IPMU International Conference

Dark Energy: Lighting up the Darkness

http://member.ipmu.jp/darkenergy09/welcome.html

June 22 – 26, 2009
At IPMU (i.e., here)

Primordial Non-Gaussianity and Galaxy Bispectrum (and Conference Summary)

Eiichiro Komatsu (Texas Cosmology Center, Univ. of Texas at Austin)

April 10, 2009

Effects of f_{NL} on the statistics of PEAKS

- You heard talks on the effects of f_{NL} on the power spectrum of peaks (i.e., galaxies)
- **How about the bispectrum of galaxies?**

Previous Calculation

- Sefusatti & Komatsu (2007)
  - Treated the distribution of galaxies as a *continuous distribution*, biased relative to the matter distribution:
    - $\delta_g = b_1\delta_m + (b_2/2)(\delta_m)^2 + \ldots$
  - Then, the calculation is straightforward. Schematically:
    - $\langle\delta_g^3\rangle = (b_1)^3\langle\delta_m^3\rangle + (b_1^2 b_2/2)\langle\delta_m^4\rangle + \ldots$

Non-linear Gravity *Non-linear Bias Bispectrum*

Primordial NG

Previous Calculation

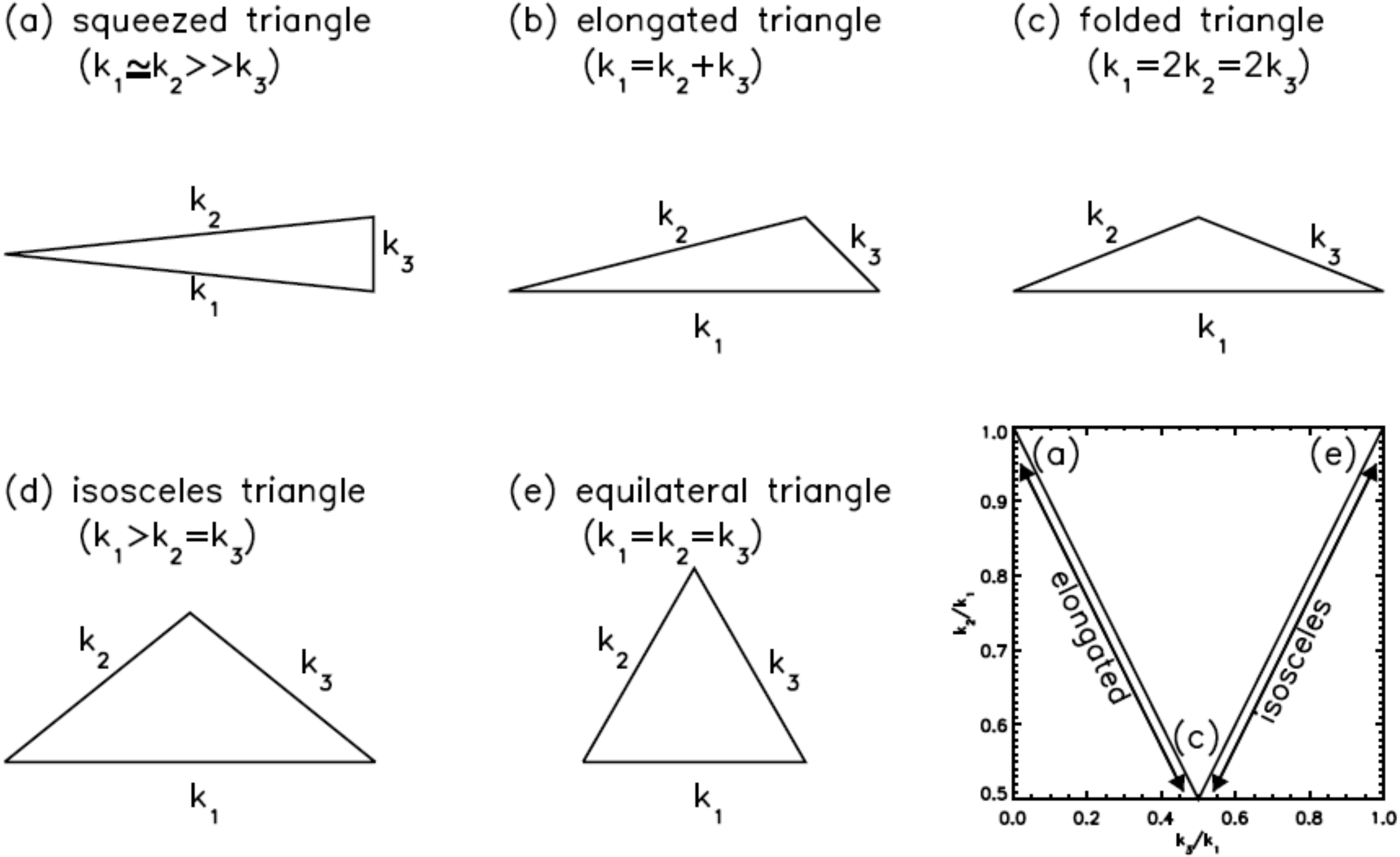
$$B_g(k_1,k_2,k_3,z)$$
$$= 3b_1^3 f_{\rm NL}\Omega_m H_0^2 \left[\frac{P_m(k_1,z)}{k_1^2 T(k_1)}\frac{P_m(k_2,z)}{k_2^2 T(k_2)}\frac{k_3^2 T(k_3)}{D(z)} + ({\rm cyclic})\right]$$
$$+ 2b_1^3\left[F_2^{(s)}(\boldsymbol{k}_1,\boldsymbol{k}_2)P_m(k_1,z)P_m(k_2,z) + ({\rm cyclic})\right]$$
$$+ b_1^2 b_2\left[P_m(k_1,z)P_m(k_2,z) + ({\rm cyclic})\right]$$

Primordial NG

Non-linear Gravity

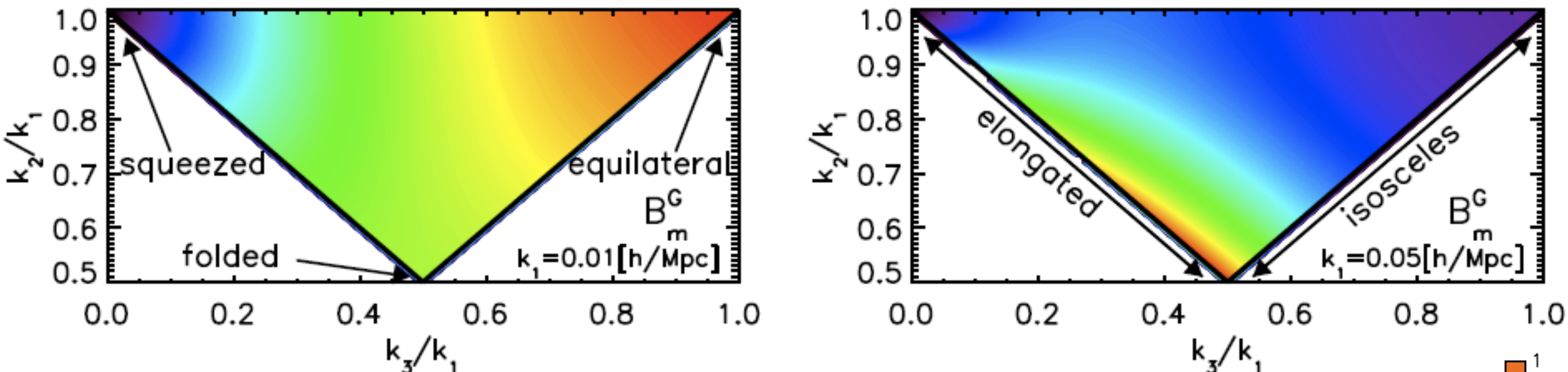
Non-linear Bias

- We find that this formula captures only a part of the full contributions. In fact, this formula is sub-dominant in the squeezed configuration, and the new terms are dominant.

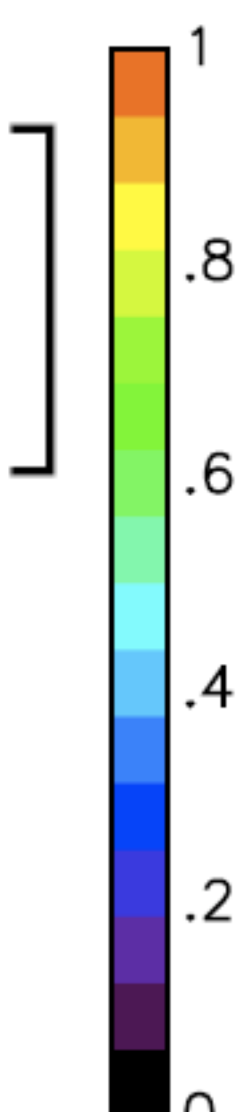


(a) squeezed triangle
($k_1 \simeq k_2 >> k_3$)
(b) elongated triangle
($k_1 = k_2 + k_3$)
(c) folded triangle
($k_1 = 2k_2 = 2k_3$)
(d) isosceles triangle
($k_1 > k_2 = k_3$)
(e) equilateral triangle
($k_1 = k_2 = k_3$)
k_1
k_2
k_3
elongated
isosceles
(a)
(c)
(e)
k_2/k_1
k_3/k_1
0.0
0.2
0.4
0.6
0.8
1.0
0.5
0.6
0.7
0.8
0.9
1.0

