Non-Gaussianity

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Thank You, I'm Honored To Receive the Prize.

Center for Cosmology, The University of Texas Austin

- The new Center for Cosmology will be founded in January 2009, at the University of Texas at Austin!

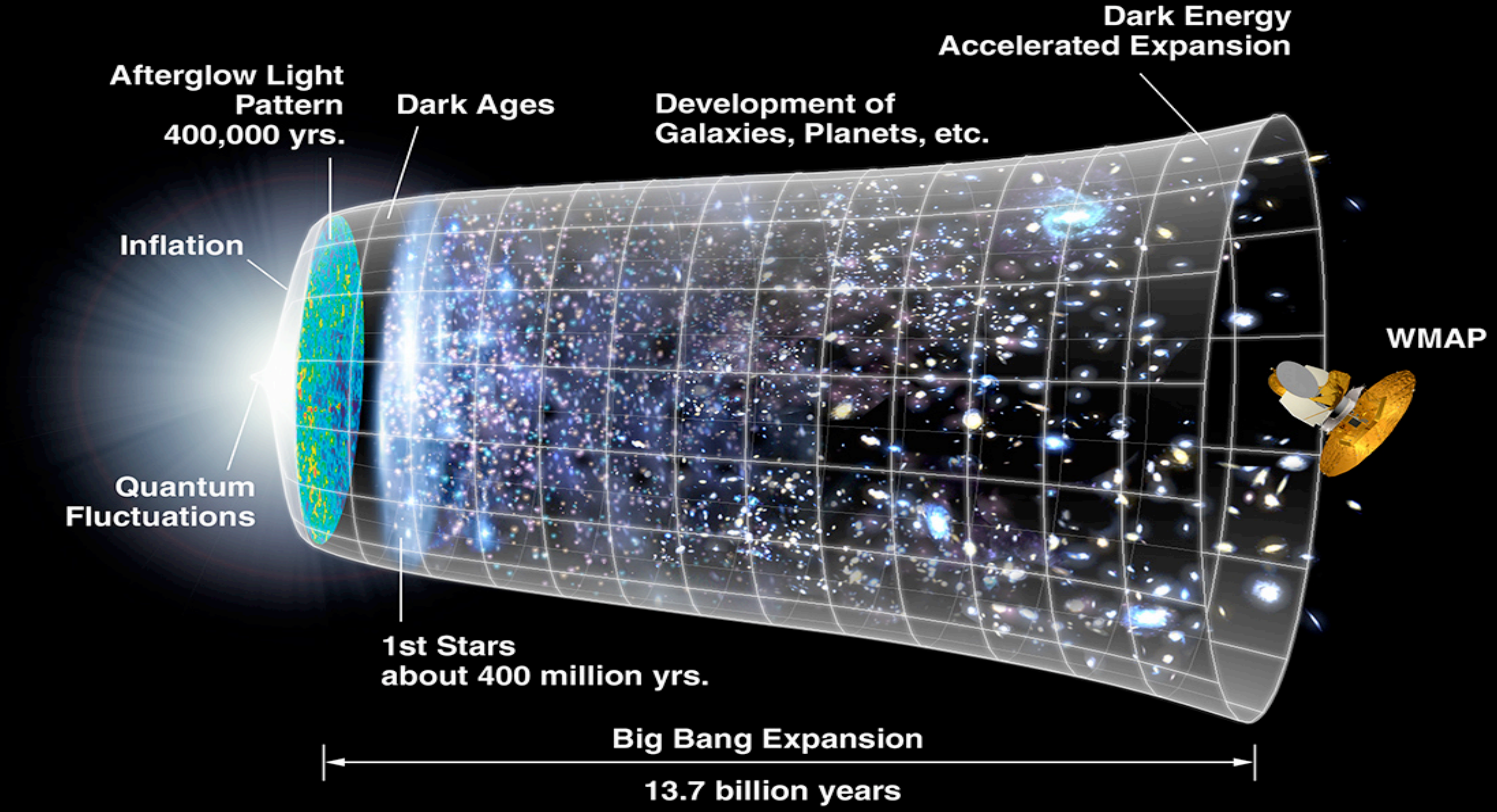
Research Unit, Center for Cosmology

Astronomy	*Physics*
Volker Bromm	Duane Dicus
Karl Gebhardt	Jacques Distler
Gary Hill	Willy Fischler
Eiichiro Komatsu(Director)	Vadim Kaplunovsky
Milos Milosavljevic	Sonia Paban
Paul Shapiro	Steven Weinberg

Why Study Non-Gaussianity?

- What do I mean by "*non-Gaussianity*"?
    - Non-Gaussianity = **Not** a Gaussian Distribution
- Distribution of *what*?
    - Distribution of primordial fluctuations.
- *How* do we observe primordial fluctuations?
    - In several ways: believe me, we can do that.
- *What is non-Gaussianity good for?*
    - **Probing the Primordial Universe**

Messages From the Primordial Universe...



NASA/WMAP Science Team

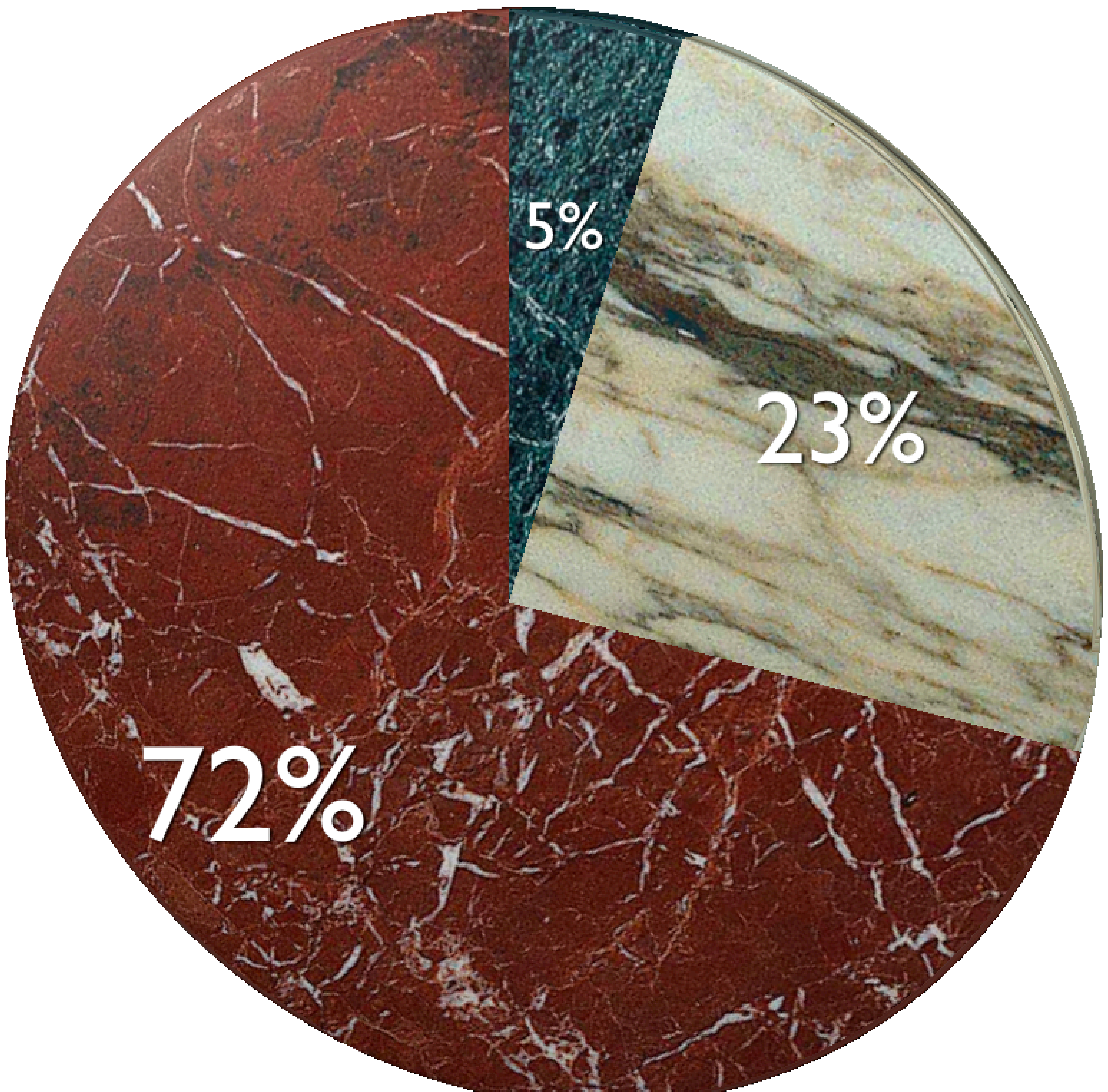
Komatsu et al. (2008)

Observations I: Homogeneous Universe

- $H^2(z) = H^2(0)[\Omega_r(1+z)^4+\Omega_m(1+z)^3+\Omega_k(1+z)^2+\Omega_{de}(1+z)^{3(1+w)}]$
  - (expansion rate) $H(0) = 70.5 \pm 1.3$ km/s/Mpc
  - (radiation) $\Omega_r = (8.4\pm0.3)\times10^{-5}$
  - (matter) $\Omega_m = 0.274\pm0.015$
  - **(curvature) Ω_k < 0.008 (95%CL) -> Inflation**
  - (dark energy) $\Omega_{de} = 0.726\pm0.015$
  - (DE equation of state) $1+w = -0.006\pm0.068$

Composition of our Universe

Cosmic Pie Chart



- WMAP 5-Year Data, combined with the local distance measurements from Type Ia Supernovae and Large-scale structure (BAOs).

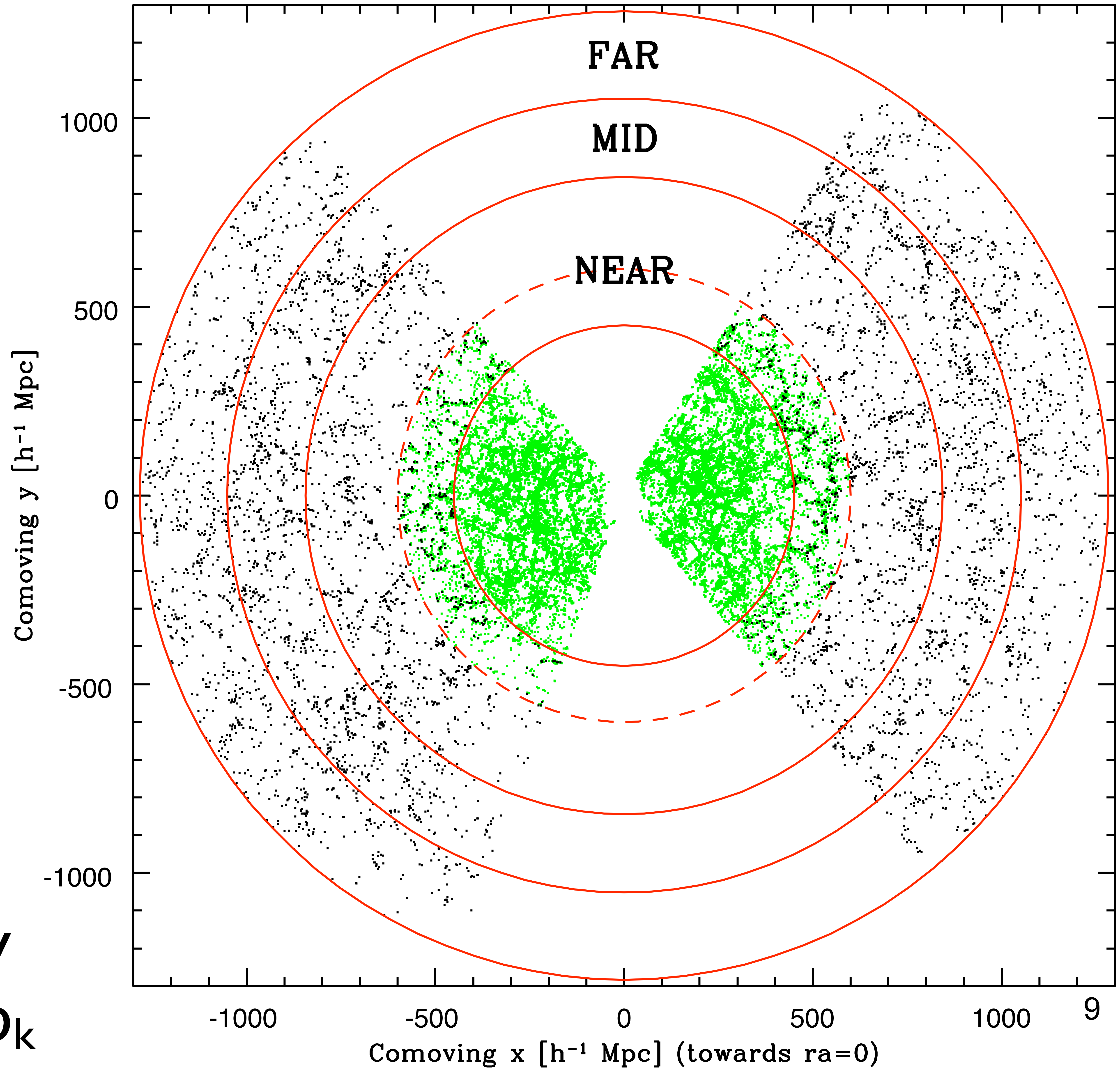
H, He
Dark Matter
Dark Energy

Observations II: Density Fluctuations, $\delta(x)$

- In Fourier space, $\delta(k) = A(k)\exp(i\varphi_k)$
  - **Power**: $P(k) = \langle|\delta(k)|^2\rangle = A^2(k)$
  - **Phase**: φ_k
- We can use the observed distribution of...
  - matter (e.g., galaxies, gas)
  - radiation (e.g., Cosmic Microwave Background)
- to learn about both $P(k)$ and φ_k.

