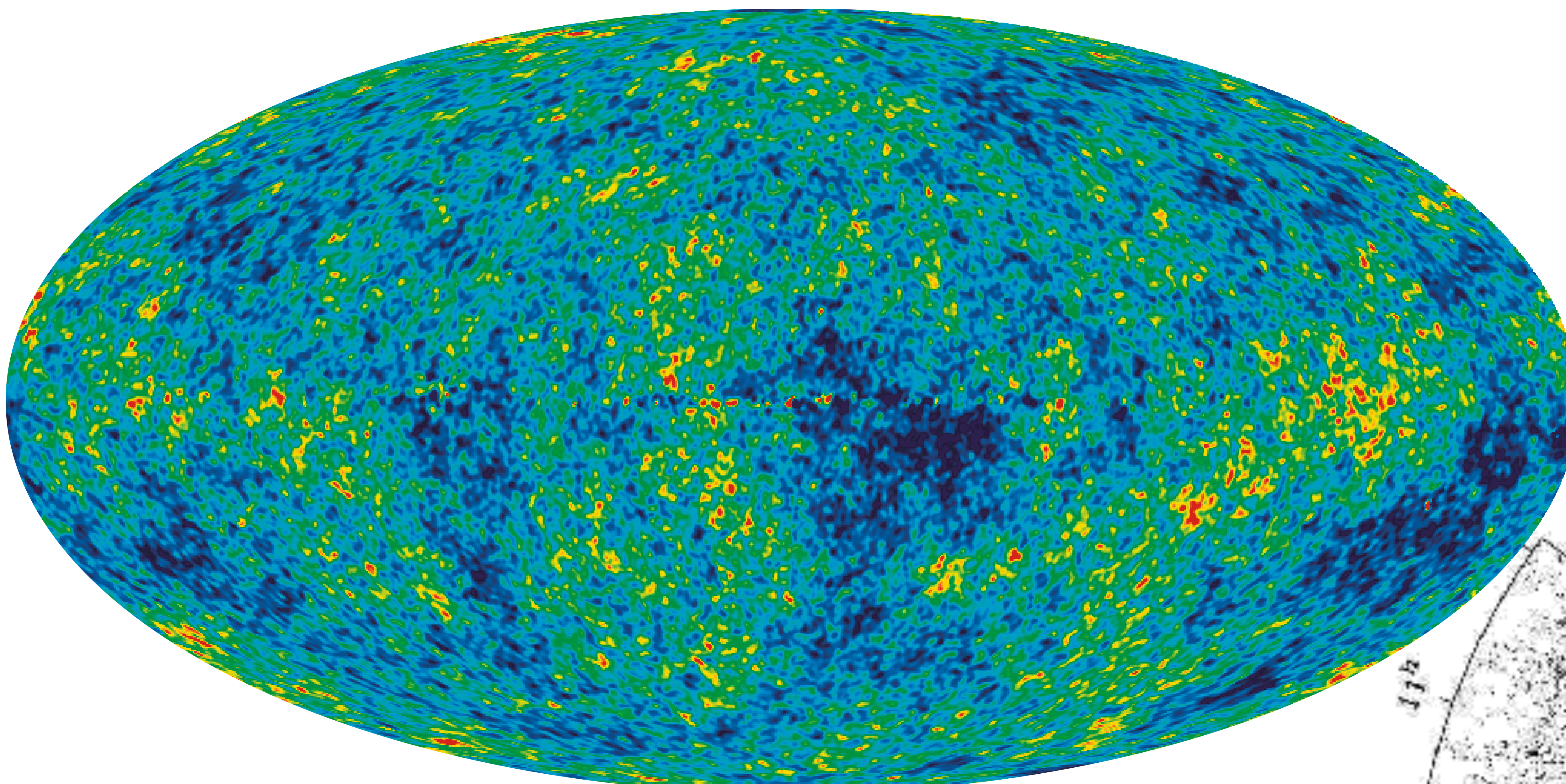
Quantum fluctuations on microscopic scales

INFLATION!



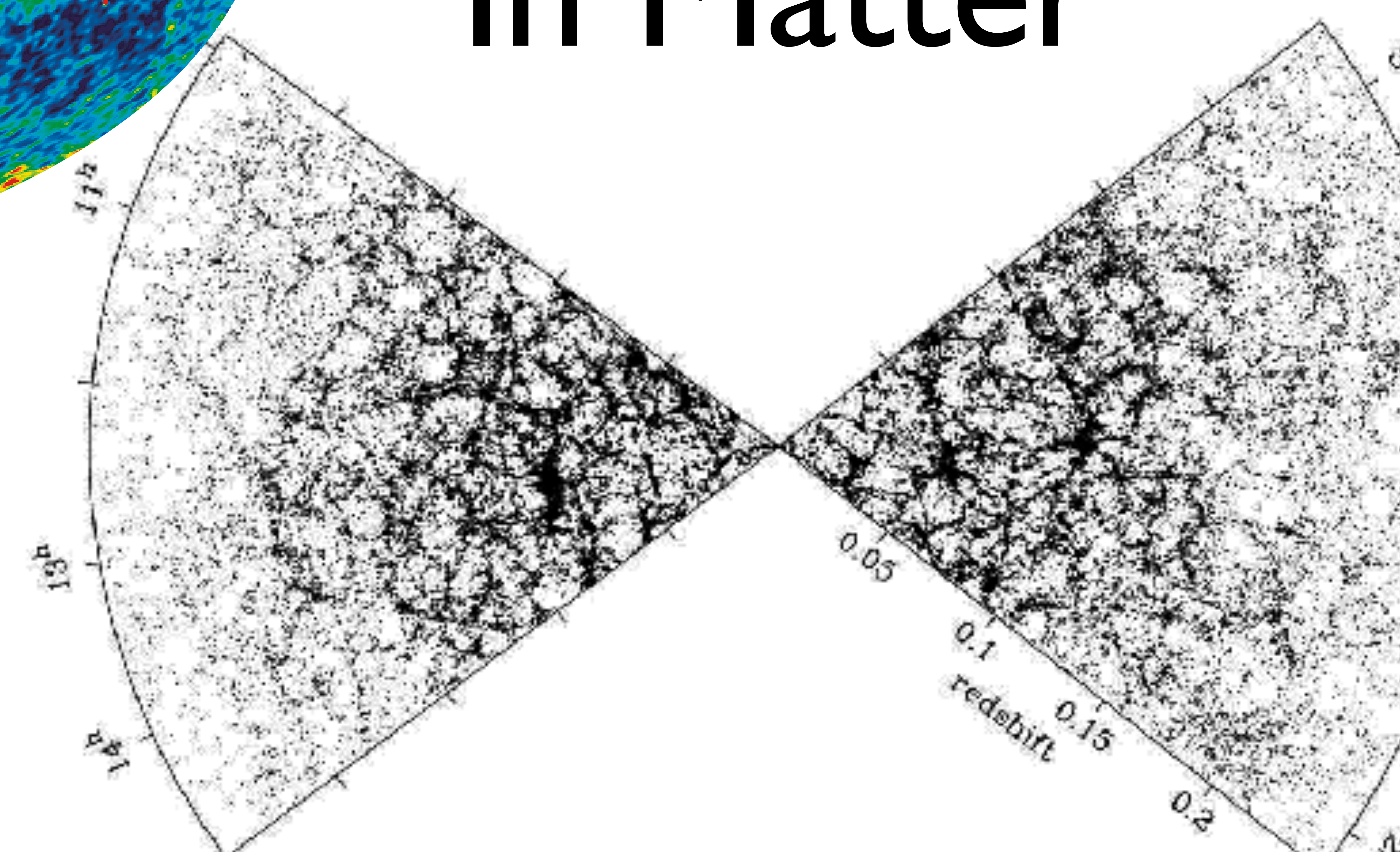
Quantum fluctuations cease to be quantum, and become observable

And, they look like these



In Photon

In Matter



Inflation produces:

- **Curvature perturbation, ζ.**
- For the metic of

$$ds^2 = -[1 + 2\Psi(t, \vec{x})]dt^2 + a^2(t)[1 + 2\Phi(t, \vec{x})]d\vec{x} \cdot d\vec{x}$$

- We define
  - ζ = Φ – Hδφ/(dφ/dt)
- It is "curvature perturbation" because it has Φ in it.
  - ζ is a gauge-invariant quantity. It is precisely the curvature perturbation in the so-called "comoving gauge" in which δφ vanishes (for a single-field model)

And ζ produces:

- Temperature anisotropy (on very large scales):
  - **$\delta T/T = -(1/5)\zeta$ [Sachs-Wolfe Effect]**
- Density fluctuation (on very large scales):
  - **$\delta = -\Delta\zeta \,/\, (4\pi G a^2 \rho)$ [Poisson Equation]**
- Therefore, the statistical properties of the observed quantities such as the temperature anisotropy of the cosmic microwave background and the density fluctuations of matter distribution tell us something about inflation!

Inflation also produces:

- **Tensor perturbations, h_{ij}^{TT}.**
- For the metic of

$$ds^2=-dt^2+a^2(t)[\delta_{ij}+h_{ij}^{TT}]dx^idx^j$$

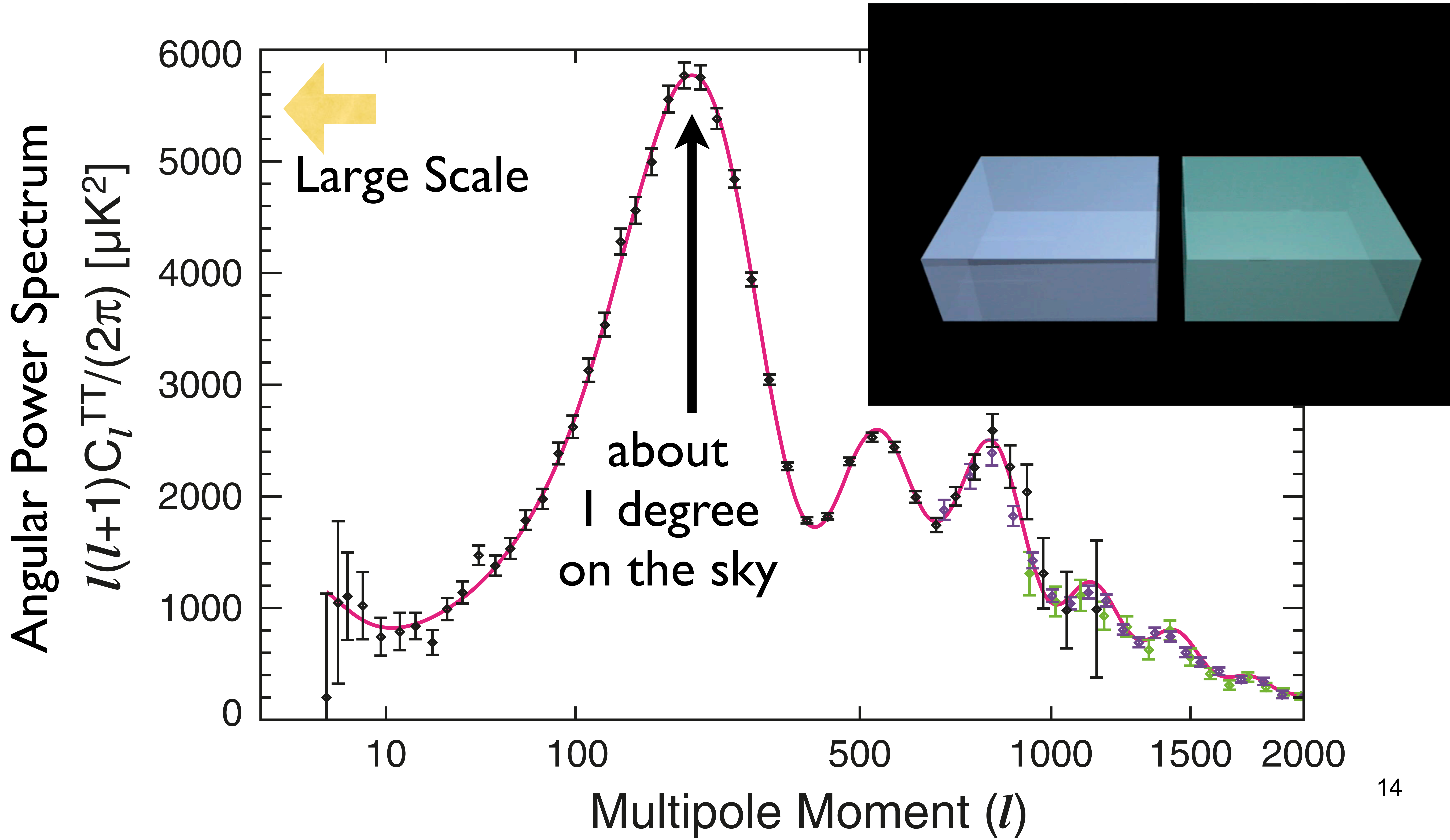
- For a tensor perturbation (gravitational waves) propagating in z direction (in the so-called transverse&traceless gauge),
    - $h_+ = h_{11}^{TT} = h_{22}^{TT}$ ["+" mode]
    - $h_x = h_{12}^{TT} = h_{21}^{TT}$ ["x" mode]

Scalar Perturbations (Density Fluctuations)

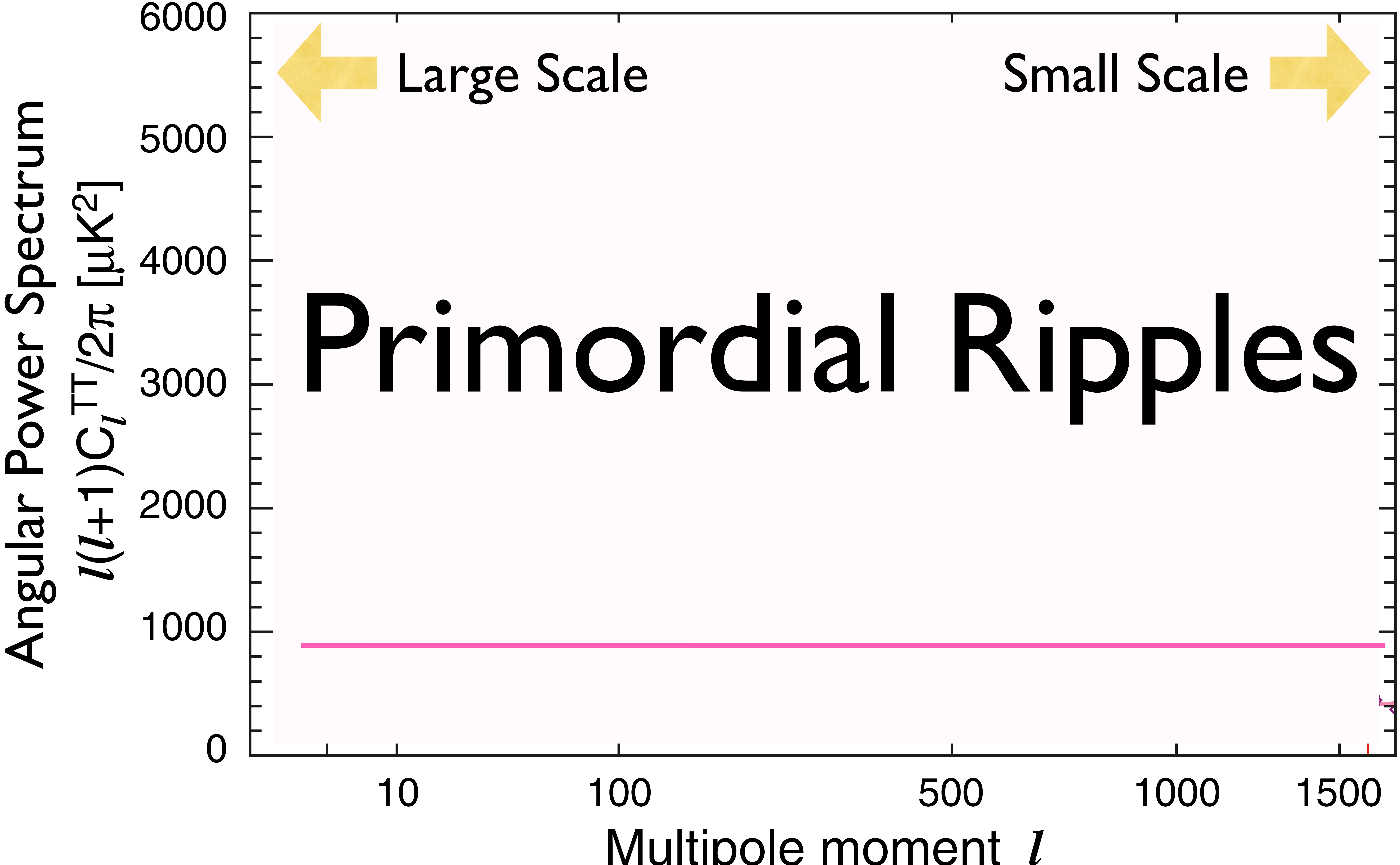
Power Spectrum of ζ

- A very successful explanation (Mukhanov & Chibisov; Guth & Pi; Hawking; Starobinsky; Bardeen, Steinhardt & Turner) is:
  - *Primordial fluctuations were generated by quantum fluctuations of the scalar field that drove inflation.*
  - The prediction: a nearly scale-invariant power spectrum in the curvature perturbation:
    - $P_\zeta(k) = \langle|\zeta_k|^2\rangle = A/k^{4-n_s} \sim A/k^3$
      - where $n_s \sim 1$ and A is a normalization.

