New Probes of Initial State of Quantum Fluctuations during Inflation

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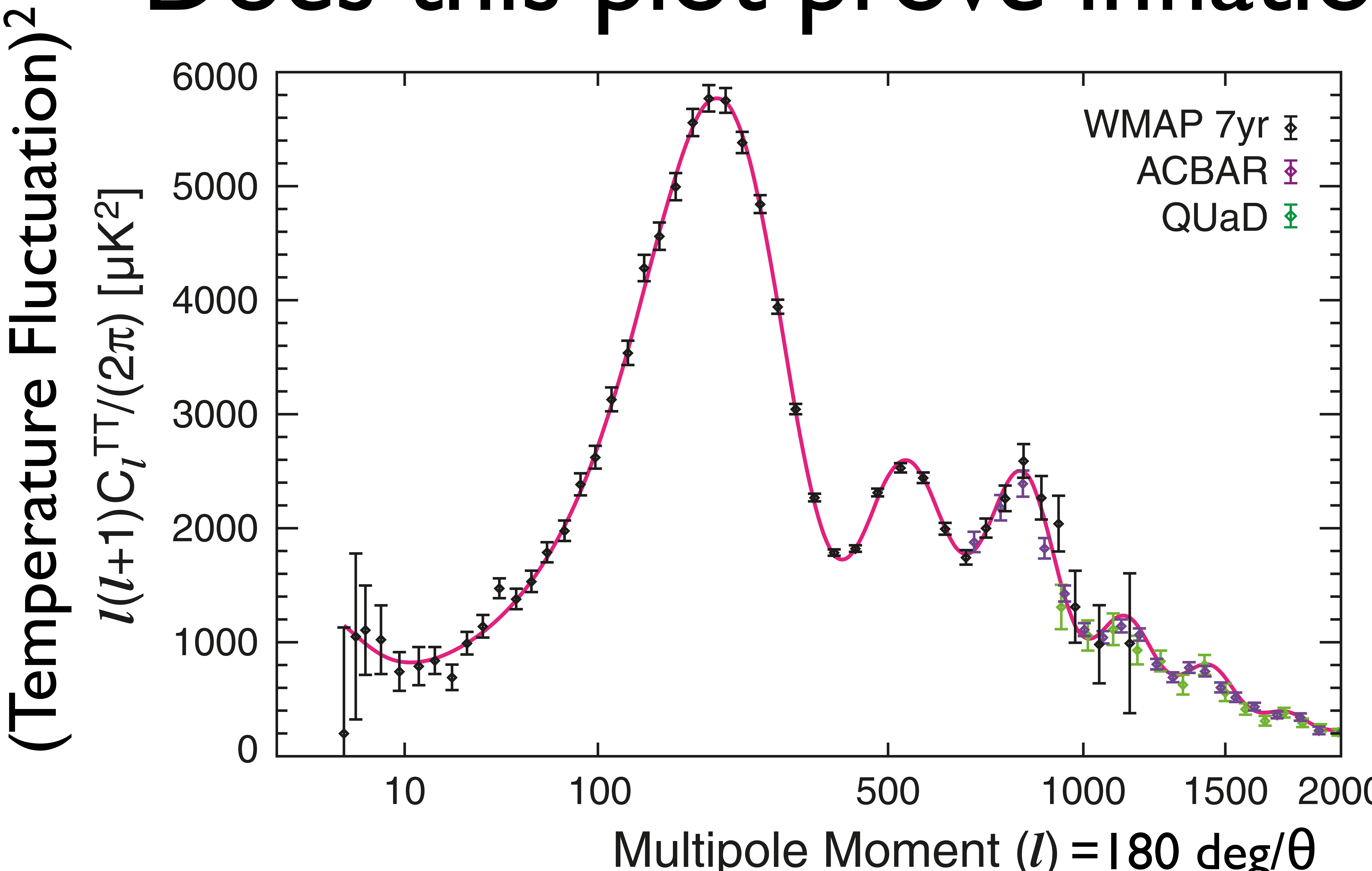
Perimeter Institute, May 22, 2012

This talk is based on...