Galaxy Distribution

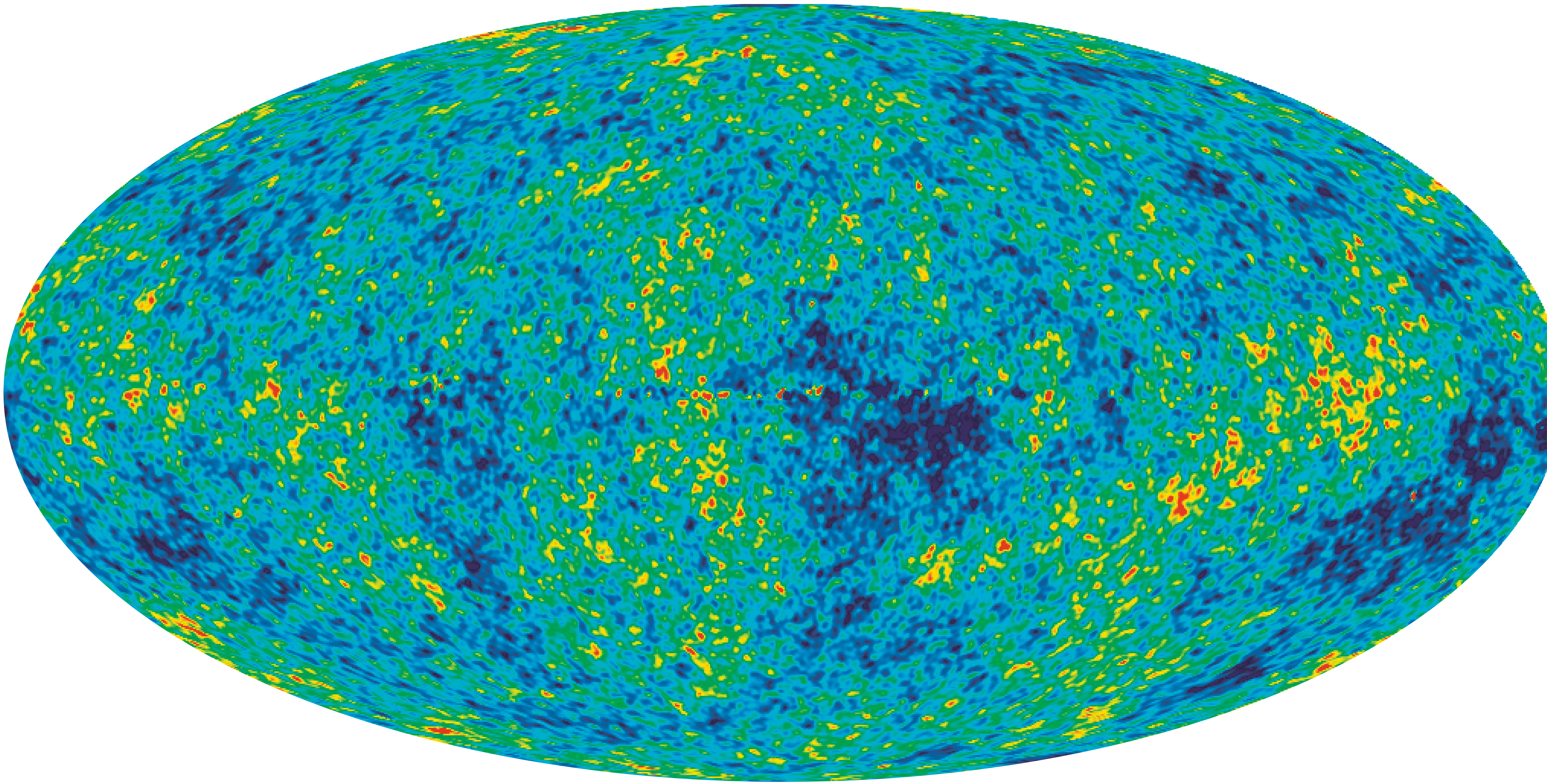
SDSS



- Matter distribution today (z=0~0.2): P(k), φ_k

Radiation Distribution

WMAP5

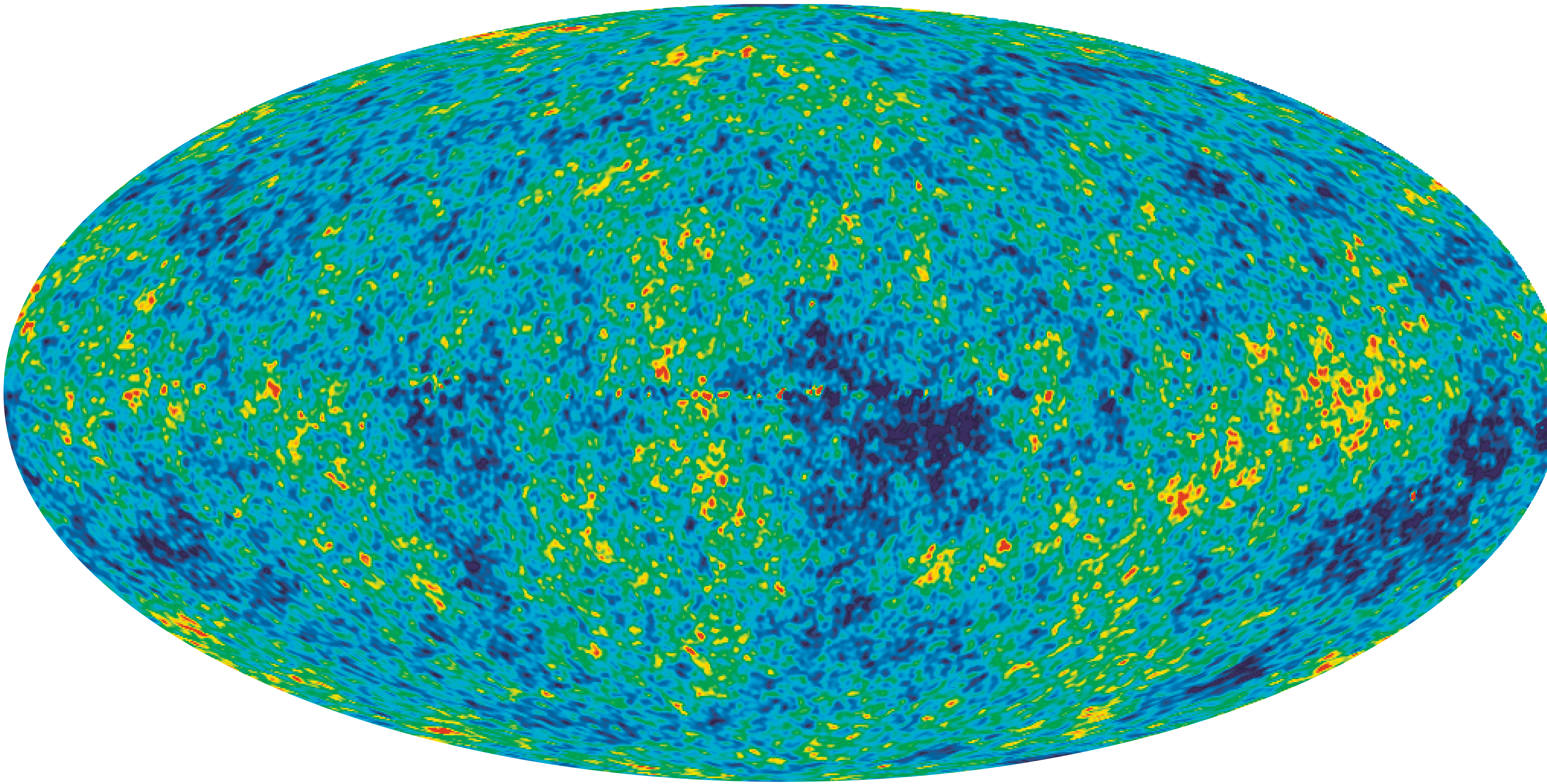


- Matter distribution at z=1090: P(k), φ_k

P(k): There were expectations

- Metric perturbations in g_{ij} (let's call that "curvature perturbations" Φ) is related to δ via
    - $k^2\Phi(k)=4\pi G\rho a^2\delta(k)$
- Variance of $\Phi(x)$ in position space is given by
    - $<\Phi^2(x)>=\int \ln k \, \mathbf{k^3|\Phi(k)|^2}$
    - In order to avoid the situation in which curvature (geometry) diverges on small or large scales, a "scale-invariant spectrum" was proposed: $\mathbf{k^3|\Phi(k)|^2 = const.}$
    - This leads to the expectation: $\mathbf{P(k)}=|\delta(k)|^2\mathbf{=k^{ns}}$ $\mathbf{(n_s=1)}$
        - *Harrison 1970; Zel'dovich 1972; Peebles&Yu 1970*