Non-linear Gravity

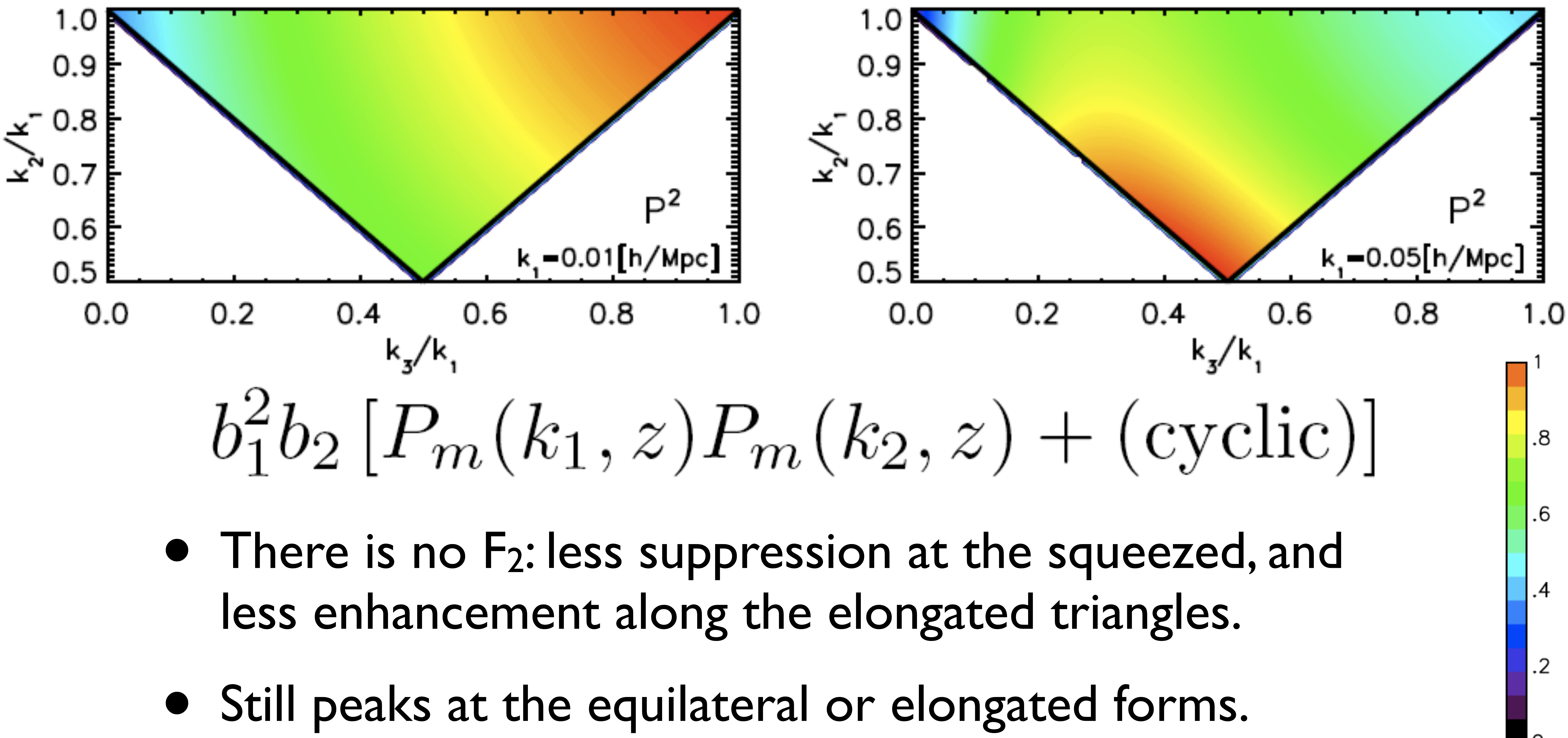


$$2b_1^3\left[F_2^{(s)}(\boldsymbol{k}_1,\boldsymbol{k}_2)P_m(k_1,z)P_m(k_2,z)+(\mathrm{cyclic})\right]$$



- For a given k_1, vary k_2 and k_3, with $k_3 \leq k_2 \leq k_1$
- $F_2(k_2,k_3)$ vanishes in the squeezed limit, and peaks at the elongated triangles.

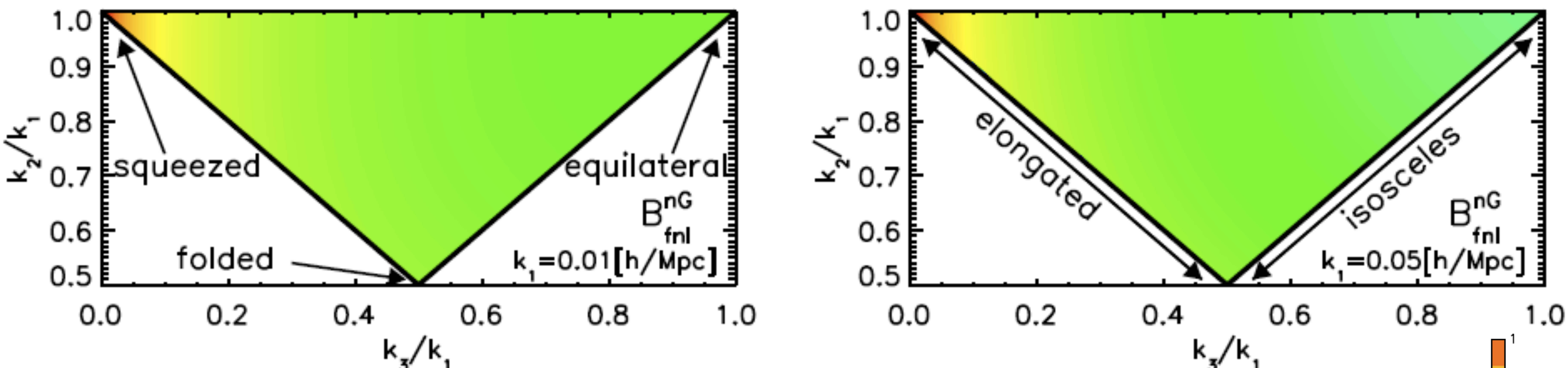
Non-linear Galaxy Bias

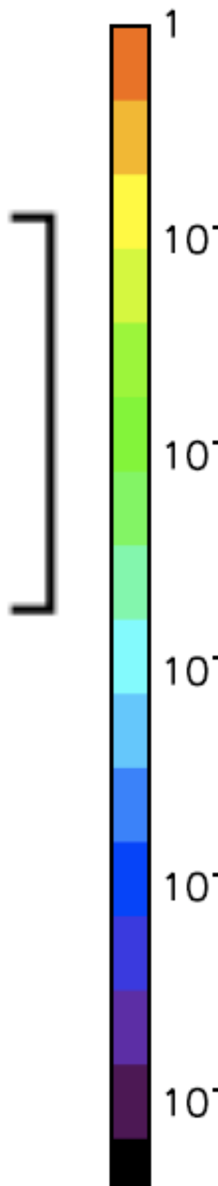


$$b_1^2 b_2 \left[P_m(k_1, z)P_m(k_2, z) + (\text{cyclic})\right]$$

- There is no F_2: less suppression at the squeezed, and less enhancement along the elongated triangles.
- Still peaks at the equilateral or elongated forms.

Primordial NG (SK07)





$$3b_1^3 f_{\rm NL}\Omega_m H_0^2\left[\frac{P_m(k_1,z)}{k_1^2T(k_1)}\frac{P_m(k_2,z)}{k_2^2T(k_2)}\frac{k_3^2T(k_3)}{D(z)}+({\rm cyclic})\right]$$

- Notice the factors of k^2 in the denominator.
- This gives the peaks at the squeezed configurations.

New Terms

- But, it turns our that Sefusatti & Komatsu's calculation, which is valid only for the continuous field, misses the dominant terms that come from the statistics of PEAKS.
- Jeong & Komatsu, arXiv:0904.0497

Donghui Jeong

Matarrese, Lucchin & Bonometto (1986)

MLB Formula

$$1+\xi_h(x_{12})+\xi_h(x_{23})+\xi_h(x_{31})+\zeta_h(\boldsymbol{x}_1,\boldsymbol{x}_2,\boldsymbol{x}_3)$$

$$=\exp\left[\frac{1}{2}\frac{\nu^2}{\sigma_R^2}\sum_{i\neq j}\xi_R^{(2)}(x_{ij})+\sum_{n=3}^{\infty}\left\{\sum_{m_1=0}^{n}\sum_{m_2=0}^{n-m_1}\frac{\nu^n\sigma_R^{-n}}{m_1!m_2!m_3!}\right.\right.$$

$$\times\xi_R^{(n)}\begin{pmatrix}\boldsymbol{x}_1,\cdots,\boldsymbol{x}_1, & \boldsymbol{x}_2,\cdots,\boldsymbol{x}_2, & \boldsymbol{x}_3,\cdots,\boldsymbol{x}_3\\ m_1\,\text{times} & m_2\,\text{times} & m_3\,\text{times}\end{pmatrix}$$

$$\left.\left.-3\frac{\nu^n\sigma_R^{-n}}{n!}\xi_R^{(n)}\begin{pmatrix}\boldsymbol{x},\cdots,\boldsymbol{x}\\ n\,\text{times}\end{pmatrix}\right\}\right]$$

- N-point correlation function of peaks is the sum of M-point correlation functions, where M≥N.