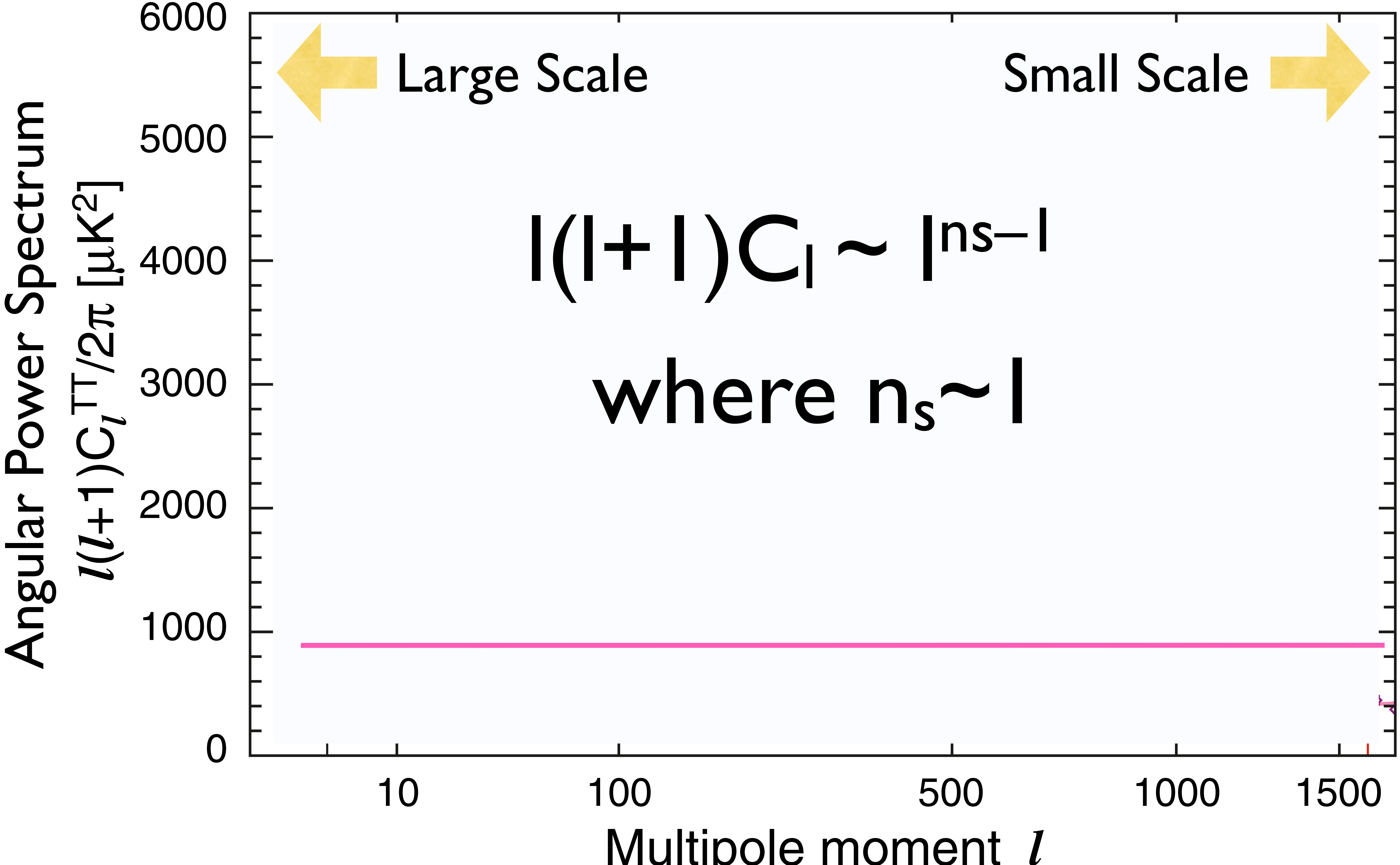
WMAP Power Spectrum



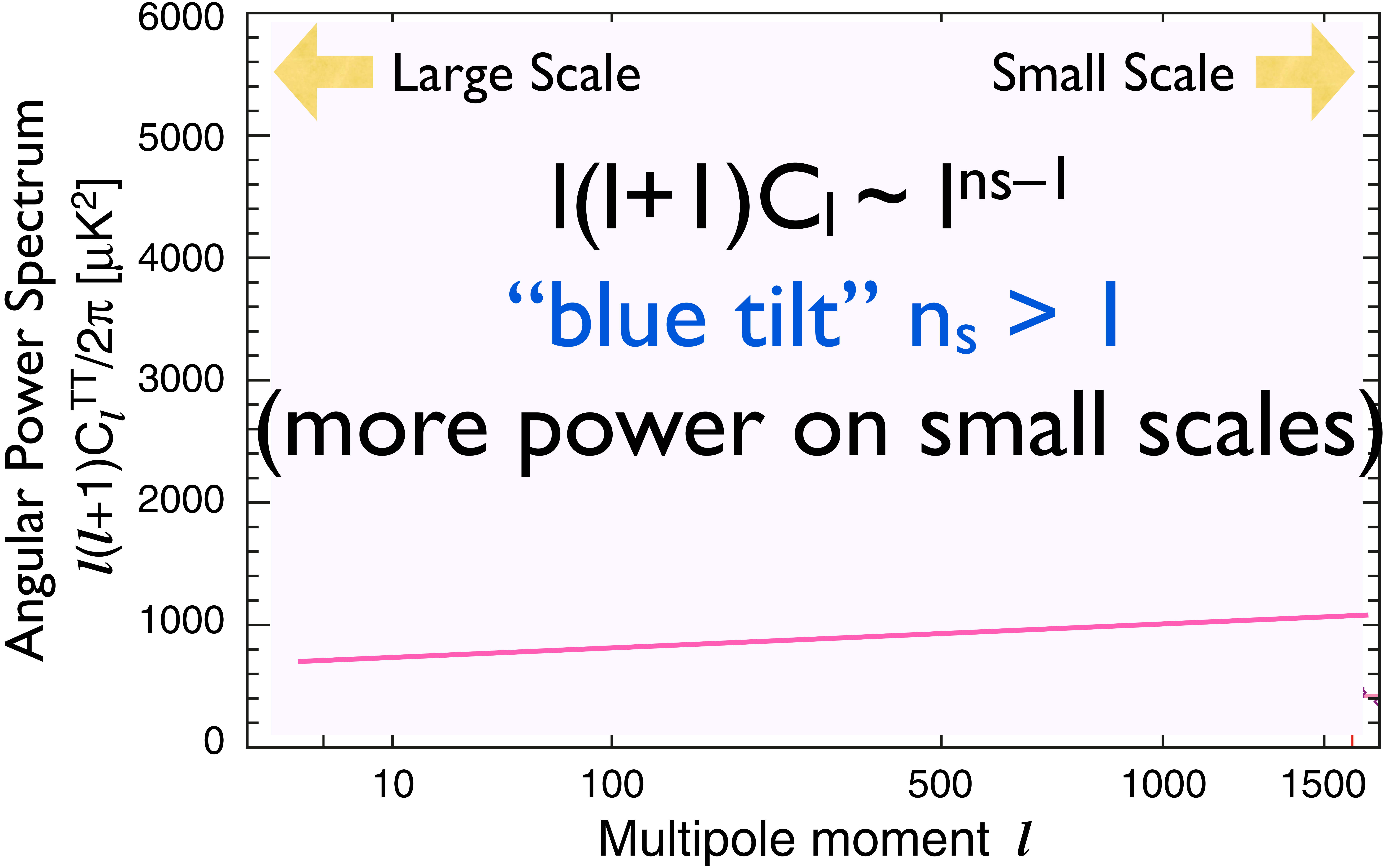
Getting rid of the Sound Waves



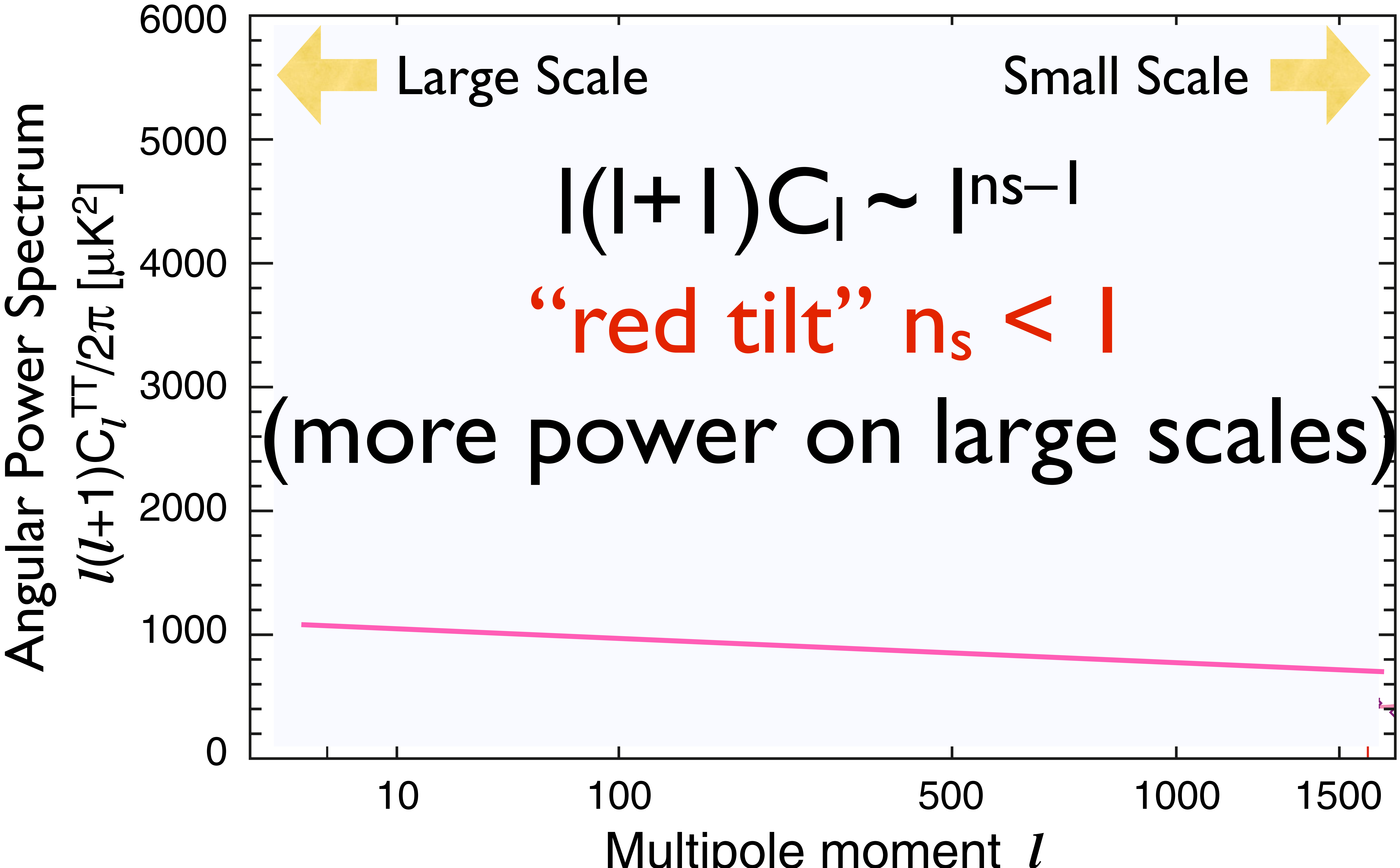
Inflation Predicts:



Inflation may do this

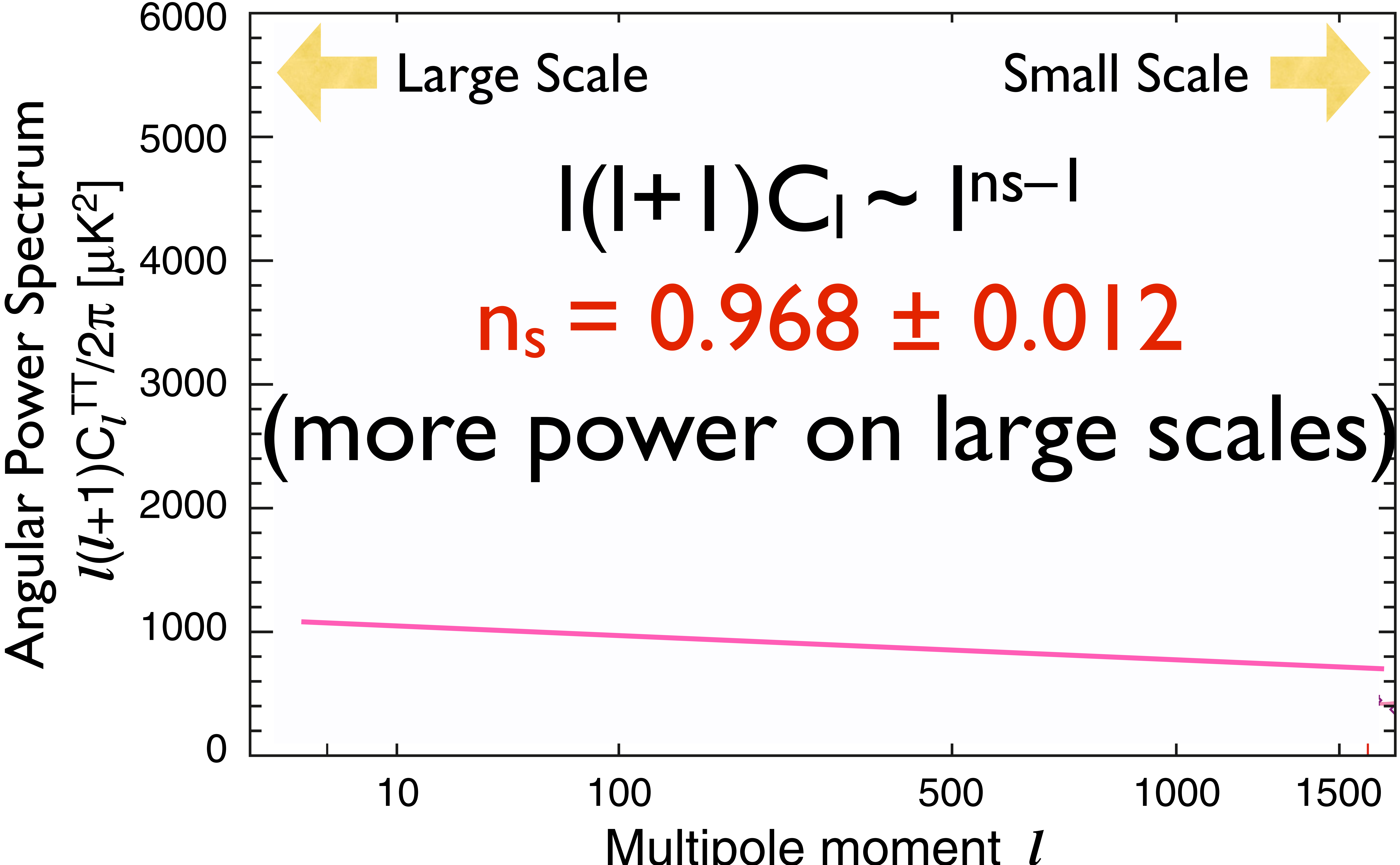


...or this



6000
5000
4000
3000
2000
1000
0
Angular Power Spectrum
$l(l+1)C_l^{TT}/2\pi$ [μK²]
10
100
500
1000
1500
Multipole moment l
Large Scale
Small Scale
$l(l+1)C_l \sim l^{ns-1}$
“red tilt” $n_s < 1$
(more power on large scales)

WMAP 7-year Measurement (Komatsu et al. 2011)



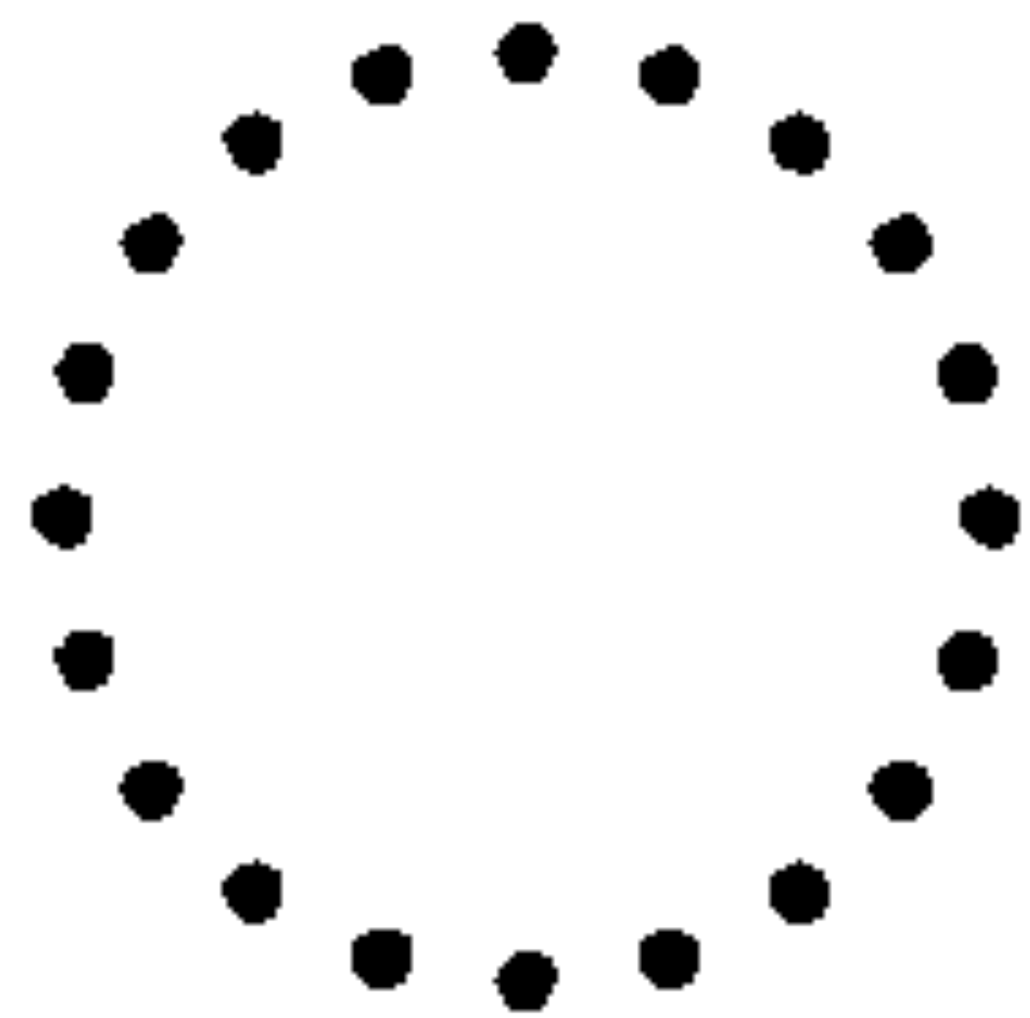
$$l(l+1)C_l \sim l^{n_s - 1}$$

$$n_s = 0.968 \pm 0.012$$

(more power on large scales)

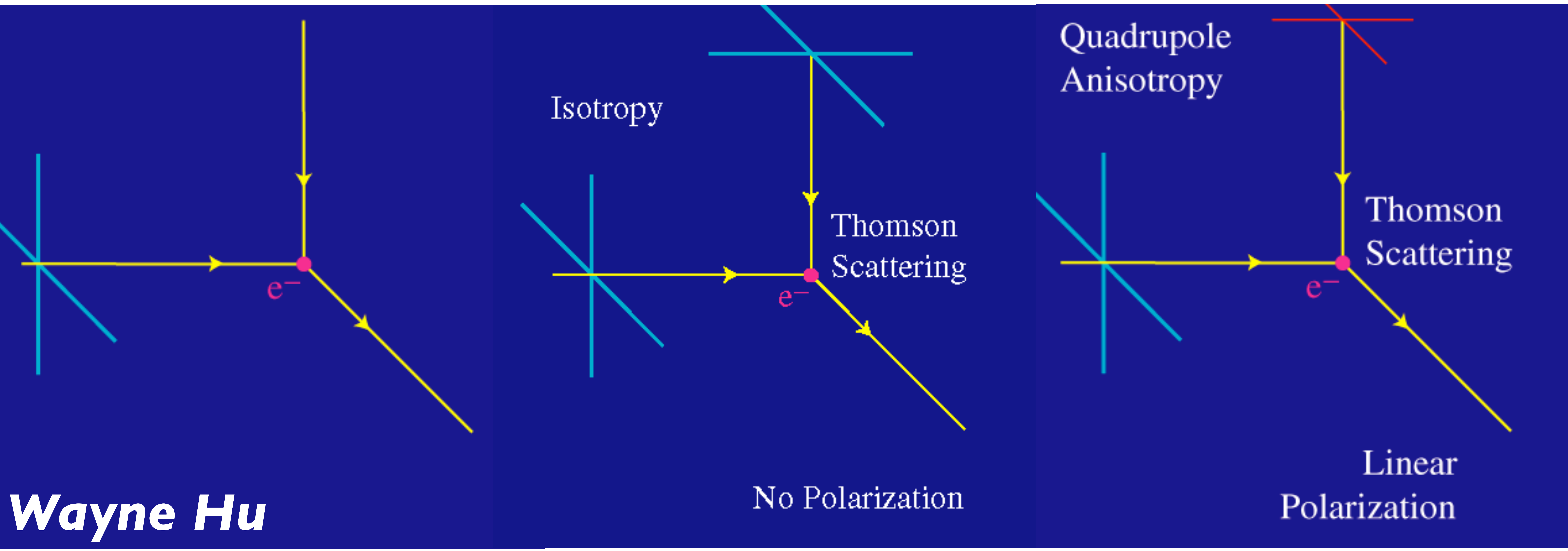
Tensor Perturbations (Gravitational Waves)

Gravitational waves are coming toward you... What do you do?



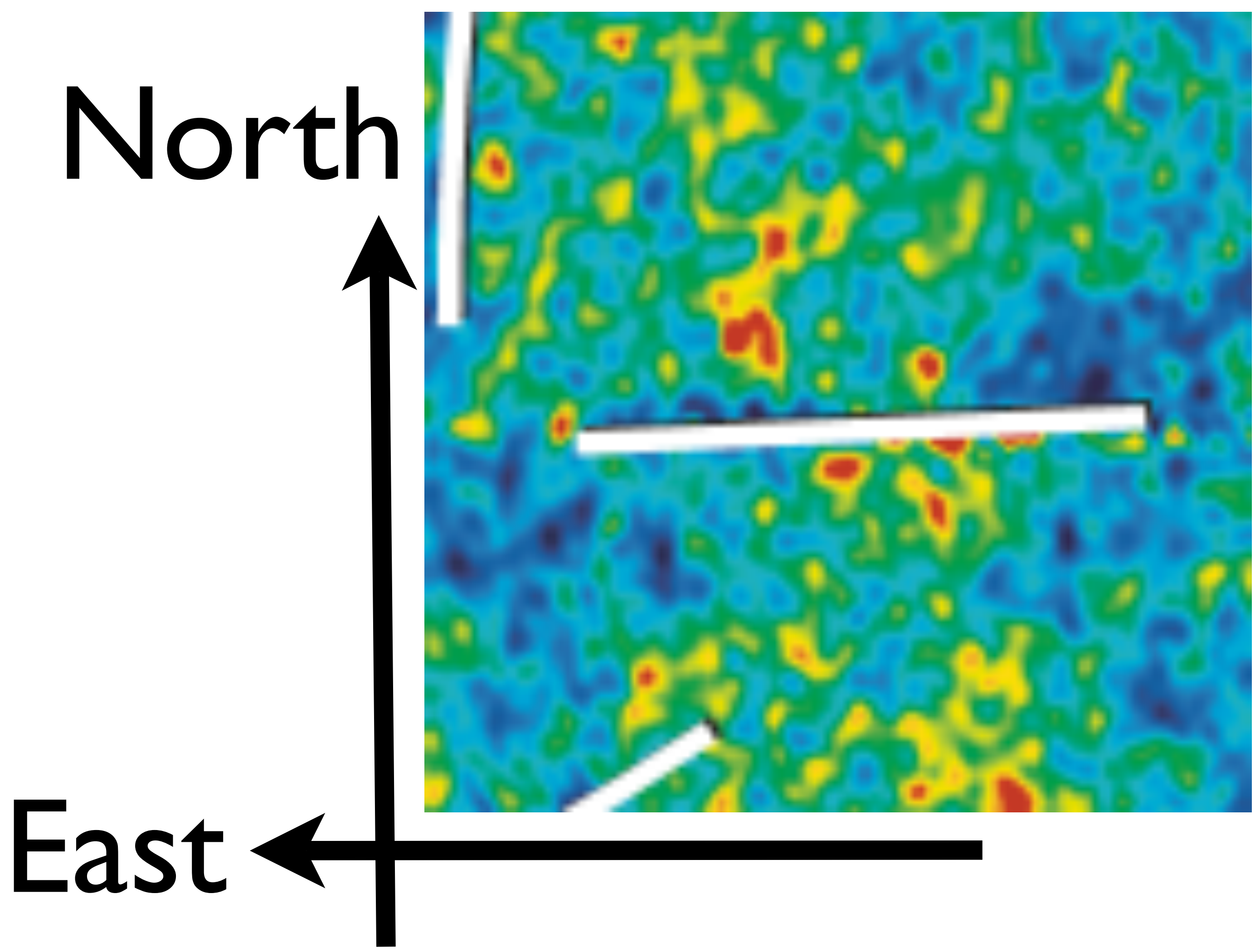
- Gravitational waves stretch space, causing particles to move.