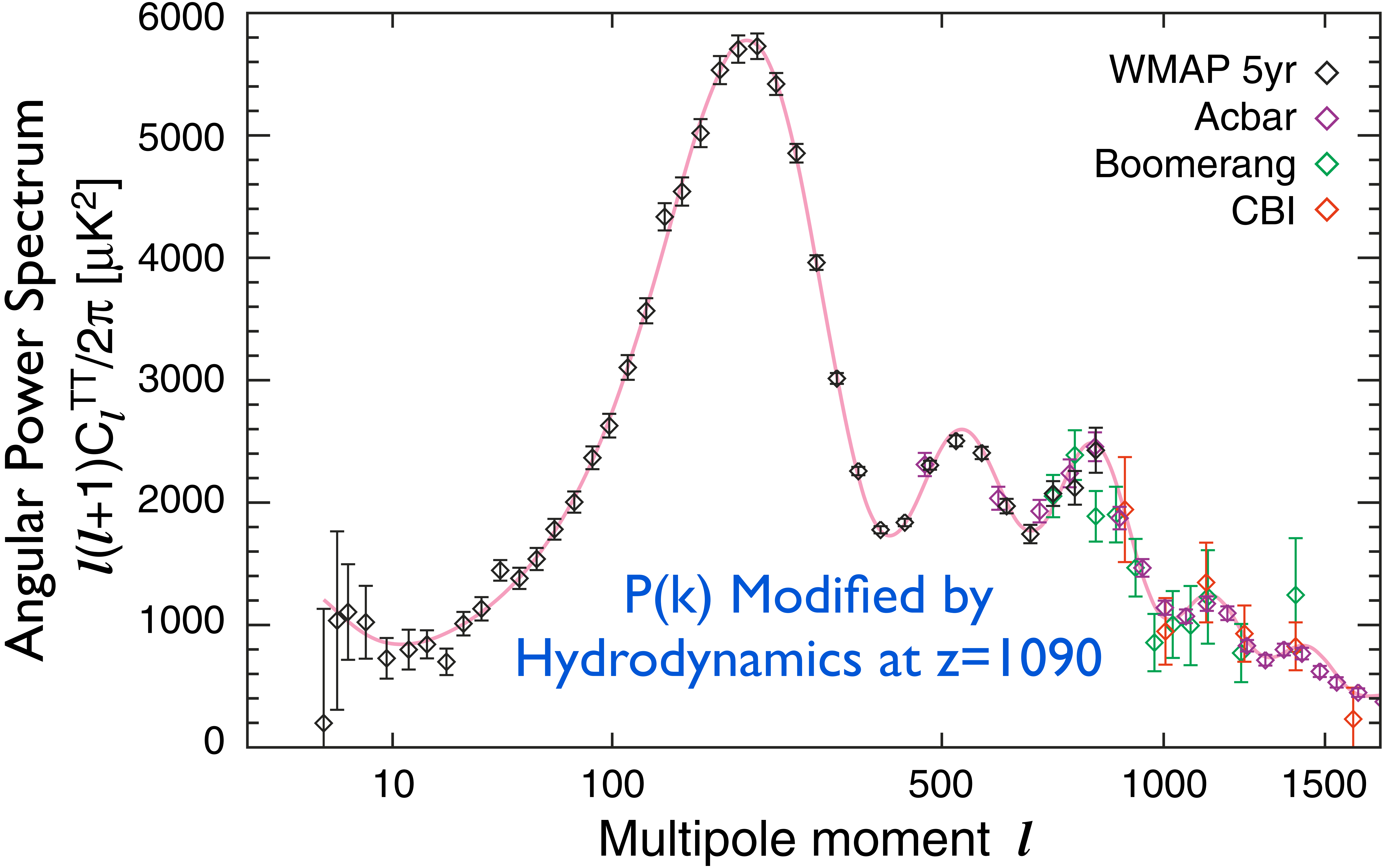
Take Fourier Transform of *WMAP5*



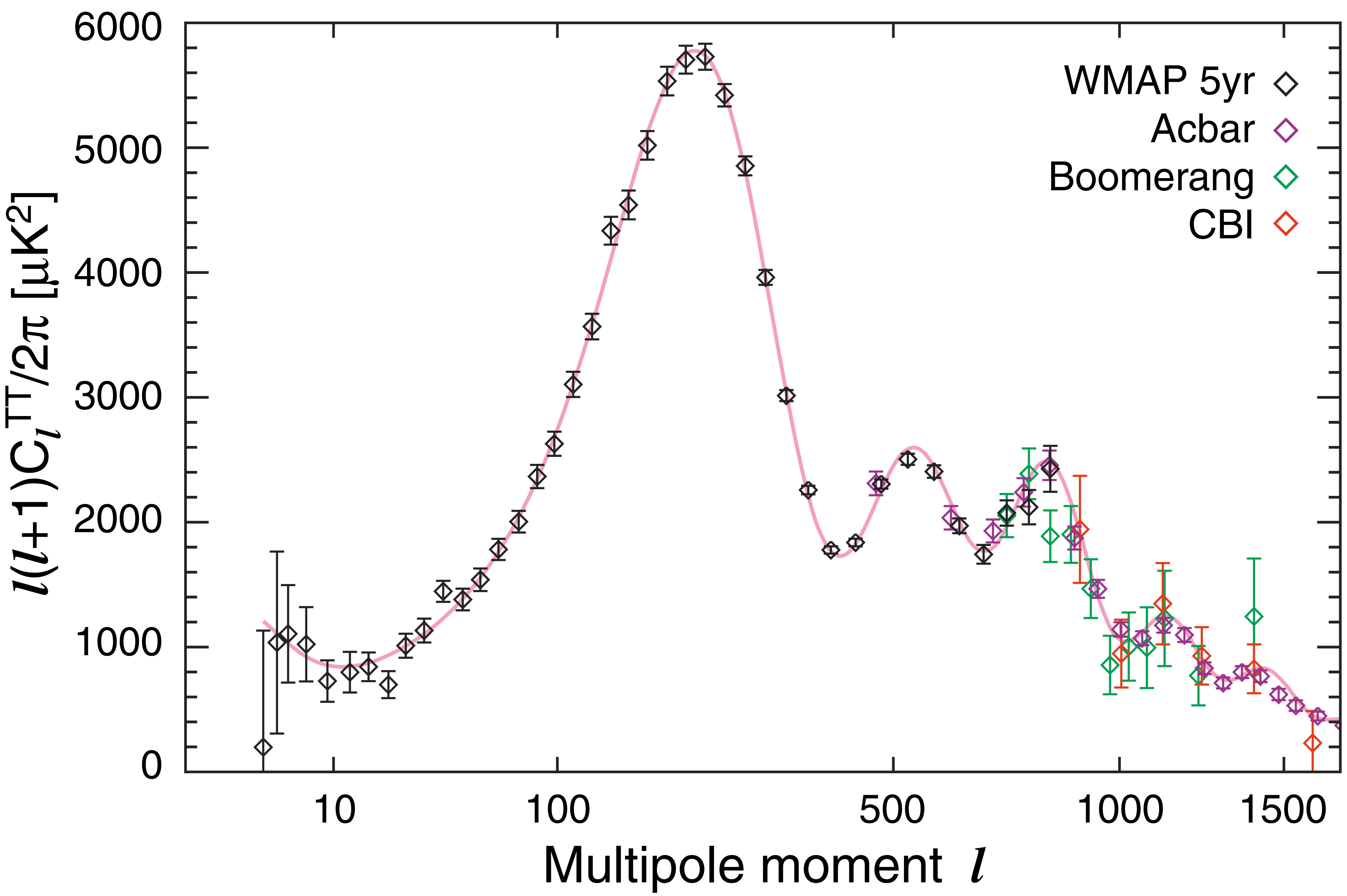
- ...and, square it in your head...

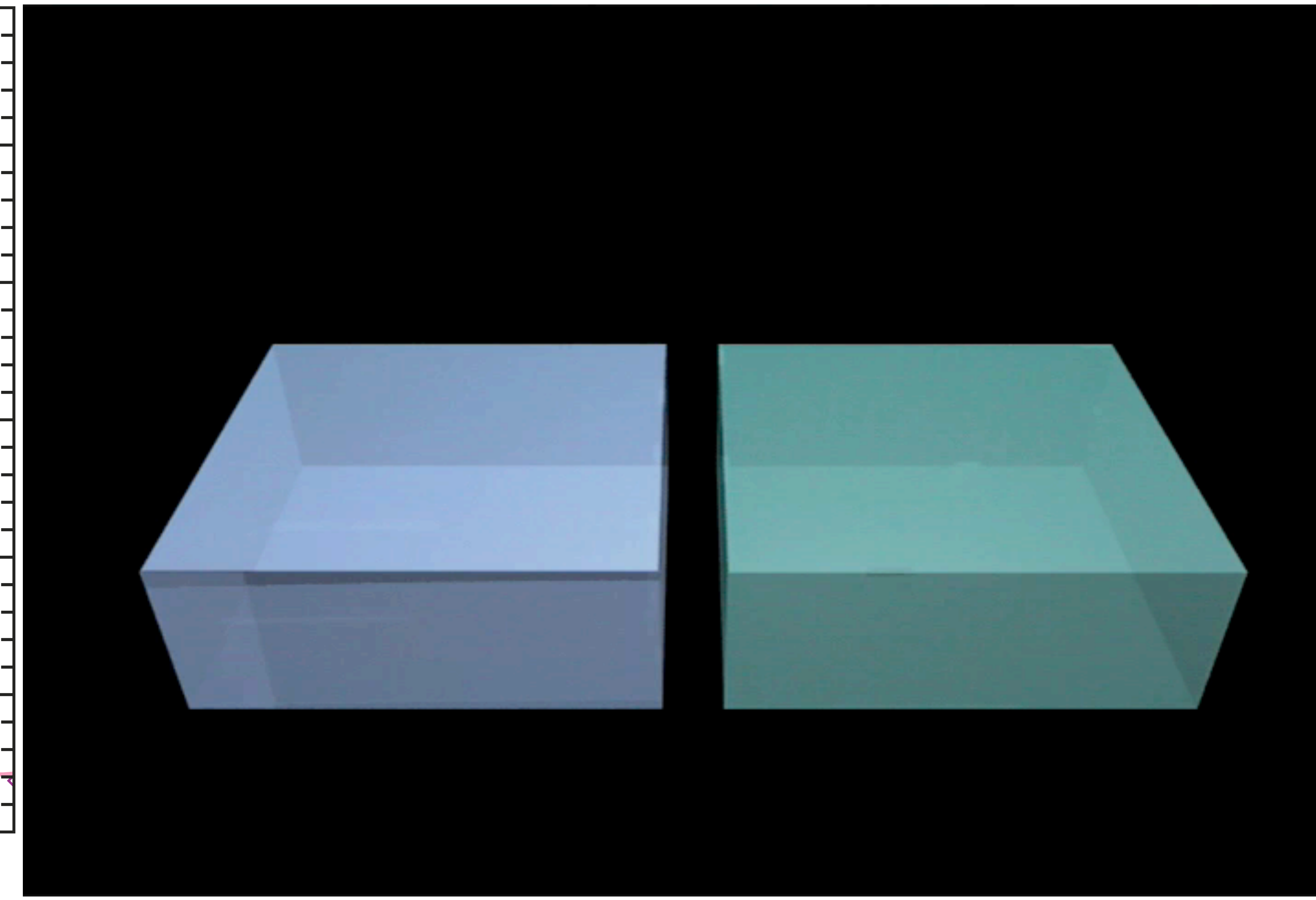
...and decode it.

Nolta et al. (2008)