Bottom Line

- **The bottom line is:**
- The power spectrum (2-pt function) of peaks is sensitive to the power spectrum of the underlying mass distribution, and the bispectrum, and the trispectrum, etc.
  - Truncate the sum at the bispectrum: sensitivity to f_{NL}
  - Dalal et al.; Matarrese&Verde; Slosar et al.; Afshordi&Tolley

Bottom Line

- **The bottom line is:**
- The bispectrum (3-pt function) of peaks is sensitive to the bispectrum of the underlying mass distribution, and the trispectrum, and the quadspectrum, etc.
  - Truncate the sum at the trispectrum: sensitivity to τ_{NL} ($\sim f_{NL}^2$) and g_{NL}!
  - This is the new effect that was missing in Sefusatti & Komatsu (2007).

Real-space 3pt Function

$$\zeta_h(\boldsymbol{x}_1, \boldsymbol{x}_2, \boldsymbol{x}_3) = \frac{\nu^3}{\sigma_R^3}\xi_R^{(3)}(\boldsymbol{x}_1, \boldsymbol{x}_2, \boldsymbol{x}_3)$$
$$+ \frac{\nu^4}{\sigma_R^4}\left[\xi_R^{(2)}(x_{12})\xi_R^{(2)}(x_{23}) + (\mathrm{cyclic})\right]$$
$$+ \frac{\nu^4}{2\sigma_R^4}\left[\xi_R^{(4)}(\boldsymbol{x}_1, \boldsymbol{x}_1, \boldsymbol{x}_2, \boldsymbol{x}_3) + (\mathrm{cyclic})\right]$$

- Plus 5-pt functions, etc...

New Bispectrum Formula

$$B_h(\boldsymbol{k}_1, \boldsymbol{k}_2, \boldsymbol{k}_3)$$

$$= b_1^3 \Bigg[B_R(\boldsymbol{k}_1, \boldsymbol{k}_2, \boldsymbol{k}_3) + \frac{b_2}{b_1} \{ P_R(k_1) P_R(k_2) + (\text{cyclic}) \}$$

$$+ \frac{\delta_c}{2\sigma_R^2} \int \frac{d^3 q}{(2\pi)^3} T_R(\boldsymbol{q}, \boldsymbol{k}_1 - \boldsymbol{q}, \boldsymbol{k}_2, \boldsymbol{k}_3) + (\text{cyclic}) \Bigg].$$

- First: bispectrum of the underlying mass distribution.
- Second: non-linear bias
- Third: trispectrum of the underlying mass distribution.

Local Form Trispectrum

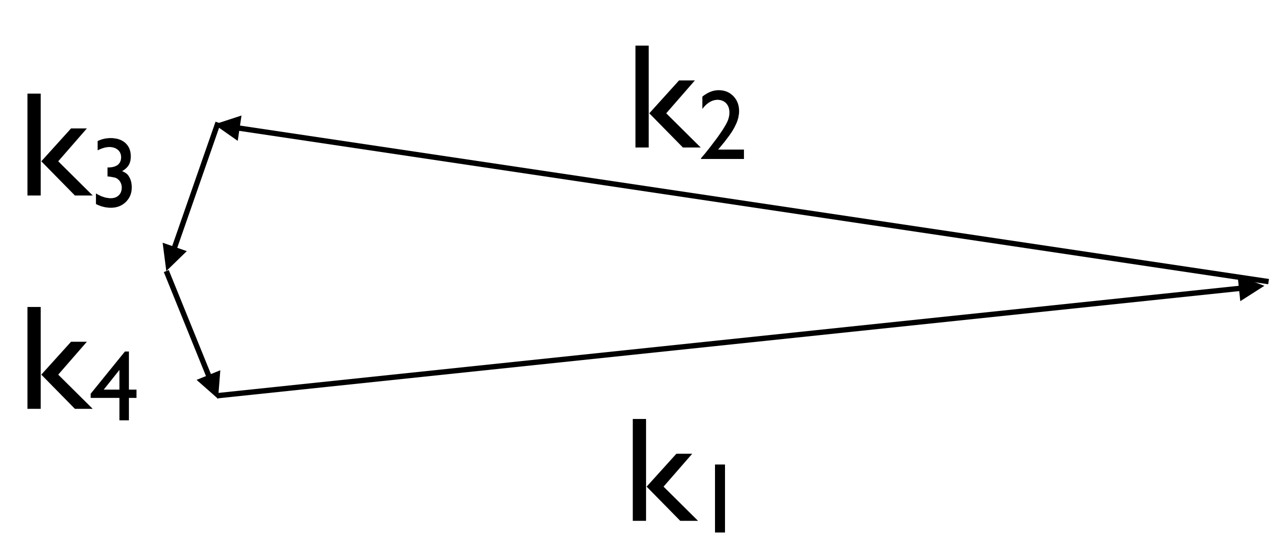
$$\Phi(\boldsymbol{x}) = \phi(\boldsymbol{x}) + f_{\mathrm{NL}}\left[\phi^2(\boldsymbol{x}) - \langle\phi^2\rangle\right] + g_{\mathrm{NL}}\phi^3(\boldsymbol{x})$$

$$\begin{aligned} &T_\Phi(\boldsymbol{k}_1, \boldsymbol{k}_2, \boldsymbol{k}_3, \boldsymbol{k}_4) \\ =\, &6g_{\mathrm{NL}}\left[P_\phi(k_1)P_\phi(k_2)P_\phi(k_3) + (\mathrm{cyclic})\right] + \underline{2f_{\mathrm{NL}}^2} \\ &\times\left[P_\phi(k_1)P_\phi(k_2)\left\{P_\phi(k_{13}) + P_\phi(k_{14})\right\} + (\mathrm{cyclic})\right] \end{aligned}$$

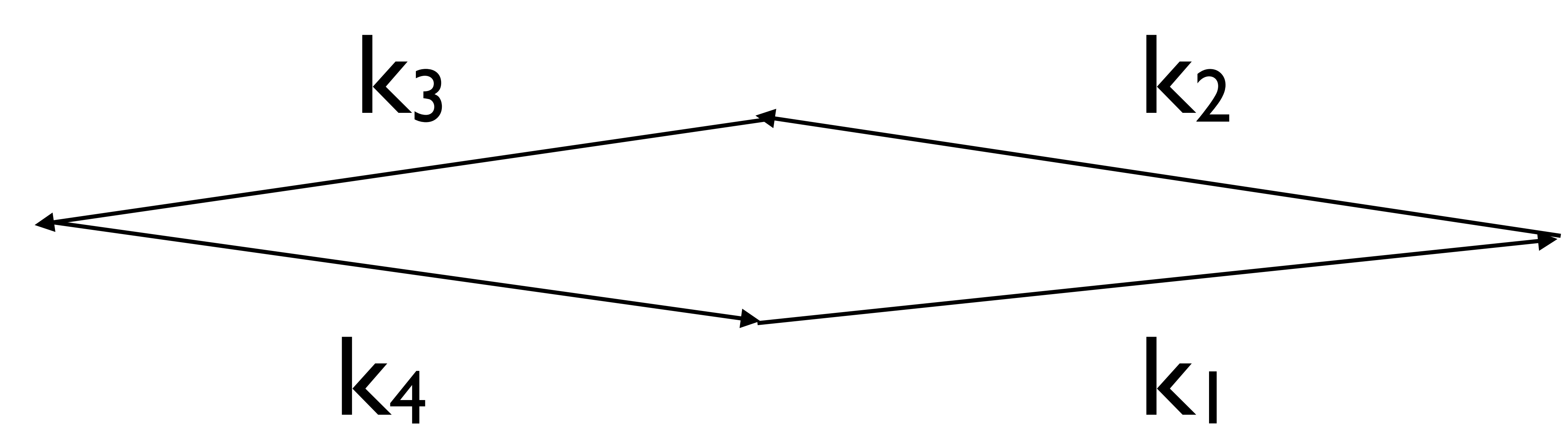
- For general multi-field models, f_{NL}^2 can be more generic: often called τ_{NL}.
- Exciting possibility for testing more about inflation!