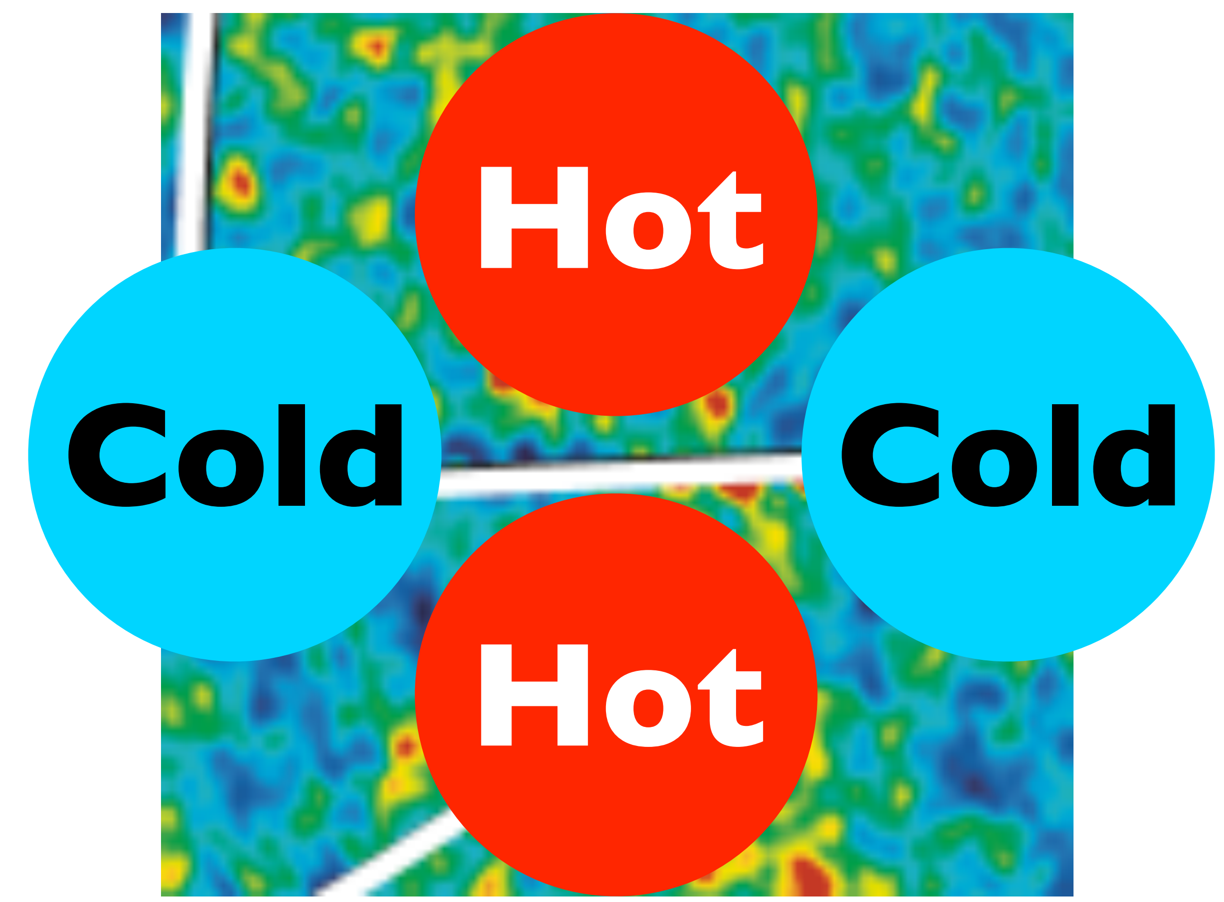
Physics of CMB Polarization



- CMB Polarization is created by a local temperature **quadrupole** anisotropy.

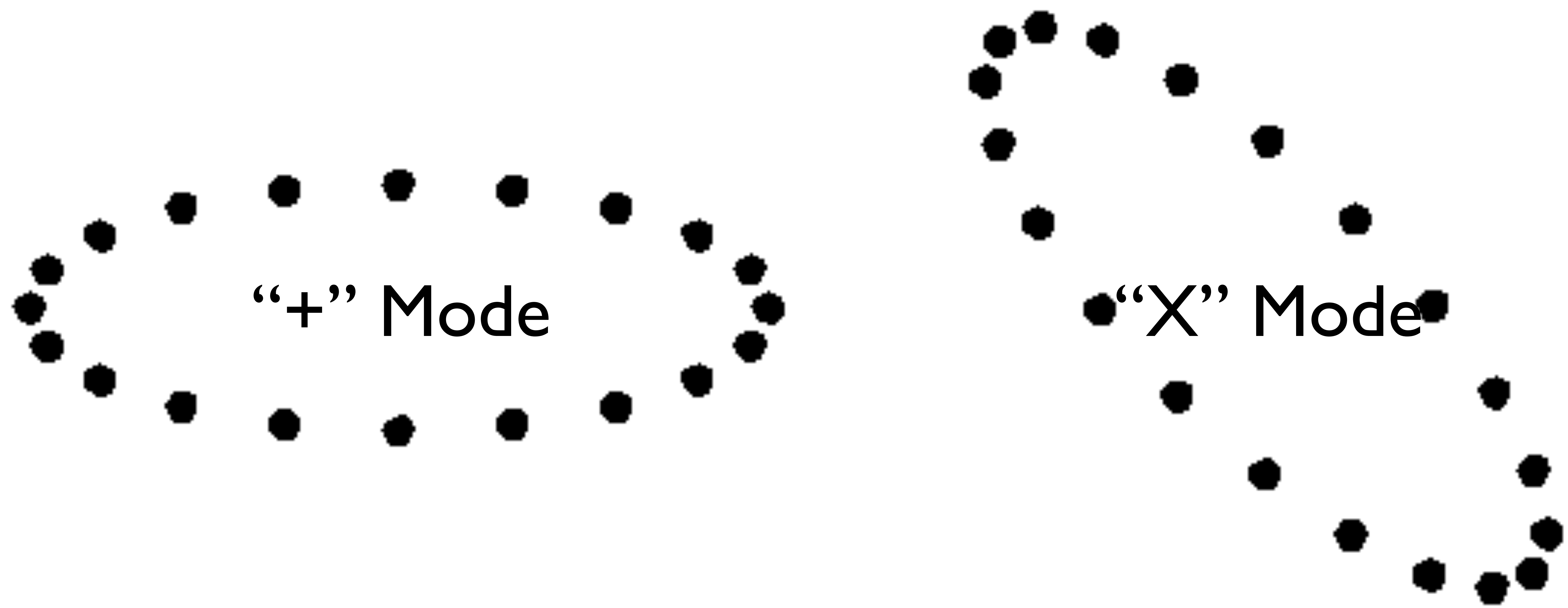
Principle





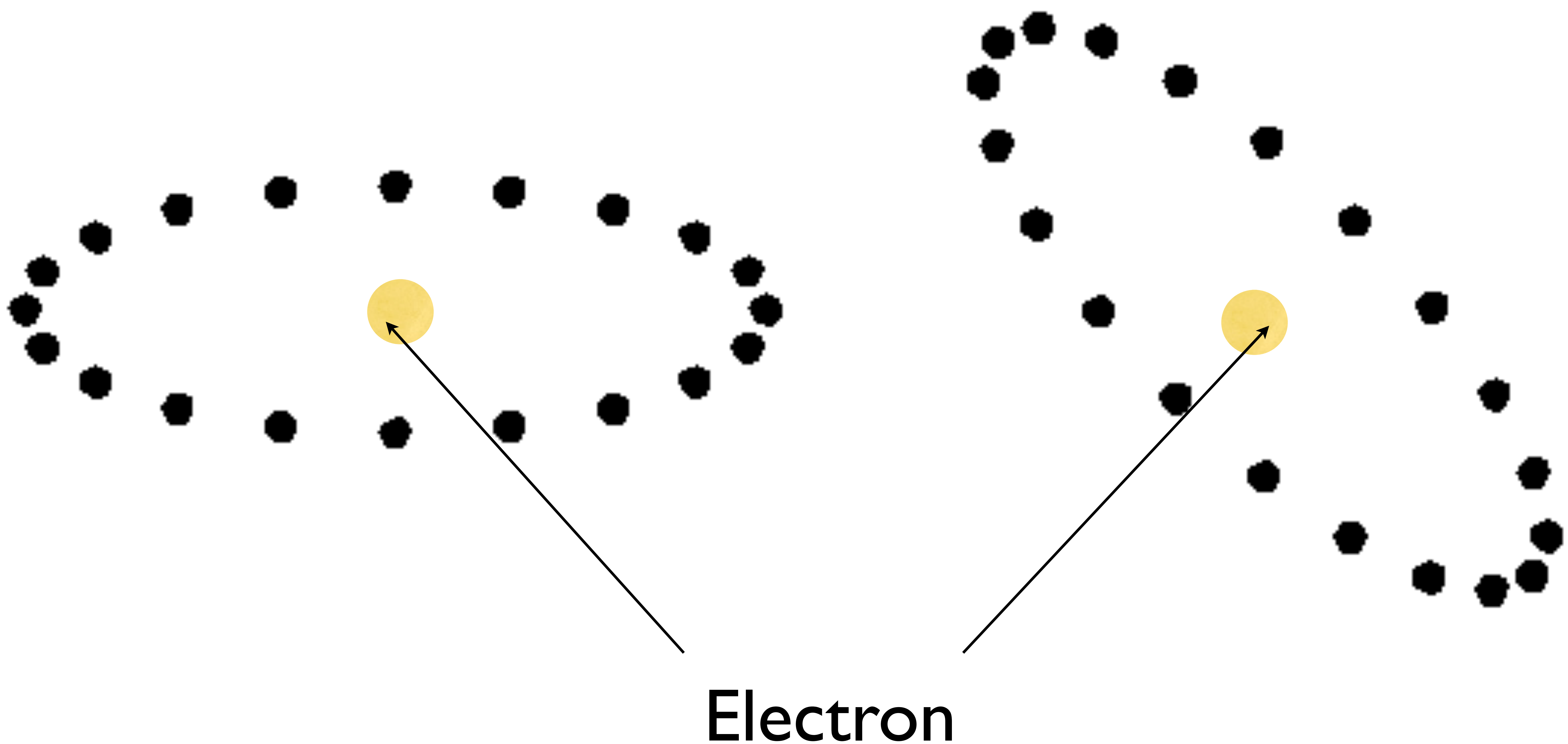
- **Polarization direction is parallel to “hot.”**

Two Polarization States of GW

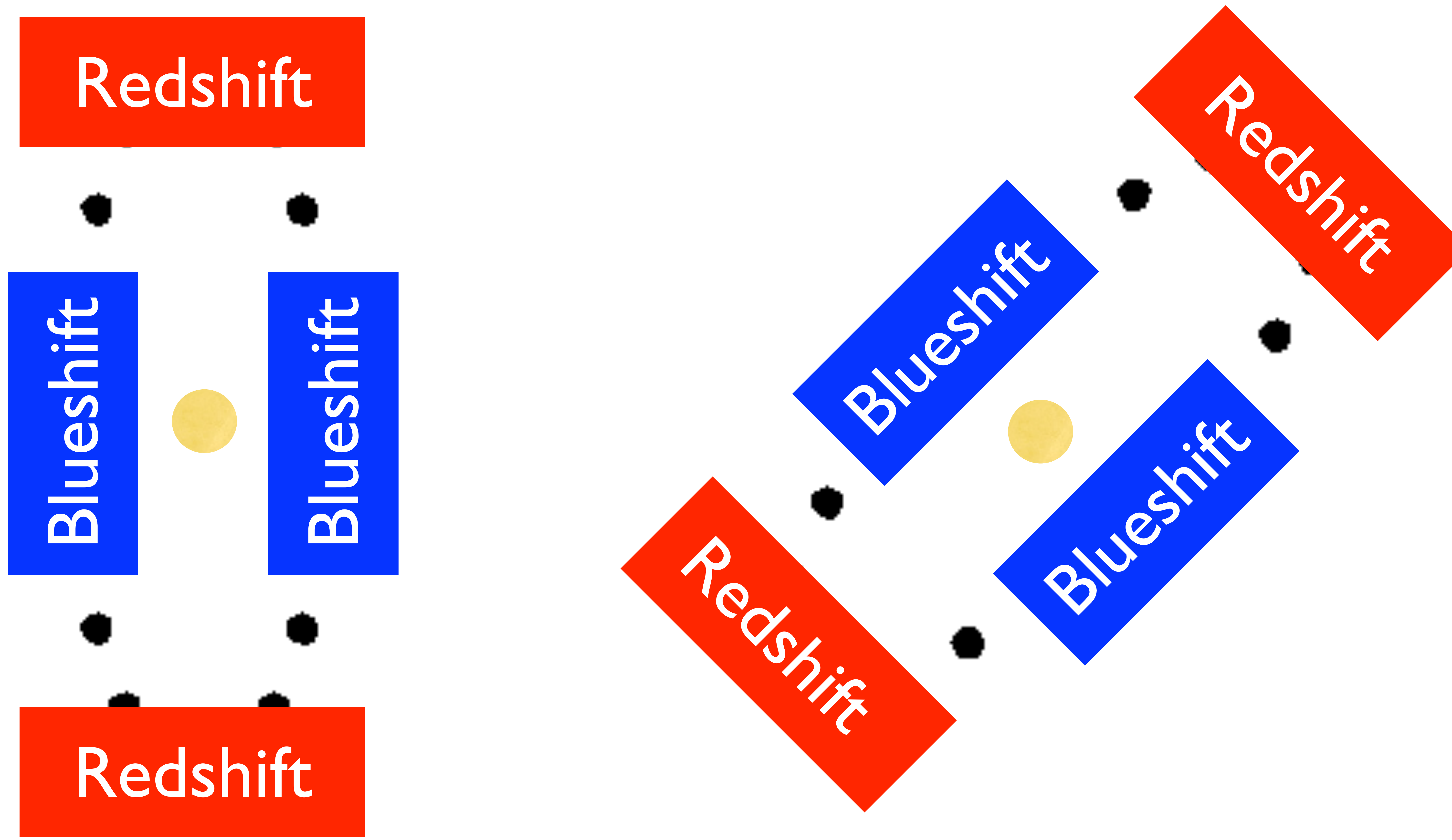


- This is great - this will automatically generate quadrupolar temperature anisotropy around electrons!

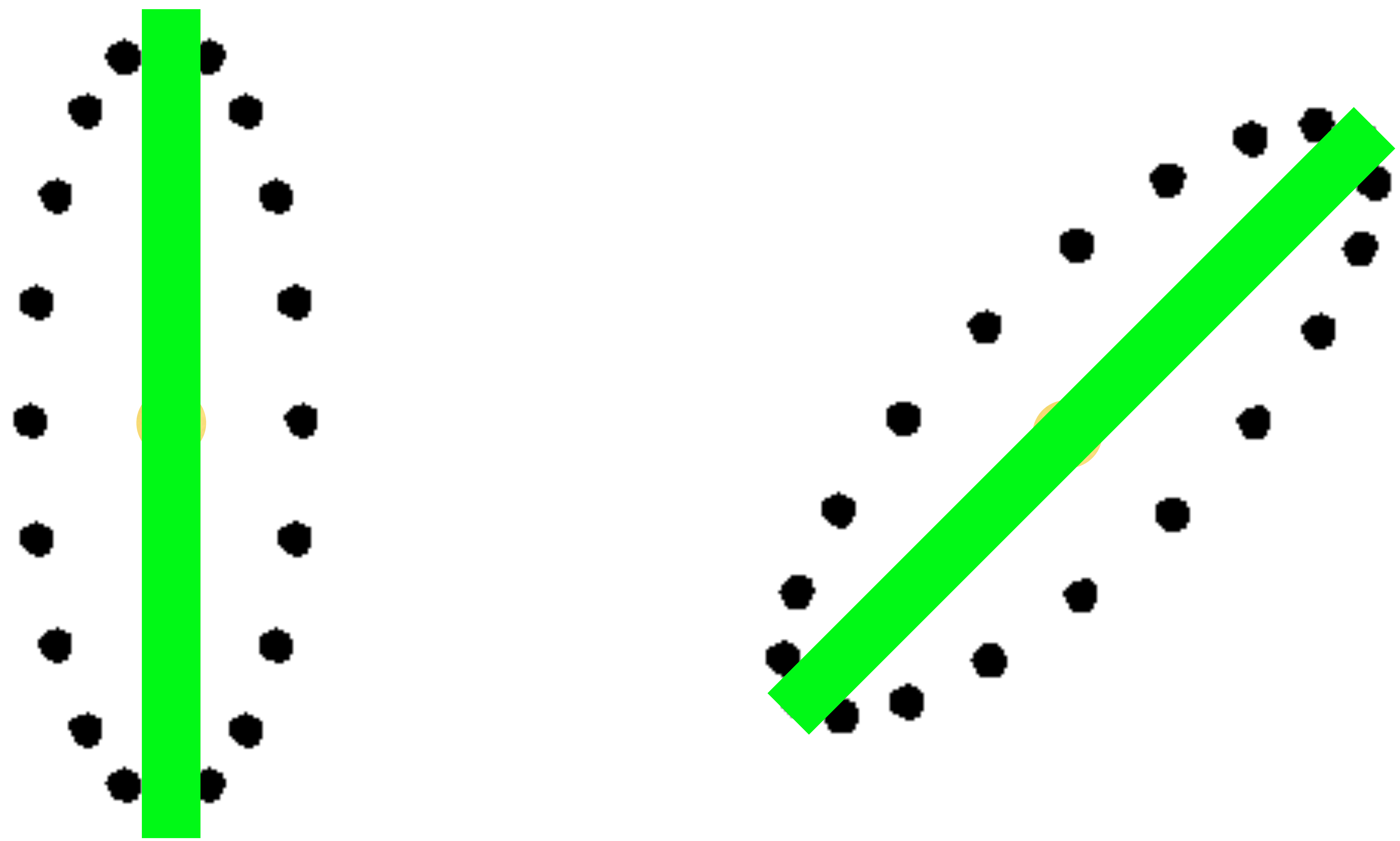
From GW to CMB Polarization



From GW to CMB Polarization



From GW to CMB Polarization



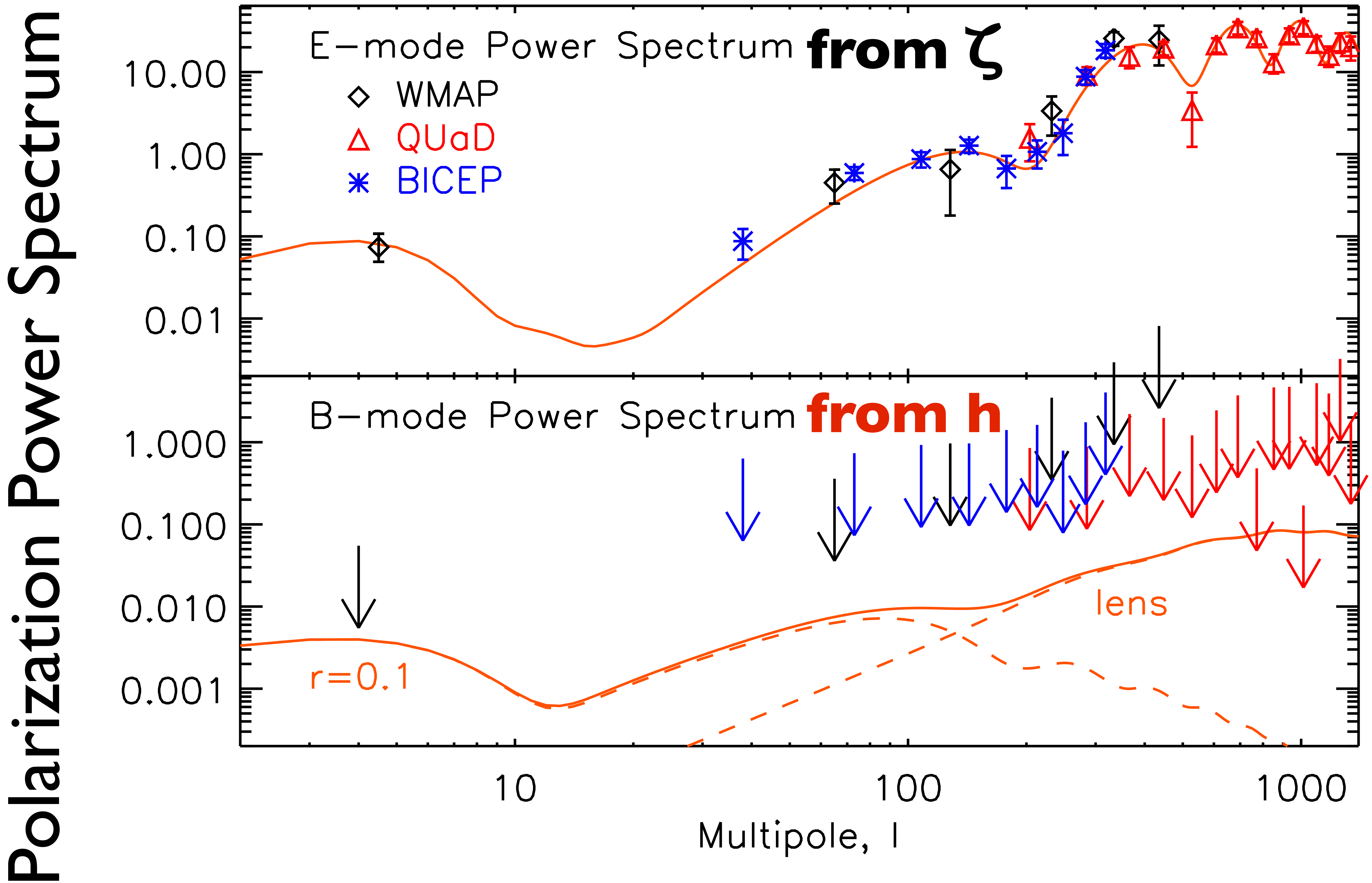
“Tensor-to-scalar Ratio,” r

$$r \equiv \frac{2\langle |h_{\mathbf{k}}^{+}|^2 + |h_{\mathbf{k}}^{\times}|^2 \rangle}{\langle |\zeta_{\mathbf{k}}|^2 \rangle}$$