- Squeezed-limit bispectrum
  - *Ganc & Komatsu, JCAP, 12, 009 (2010)*
- Non-Bunch-Davies vacuum and CMB
  - *Ganc, PRD 84, 063514 (2011)*
- Scale-dependent bias and μ-distortion
  - *Ganc & Komatsu, arXiv:1204.4241*

Does this plot prove inflation?



Motivation

- Can we falsify inflation?

Falsifying “inflation”

- We still need inflation to explain the flatness problem!
  - (Homogeneity problem can be explained by a bubble nucleation.)
- However, the observed fluctuations may come from different sources.
- So, what I ask is, “can we rule out inflation as a mechanism for generating the observed fluctuations?”

First Question:

- Can we falsify **single-field** inflation?

**I will not be talking about multi-field inflation today:*
for potentially ruling out multi-field inflation, see
Sugiyama, Komatsu & Futamase, PRL, 106, 251301 (2011)

An Easy One: Adiabaticity

- Single-field inflation = One degree of freedom.
  - Matter and radiation fluctuations originate from a single source.

$$\mathcal{S}_{c,\gamma} \equiv \underbrace{\frac{\delta\rho_c}{\rho_c}}_{\text{Cold Dark Matter}} - \underbrace{\frac{3\delta\rho_\gamma}{4\rho_\gamma}}_{\text{Photon}} = 0$$

* A factor of 3/4 comes from the fact that, in thermal equilibrium, $\rho_c \sim \rho_\gamma^{3/4}$

Komatsu et al. (2011)

Non-adiabatic Fluctuations

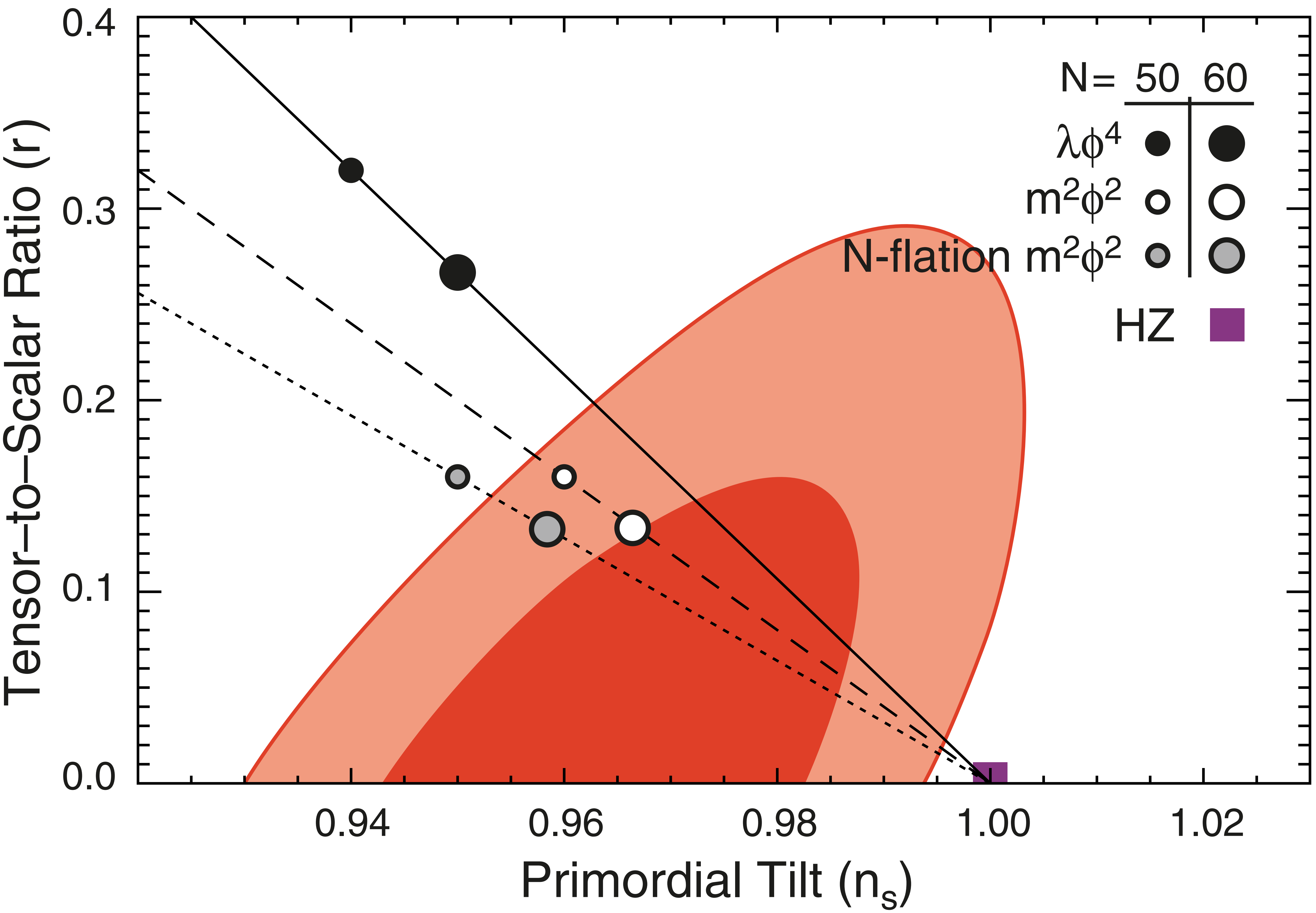
- Detection of non-adiabatic fluctuations immediately rule out single-field inflation models.