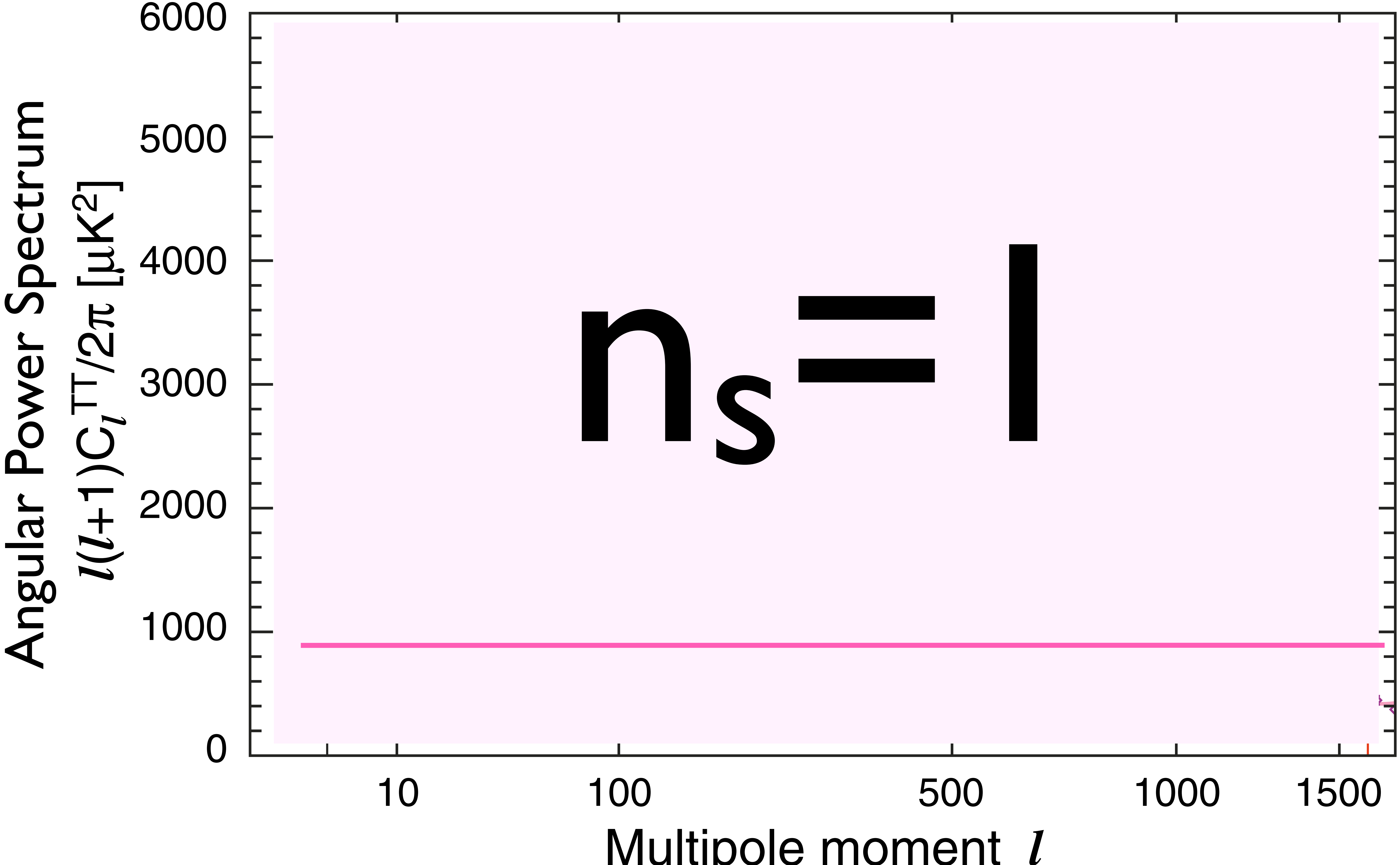
The Cosmic Sound Wave



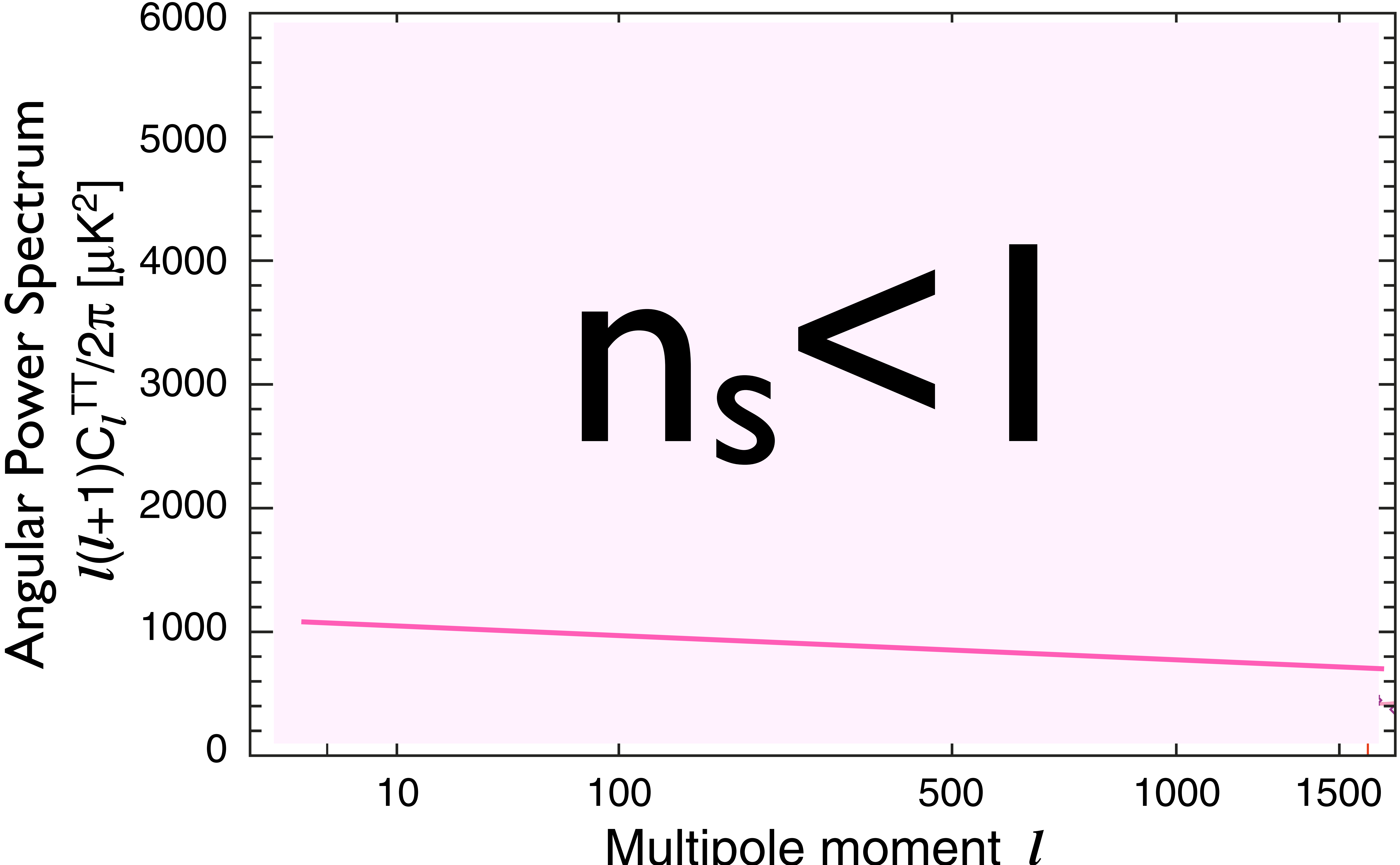


- Hydrodynamics in the early universe (z>1090) created sound waves in the matter and radiation distribution

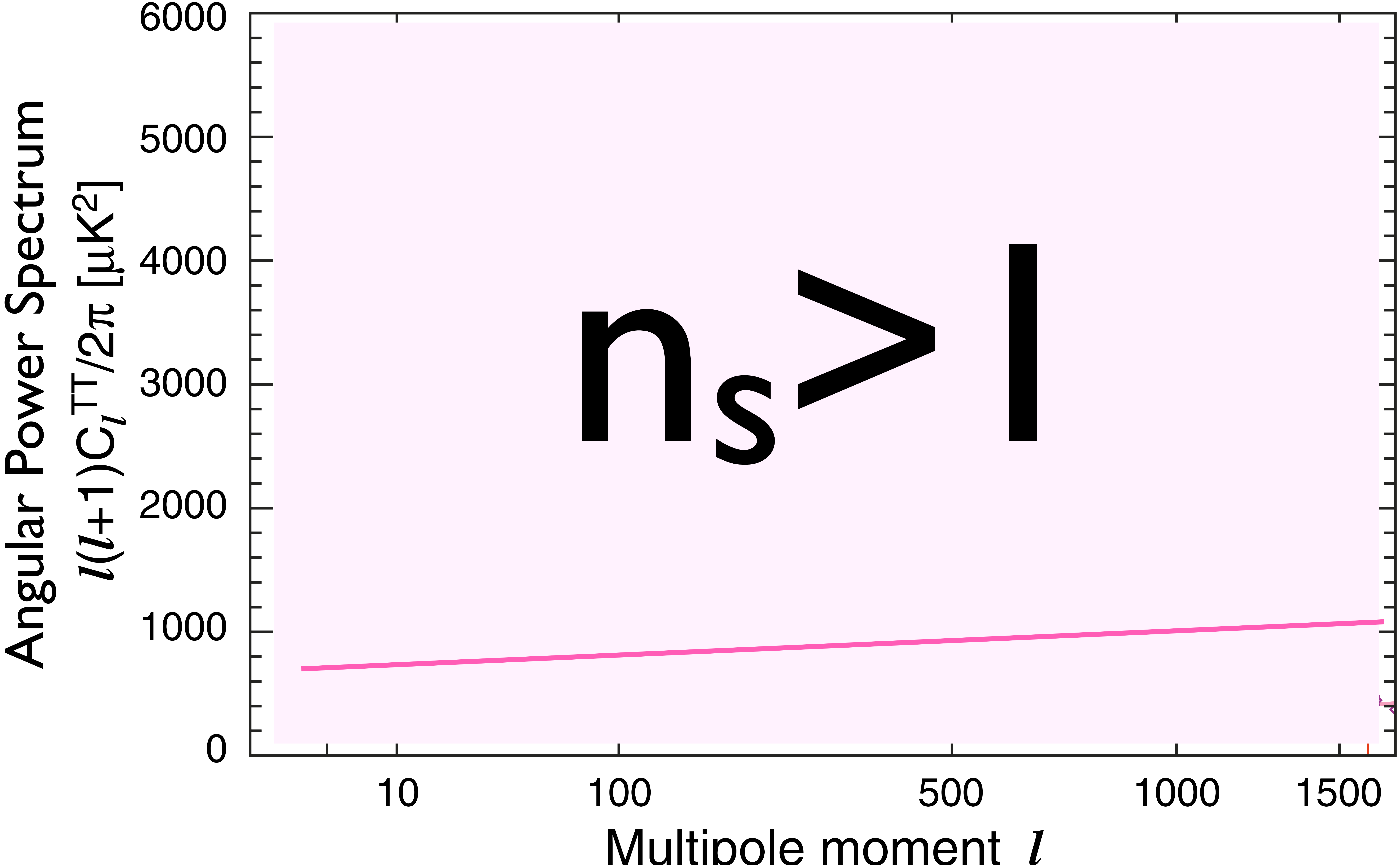
If there were no hydrodynamics...



If there were no hydrodynamics...

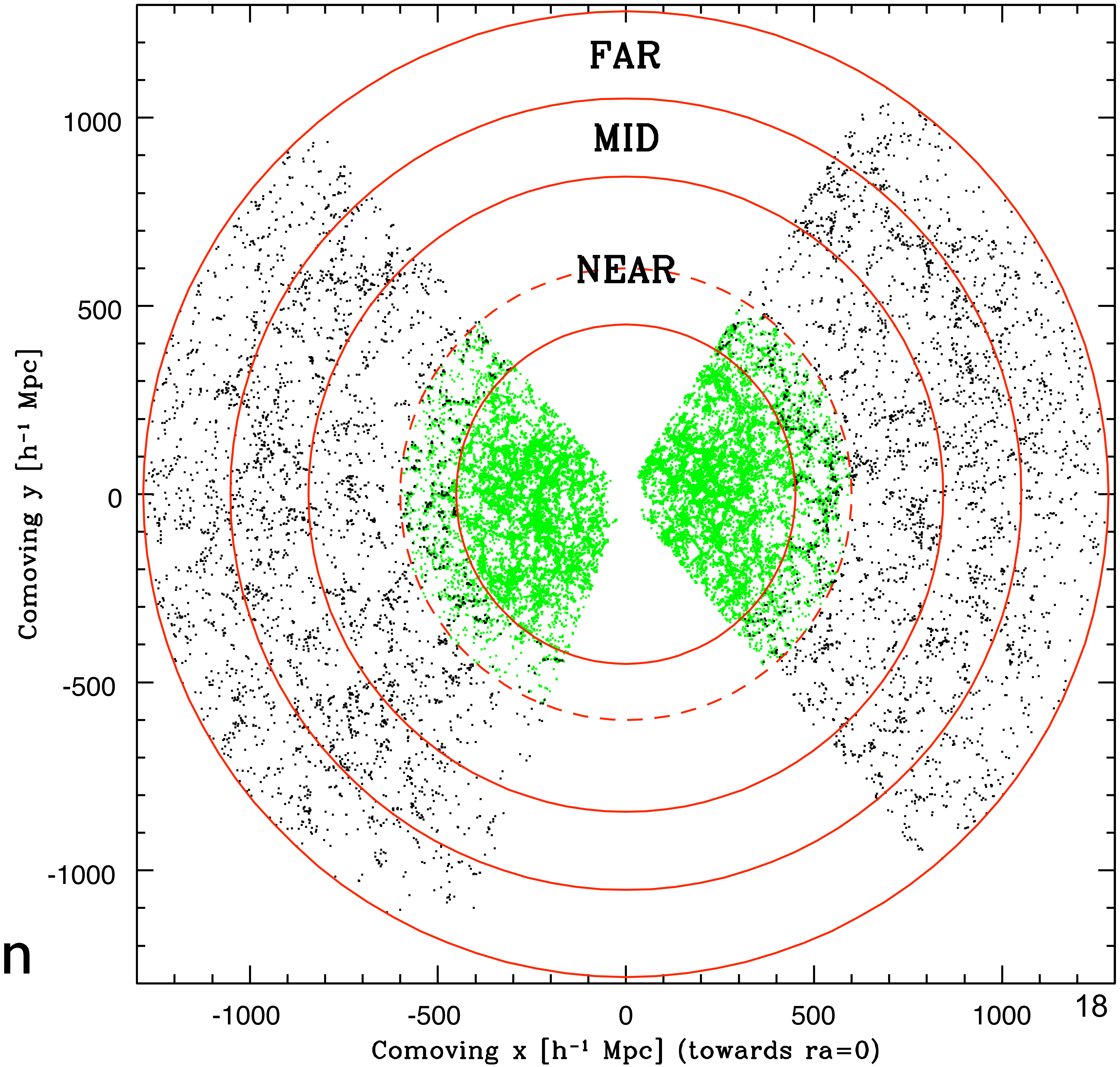


If there were no hydrodynamics...