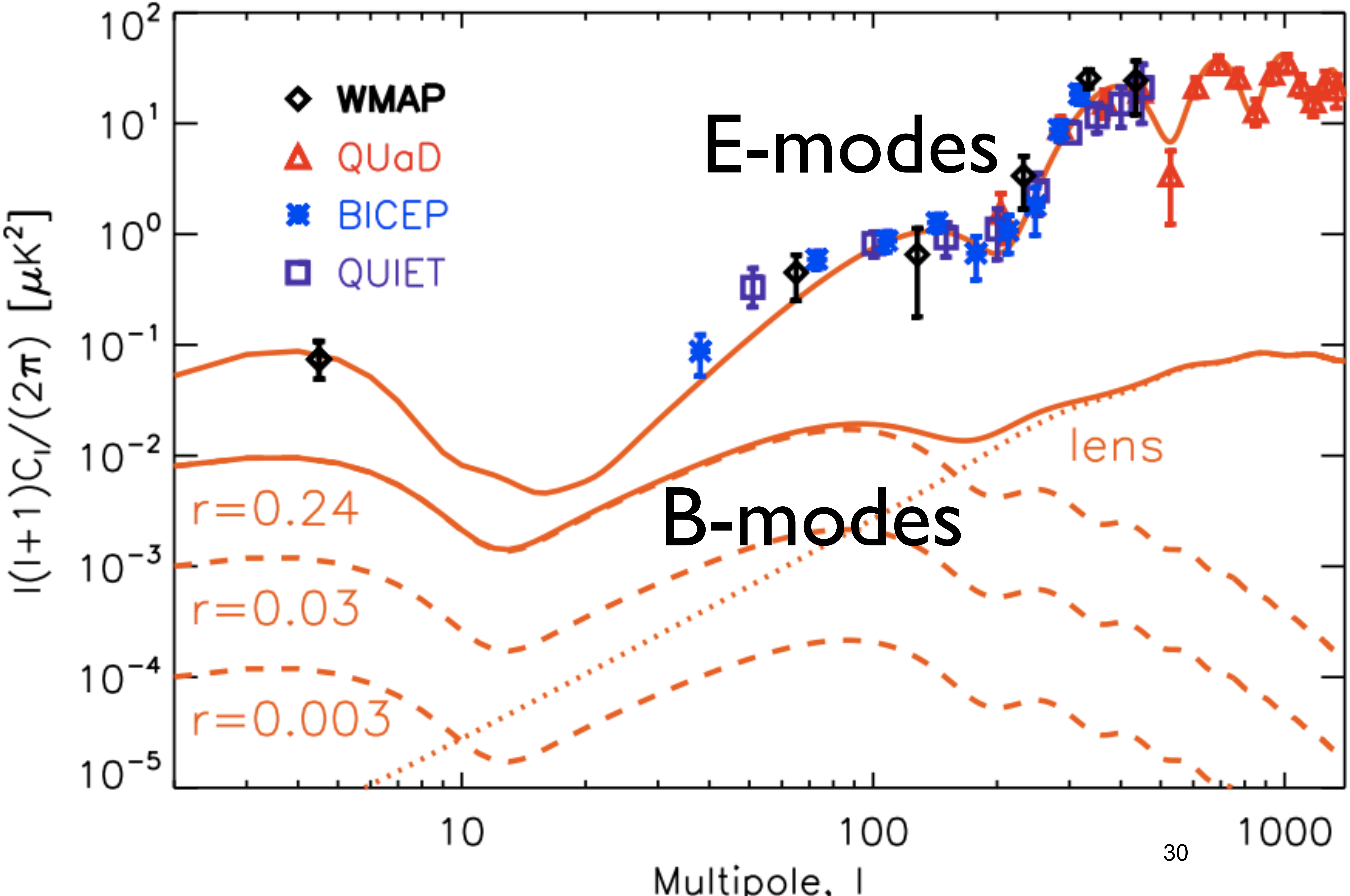
In terms of the slow-roll parameter:

$$r = 16\varepsilon$$

where $\varepsilon = -(dH/dt)/H^2 = 4\pi G(d\varphi/dt)^2/H^2 \approx (16\pi G)^{-1}(dV/d\varphi)^2/V^2$



- No detection of polarization from gravitational waves (B-mode polarization) yet.


WMAP
QUaD
BICEP
QUIET
E-modes
B-modes
lens
r=0.24
r=0.03
r=0.003
10^2
10^1
10^0
10^{-1}
10^{-2}
10^{-3}
10^{-4}
10^{-5}
$l(l+1)C_l/(2\pi)$ [μK^2]
10
100
1000
Multipole, l

Proof: A Punch Line

- Detection of the primordial gravitational wave (i.e., the tensor-to-scalar ratio, "r") with the expected shape of the spectrum provides an unambiguous proof that inflation did occur in the early universe!

How can we falsify inflation?

How can we falsify **single-field** inflation?

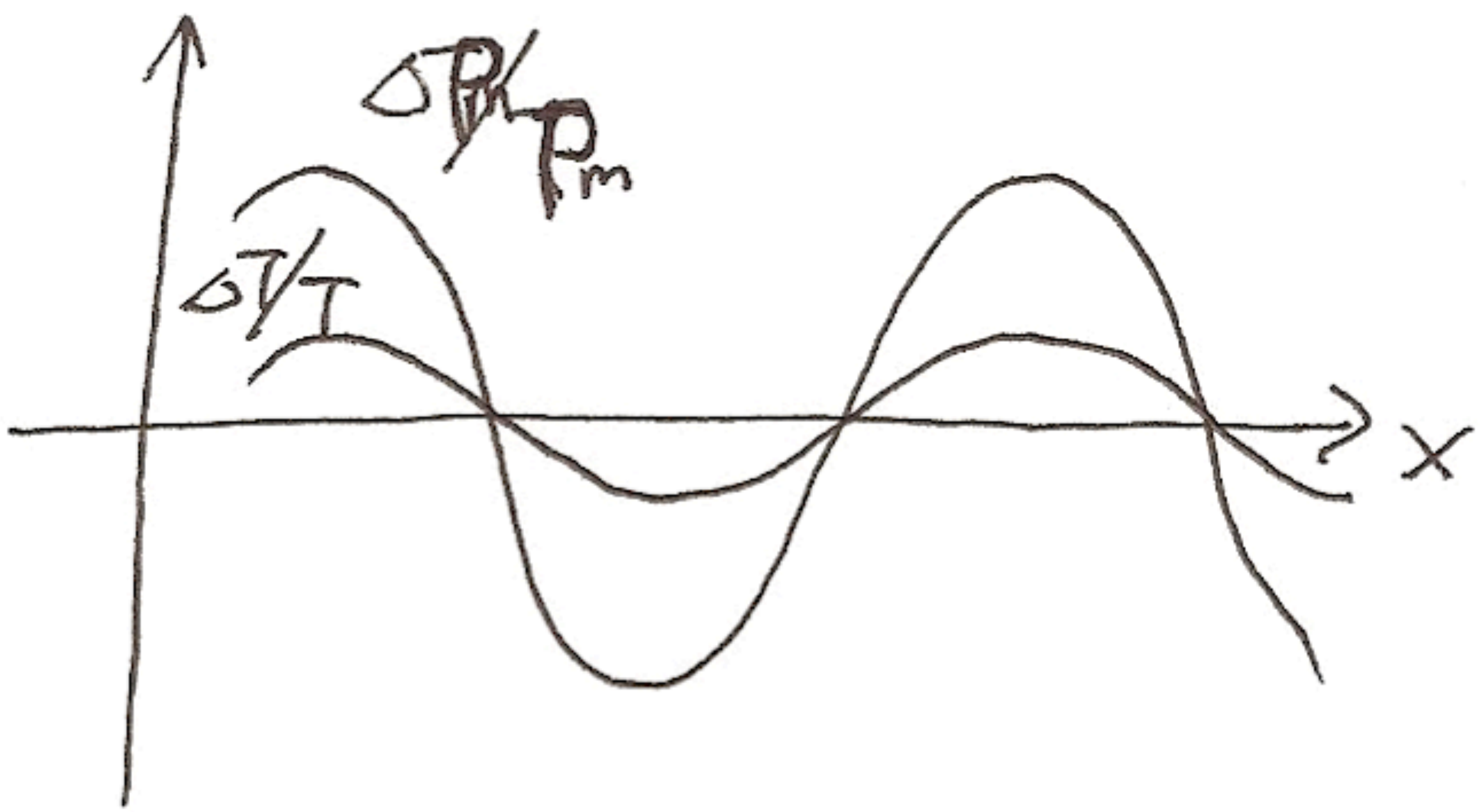
Single Field = Adiabatic fluctuations

- Single-field inflation = One degree of freedom.
  - Matter and radiation fluctuations originate from a single source.

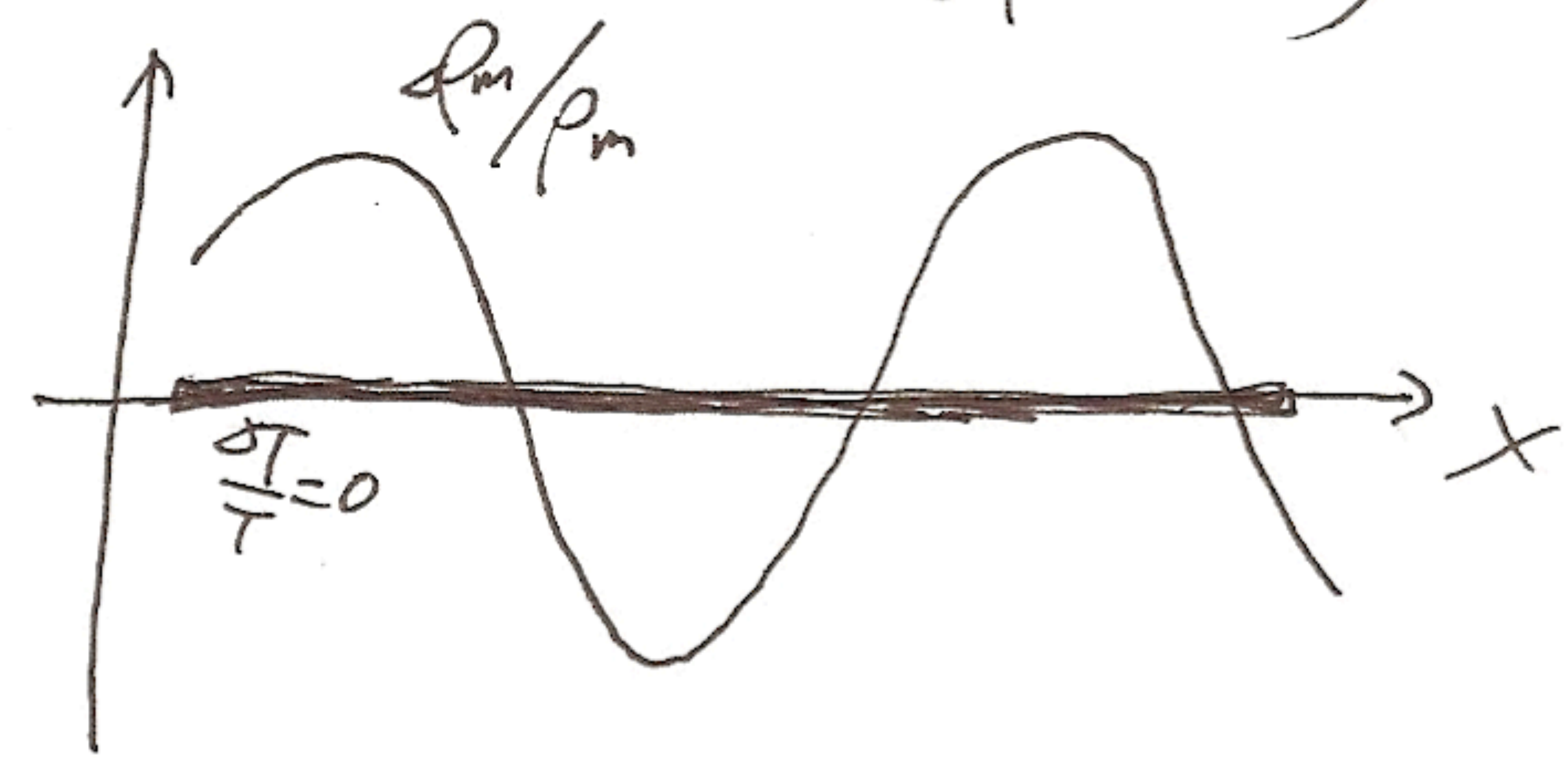
$$\mathcal{S}_{c,\gamma} \equiv \underbrace{\frac{\delta\rho_c}{\rho_c}}_{\text{Cold Dark Matter}} - \underbrace{\frac{3\delta\rho_\gamma}{4\rho_\gamma}}_{\text{Photon}} = 0$$

* A factor of 3/4 comes from the fact that, in thermal equilibrium, $\rho_c \sim (1+z)^3$ and $\rho_\gamma \sim (1+z)^4$.

Adiabatic $\left(\frac{\delta T}{T} = \frac{1}{3}\frac{\delta \rho_m}{\rho_m}\right)$



Example of non-Adiabatic :
Isothermal $\left(\frac{\delta T}{T} = 0\right)$



Komatsu et al. (2011)

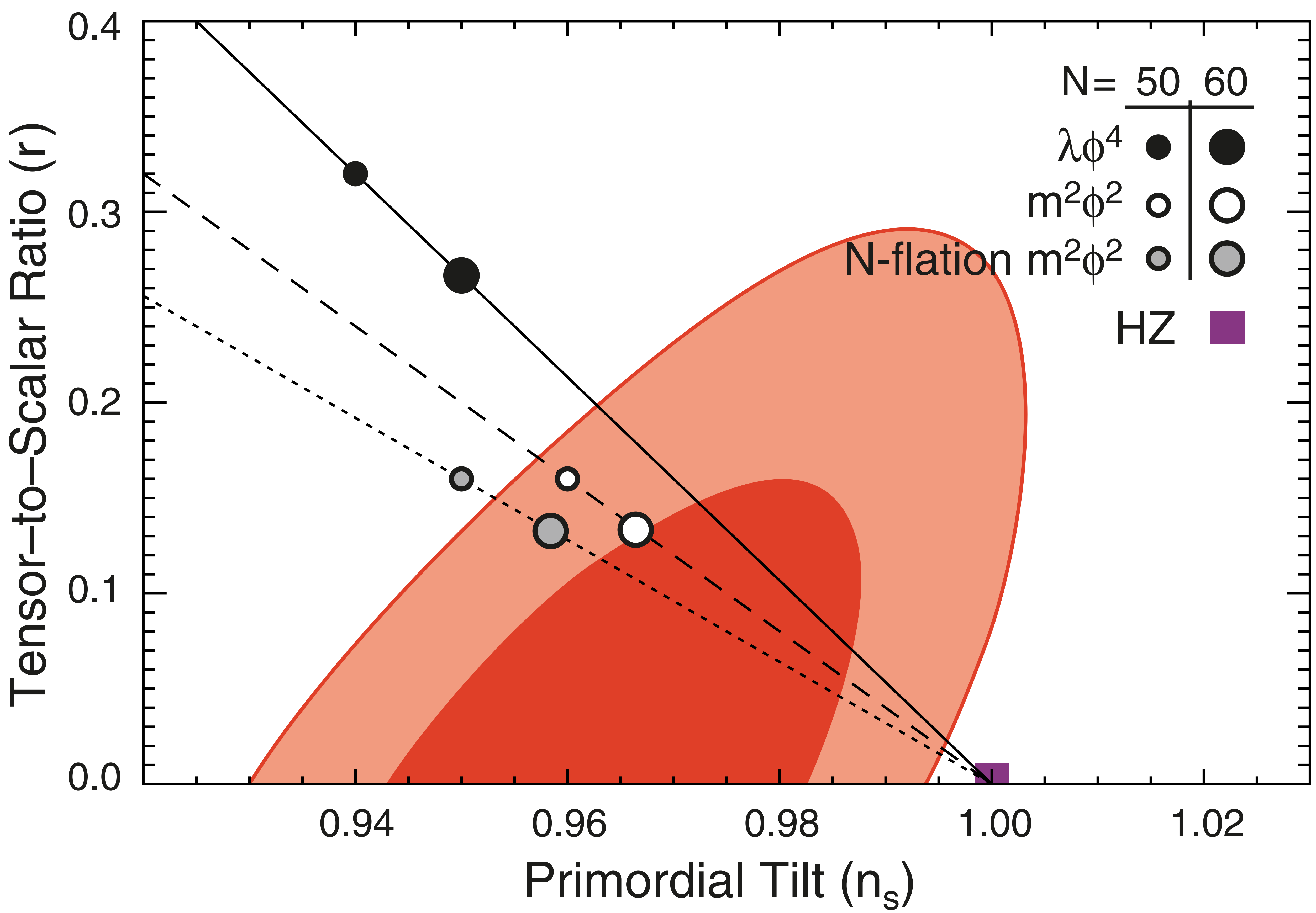
Non-adiabatic Fluctuations

- Detection of non-adiabatic fluctuations immediately rule out single-field inflation models.