Local Form Trispectrum

$$T_\Phi(\boldsymbol{k}_1, \boldsymbol{k}_2, \boldsymbol{k}_3, \boldsymbol{k}_4)$$
$$= 6 g_{\rm NL} \left[P_\phi(k_1) P_\phi(k_2) P_\phi(k_3) + ({\rm cyclic}) \right] + 2 f_{\rm NL}^2$$
$$\times \left[P_\phi(k_1) P_\phi(k_2) \left\{ P_\phi(k_{13}) + P_\phi(k_{14}) \right\} + ({\rm cyclic}) \right]$$



g_{NL}



f_{NL}^2 (or τ_{NL})

Trispectrum Term

$$\frac{\delta_c}{2\sigma_R^2}\int\frac{d^3q}{(2\pi)^3}\left[T_R(\boldsymbol{q},\boldsymbol{k}_1-\boldsymbol{q},\boldsymbol{k}_2,\boldsymbol{k}_3)+(\text{cyclic})\right]$$
$$=g_{\rm NL}B^{nG}_{g_{\rm NL}}(k_1,k_2,k_3)+f_{\rm NL}^2B^{nG}_{f_{\rm NL}^2}(k_1,k_2,k_3),$$

$$B^{nG}_{g_{\rm NL}}(k_1,k_2,k_3)\equiv\frac{\delta_c}{2\sigma_R^2}\Bigg[6\mathcal{M}_R(k_2)\mathcal{M}_R(k_3)\left[P_\phi(k_2)+P_\phi(k_3)\right]\int\frac{d^3q}{(2\pi)^3}\mathcal{M}_R(q)\mathcal{M}_R(|\boldsymbol{k}_1-\boldsymbol{q}|)P_\phi(q)P_\phi(|\boldsymbol{k}_1-\boldsymbol{q}|)+(\text{cyclic})$$
$$+12\mathcal{M}_R(k_2)\mathcal{M}_R(k_3)P_\phi(k_2)P_\phi(k_3)\int\frac{d^3q}{(2\pi)^3}\mathcal{M}_R(q)\mathcal{M}_R(|\boldsymbol{k}_1-\boldsymbol{q}|)P_\phi(q)+(\text{cyclic})\Bigg].\qquad(20)$$

$$B^{nG}_{f_{\rm NL}^2}(k_1,k_2,k_3)\approx\frac{\delta_c}{2\sigma_R^2}\Bigg[8\mathcal{M}_R(k_2)\mathcal{M}_R(k_3)P_\phi(k_1)\left[P_\phi(k_2)+P_\phi(k_3)\right]\int\frac{d^3q}{(2\pi)^3}\mathcal{M}_R(q)\mathcal{M}_R(|\boldsymbol{k}_1-\boldsymbol{q}|)P_\phi(q)+(\text{cyclic})$$
$$+4\mathcal{M}_R(k_2)\mathcal{M}_R(k_3)P_\phi(k_2)P_\phi(k_3)\int\frac{d^3q}{(2\pi)^3}\mathcal{M}_R(q)\mathcal{M}_R(|\boldsymbol{k}_1-\boldsymbol{q}|)$$
$$\times\left[P_\phi(|\boldsymbol{k}_2+\boldsymbol{q}|)+P_\phi(|\boldsymbol{k}_3+\boldsymbol{q}|)\right]+(\text{cyclic})\Bigg].\qquad(21)$$

Trispectrum Term

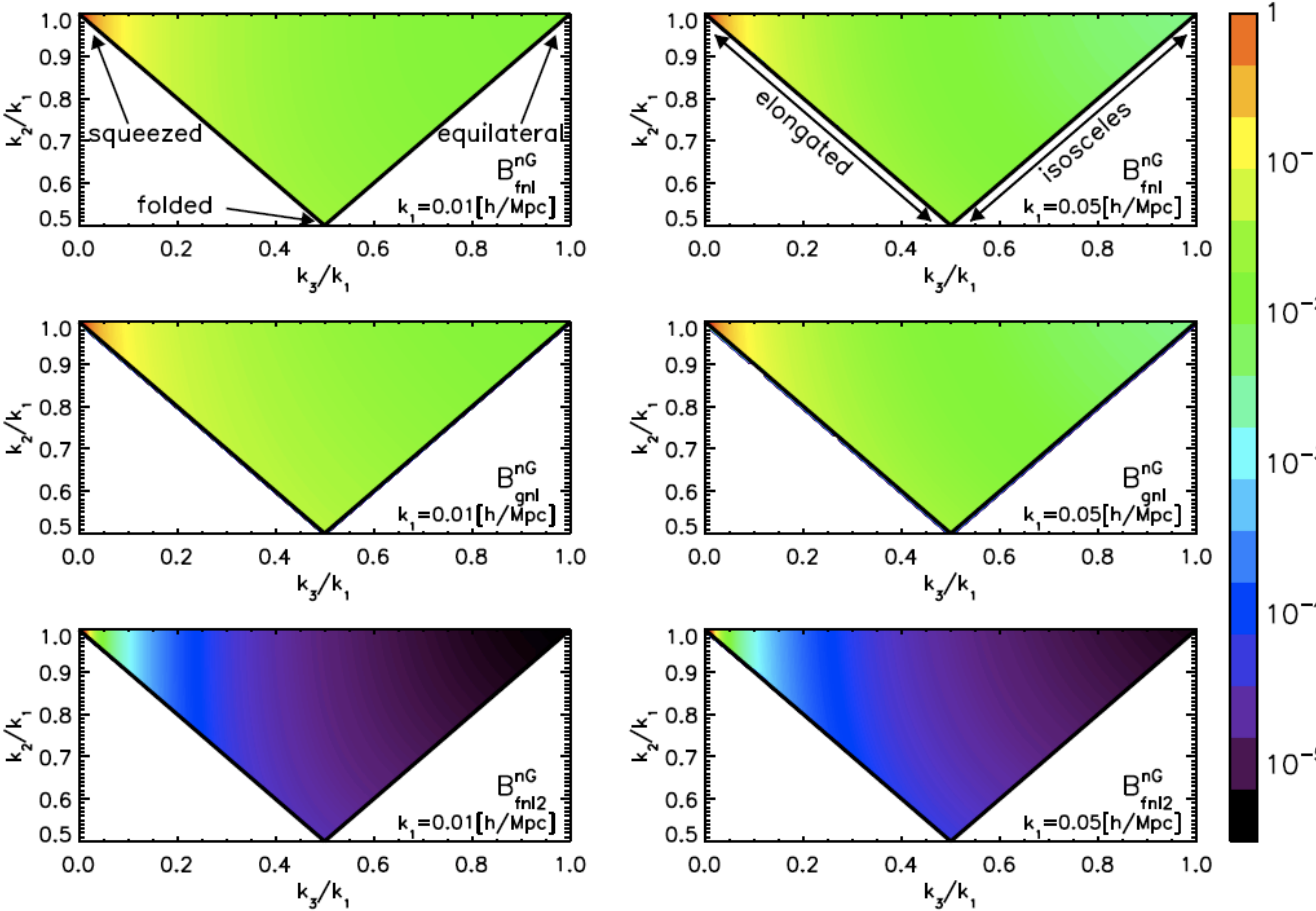
$$\frac{\delta_c}{2\sigma_R^2}\int\frac{d^3q}{(2\pi)^3}\left[T_R(\boldsymbol{q},\boldsymbol{k}_1-\boldsymbol{q},\boldsymbol{k}_2,\boldsymbol{k}_3)+(\text{cyclic})\right]$$

$$=g_{\rm NL}B^{nG}_{g_{\rm NL}}(k_1,k_2,k_3)+f_{\rm NL}^2B^{nG}_{f^2_{\rm NL}}(k_1,k_2,k_3),$$

$$B^{nG}_{g_{\rm NL}}(k_1,k_2,k_3)\equiv\frac{\delta_c}{2\sigma_R^2}\Bigg[6\mathcal{M}_R(k_2)\mathcal{M}_R(k_3)\left[P_\phi(k_2)+P_\phi(k_3)\right]\int\frac{d^3q}{(2\pi)^3}\mathcal{M}_R(q)\mathcal{M}_R(|\boldsymbol{k}_1-\boldsymbol{q}|)P_\phi(q)P_\phi(|\boldsymbol{k}_1-\boldsymbol{q}|)+(\text{cyclic})$$
$$+12\mathcal{M}_R(k_2)\mathcal{M}_R(k_3)P_\phi(k_2)P_\phi(k_3)\int\frac{d^3q}{(2\pi)^3}\mathcal{M}_R(q)\mathcal{M}_R(|\boldsymbol{k}_1-\boldsymbol{q}|)P_\phi(q)+(\text{cyclic})\Bigg]. \quad (20)$$

$$B^{nG}_{f^2_{\rm NL}}(k_1,k_2,k_3)\approx\frac{\delta_c}{2\sigma_R^2}\Bigg[8\mathcal{M}_R(k_2)\mathcal{M}_R(k_3)P_\phi(k_1)\left[P_\phi(k_2)+P_\phi(k_3)\right]\int\frac{d^3q}{(2\pi)^3}\mathcal{M}_R(q)\mathcal{M}_R(|\boldsymbol{k}_1-\boldsymbol{q}|)P_\phi(q)+(\text{cyclic})$$
$$+4\mathcal{M}_R(k_2)\mathcal{M}_R(k_3)P_\phi(k_2)P_\phi(k_3)\int\frac{d^3q}{(2\pi)^3}\mathcal{M}_R(q)\mathcal{M}_R(|\boldsymbol{k}_1-\boldsymbol{q}|)$$
$$\times\left[P_\phi(|\boldsymbol{k}_2+\boldsymbol{q}|)+P_\phi(|\boldsymbol{k}_3+\boldsymbol{q}|)\right]+(\text{cyclic})\Bigg]. \quad (21)$$