Take Fourier Transform of

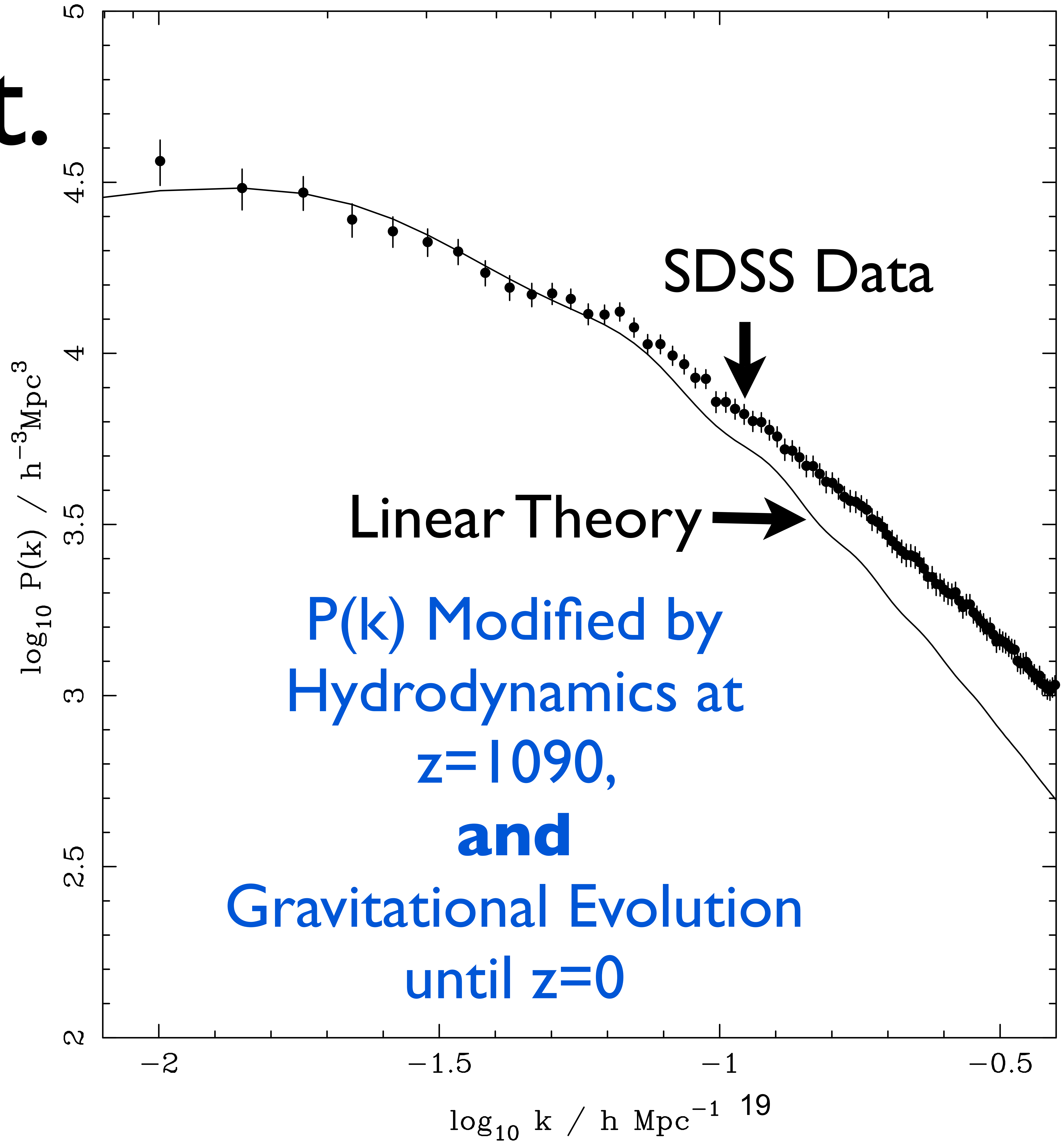
SDSS



- ...and square it in your head...

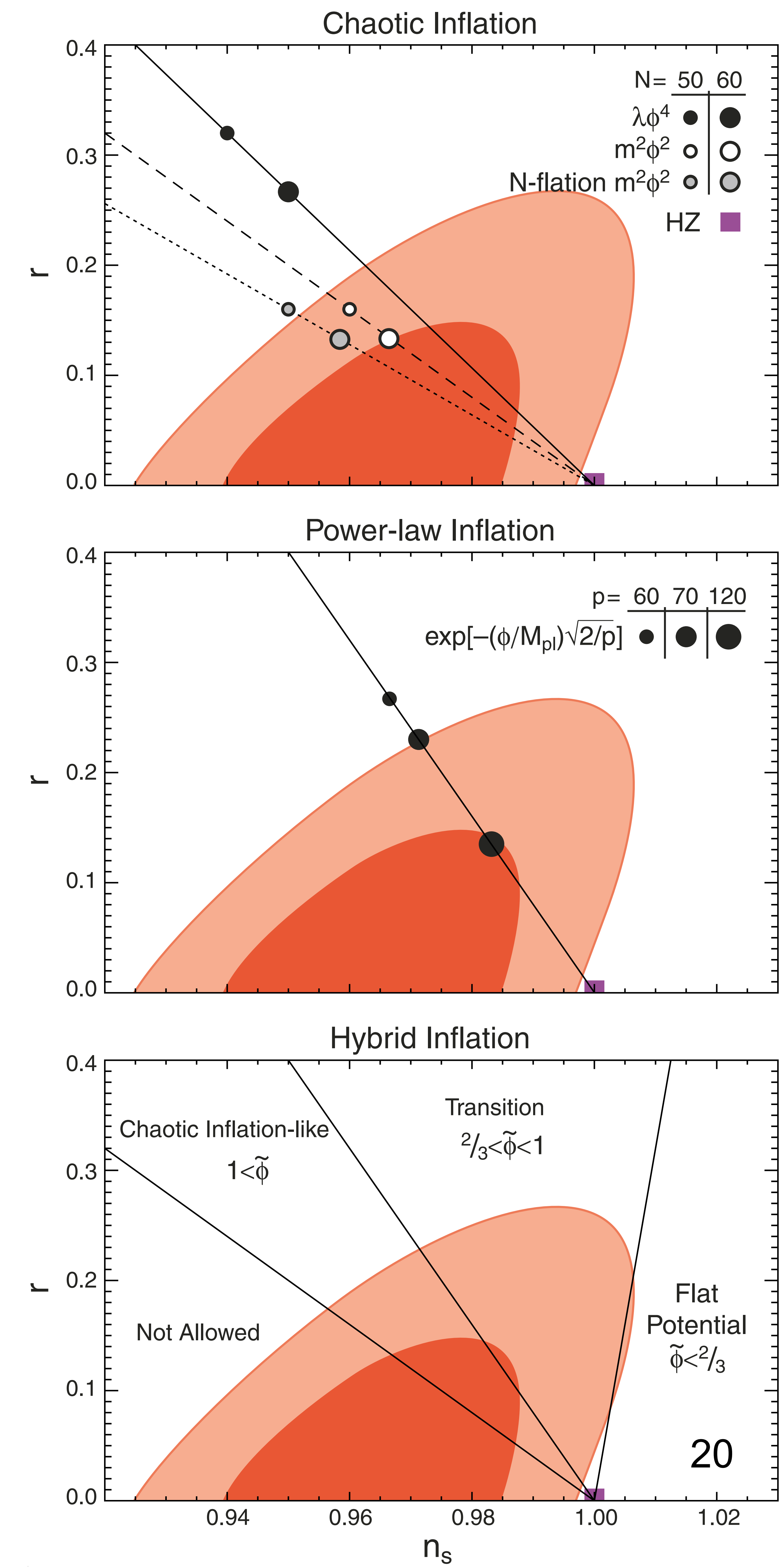
...and decode it.

- Decoding is complex, but you can do it.
- The latest result (from WMAP+: *Komatsu et al.*)
    - $P(k)=k^{ns}$
    - **$n_s=0.960\pm0.013$**
    - 3.1σ away from scale-invariance, $n_s=1$!



Deviation from n_s=1

- **This was expected by many inflationary models**
- In n_s–r plane (where r is called the "tensor-to-scalar ratio," which is P(k) of gravitational waves divided by P(k) of density fluctuations) **many inflationary models are compatible with the current data**
- Many models have been excluded also



Searching for Primordial Gravitational Waves in CMB

- Not only do inflation models produce density fluctuations, but also primordial gravitational waves
- Some predict the observable amount ($r>0.01$), some don't
  - **Current limit: $r<0.22$ (95%CL)** (Komatsu et al.)
- Alternative scenarios (e.g., New Ekpyrotic) don't
- A powerful probe for testing inflation and testing specific models: next "Holy Grail" for CMBist

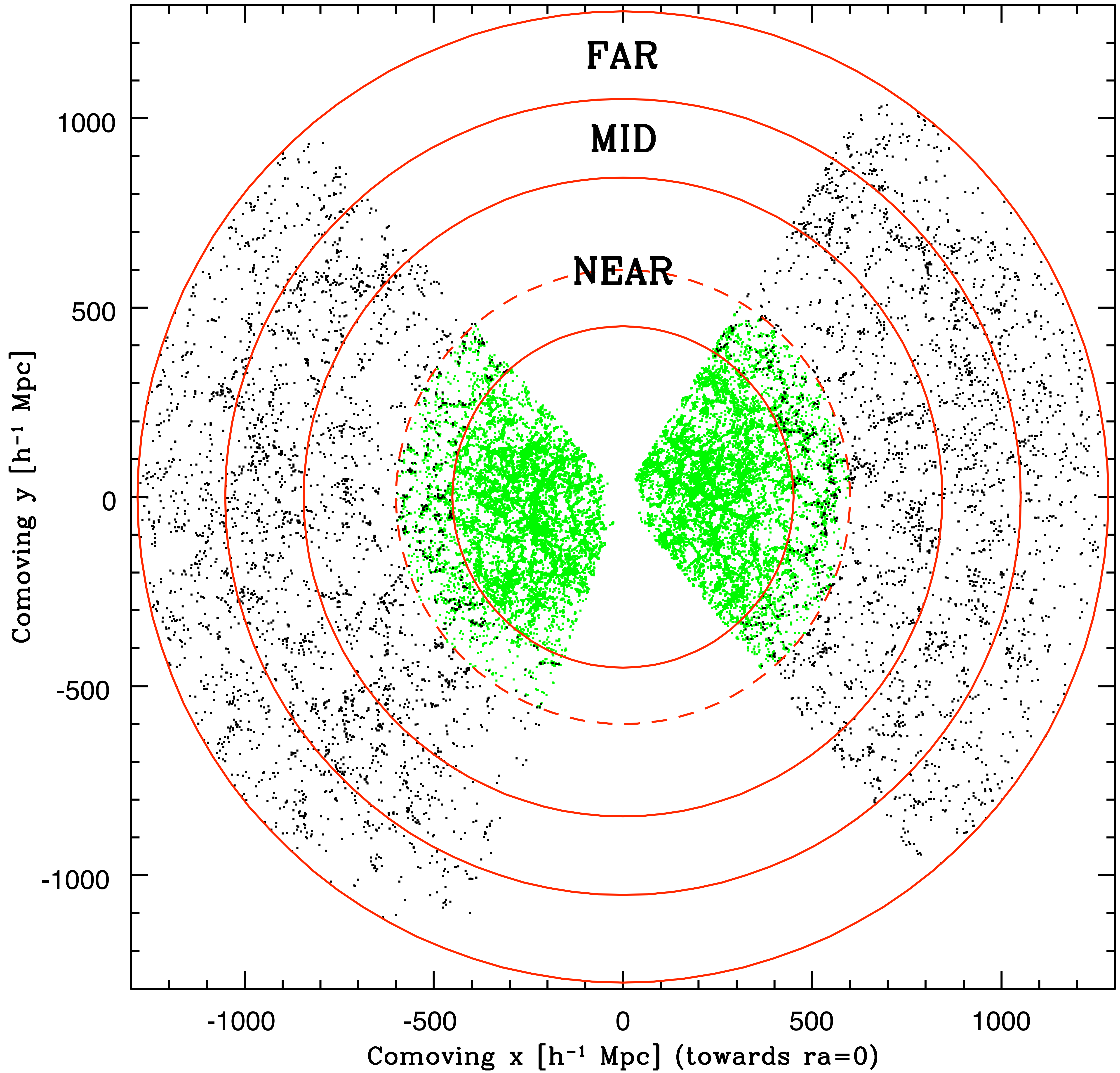
What About Phase, φ_k

- There were expectations also:
  - Random phases! (Peebles, ...)
- Collection of random, uncorrelated phases leads to the most famous probability distribution of δ:

Gaussian Distribution

Gaussian?