The data are consistent with adiabatic fluctuations:

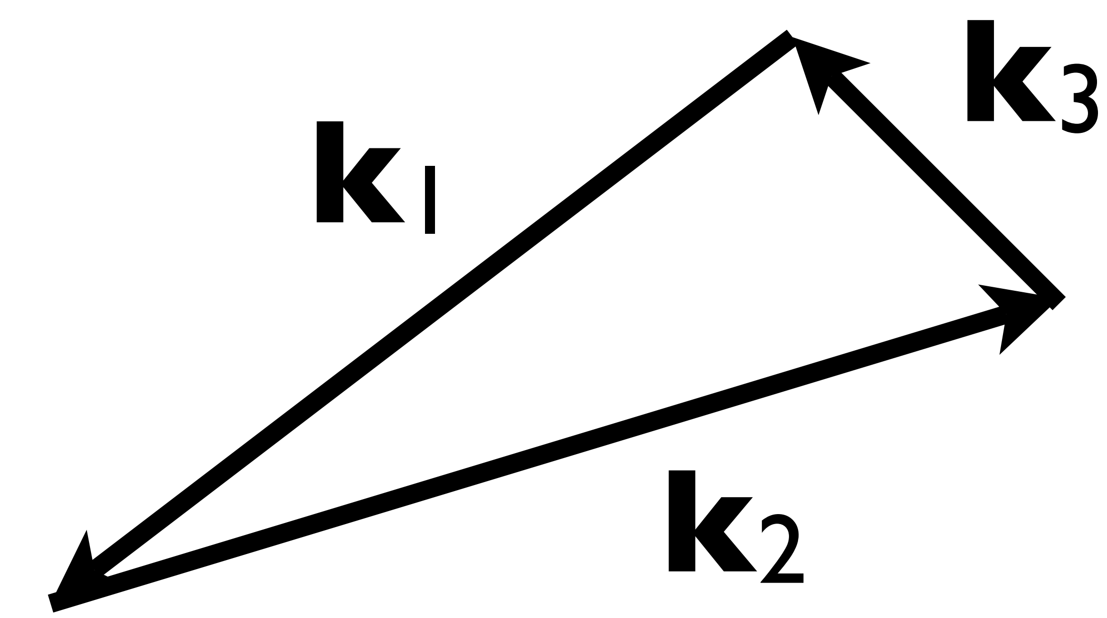
$$\frac{|\delta\rho_c/\rho_c - 3\delta\rho_\gamma/(4\rho_\gamma)|}{\frac{1}{2}[\delta\rho_c/\rho_c + 3\delta\rho_\gamma/(4\rho_\gamma)]} < 0.09 \quad (95\%\ \mathrm{CL})$$

Inflation looks good (in 2-point function)



- Joint constraint on the primordial tilt, n_s, and the tensor-to-scalar ratio, r.
  - **r < 0.24** (95%CL; WMAP7+BAO+H_0)

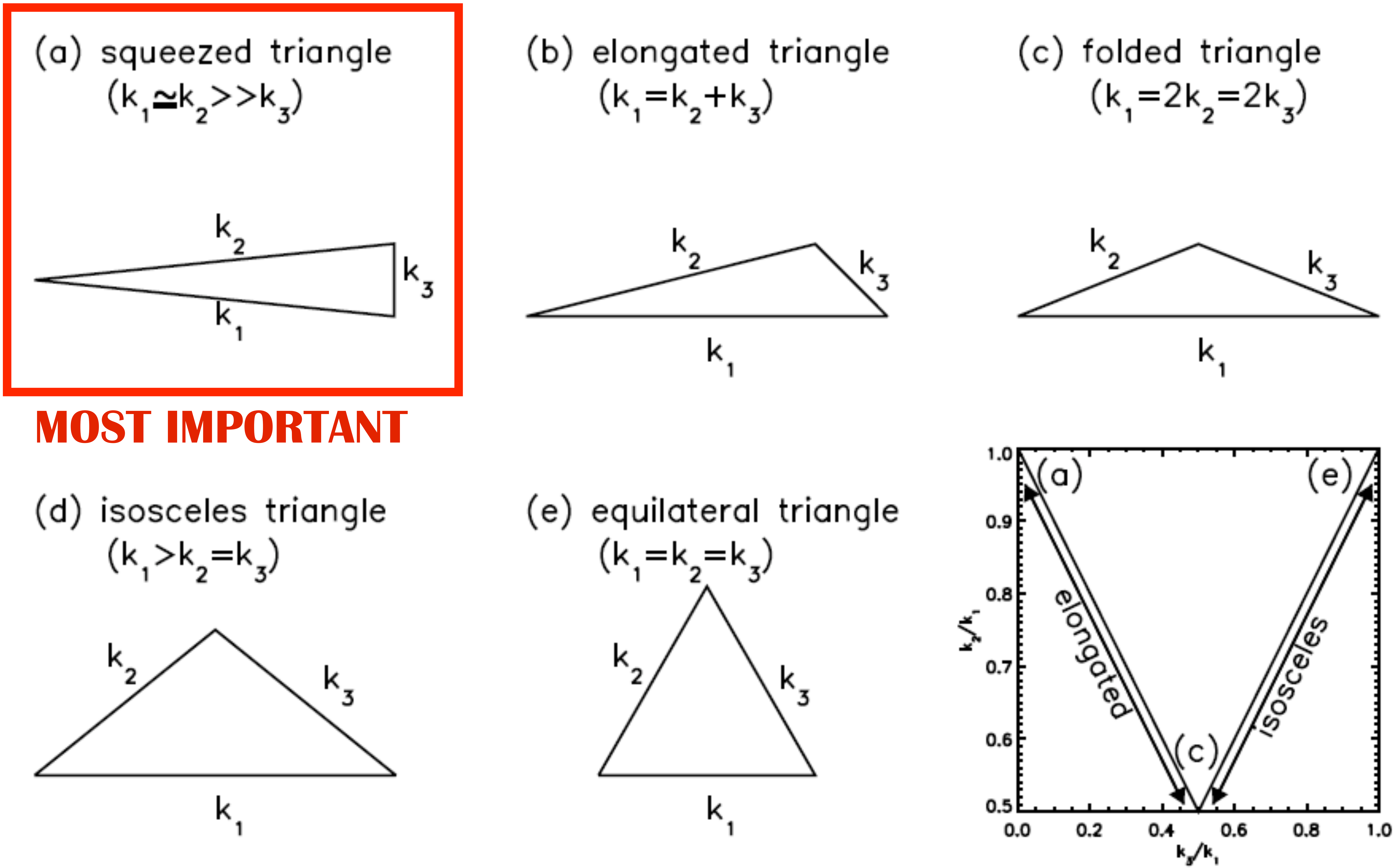
Bispectrum



- Three-point function!
- $B_\zeta(\mathbf{k}_1,\mathbf{k}_2,\mathbf{k}_3)$
  $= \langle \zeta_{\mathbf{k}1}\zeta_{\mathbf{k}2}\zeta_{\mathbf{k}3} \rangle = (\text{amplitude}) \times (2\pi)^3\delta(\mathbf{k}_1+\mathbf{k}_2+\mathbf{k}_3)b(k_1,k_2,k_3)$
  model-dependent function

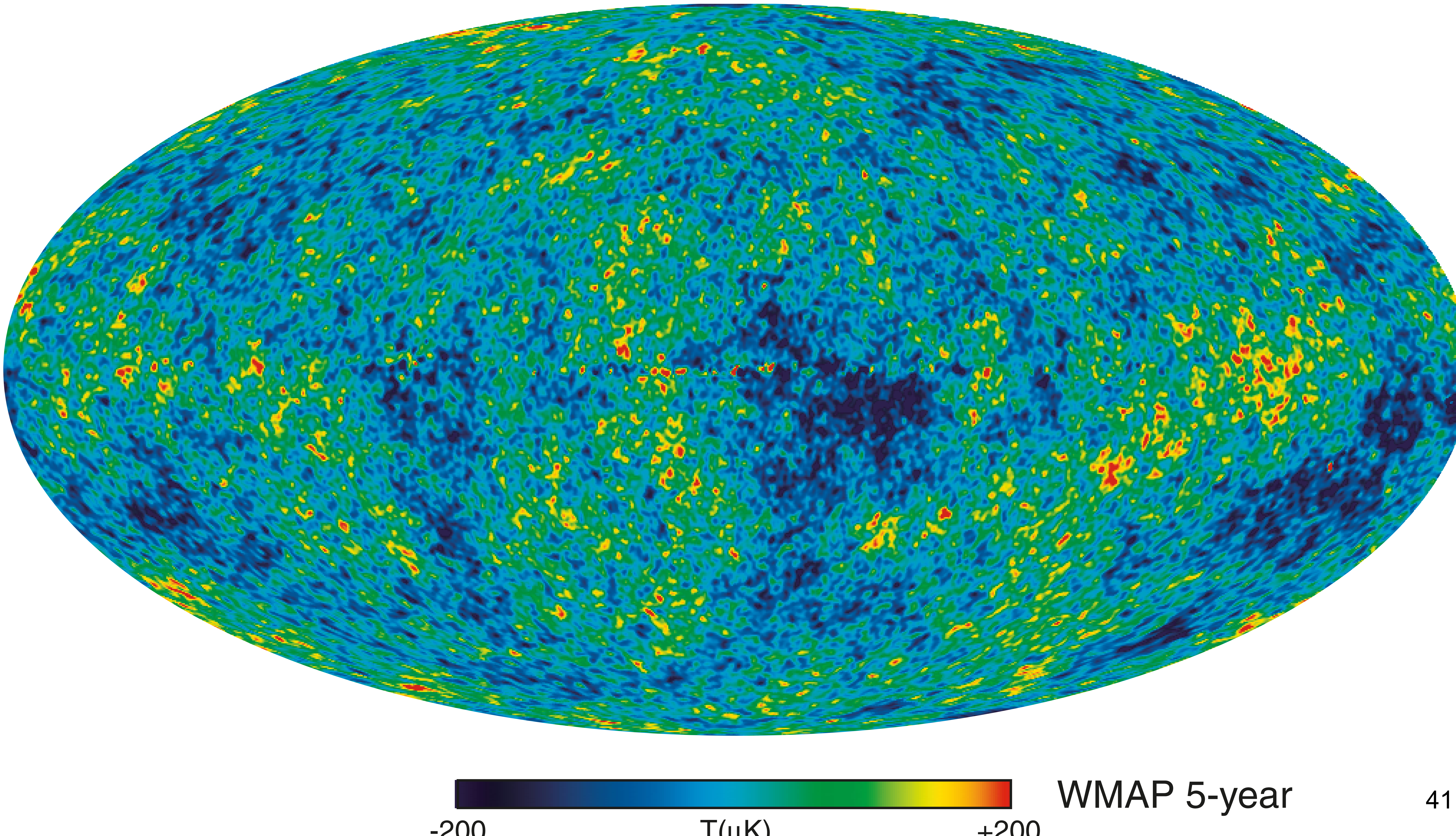
Single Field Theorem

= Negligible “Local-form”
Three-point Function


(a) squeezed triangle
(k1≃k2>>k3)
k2
k3
k1
MOST IMPORTANT
(b) elongated triangle
(k1=k2+k3)
k2
k3
k1
(c) folded triangle
(k1=2k2=2k3)
k2
k3
k1
(d) isosceles triangle
(k1>k2=k3)
k2
k3
k1
(e) equilateral triangle
(k1=k2=k3)
k2
k3
k1
1.0
0.9
0.8
0.7
0.6
0.5
0.0
0.2
0.4
0.6
0.8
1.0
k2/k1
k3/k1
(a)
(e)
(c)
elongated
isosceles

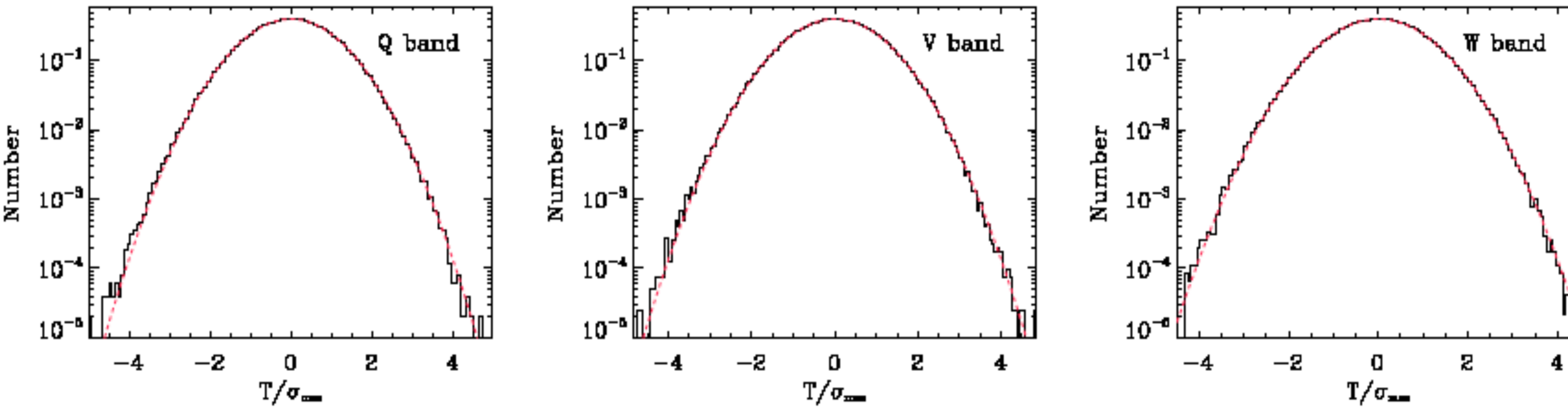
Gaussian?

WMAP5



Spergel et al. (2008)

Take One-point Distribution Function



- The one-point distribution of WMAP map looks pretty Gaussian.
  - –Left to right: Q (41GHz), V (61GHz), W (94GHz).
- Deviation from Gaussianity is small, if any.

Inflation Likes This Result

- According to inflation (Mukhanov & Chibisov; Guth & Yi; Hawking; Starobinsky; Bardeen, Steinhardt & Turner), CMB anisotropy was created from **quantum fluctuations of a scalar field in Bunch-Davies vacuum** during inflation
- Successful inflation (with the expansion factor more than e^{60}) *demands* the scalar field be almost interaction-free
- The wave function of free fields in the ground state is a Gaussian!

But, Not Exactly Gaussian

- Of course, there are always corrections to the simplest statement like this.
- For one, inflaton field **does** have interactions. They are simply weak – they are suppressed by the so-called slow-roll parameter, $\varepsilon \sim O(0.01)$, relative to the free-field action.

A Non-linear Correction to Temperature Anisotropy

- The CMB temperature anisotropy, $\Delta T/T$, is given by the curvature perturbation in the matter-dominated era, Φ.
  - One large scales (the Sachs-Wolfe limit), $\Delta T/T = -\Phi/3$.

For the Schwarzschild metric, $\Phi = +GM/R$.

- Add a non-linear correction to Φ:
  - $\Phi(\mathbf{x}) = \Phi_g(\mathbf{x}) + f_{NL}[\Phi_g(\mathbf{x})]^2$ (Komatsu & Spergel 2001)
  - f_{NL} was predicted to be small (~0.01) for slow-roll models (Salopek & Bond 1990; Gangui et al. 1994)

f_{NL}: Form of B_ζ

- Φ is related to the primordial curvature perturbation, ζ, as $\Phi=(3/5)\zeta$.

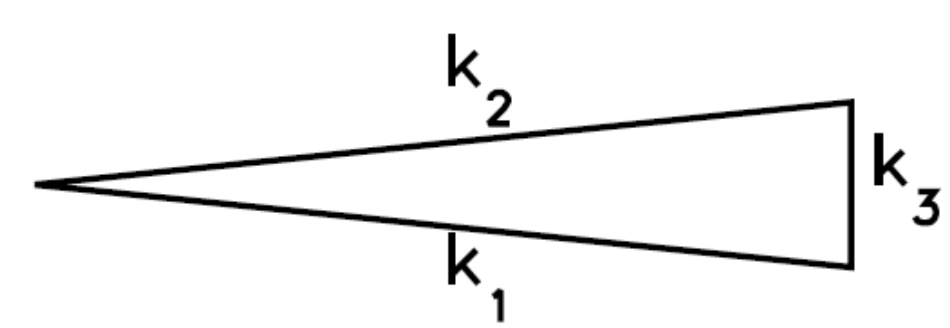
- $\zeta(\mathbf{x}) = \zeta_g(\mathbf{x}) + (3/5)f_{NL}[\zeta_g(\mathbf{x})]^2$

- $B_\zeta(\mathbf{k}_1,\mathbf{k}_2,\mathbf{k}_3)=(6/5)f_{NL} \times (2\pi)^3\delta(\mathbf{k}_1+\mathbf{k}_2+\mathbf{k}_3) \times [P_\zeta(k_1)P_\zeta(k_2) + P_\zeta(k_2)P_\zeta(k_3) + P_\zeta(k_3)P_\zeta(k_1)]$

f_{NL}: Shape of Triangle

- For a scale-invariant spectrum, $P_\zeta(k)=A/k^3$,
  - $B_\zeta(\mathbf{k}_1,\mathbf{k}_2,\mathbf{k}_3)=(6A^2/5)f_{NL} \times (2\pi)^3\delta(\mathbf{k}_1+\mathbf{k}_2+\mathbf{k}_3) \times [1/(k_1k_2)^3 + 1/(k_2k_3)^3 + 1/(k_3k_1)^3]$
- Let's order k_i such that $k_3 \le k_2 \le k_1$. For a given k_1, one finds the largest bispectrum when the smallest k, i.e., k_3, is very small.
  - $B_\zeta(\mathbf{k}_1,\mathbf{k}_2,\mathbf{k}_3)$ peaks when $k_3 << k_2 \sim k_1$
  - Therefore, the shape of f_{NL} bispectrum is the squeezed triangle!

(Babich et al. 2004)



B_ζ in the Squeezed Limit

- In the squeezed limit, the f_{NL} bispectrum becomes: $B_\zeta(\mathbf{k}_1,\mathbf{k}_2,\mathbf{k}_3) \approx (12/5)f_{NL} \times (2\pi)^3\delta(\mathbf{k}_1+\mathbf{k}_2+\mathbf{k}_3) \times P_\zeta(k_1)P_\zeta(k_3)$

Maldacena (2003); Seery & Lidsey (2005); Creminelli & Zaldarriaga (2004)

Single-field Theorem (Consistency Relation)

- For **ANY** single-field models*, the bispectrum in the squeezed limit is given by
  - $B_\zeta(\mathbf{k}_1,\mathbf{k}_2,\mathbf{k}_3) \approx (1-n_s) \times (2\pi)^3\delta(\mathbf{k}_1+\mathbf{k}_2+\mathbf{k}_3) \times P_\zeta(k_1)P_\zeta(k_3)$
  - Therefore, all single-field models predict $f_{NL} \approx (5/12)(1-n_s)$.
  - With the current limit $n_s=0.96$, f_{NL} is predicted to be 0.017.

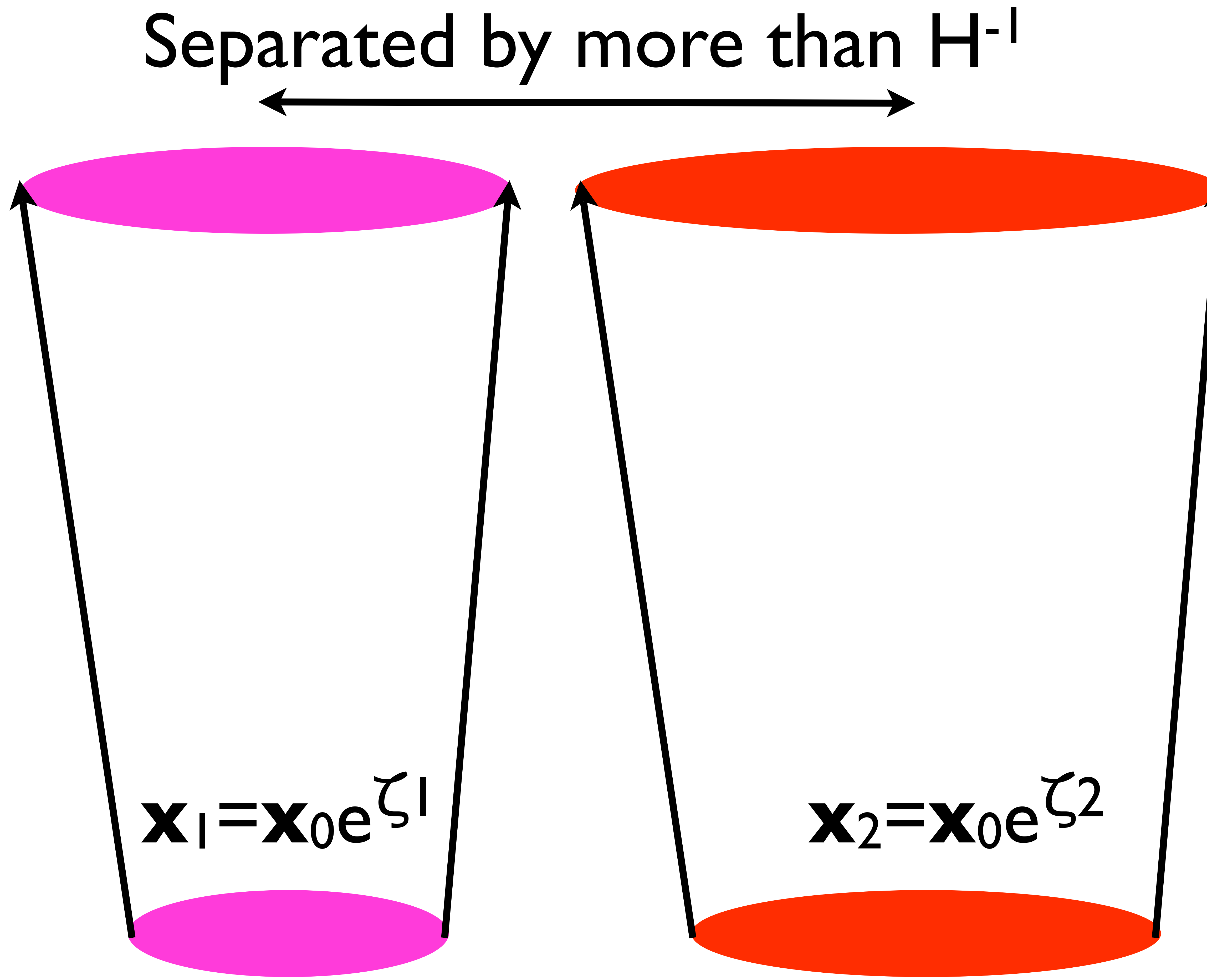
* for which the single field is solely responsible for driving inflation and generating observed fluctuations.