Most Dominant in the Squeezed Limit

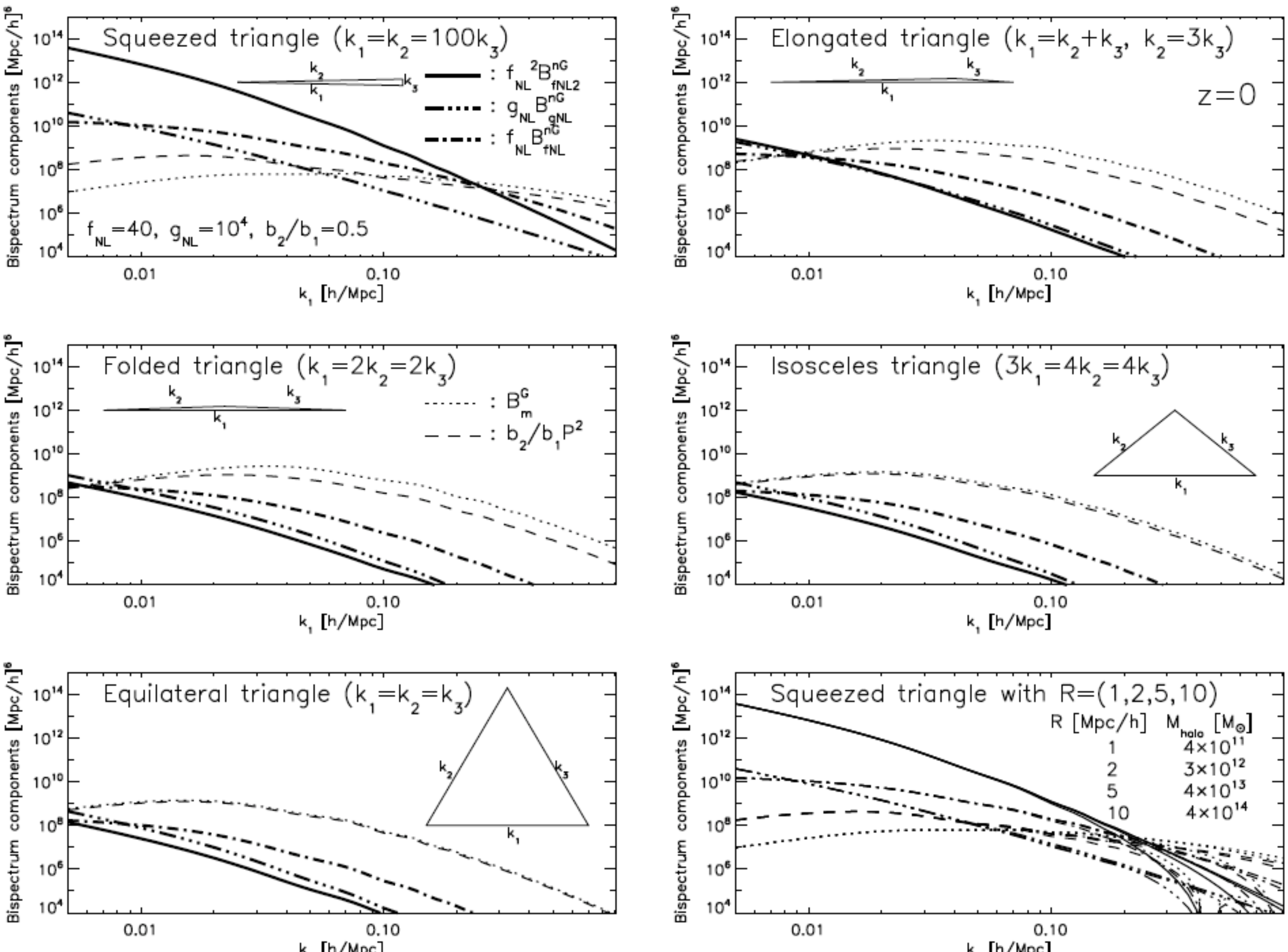

squeezed
equilateral
folded
elongated
isosceles
B^{nG}_{fnl}
k_1=0.01[h/Mpc]
k_1=0.05[h/Mpc]
B^{nG}_{gnl}
B^{nG}_{fnl2}
k_2/k_1
k_3/k_1
1
10^{-1}
10^{-2}
10^{-3}
10^{-4}
10^{-5}

Shape Results

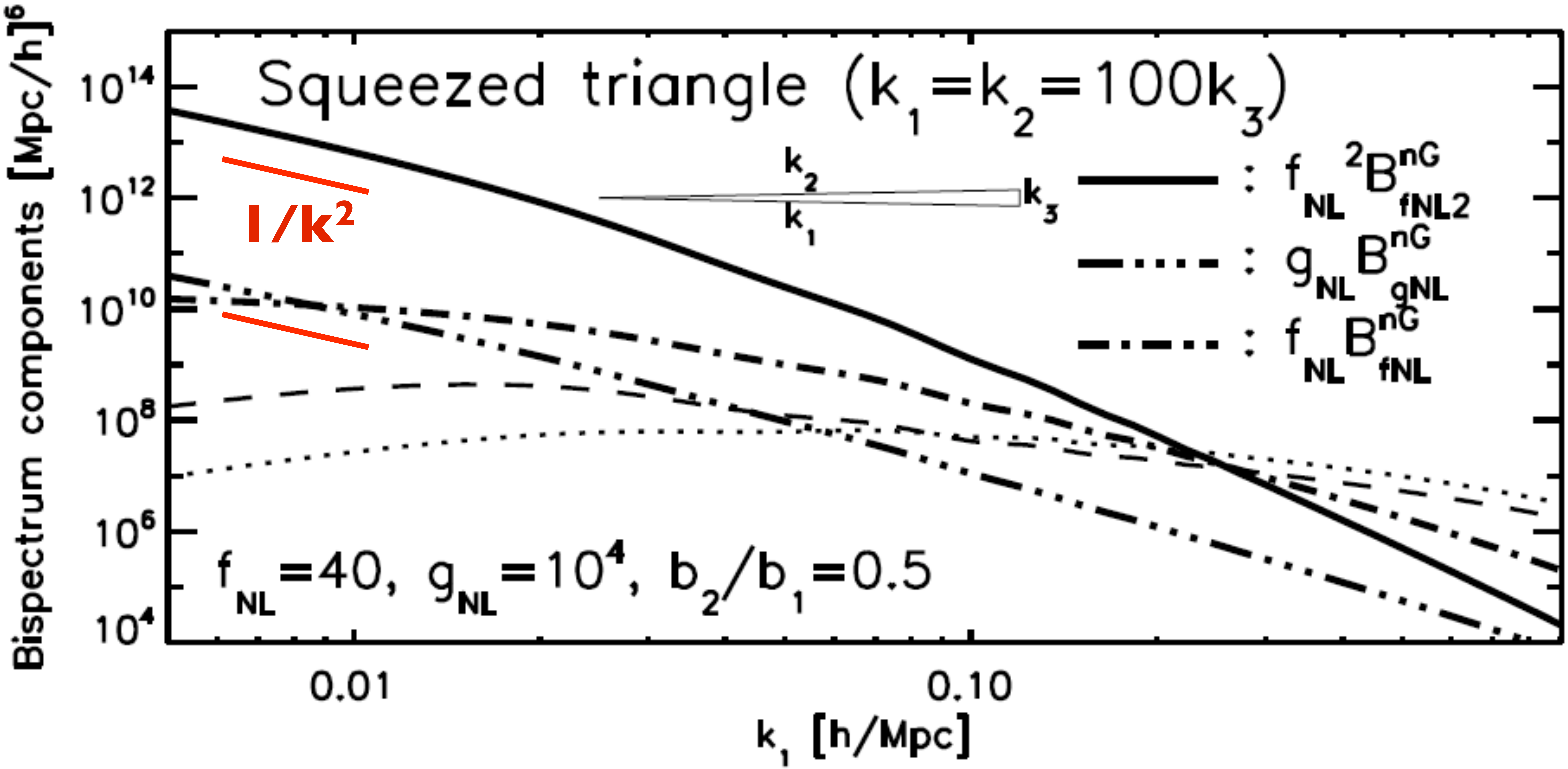
- The primordial non-Gaussianity terms peak at the squeezed triangle.
- f_{NL} and g_{NL} terms have the same shape dependence:
  - For $k_1=k_2=\alpha k_3$, (f_{NL} term)~α and (g_{NL} term)~α
- f_{NL}^2 (τ_{NL}) is more sharply peaked at the squeezed:
  - (f_{NL}^2 term)~α^3

Key Question

- Are g_{NL} or τ_{NL} terms important?