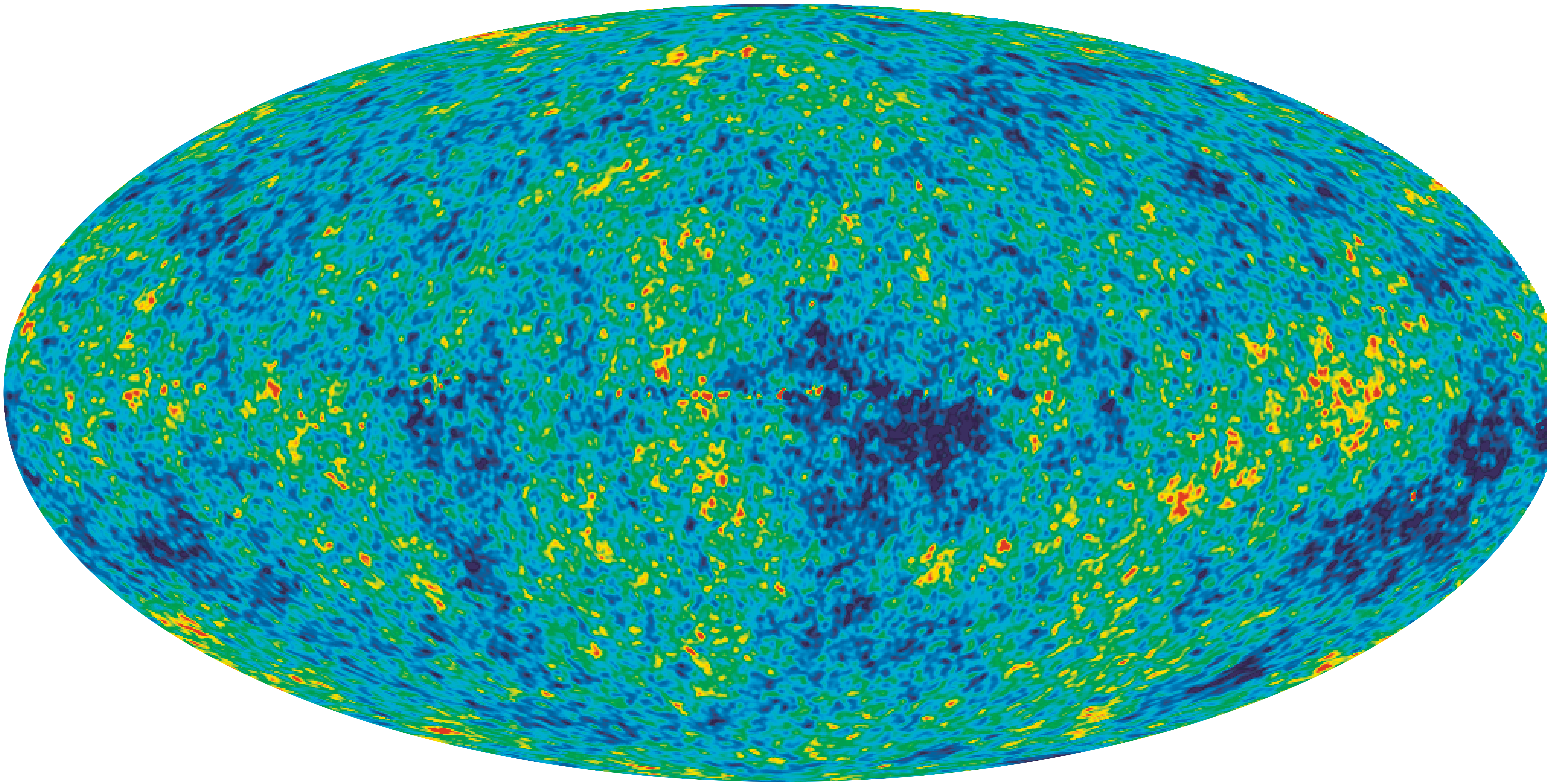
SDSS



- Phases are not random, due to non-linear gravitational evolution

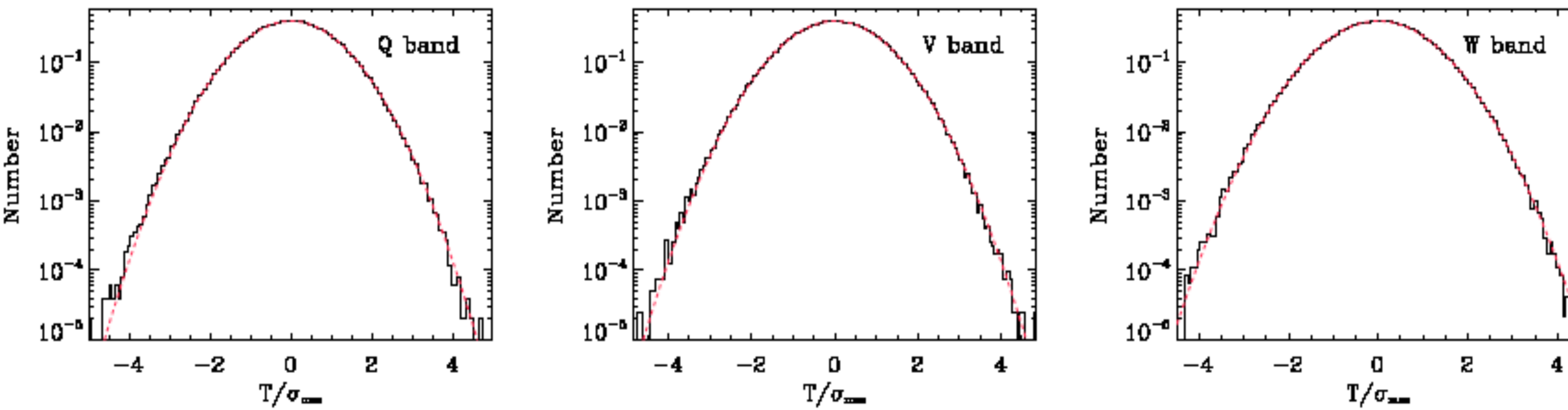
Gaussian?

WMAP5



- Promising probe of Gaussianity – fluctuations still linear!

Take One-point Distribution Function



- The one-point distribution of WMAP map looks pretty Gaussian.
  - –Left to right: Q (41GHz), V (61GHz), W (94GHz).
- Deviation from Gaussianity is small, if any.

Inflation Likes This Result

- According to inflation (Guth & Yi; Hawking; Starobinsky; Bardeen, Steinhardt & Turner), CMB anisotropy was created from **quantum fluctuations of a scalar field in Bunch-Davies vacuum** during inflation
- Successful inflation (with the expansion factor more than e^{60}) *demands* the scalar field be almost interaction-free
- The wave function of free fields in the ground state is a Gaussian!

But, Not Exactly Gaussian

- Of course, there are always corrections to the simplest statement like this
- For one, inflaton field **does** have interactions. They are simply weak – of order the so-called slow-roll parameters, ε and η, which are O(0.01)

Non-Gaussianity from Inflation

- You need cubic interaction terms (or higher order) of fields.
  - $V(\phi)\sim\phi^3$: *Falk, Rangarajan & Srendnicki (1993)* [gravity not included yet]
  - Full expansion of the action, including gravity action, to cubic order was done a decade later by *Maldacena (2003)*

$$\phi = \phi(t) + \varphi(t,x)$$

$$\partial^2\chi = \frac{\dot\phi^2}{2\dot\rho^2}\frac{d}{dt}\left(-\frac{\dot\rho}{\dot\phi}\varphi\right)$$

$$h_{ij} = e^{2\rho}\hat h_{ij}$$

$$S_3 = \int e^{3\rho}\left(-\frac{\dot\phi}{4\dot\rho}\varphi\dot\varphi^2 - e^{-2\rho}\frac{\dot\phi}{4\dot\rho}\varphi(\partial\varphi)^2 - \dot\varphi\partial_i\chi\partial_i\varphi + \right.$$
$$+\frac{3\dot\phi^3}{8\dot\rho}\varphi^3 - \frac{\dot\phi^5}{16\dot\rho^3}\varphi^3 - \frac{\dot\phi V''}{4\dot\rho}\varphi^3 - \frac{V'''}{6}\varphi^3 + \frac{\dot\phi^3}{4\dot\rho^2}\varphi^2\dot\varphi + \frac{\dot\phi^2}{4\dot\rho}\varphi^2\partial^2\chi$$
$$\left. + \frac{\dot\phi}{4\dot\rho}(-\varphi\partial_i\partial_j\chi\partial_i\partial_j\chi + \varphi\partial^2\chi\partial^2\chi)\right)$$

Computing Primordial Bispectrum

- Three-point function, using in-in formalism (*Maldacena 2003; Weinberg 2005*)

$$\text{3-point function}(\mathbf{x}_1, \mathbf{x}_2, \mathbf{x}_3) = \left\langle \text{in} \left| \tilde{T} e^{i \int_{-\infty}^{t} H_I(t')dt'} \Phi(\mathbf{x}_1)\Phi(\mathbf{x}_2)\Phi(\mathbf{x}_3) T e^{-i \int_{-\infty}^{t} H_I(t')dt'} \right| \text{in} \right\rangle$$

- $H_I(t)$: Hamiltonian in interaction picture
  - Model-dependent: this determines which triangle shapes will dominate the signal
- $\Phi(x)$: operator representing curvature perturbations in interaction picture

Simplified Treatment

- Let's try to capture field interactions, or whatever non-linearities that might have been there during inflation, by the following simple, order-of-magnitude form (*Komatsu & Spergel 2001*):
  - $\mathbf{\Phi(x) = \Phi_{gaussian}(x) + f_{NL}[\Phi_{gaussian}(x)]^2}$
  - One finds f_{NL}=O(0.01) from inflation (*Maldacena 2003*; *Acquaviva et al. 2003*)
- **This is a powerful prediction of inflation**

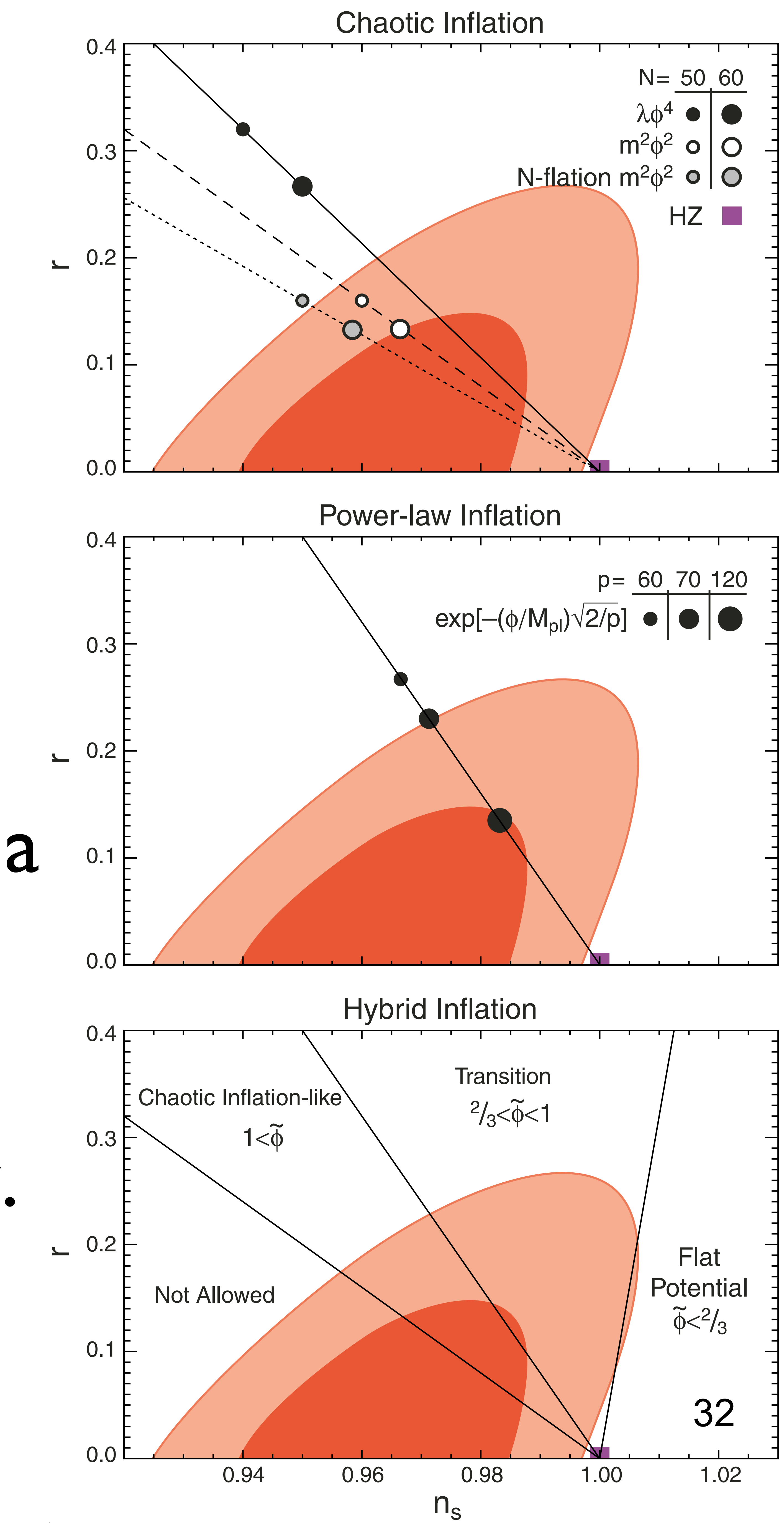
Earlier work on this form: Salopek&Bond (1990); Gangui et al. (1994); Verde et al. (2000); Wang&Kamionkowski (2000)

Why Study Non-Gaussianity?