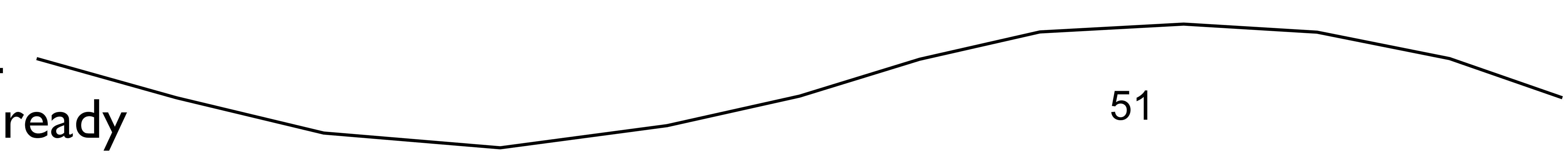
Understanding the Theorem

- First, the squeezed triangle correlates one very long-wavelength mode, k_L ($=k_3$), to two shorter wavelength modes, k_S ($=k_1 \approx k_2$):
  - $<\zeta_{\mathbf{k}1}\zeta_{\mathbf{k}2}\zeta_{\mathbf{k}3}> \approx <(\zeta_{\mathbf{k}S})^2\zeta_{\mathbf{k}L}>$
- Then, the question is: "why should $(\zeta_{\mathbf{k}S})^2$ ever care about $\zeta_{\mathbf{k}L}$?"
  - The theorem says, "it doesn't care, if $\zeta_{\mathbf{k}}$ is exactly scale invariant."

$\zeta_{\mathbf{k}L}$ rescales coordinates

- The long-wavelength curvature perturbation rescales the spatial coordinates (or changes the expansion factor) within a given Hubble patch:
  - $ds^2 = -dt^2 + [a(t)]^2 e^{2\zeta} (d\mathbf{x})^2$

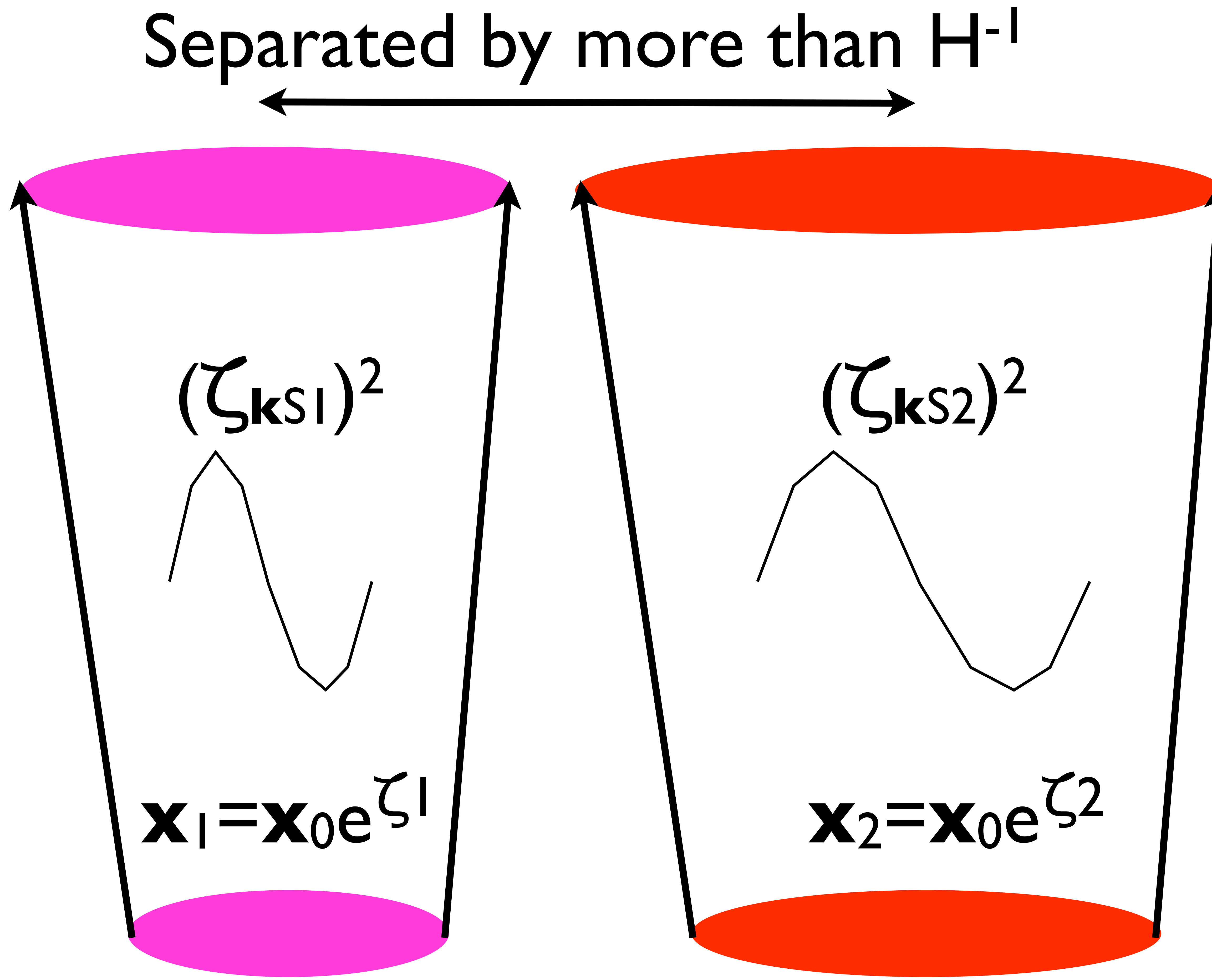


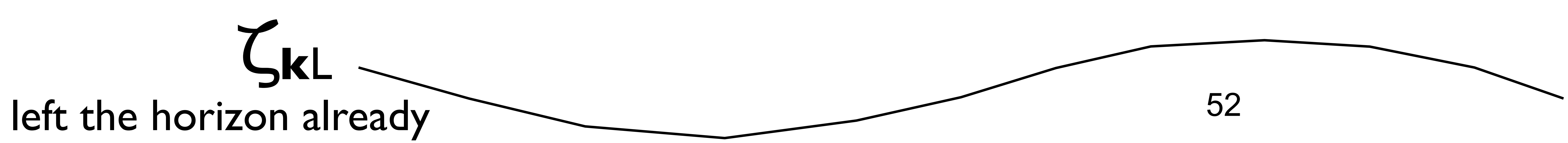


$\zeta_{\mathbf{k}L}$
left the horizon already

$\zeta_{\mathbf{k}L}$ rescales coordinates

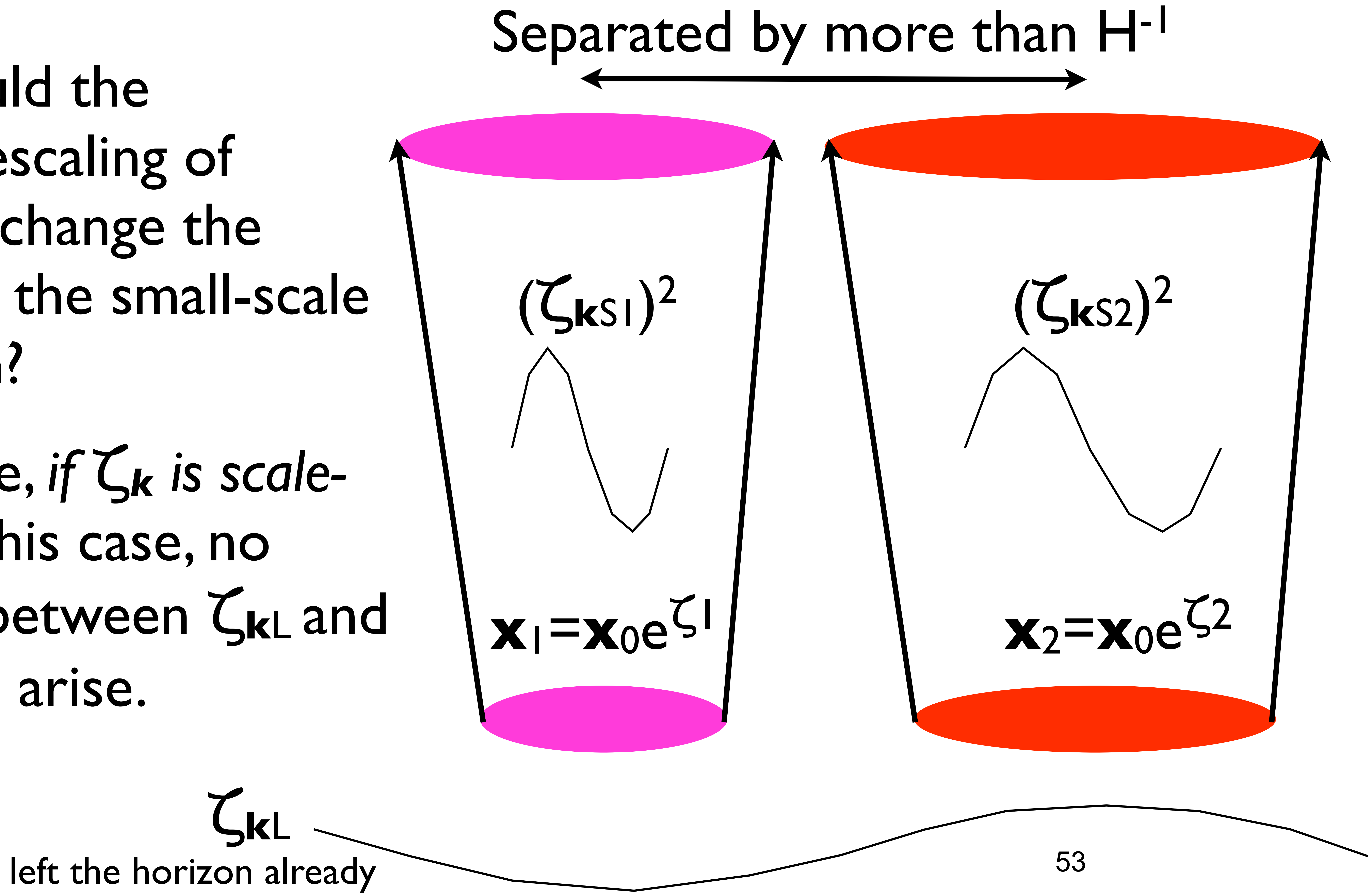
- Now, let's put small-scale perturbations in.
- Q. How would the conformal rescaling of coordinates change the amplitude of the small-scale perturbation?





$\zeta_{\mathbf{k}L}$ rescales coordinates

- Q. How would the conformal rescaling of coordinates change the amplitude of the small-scale perturbation?
- A. No change, *if $\zeta_{\mathbf{k}}$ is scale-invariant*. In this case, no correlation between $\zeta_{\mathbf{k}L}$ and $(\zeta_{\mathbf{k}S})^2$ would arise.



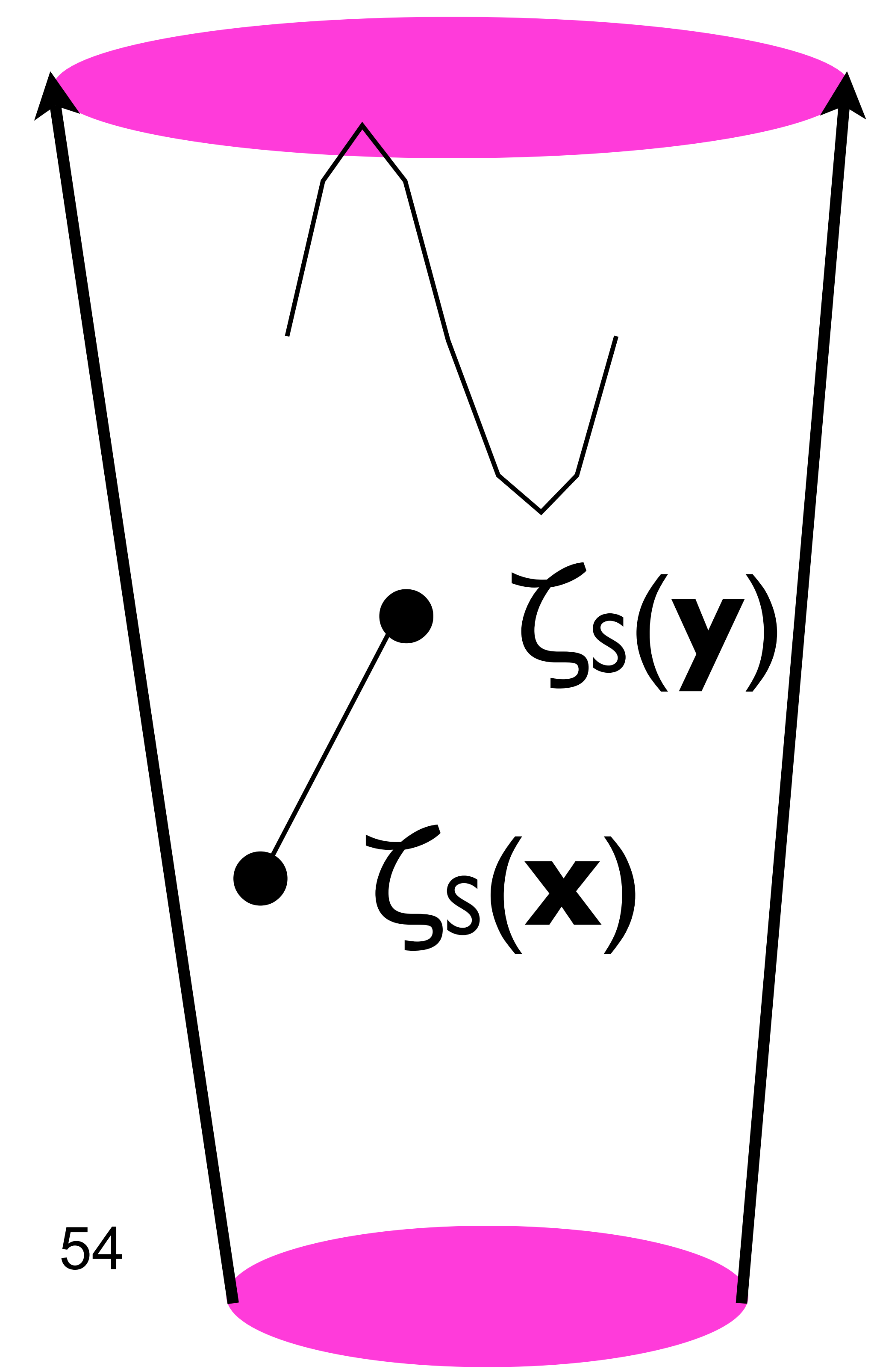
Creminelli & Zaldarriaga (2004); Cheung et al. (2008)

Real-space Proof

- The 2-point correlation function of short-wavelength modes, $\xi=<\zeta_S(\mathbf{x})\zeta_S(\mathbf{y})>$, within a given Hubble patch can be written in terms of its vacuum expectation value (in the absence of ζ_L), ξ_0, as:

  - $\xi_{\zeta L} \approx \xi_0(|\mathbf{x}-\mathbf{y}|) + \zeta_L\,[d\xi_0(|\mathbf{x}-\mathbf{y}|)/d\zeta_L]$
  - $\xi_{\zeta L} \approx \xi_0(|\mathbf{x}-\mathbf{y}|) + \zeta_L\,[d\xi_0(|\mathbf{x}-\mathbf{y}|)/d\ln|\mathbf{x}-\mathbf{y}|]$
  - $\xi_{\zeta L} \approx \xi_0(|\mathbf{x}-\mathbf{y}|) + \zeta_L\,(1-n_s)\xi_0(|\mathbf{x}-\mathbf{y}|)$

3-pt func. $= <(\zeta_S)^2\zeta_L> = <\xi_{\zeta L}\zeta_L>$
$= (1-n_s)\xi_0(|\mathbf{x}-\mathbf{y}|)<\zeta_L{}^2>$



Where was "Single-field"?

- Where did we assume "single-field" in the proof?
- For this proof to work, it is crucial that there is only one dynamical degree of freedom, i.e., it is only ζ_L that modifies the amplitude of short-wavelength modes, and nothing else modifies it.
- Also, ζ must be constant outside of the horizon (otherwise anything can happen afterwards). This is also the case for single-field inflation models.

Probing Inflation (3-point Function)

- No detection of this form of 3-point function of primordial curvature perturbations. The 95% CL limit is:
  - $-10 < f_{NL}^{local} < 74$
  - $f_{NL}^{local} = 32 \pm 21$ (68% CL)

After 9 years of observations...

WMAP taught us:

- **All of the basic predictions of single-field and slow-roll inflation models are consistent with the data** ($1-n_s \approx r \approx f_{NL}$)
  - But, not all models are consistent (i.e., $\lambda\varphi^4$ is out unless you introduce a non-minimal coupling)

However

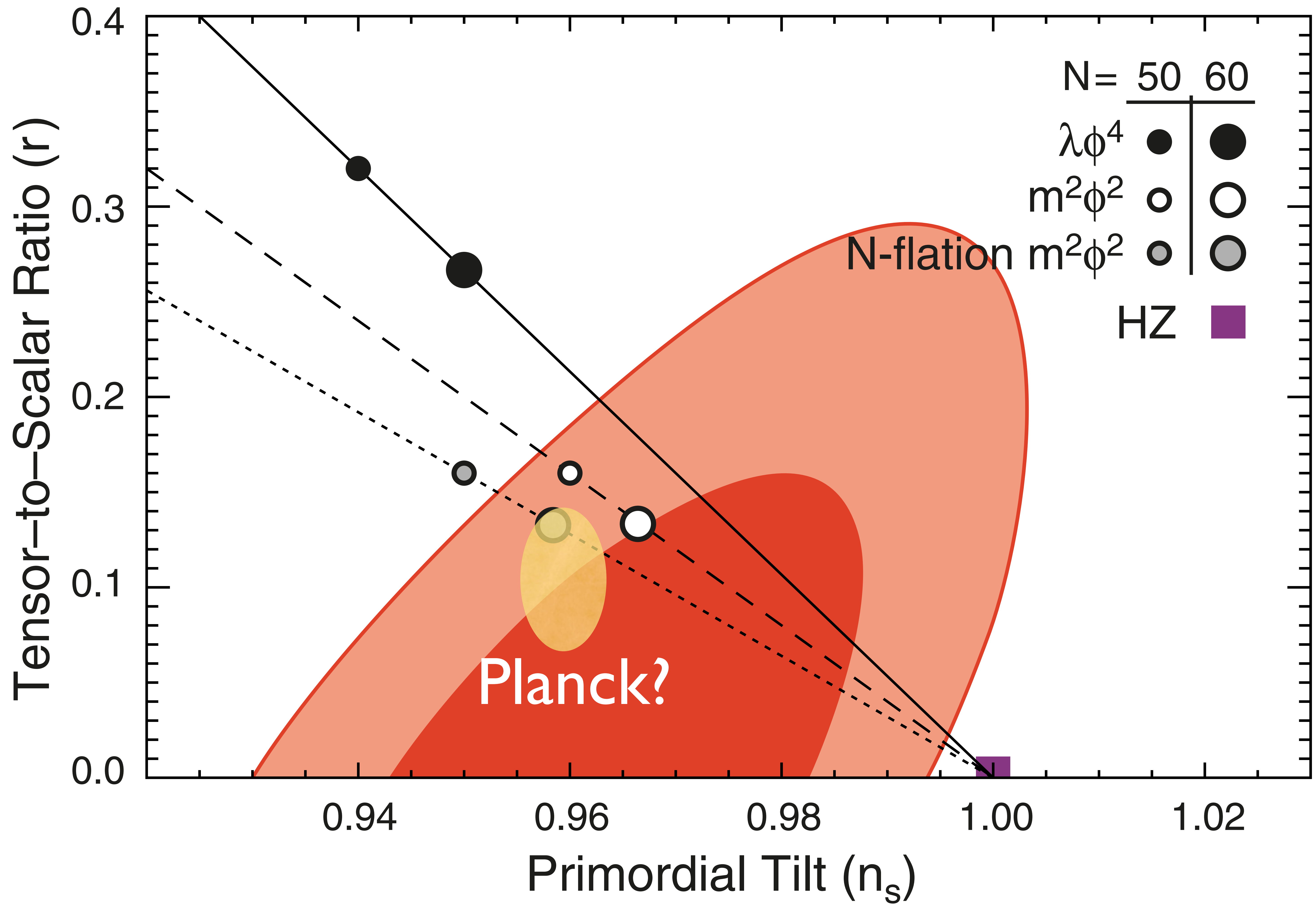
- We cannot say, just yet, that we have definite evidence for inflation.
- *Can we ever prove, or disprove, inflation?*

Planck may:

- Prove inflation by detecting the effect of primordial gravitational waves on polarization of the cosmic microwave background (i.e., detection of *r*)
- Rule out single-field inflation by detecting a particular form of the 3-point function called the "local form" (i.e., detection of f_{NL}^{local})

- Challenge the inflation paradigm by detecting a violation of inequality that should be satisfied between the local-form 3-point and 4-point functions

Planck might find gravitational waves (if r~0.1)



If found, this would give us a pretty convincing proof that inflation did indeed happen.