The current CMB data are consistent with adiabatic fluctuations:

$$\frac{|\delta\rho_c/\rho_c - 3\delta\rho_\gamma/(4\rho_\gamma)|}{\frac{1}{2}[\delta\rho_c/\rho_c + 3\delta\rho_\gamma/(4\rho_\gamma)]} < 0.09 \quad \text{(95\% CL)}$$

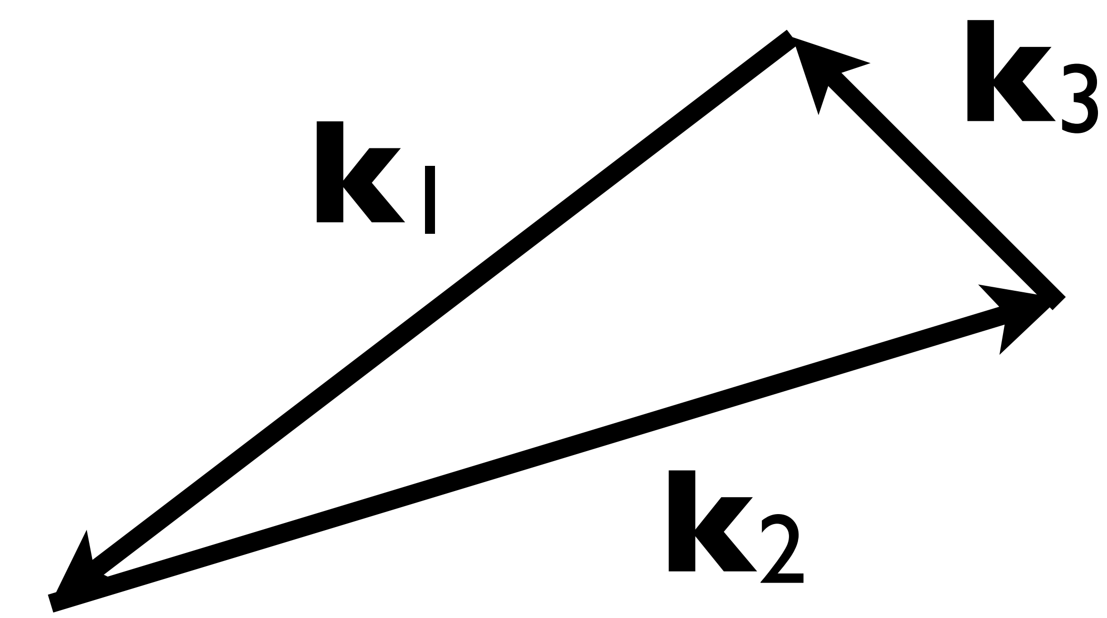
Komatsu et al. (2011)

Single-field inflation looks good (in 2-point function)