- Because a detection of f_{NL} has a best chance of **ruling out the largest class of inflation models**.
- Namely, it will rule out inflation models based upon
  - a single scalar field <u>with</u>
  - the canonical kinetic term <u>that</u>
  - rolled down a smooth scalar potential slowly, <u>and</u>
  - was initially in the Bunch-Davies vacuum.
- ***Detection of non-Gaussianity would be a major breakthrough in cosmology.***

We have *r* and n_s. Why Bother?

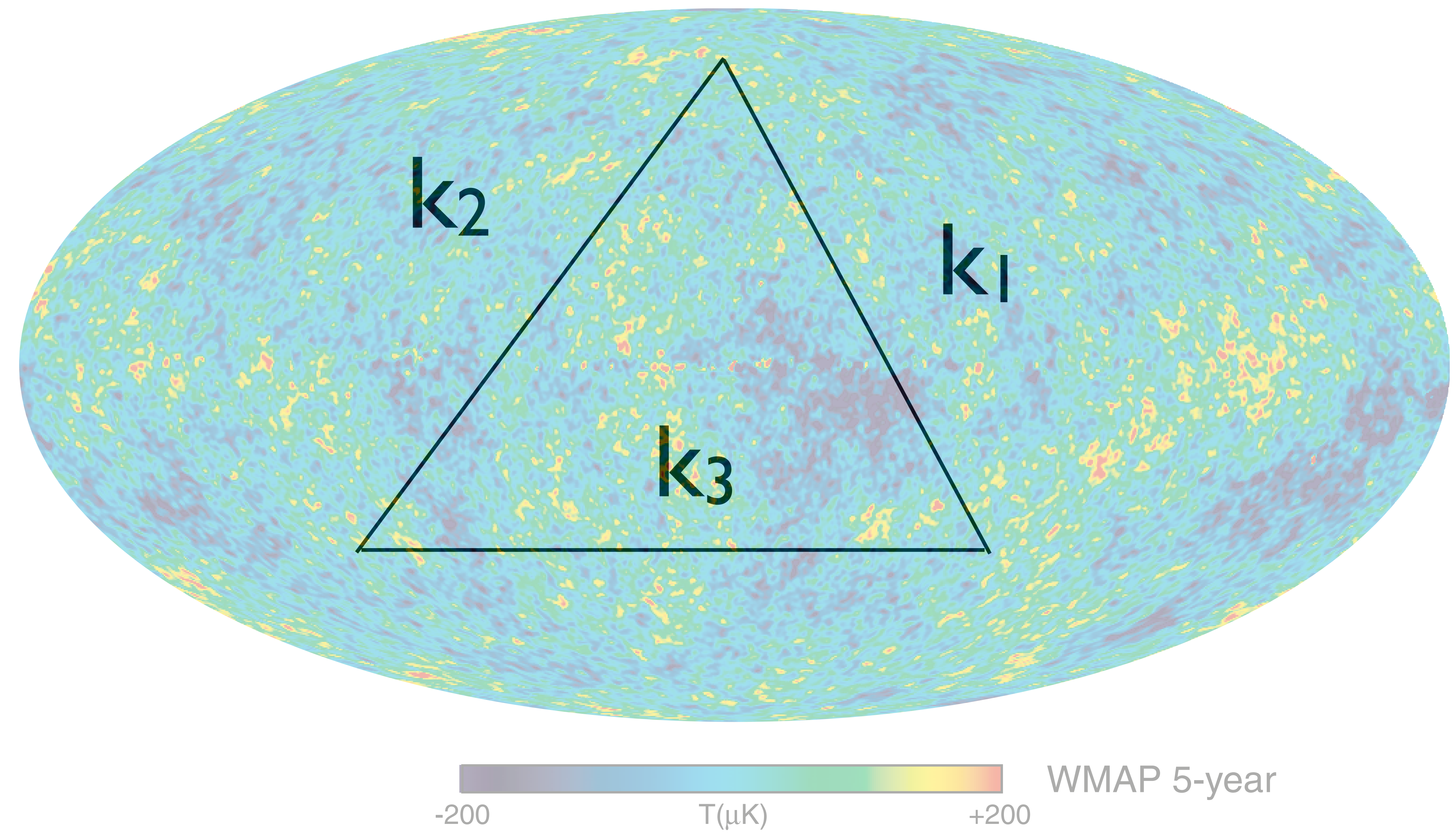
- While the current limit on the power-law index of the primordial power spectrum, **n_s**, and the amplitude of gravitational waves, **r**, have ruled out many inflation models already, many still survive (which is a good thing!)
- A convincing detection of f_{NL} would rule out most of them **regardless of n_s or r**.
- f_{NL} offers more ways to test various early universe models!



Tool: Bispectrum

- **Bispectrum = Fourier Trans. of 3-pt Function**
- **The bispectrum <u>vanishes</u>** for Gaussian fluctuations with random phases.
- Any non-zero detection of the bispectrum indicates the presence of (some kind of) non-Gaussianity.
- A sensitive tool for finding non-Gaussianity.

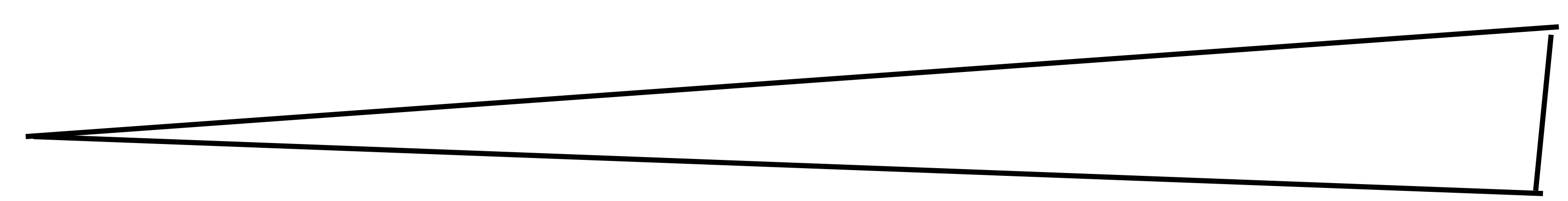
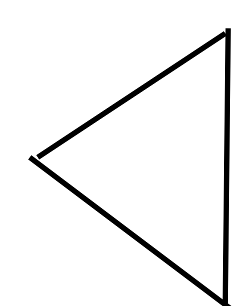
f_{NL} Generalized



- **f_{NL} = the amplitude of bispectrum**, which is
  - $=\langle\Phi(k_1)\Phi(k_2)\Phi(k_3)\rangle=f_{NL}(2\pi)^3\delta^3(k_1+k_2+k_3)b(k_1,k_2,k_3)$
  - where $\Phi(k)$ is the Fourier transform of the curvature perturbation, and $b(k_1,k_2,k_3)$ is a model-dependent function that defines the shape of triangles predicted by various models.

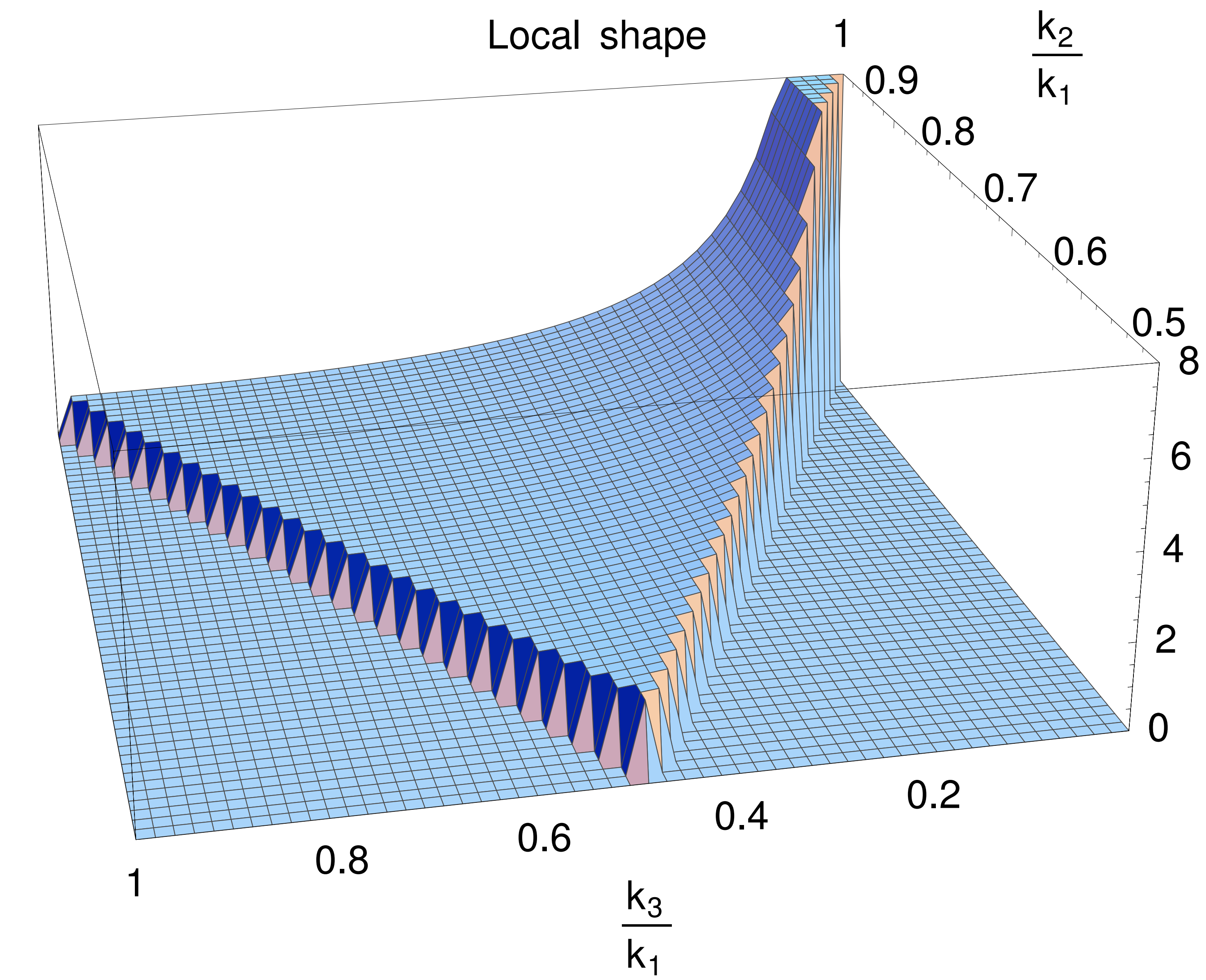
Two f_{NL}'s

There are more than two; I will come back to that later.