But...

- Can you falsify inflation (not just single-field models)?

Maybe!

- Using the consistency relation between the ***local-form 3- and 4-point functions***.
  - Sugiyama, Komatsu & Futamase, PRL, 106, 251301(2011)
    - Generalization of the "**Suyama-Yamaguchi inequality**" (2008)

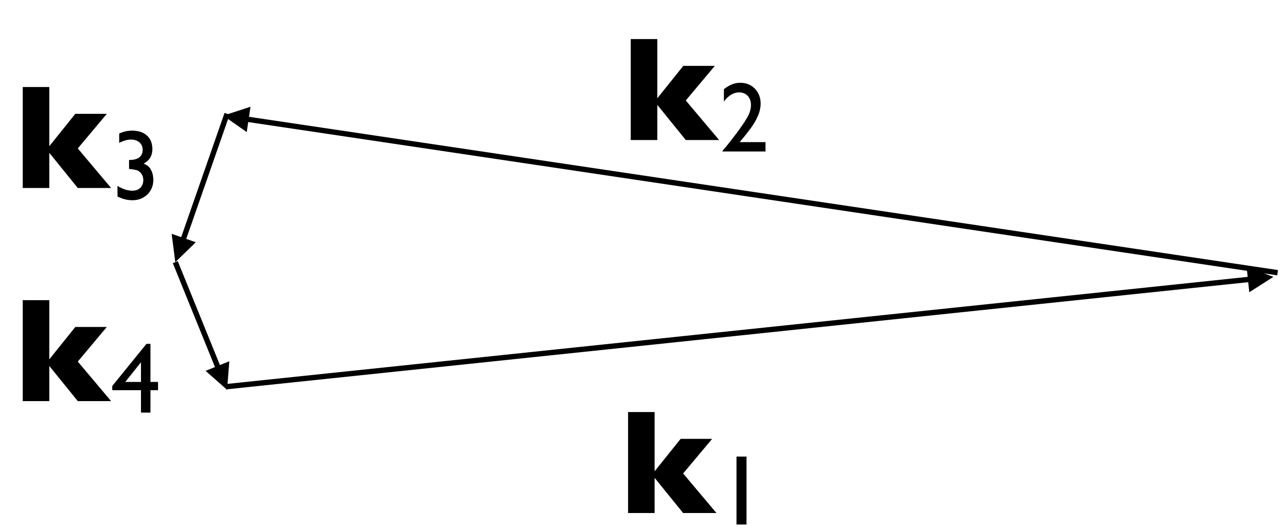
Which Local-form Trispectrum?

- The local-form bispectrum:
  - $B_\zeta(\mathbf{k}_1,\mathbf{k}_2,\mathbf{k}_3)=(2\pi)^3\delta(\mathbf{k}_1+\mathbf{k}_2+\mathbf{k}_3)f_{NL}[(6/5)P_\zeta(k_1)P_\zeta(k_2)+\text{cyc.}]$
- can be produced by a curvature perturbation in position space in the form of:
  - $\zeta(\mathbf{x})=\zeta_g(\mathbf{x}) + (3/5)f_{NL}[\zeta_g(\mathbf{x})]^2$
- This can be extended to higher-order:
  - $\zeta(\mathbf{x})=\zeta_g(\mathbf{x}) + (3/5)f_{NL}[\zeta_g(\mathbf{x})]^2 + (9/25)g_{NL}[\zeta_g(\mathbf{x})]^3$

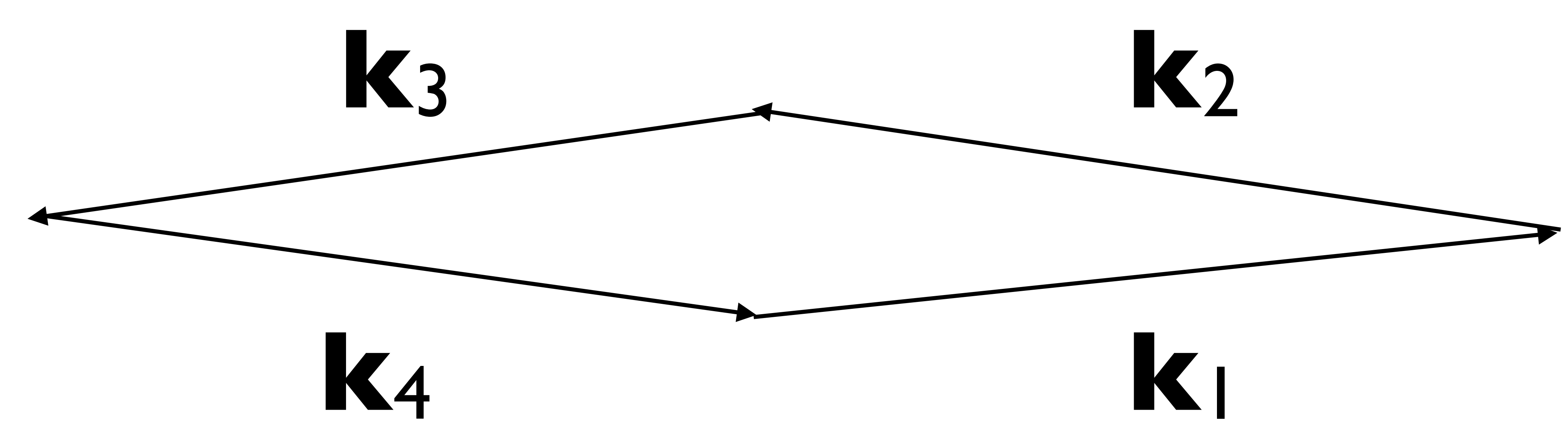
This term (ζ^3) is too small to see, so I will ignore this in this talk.

Two Local-form Shapes

- For $\zeta(\mathbf{x})=\zeta_g(\mathbf{x}) + (3/5)f_{NL}[\zeta_g(\mathbf{x})]^2 + (9/25)g_{NL}[\zeta_g(\mathbf{x})]^3$, we obtain the trispectrum:
  - $T_\zeta(\mathbf{k}_1,\mathbf{k}_2,\mathbf{k}_3,\mathbf{k}_4)=(2\pi)^3\delta(\mathbf{k}_1+\mathbf{k}_2+\mathbf{k}_3+\mathbf{k}_4)$ $\{\mathbf{g_{NL}}[(54/25)P_\zeta(k_1)P_\zeta(k_2)P_\zeta(k_3)+\text{cyc.}] +\mathbf{(f_{NL})^2}[(18/25)P_\zeta(k_1)P_\zeta(k_2)(P_\zeta(|\mathbf{k}_1+\mathbf{k}_3|)+P_\zeta(|\mathbf{k}_1+\mathbf{k}_4|))+\text{cyc.}]\}$



g_{NL}

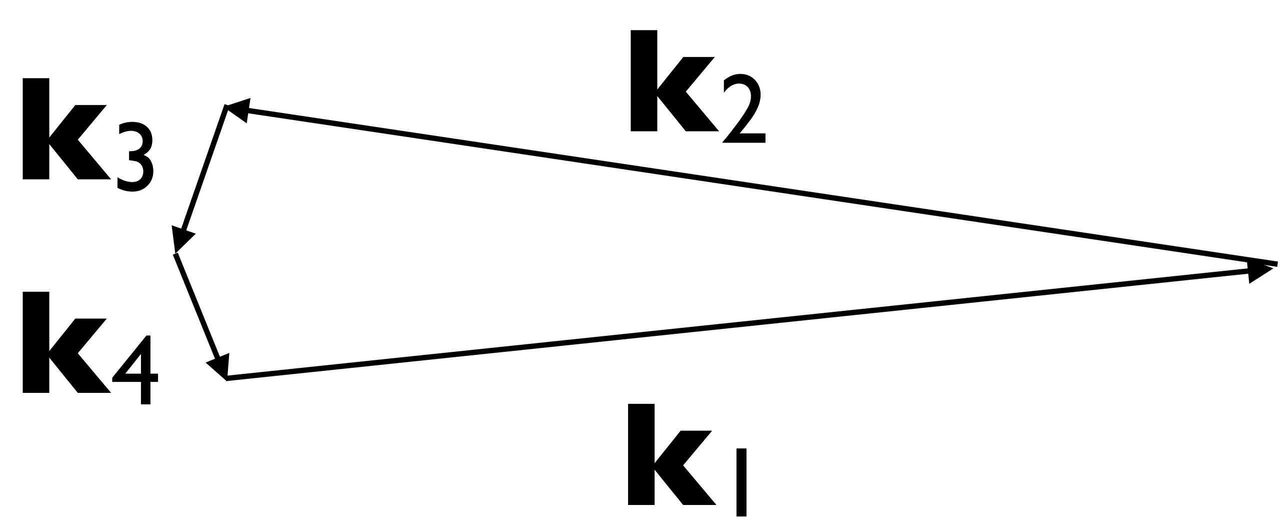


f_{NL}^2

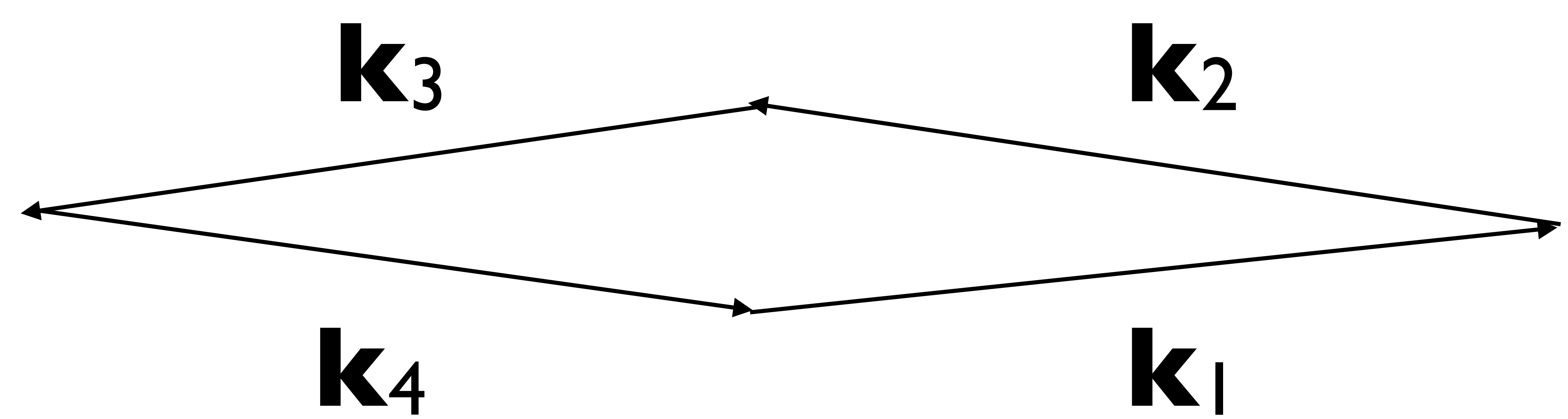
Generalized Trispectrum

- $T_\zeta(\mathbf{k}_1,\mathbf{k}_2,\mathbf{k}_3,\mathbf{k}_4)=(2\pi)^3\delta(\mathbf{k}_1+\mathbf{k}_2+\mathbf{k}_3+\mathbf{k}_4)$ $\{$**g_{NL}**$[(54/25) P_\zeta(k_1)P_\zeta(k_2)P_\zeta(k_3)+\text{cyc.}]$ $+$**τ_{NL}**$[P_\zeta(k_1)P_\zeta(k_2)(P_\zeta(|\mathbf{k}_1+\mathbf{k}_3|)+P_\zeta(|\mathbf{k}_1+\mathbf{k}_4|))+\text{cyc.}]\}$

The single-source local form consistency relation,
$\tau_{NL}=(6/5)(f_{NL})^2$, may not be respected –
additional test of multi-field inflation!



g_{NL}

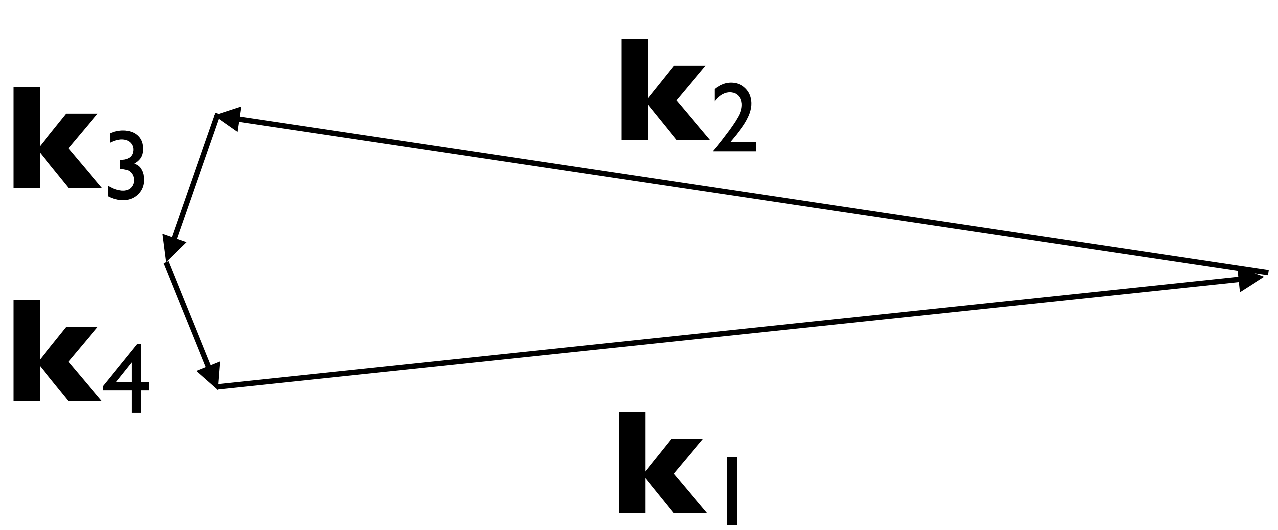


τ_{NL}

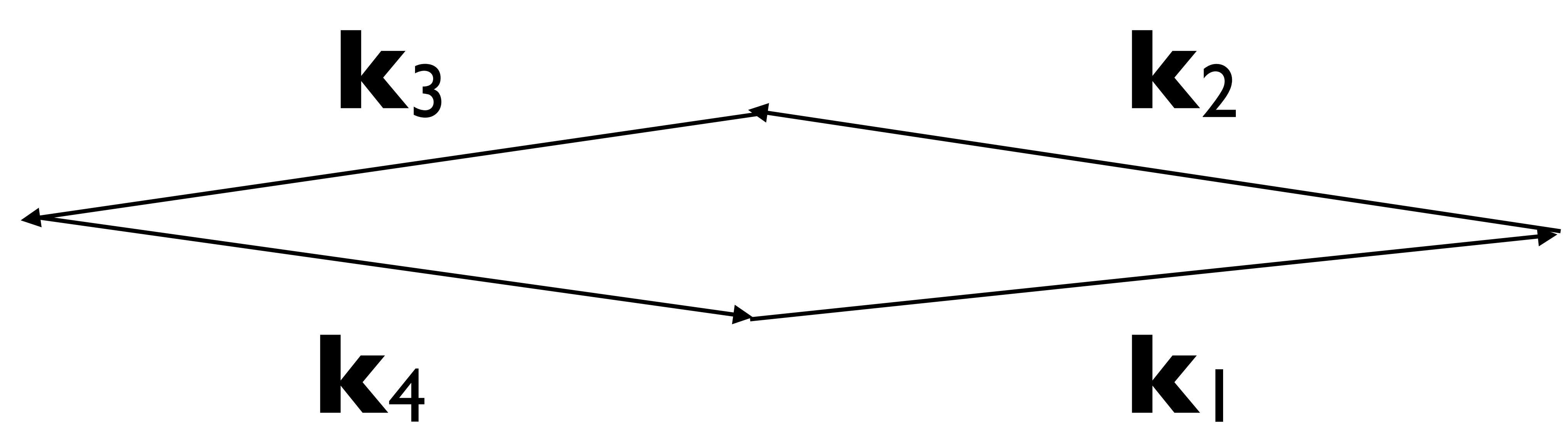
(Slightly) Generalized Trispectrum

- $T_\zeta(\mathbf{k}_1,\mathbf{k}_2,\mathbf{k}_3,\mathbf{k}_4)=(2\pi)^3\delta(\mathbf{k}_1+\mathbf{k}_2+\mathbf{k}_3+\mathbf{k}_4)\ \{g_{NL}[(54/25) P_\zeta(k_1)P_\zeta(k_2)P_\zeta(k_3)+\text{cyc.}] +\tau_{NL}[P_\zeta(k_1)P_\zeta(k_2)(P_\zeta(|\mathbf{k}_1+\mathbf{k}_3|)+P_\zeta(|\mathbf{k}_1+\mathbf{k}_4|))+\text{cyc.}]\}$

The single-source local form consistency relation, $\tau_{NL}=(6/5)(f_{NL})^2$, may not be respected – additional test of multi-field inflation!