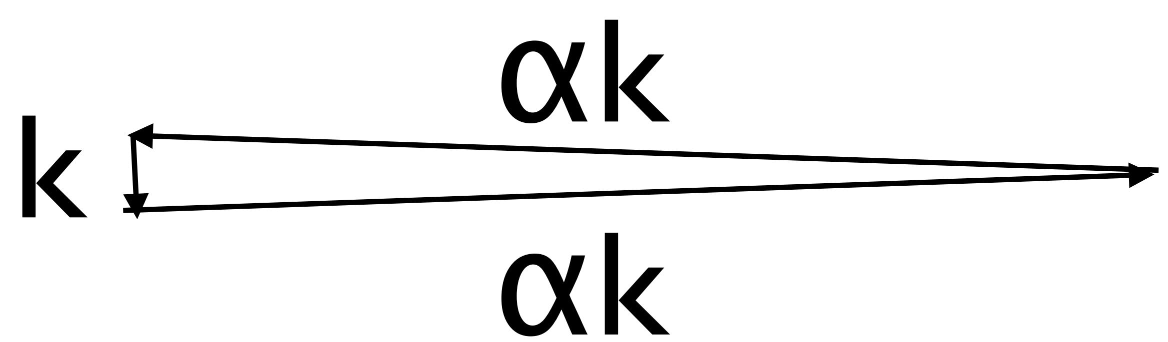


Squeezed triangle ($k_1=k_2=100k_3$)
$f_{NL}{}^2 B^{nG}_{fNL2}$
$g_{NL} B^{nG}_{gNL}$
$f_{NL} B^{nG}_{fNL}$
$f_{NL}=40$, $g_{NL}=10^4$, $b_2/b_1=0.5$
Elongated triangle ($k_1=k_2+k_3$, $k_2=3k_3$)
z=0
Folded triangle ($k_1=2k_2=2k_3$)
B^G_m
$b_2/b_1 P^2$
Isosceles triangle ($3k_1=4k_2=4k_3$)
Equilateral triangle ($k_1=k_2=k_3$)
Squeezed triangle with R=(1,2,5,10)
R [Mpc/h] M_{halo} [$M_\odot$]
1 4×10^{11}
2 3×10^{12}
5 4×10^{13}
10 4×10^{14}
Bispectrum components [Mpc/h]^6
k_1 [h/Mpc]



Squeezed triangle (k_1=k_2=100k_3)
$1/k^2$
k_2
k_1
k_3
: $f_{NL}{}^2B^{nG}_{fNL2}$
: $g_{NL}B^{nG}_{gNL}$
: $f_{NL}B^{nG}_{fNL}$
f_{NL}=40, g_{NL}=10^4, b_2/b_1=0.5
Bispectrum components [Mpc/h]^6
10^{14}
10^{12}
10^{10}
10^8
10^6
10^4
0.01
0.10
k_1 [h/Mpc]

Importance Ratios



$$\frac{f_{\rm NL} B^{nG}_{f_{\rm NL}}}{f^2_{\rm NL} B^{nG}_{f^2_{\rm NL}}} \simeq \frac{1}{f_{\rm NL}\alpha^2} \frac{\mathcal{M}_0 k^2}{2\mathcal{I}_0 \delta_c/\sigma_R^2}$$

$$\simeq 0.0016 \left(\frac{100}{\alpha}\right)^2 \frac{40}{f_{\rm NL}} \left(\frac{k}{0.01\ h\ {\rm Mpc}^{-1}}\right)^2 \quad (29)$$

$$\frac{g_{\rm NL} B^{nG}_{g_{\rm NL}}}{f^2_{\rm NL} B^{nG}_{f^2_{\rm NL}}} \simeq \frac{4}{3\alpha^2} \frac{g_{\rm NL}}{f^2_{\rm NL}}$$

$$\simeq 0.0008 \left(\frac{100}{\alpha}\right)^2 \left(\frac{40}{f_{\rm NL}}\right)^2 \frac{g_{\rm NL}}{10^4}, \quad (30)$$

$$\frac{f_{\rm NL} B^{nG}_{f_{\rm NL}}}{g_{\rm NL} B^{nG}_{g_{\rm NL}}} \simeq \frac{f_{\rm NL}}{g_{\rm NL}} \frac{\mathcal{M}_0 k^2}{3\mathcal{I}_0 \delta_c/\sigma_R^2}$$

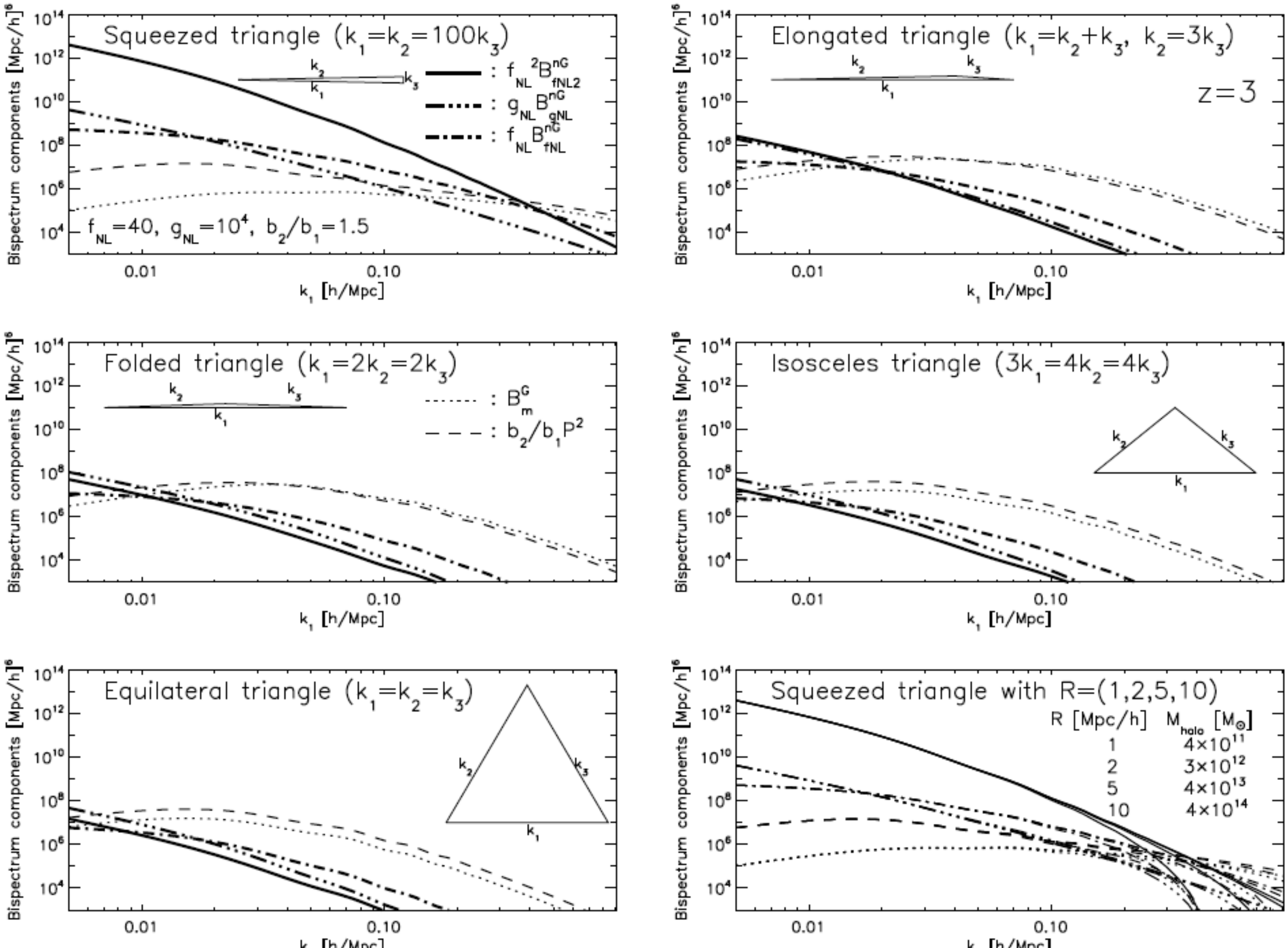
$$\simeq 1.7 \frac{f_{\rm NL}}{40} \frac{10^4}{g_{\rm NL}} \left(\frac{k}{0.01\ h\ {\rm Mpc}^{-1}}\right)^2 . \quad (31)$$

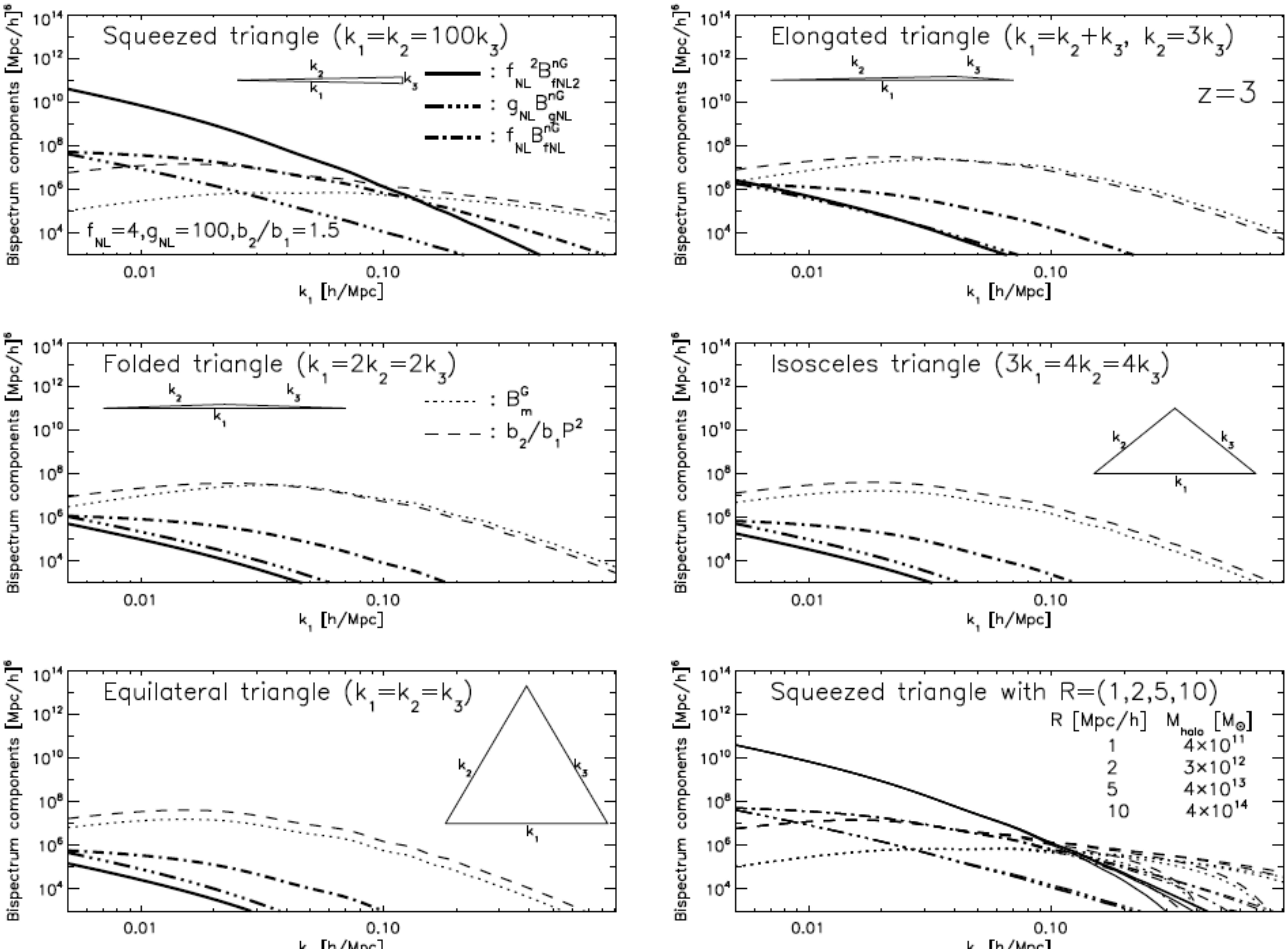
- **f_{NL}^2 dominates over f_{NL} term easily for $f_{NL}>1$!**