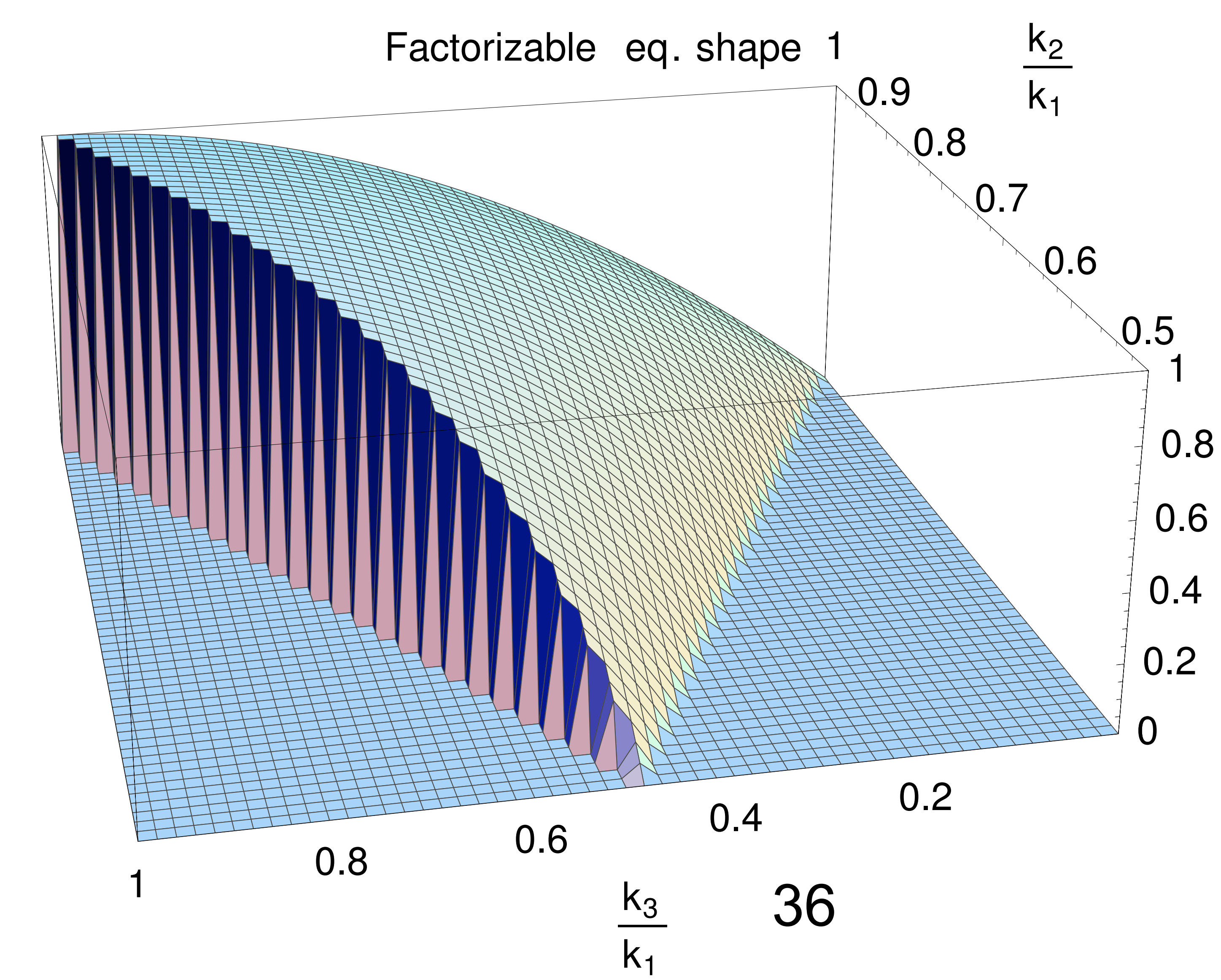
- Depending upon the shape of triangles, one can define various f_{NL}'s:
- "Local" form 
  - which generates non-Gaussianity locally in position space via $\Phi(x)=\Phi_{gaus}(x)+f_{NL}^{local}[\Phi_{gaus}(x)]^2$
- "Equilateral" form 
  - which generates non-Gaussianity locally in momentum space (e.g., k-inflation, DBI inflation)

Forms of $b(k_1,k_2,k_3)$

- Local form (*Komatsu & Spergel 2001*)
  - $b^{\mathrm{local}}(k_1,k_2,k_3) = 2[P(k_1)P(k_2)+\mathrm{cyc.}]$

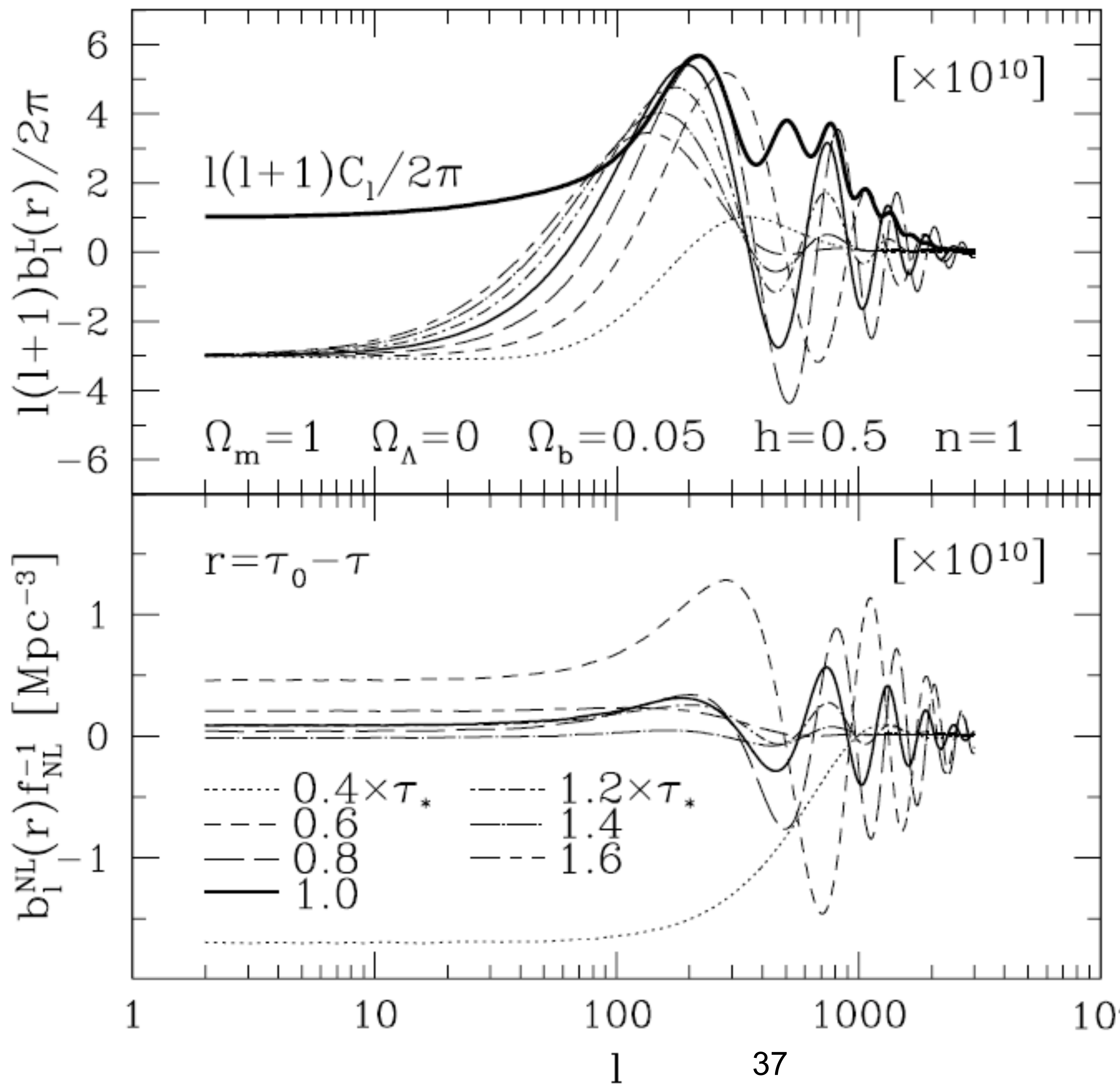


- Equilateral form (*Babich, Creminelli & Zaldarriaga 2004*)
  - $b^{\mathrm{equilateral}}(k_1,k_2,k_3) = 6\{-[P(k_1)P(k_2)+\mathrm{cyc.}] - 2[P(k_1)P(k_2)P(k_3)]^{2/3} + [P(k_1)^{1/3}P(k_2)^{2/3}P(k_3)+\mathrm{cyc.}]\}$



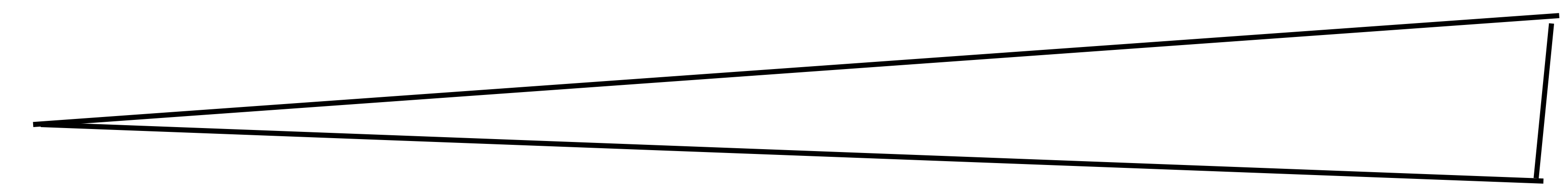
Decoding Bispectrum

- Hydrodynamics at z=1090 generates acoustic oscillations in the bispectrum
- Well understood at the linear level (*Komatsu & Spergel 2001*)
- Non-linear extension?
  - *Nitta, Komatsu, Bartolo, Matarrese & Riotto* in prep.

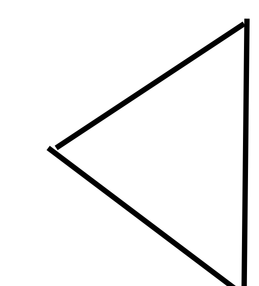


What if f_{NL} is detected?

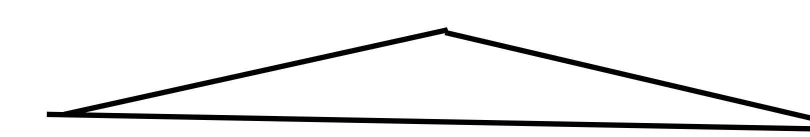
- A single field, canonical kinetic term, slow-roll, and/or Banch-Davies vacuum, must be modified.

Local • Multi-field (curvaton); 

Preheating (e.g., *Chambers & Rajantie 2008*)

Equil. • Non-canonical kinetic term (k-inflation, DBI) 

Bump +Osci. • Temporary fast roll (features in potential)

Folded • Departures from the Bunch-Davies vacuum 

- *It will give us a lot of clues as to what the correct early universe models should look like.*

...or, simply not inflation?

- It has been pointed out recently that New Ekpyrotic scenario generates $f_{NL}^{local} \sim 100$ generically
  - *Creminelli & Senatore; Koyama et al.; Buchbinder et al.; Lehners & Steinhardt*

Measurement

- Use everybody's favorite: χ^2 minimization.
  - Minimize:

$$\chi^2 \equiv \sum_{2\leq l_1\leq l_2\leq l_3} \frac{\left(B^{obs}_{l_1 l_2 l_3} - \sum_i A_i B^{(i)}_{l_1 l_2 l_3}\right)^2}{\sigma^2_{l_1 l_2 l_3}}$$

  - with respect to $A_i=(f_{NL}^{local}, f_{NL}^{equilateral}, b_{src})$
  - B^{obs} is the observed bispectrum
  - $B^{(i)}$ is the theoretical template from various predictions

Journal on f_{NL}

- Local
  - $-3500 < f_{NL}^{local} < 2000$ [COBE 4yr, l_{max}=20] Komatsu et al. (2002)
  - $-58 < f_{NL}^{local} < 134$ [WMAP 1yr, l_{max}=265] Komatsu et al. (2003)
  - $-54 < f_{NL}^{local} < 114$ [WMAP 3yr, l_{max}=350] Spergel et al. (2007)
  - **$-9 < f_{NL}^{local} < 111$ [WMAP 5yr, l_{max}=500]** Komatsu et al. (2008)
- Equilateral
  - $-366 < f_{NL}^{equil} < 238$ [WMAP 1yr, l_{max}=405] Creminelli et al. (2006)
  - $-256 < f_{NL}^{equil} < 332$ [WMAP 3yr, l_{max}=475] Creminelli et al. (2007)
  - **$-151 < f_{NL}^{equil} < 253$ [WMAP 5yr, l_{max}=700]** Komatsu et al. (2008)

What does f_{NL}~100 mean?

- Recall this form: $\Phi(x)=\Phi_{gaus}(x)+f_{NL}^{local}[\Phi_{gaus}(x)]^2$
  - Φ_{gaus} is small, of order 10^{-5}; thus, the second term is 10^{-3} times the first term, if f_{NL}~100
  - Precision test of inflation: **non-Gaussianity term is less than 0.1% of the Gaussian term**
    - cf: flatness tests inflation at 1% level

Non-Gaussianity Has Not Been Discovered Yet, but...

- At 68% CL, we have $\mathbf{f_{NL}=51\pm30}$ (positive 1.7σ)
  - Shift from Yadav & Wandelt's 2.8σ "hint" (f_{NL}~80) from the 3-year data can be explained largely by adding more years of data, i.e., statistical fluctuation, and a new 5-year Galaxy mask that is 10% larger than the 3-year mask
- There is a room for improvement
  - More years of data (WMAP 9-year survey funded!)
  - Better statistical analysis (*Smith & Zaldarriaga 2006*)
  - IF (big if) f_{NL}=50, we would see it at **3σ** in the 9-year data

Exciting Future Prospects

- Planck satellite (to be launched in March 2009)
  - will see f_{NL}^{local} at **17σ**, IF (big if) f_{NL}^{local}=50

Summary

- **Non-Gaussianity is a *new, powerful* probe of physics of the early universe**
  - It has a best chance of ruling out the largest class of inflation models — could even rule out the inflationary paradigm, and support alternatives
- Various forms of f_{NL} available today — 1.7σ at the moment, wait for WMAP 9-year (2011) and Planck (2012) for >3σ
- To convince ourselves of detection, we need to see the acoustic oscillations, and the same signal in bispectrum, trispectrum, Minkowski functionals, of both CMB and large-scale structure of the universe

- **New "industry" — active field!**