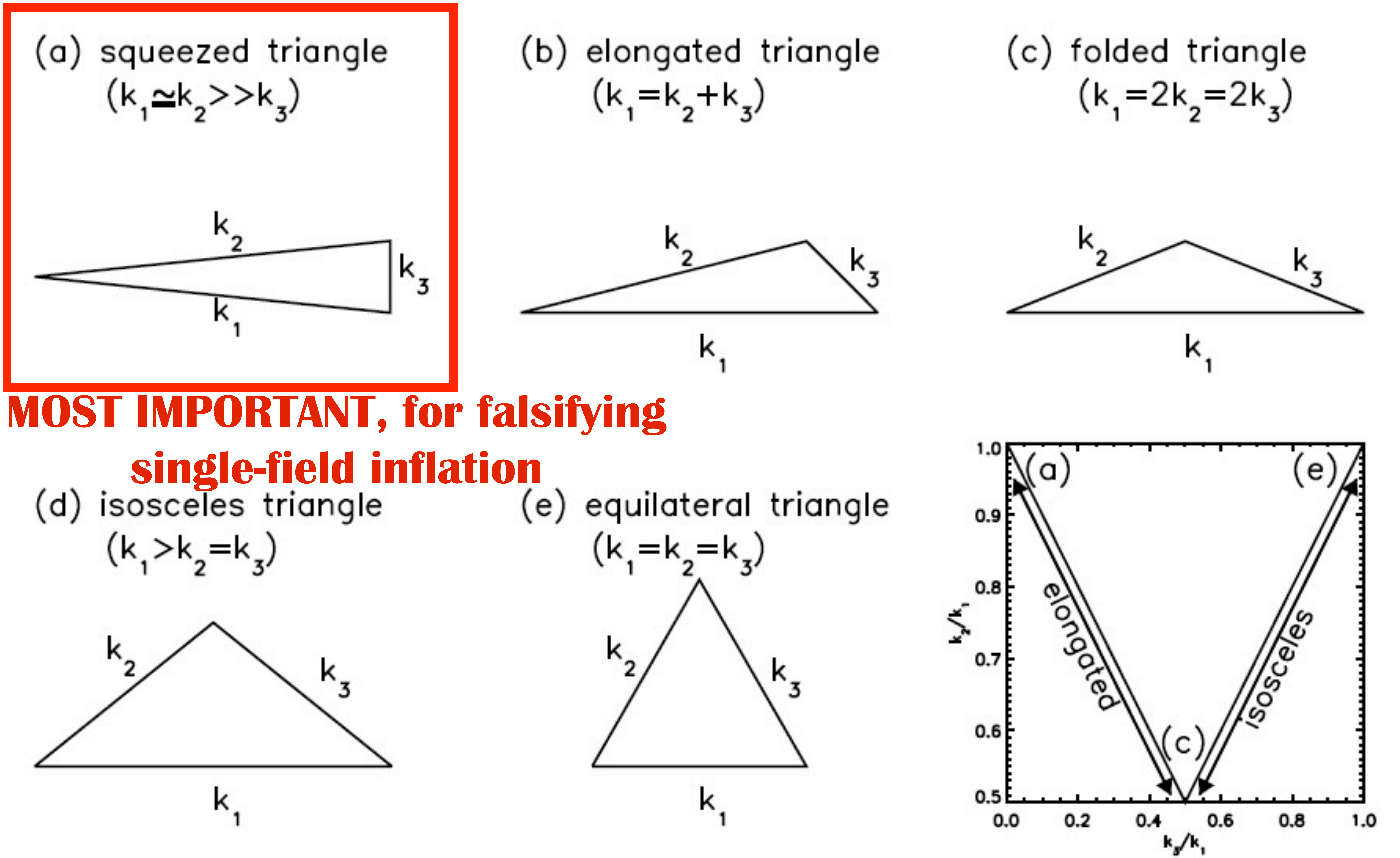


- $P_{scalar}(k) \sim k^{4-\mathbf{ns}}$
  - **$\mathbf{n_s}$=0.968±0.012** (68%CL; WMAP7+BAO+H_0)
- **r**=$4P_{tensor}(k)/P_{scalar}(k)$
  - **r < 0.24** (95%CL; WMAP7+BAO+H_0)