Redshift Dependence

$$B_h(k_1,k_2,k_3,z) = b_1^3(z)D^4(z)\left[B_m^G(k_1,k_2,k_3) + \frac{b_2(z)}{b_1(z)}\{P_R(k_1)P_R(k_2) + (\text{cyclic})\} + f_{\rm NL}\frac{B^{nG}_{f_{\rm NL}}(k_1,k_2,k_3)}{D(z)}\right.$$
$$\left. + f_{\rm NL}^2\frac{B^{nG}_{f^2_{\rm NL}}(k_1,k_2,k_3)}{D^2(z)} + g_{\rm NL}\frac{B^{nG}_{g_{\rm NL}}(k_1,k_2,k_3)}{D^2(z)}\right],$$

- Primordial non-Gaussianity terms are more important at higher redshifts.
- The new trispectrum terms are even more important.


Squeezed triangle (k1=k2=100k3)
Elongated triangle (k1=k2+k3, k2=3k3)
Folded triangle (k1=2k2=2k3)
Isosceles triangle (3k1=4k2=4k3)
Equilateral triangle (k1=k2=k3)
Squeezed triangle with R=(1,2,5,10)
z=3
fNL=40, gNL=10^4, b2/b1=1.5
: fNL^2 B^nG_fNL2
: gNL B^nG_gNL
: fNL B^nG_fNL
: B^G_m
: b2/b1 P^2
R [Mpc/h] Mhalo [M⊙]
1 4×10^11
2 3×10^12
5 4×10^13
10 4×10^14
Bispectrum components [Mpc/h]^6
k1 [h/Mpc]
k1
k2
k3



Squeezed triangle ($k_1=k_2=100k_3$)
$f_{NL}{}^2 B^{nG}_{fNL2}$
$g_{NL} B^{nG}_{gNL}$
$f_{NL} B^{nG}_{fNL}$
$f_{NL}=4, g_{NL}=100, b_2/b_1=1.5$
Elongated triangle ($k_1=k_2+k_3$, $k_2=3k_3$)
z=3
Folded triangle ($k_1=2k_2=2k_3$)
B^G_m
$b_2/b_1 P^2$
Isosceles triangle ($3k_1=4k_2=4k_3$)
Equilateral triangle ($k_1=k_2=k_3$)
Squeezed triangle with R=(1,2,5,10)
R [Mpc/h] M_{halo} [$M_\odot$]
1 4×10^{11}
2 3×10^{12}
5 4×10^{13}
10 4×10^{14}
Bispectrum components [Mpc/h]^6
k_1 [h/Mpc]

Summary

- We have shown that the bispectrum of peaks is not only sensitive to the bispectrum of underlying matter density field, but also to the **trispectrum**.
- This gives us a chance of:
  - improving the limit on f_{NL} significantly, much better than previously recognized in Sefusatti & Komatsu,
  - measuring the next-to-leading order term, g_{NL}, and
  - testing more details of the physics of inflation! Discovery of $\tau_{NL} \neq f_{NL}^2$ would be very exciting...

Conference Summary

Past Decade and Coming Decade

Salopek-Bond (1990)

δN (1996)

- We are following the bold paths taken by the giants
- Now, a lot of young people are contributing to push this field forward

Past Decade and Coming Decade

"I do not think that it is worth spending my time on non-Gaussianism."

Bond (Feb 2002, Toronto)

Salopek-Bond (1990)

δN (1996)

- We are following the bold paths taken by the giants
- Now, a lot of young people are contributing to push this field forward

Past Decade and Coming Decade

Salopek-Bond (1990)

"For someone who understands inflation, it was obvious that non-Gaussianity should be completely negligible."

Sasaki (Oct 2008, Munich)

δN (1996)

- We are following the bold paths taken by the giants
- Now, a lot of young people are contributing to push this field forward

Multi-field Paradise

- Detection of the local-form f_{NL} is a smoking-gun for multi-field inflation.
- Very rich phenomenology, e.g., "preheating surprise"
  - Different observational consequences, **especially for signatures on non-Gaussianity**
  - Other signatures, e.g., tilt, tensor modes, isocurvature, are not as powerful or rich as non-Gaussianity
- Dick and Misao are now convinced ;-)

“Why Constant f_{NL}?” Dick Asked

- As many people have repeatedly shown during this workshop, a constant f_{NL} is merely one of MANY possibilities.

F_{NL}, f_{NL}, and F_{NL} again

- Pre-f_{NL} Era (<2001)
- Gaussianity Tests = "Blind Test" Mode
- Basically, people assumed that the form of non-Gaussianity was a free function, and tested whether the data were consistent with Gaussianity.
- No limits on physical parameters.
- In a sense, f_{NL} was a free function, F_{NL}.

F_{NL}, f_{NL}, and F_{NL} again

Free Function
(Chaotic Situation)

f_{NL}

f_{NL}^{local} & $f_{NL}^{equilateral}$

f_{NL}^{local}, $f_{NL}^{equilateral}$, f_{NL}^{warm}, f_{NL}^{orthog}, etc

F_{NL}

Free Function Again?

Wish List (as of April 2009)

- f_{NL}^{local}
- $f_{NL}^{equilateral}$
- f_{NL}^{iso}
- $f_{NL}^{orthogonal}$
- $f_{NL}(\text{direction})$
- g_{NL}, τ_{NL}

- $R = R_c + A*\chi^2$
- $R = R_c + A*\chi + B*\chi^2$
- $R = R_c + A*R_c^2 + B*R_cS + C*S^2$
- $R = R_c + A*\chi_{\text{very-non-gaussian}}$
- $F_{NL} = \exp[-(\chi-\chi_0)^2/(2\sigma^2)]$
- $u_{NL}^{(1)}$, $u_{NL}^{(2)}$, $u_{NL}^{(3)}$

- Bumps and wiggles

Single-field Laboratory

- The "effective field theory of inflation" approach relates the observed bispectrum to the terms in the Lagrangian

$$S_\pi = \int d^4x\sqrt{-g}\left[\frac{1}{2}M_{\rm Pl}^2R - M_{\rm Pl}^2\dot{H}\left(\dot{\pi}^2 - \frac{(\partial_i\pi)^2}{a^2}\right) + 2M_2^4\left(\dot{\pi}^2 + \dot{\pi}^3 - \dot{\pi}\frac{(\partial_i\pi)^2}{a^2}\right) - \frac{4}{3}M_3^4\dot{\pi}^3 + \ldots\right]$$

- "*This is what people do for the accelerator experiment*" (L. Senatore)
- **A very strong motivation** to look for the triangles other than the local form, e.g., equilateral from the ghost condensate
  - A new shape found! ($f_{NL}^{orthogonal}$)

Observation: Current Status

- From the optimal bispectrum of WMAP5 (Senatore)
  - f_{NL}(local) = 38 ± 21 (68%CL)
  - f_{NL}(equil) = 155 ± 140 (68%CL)
  - f_{NL}(ortho) = –149 ± 110 (68%CL)
- From the large-scale structure (Seljak)
  - f_{NL}(local) = 31^{+16}_{-27} (68%CL)
- From the Minkowski Functionals (Takahashi)
  - f_{NL}(iso) = –5 ± 10 (68%CL)

Wish List (as of April 2009)

- f_{NL}^{local}
- $f_{NL}^{equilateral}$
- f_{NL}^{iso}
- $f_{NL}^{orthogonal}$
- f_{NL}(direction)
- g_{NL}, τ_{NL}