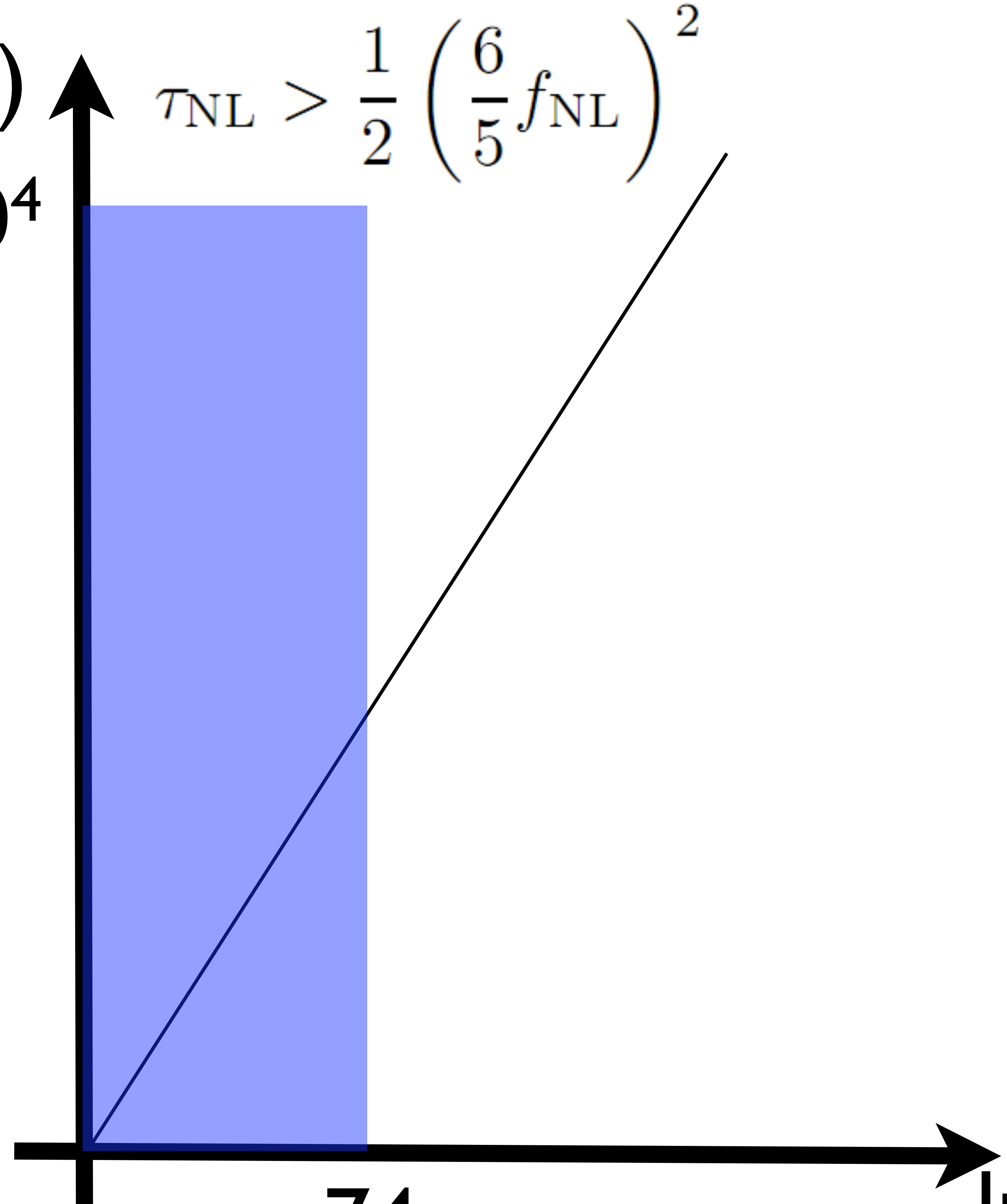
g_{NL}



τ_{NL}

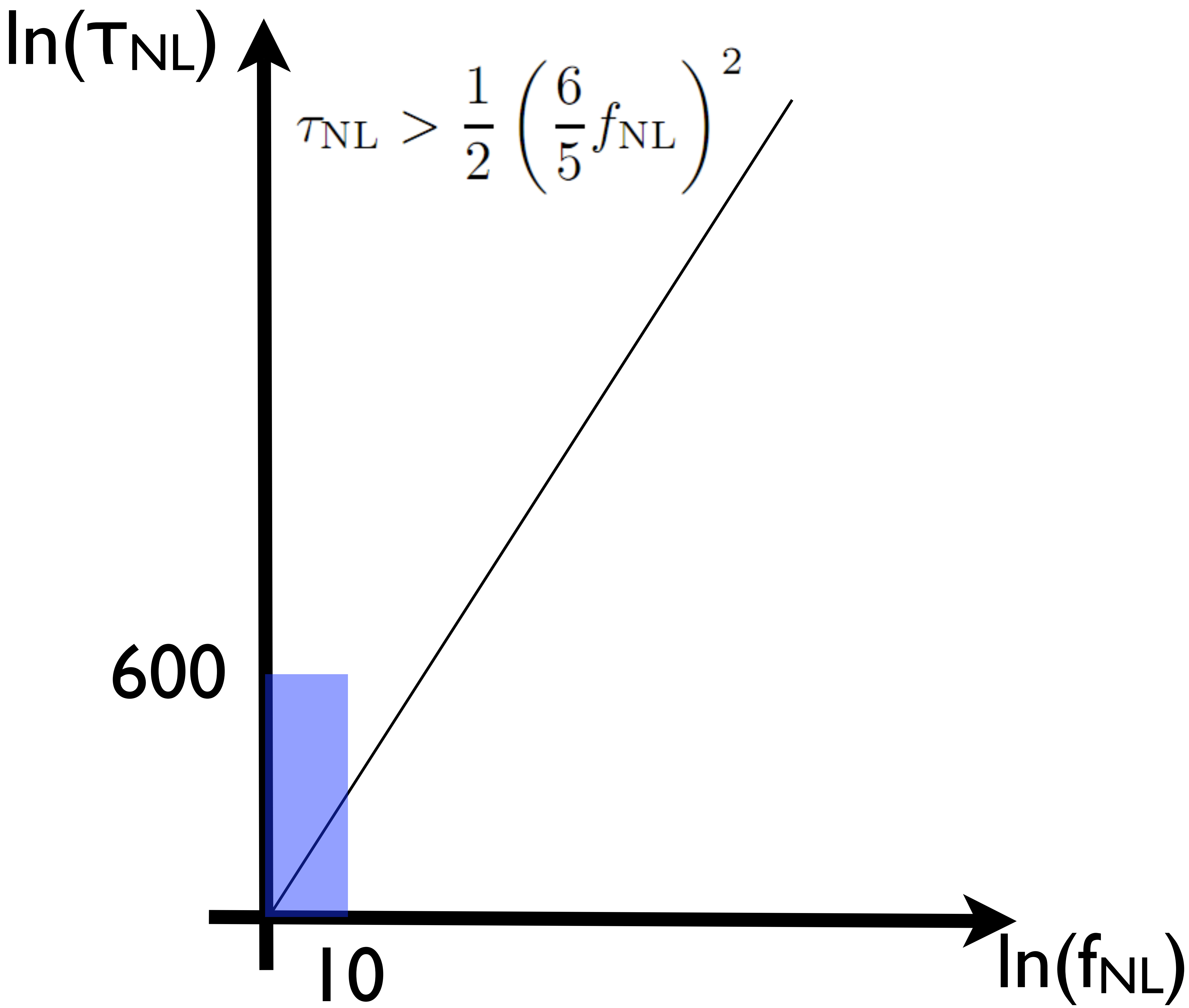
(Suyama & Yamaguchi 2008; Komatsu 2010; Sugiyama, Komatsu & Futamase 2011)

τ_{NL} >~ $(6f_{NL}/5)^2$



- The current limits from WMAP 7-year are consistent with single-field or multi-field models.
- So, let's play around with the future.

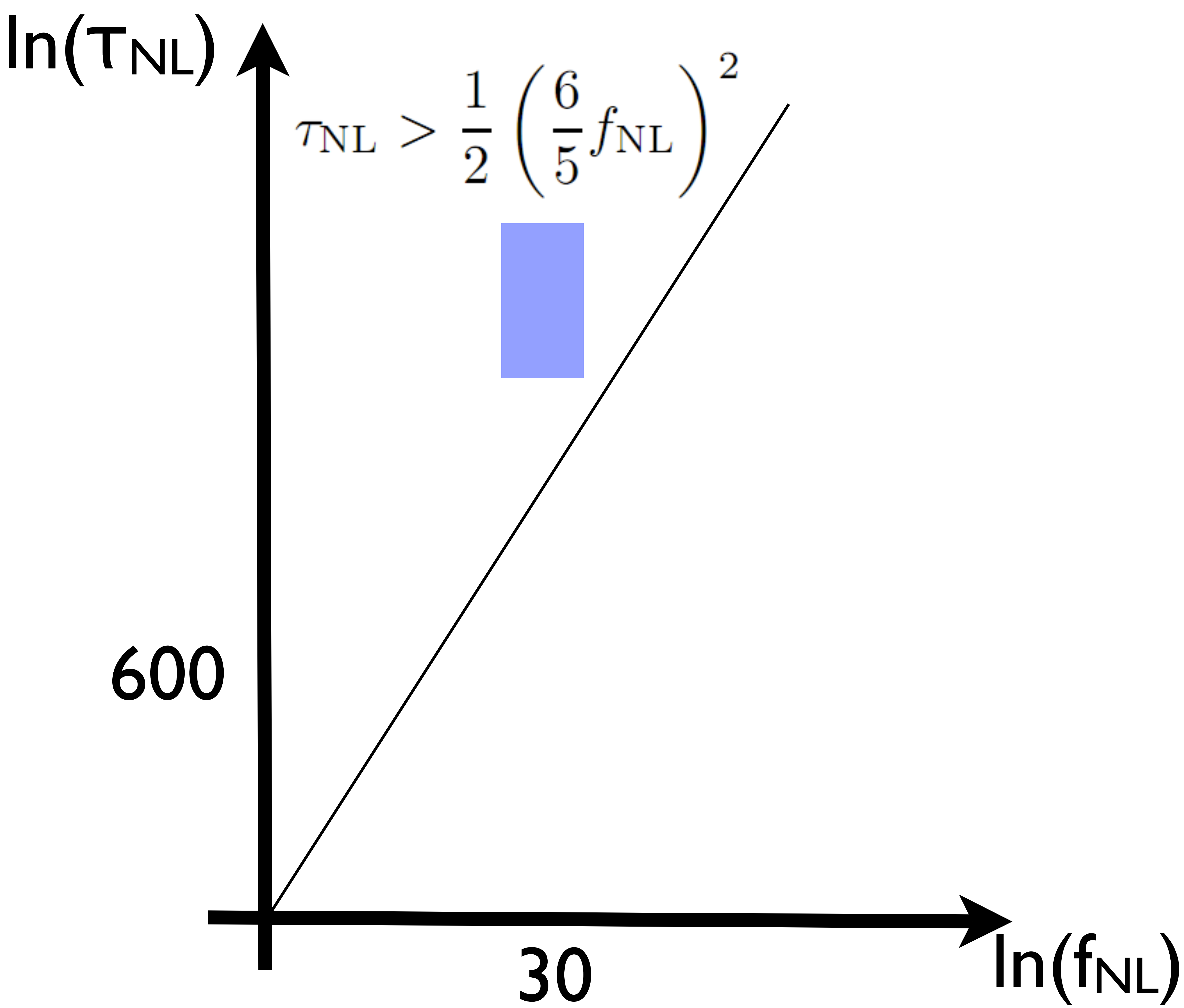
Case A: Single-field Happiness



- No detection of anything (f_{NL} or τ_{NL}) after Planck. Single-field survived the test (for the moment: the future galaxy surveys can improve the limits by a factor of ten).

(Suyama & Yamaguchi 2008; Komatsu 2010; Sugiyama, Komatsu & Futamase 2011)

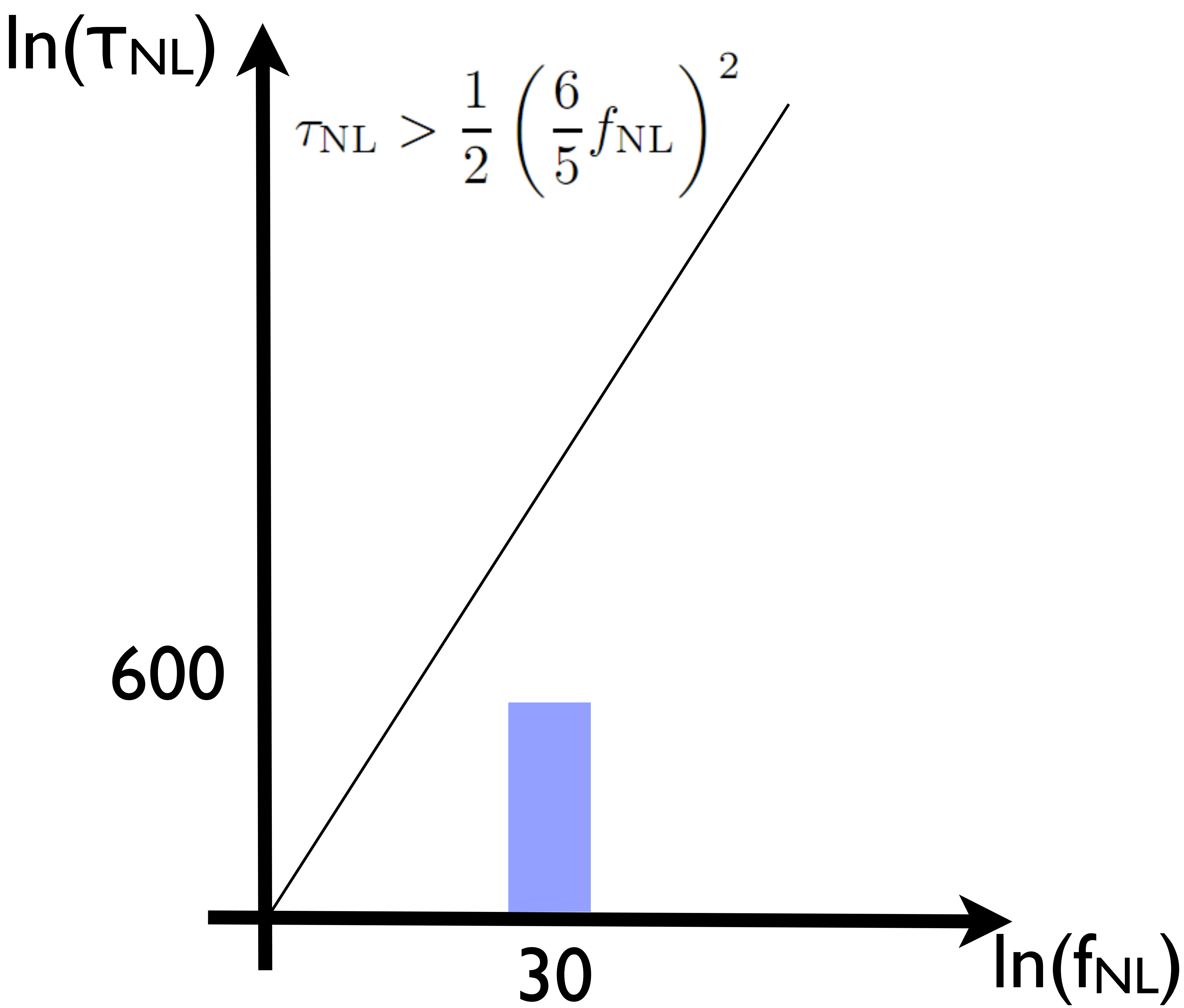
Case B: Multi-field Happiness(?)



- **f_{NL} is detected. Single-field is gone.**
- But, τ_{NL} is also detected, in accordance with **τ_{NL}>0.5(6f_{NL}/5)^2** expected from most multi-field models.

(Suyama & Yamaguchi 2008; Komatsu 2010; Sugiyama, Komatsu & Futamase 2011)

Case C: Madness



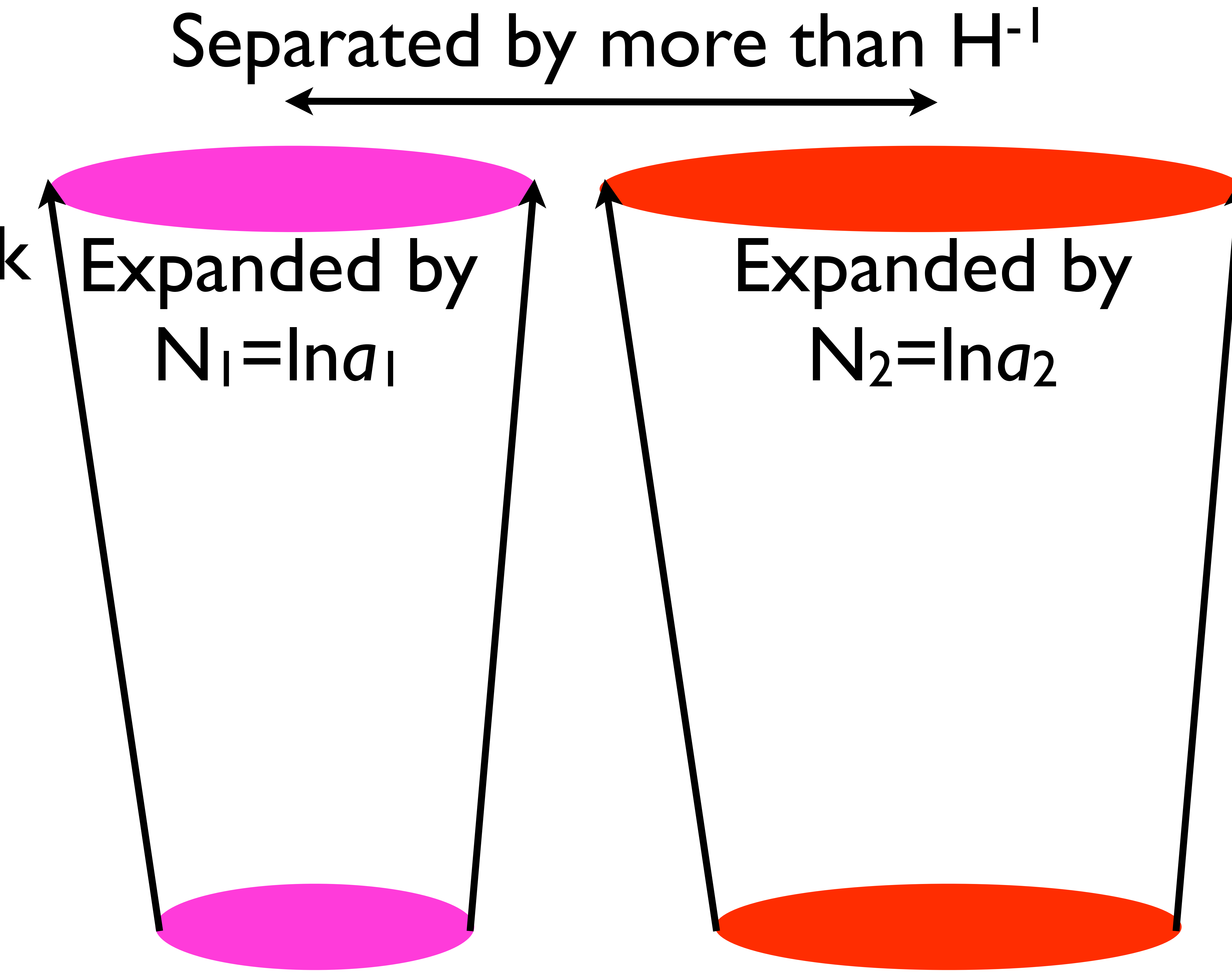
- f_{NL} is detected. Single-field is gone.
- But, τ_{NL} is not detected, or found to be negative, inconsistent with $\tau_{NL}>0.5(6f_{NL}/5)^2$.
- **Single-field AND most of multi-field models are gone.**

Cosmology in the Next Decade

- ***Inflation****, Dark Energy, Dark Matter, and Neutrinos…*
  - We may be able to prove or falsify inflation.
  - This has been regarded as *impossible* in the past, but we may be able to do that!
- Did not have time to talk about: the role of large-scale structure of the Universe on this business, and how we explore DE, DM, and neutrinos…

The δN Formalism

- The δN formalism (Starobinsky 1982; Salopek & Bond 1990; Sasaki & Stewart 1996) states that the curvature perturbation is equal to the difference in $N=\ln a$.
  - $\zeta=\delta N=N_2-N_1$
  - where $N=\int H dt$



Getting the familiar result

- Single-field example at the linear order:
  - $\zeta = \delta\{\int H dt\} = \delta\{\int (H/\varphi')d\varphi\} \approx (H/\varphi')\delta\varphi$
  - Mukhanov & Chibisov; Guth & Pi; Hawking; Starobinsky; Bardeen, Steinhardt & Turner

Extending to non-linear, multi-field cases

$$\zeta = \sum_I \frac{\partial N}{\partial \phi_I} \delta\phi_I + \frac{1}{2} \sum_{IJ} \frac{\partial^2 N}{\partial \phi_I \partial \phi_J} \delta\phi_I \delta\phi_J + \ldots$$

(Lyth & Rodriguez 2005)

- Calculating the bispectrum is then straightforward. Schematically:
    - $<\zeta^3> = <(1st)x(1st)x(2nd)> \sim <\delta\varphi^4> \neq 0$
    - $f_{NL} \sim <\zeta^3>/<\zeta^2>^2$

$$\frac{6}{5} f_{\rm NL}^{\rm local} = \frac{\sum_{IJ} N_{,IJ} N_{,I} N_{,J}}{[\sum_I (N_{,I})^2]^2}$$

Extending to non-linear, multi-field cases

$$\zeta = \sum_I \frac{\partial N}{\partial \phi_I}\delta\phi_I + \frac{1}{2}\sum_{IJ}\frac{\partial^2 N}{\partial\phi_I \partial\phi_J}\delta\phi_I\delta\phi_J + \ldots$$

(Lyth & Rodriguez 2005)

- Calculating the trispectrum is also straightforward. Schematically:
  - $<\zeta^4>=<(1\text{st})^2(2\text{nd})^2>\sim<\delta\varphi^6>\neq 0$
  - $f_{NL}\sim<\zeta^4>/<\zeta^2>^3$

$$\tau_{\rm NL} = \frac{\sum_{IJK} N_{,IJ}N_{,J}N_{,IK}N_{,K}}{[\sum_I (N_{,I})^2]^3} = \frac{\sum_I (\sum_J N_{,IJ}N_{,J})^2}{[\sum_I (N_{,I})^2]^3}$$

Now, stare at these.

$$\frac{6}{5} f_{\rm NL}^{\rm local} = \frac{\sum_{IJ} N_{,IJ} N_{,I} N_{,J}}{[\sum_I (N_{,I})^2]^2},$$

$$\tau_{\rm NL} = \frac{\sum_{IJK} N_{,IJ} N_{,J} N_{,IK} N_{,K}}{[\sum_I (N_{,I})^2]^3} = \frac{\sum_I (\sum_J N_{,IJ} N_{,J})^2}{[\sum_I (N_{,I})^2]^3}$$

Change the variable...

$$\frac{6}{5} f_{\rm NL}^{\rm local} = \frac{\sum_{IJ} N_{,IJ} N_{,I} N_{,J}}{[\sum_I (N_{,I})^2]^2},$$

$$\tau_{\rm NL} = \frac{\sum_{IJK} N_{,IJ} N_{,J} N_{,IK} N_{,K}}{[\sum_I (N_{,I})^2]^3} = \frac{\sum_I (\sum_J N_{,IJ} N_{,J})^2}{[\sum_I (N_{,I})^2]^3}$$

$$a_I = \frac{\sum_J N_{,IJ} N_{,J}}{[\sum_J (N_{,J})^2]^{3/2}}$$

$$b_I = \frac{N_{,I}}{[\sum_J (N_{,J})^2]^{1/2}}$$

$(6/5)f_{NL} = \sum_I a_I b_I$

$\tau_{NL} = (\sum_I a_I)^2 (\sum_I b_I)^2$

Then apply the Cauchy-Schwarz Inequality

$$\left(\sum_I a_I^2\right)\left(\sum_J b_J^2\right) \geq \left(\sum_I a_I b_I\right)^2$$

- Implies (Suyama & Yamaguchi 2008)

$$\tau_{\rm NL} \geq \left(\frac{6\, f_{\rm NL}^{\rm local}}{5}\right)^2$$

How generic is this inequality?