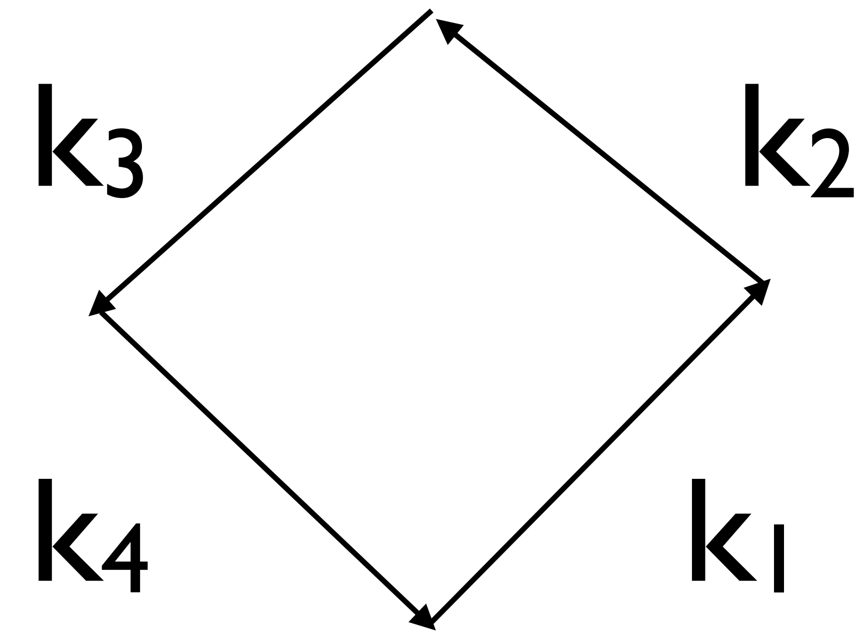
- $R = R_c + A*\chi^2$
- $R = R_c + A*\chi + B*\chi^2$
- $R = R_c + A*R_c^2 + B*R_cS + C*S^2$
- $R = R_c + A*\chi_{\text{very-non-gaussian}}$
- $F_{NL} = \exp[-(\chi-\chi_0)^2/(2\sigma^2)]$
- $u_{NL}^{(1)}$, $u_{NL}^{(2)}$, $u_{NL}^{(3)}$

- Bumps and wiggles

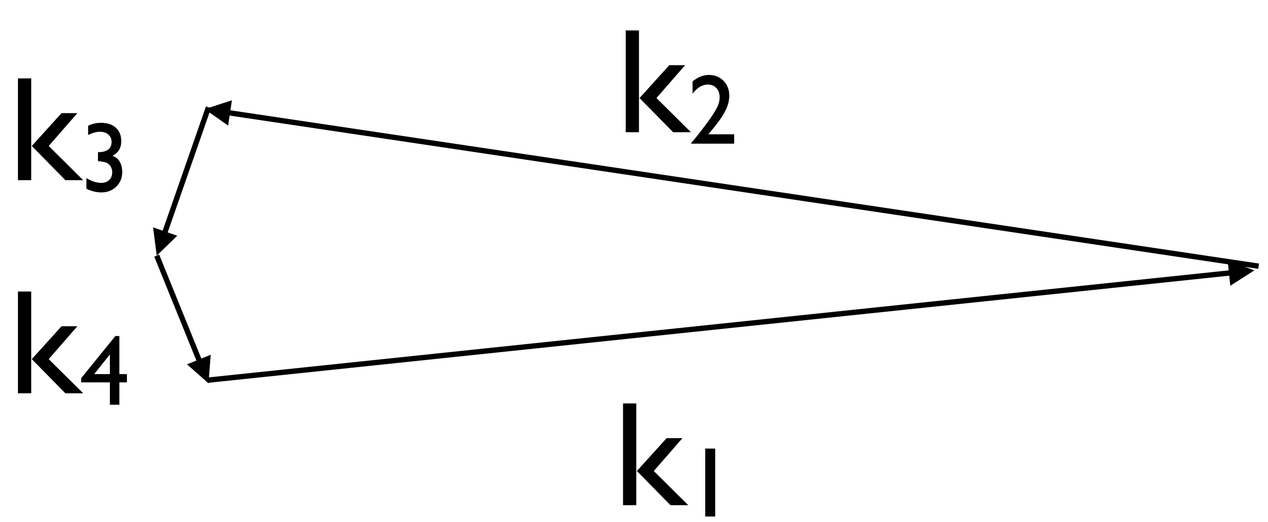
Trispectrum: Next Frontier

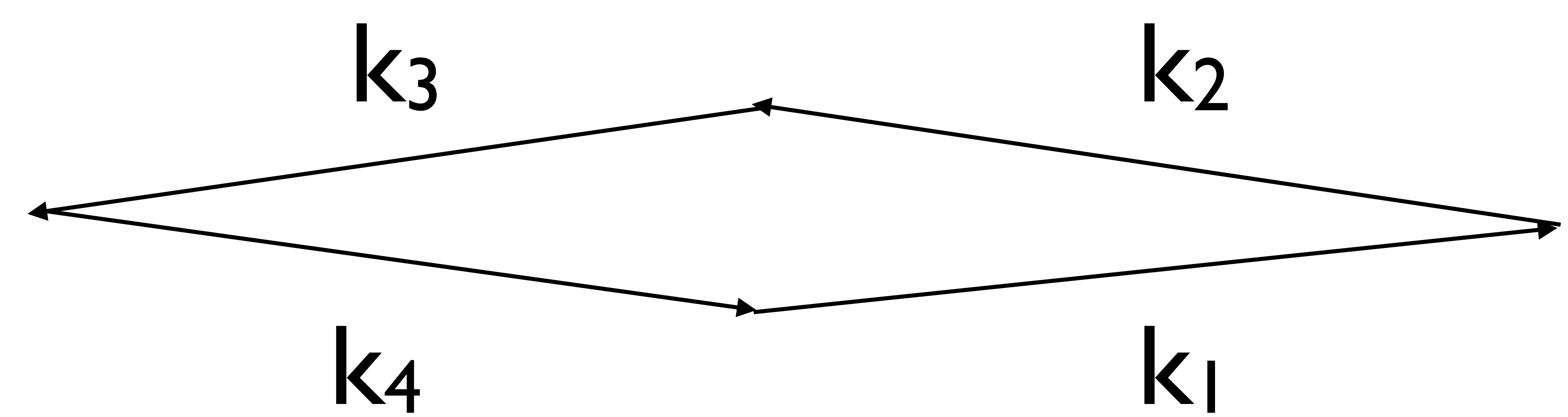
- A new phenomenon: many talks emphasized the importance of the trispectrum as a source of additional information on the physics of inflation.
- $\tau_{NL} \sim f_{NL}^2$; $\tau_{NL} \sim f_{NL}^{4/3}$; $\tau_{NL} \sim$ (isocurv.)*f_{NL}^2; $g_{NL} \sim f_{NL}$; $g_{NL} \sim f_{NL}^2$; or they are completely independent
- Shape dependence? (Squares from ghost condensate, diamonds and rectangles from multi-field, etc)

Playing with Quadrilaterals



Ghost condensate / DBI?





g_{NL} f_{NL}^2 (or τ_{NL})

BTW, how do we make plots of the trispectrum to see the shape dependence?

Beyond CMB: New Frontier

- **Galaxy Power Spectrum!**
  - $f_{NL}^{local} \sim 1$ within reach
- **Galaxy Bispectrum!**
  - τ_{NL} and g_{NL} can be probed
  - And other non-Gaussianity shapes
- **Galaxy Trispectrum?**
  - Worth doing?

Meet Mr. Seljak

- Conventional wisdom:
  - Cosmological measurements using the statistics of galaxies must, always, be affected by the **cosmic variance** and **shot noise**.
- Uros just showed that he can get rid of both: wow! Magic!

Don't Forget Real-world Issues

- **Messy second-order effects**
  - Non-linear evolution of CDM perturbations
  - Light propagation at the second order (SW, ISW, lensing, etc)
  - Crinkles in the surface of last scattering surface
    - Wandelt vs Senatore (reached an agreement?)
  - Brute-force! All the products of first-order quantities

Don't Forget Real-world Issues

- **Messy second-order effects: Goal**
  - Include ALL of the second-order effects
    - including polarization
  - Is the second-order effect detectable at all?
  - What is the contamination for f_{NL}^{local}, f_{NL}^{equil}, etc?
    - I.e., if Planck measurement gives $f_{NL}^{local}=10$, is the primordial 11? 9? 9.5?

Discovery Space

- "Targeted search" of non-Gaussianity (e.g., f_{NL}) is **powerful**, but is often limited and restricted to one's prejudice (a.k.a. theories)
- The "blind search" approach should not be abandoned
    - Lessons from the past: cold spots, violation of statistical isotropy, etc
- Planck data! The polarization data will help us clarify the situation enormously.
    - E.g., texture interpretation = lack of polarization around the Cold Spot

Summary of Summary

- Non-Gaussianity is a rapidly evolving, rich subject
- Unusually healthy interactions between observers and theorists: astronomers, cosmologists, phenomenologists, high-energy theorists
  - The list of the participants speaks for its diversity
  - Interdisciplinary efforts
- Lots of important contributions from young people
- Let our successes continue!

Now, let's pray:

- May Planck succeed!

Now, let's pray:

- May the signal be there!

Let’s thank the organizers

- Thank you Shinji and Lev for organizing such a wonderful workshop!

And, see you in late June for the IPMU Dark Energy Conference!

http://member.ipmu.jp/darkenergy09/welcome.html