So, let's use 3-point function



- Three-point function (bispectrum)
- $B_\zeta(\mathbf{k}_1,\mathbf{k}_2,\mathbf{k}_3)$
  $= \langle\zeta_{\mathbf{k}1}\zeta_{\mathbf{k}2}\zeta_{\mathbf{k}3}\rangle$ = (amplitude) x $(2\pi)^3\delta(\mathbf{k}_1+\mathbf{k}_2+\mathbf{k}_3)b(k_1,k_2,k_3)$
  model-dependent function


(a) squeezed triangle
($k_1 \simeq k_2 >> k_3$)
k_2
k_3
k_1
(b) elongated triangle
($k_1 = k_2 + k_3$)
k_2
k_3
k_1
(c) folded triangle
($k_1 = 2k_2 = 2k_3$)
k_2
k_3
k_1
MOST IMPORTANT, for falsifying single-field inflation
(d) isosceles triangle
($k_1 > k_2 = k_3$)
k_2
k_3
k_1
(e) equilateral triangle
($k_1 = k_2 = k_3$)
k_2
k_3
k_1
1.0
0.9
0.8
0.7
0.6
0.5
0.0
0.2
0.4
0.6
0.8
1.0
k_2/k_1
k_3/k_1
(a)
(e)
(c)
elongated
isosceles

Curvature Perturbation

- In the gauge where the energy density is uniform, $\delta\rho=0$, the metric on super-horizon scales ($k \ll aH$) is written as

$$ds^2 = -N^2(x,t)dt^2 + a^2(t)e^{2\zeta(x,t)}dx^2$$

  - We shall call ζ the "curvature perturbation."
  - This quantity is independent of time, $\zeta(x)$, on super-horizon scales for single-field models.
  - The lapse function, $N(x,t)$, can be found from the Hamiltonian constraint.

Action

- Einstein's gravity + a canonical scalar field:
- $S=(1/2)\int d^4x\sqrt{-g}\,[R-(\partial\Phi)^2-2V(\Phi)]$

Maldacena (2003)

Quantum-mechanical Computation of the Bispectrum

$$\langle \zeta^3(\bar{t}) \rangle = -i \int_{-(1-i\epsilon)\infty}^{\bar{t}} dt' \langle 0 | [\zeta^3(\bar{t}), H_I^{(3)}(t')] | 0 \rangle$$

$$\boxed{\begin{array}{l} \partial^2 \chi = \dfrac{\dot{\phi}^2}{2\dot{\rho}^2} \dot{\zeta} \\ H \equiv \dot{\rho} \end{array}}$$

$$S_{\rm int}^{(3)} = \int \frac{1}{4} \frac{\dot{\phi}^4}{\dot{\rho}^4} [e^{3\rho} \dot{\zeta}^2 \zeta + e^{\rho} (\partial\zeta)^2 \zeta] - \frac{\dot{\phi}^2}{\dot{\rho}^2} e^{3\rho} \dot{\zeta} \partial_i \chi \partial_i \zeta +$$

$$- \frac{1}{16} \frac{\dot{\phi}^6}{\dot{\rho}^6} e^{3\rho} \dot{\zeta}^2 \zeta + \frac{\dot{\phi}^2}{\dot{\rho}^2} e^{3\rho} \dot{\zeta} \zeta^2 \frac{d}{dt} \left[\frac{1}{2} \frac{\ddot{\phi}}{\dot{\phi}\dot{\rho}} + \frac{1}{4} \frac{\dot{\phi}^2}{\dot{\rho}^2} \right] + \frac{1}{4} \frac{\dot{\phi}^2}{\dot{\rho}^2} e^{3\rho} \partial_i \partial_j \chi \partial_i \partial_j \chi \zeta$$

$$+ f(\zeta) \left. \frac{\delta L}{\delta \zeta} \right|_1$$

Initial Vacuum State

$$\zeta_{\mathbf{k}}(t) = u_k(t)a_{\mathbf{k}} + u_k^*(t)a^\dagger_{-\mathbf{k}}$$

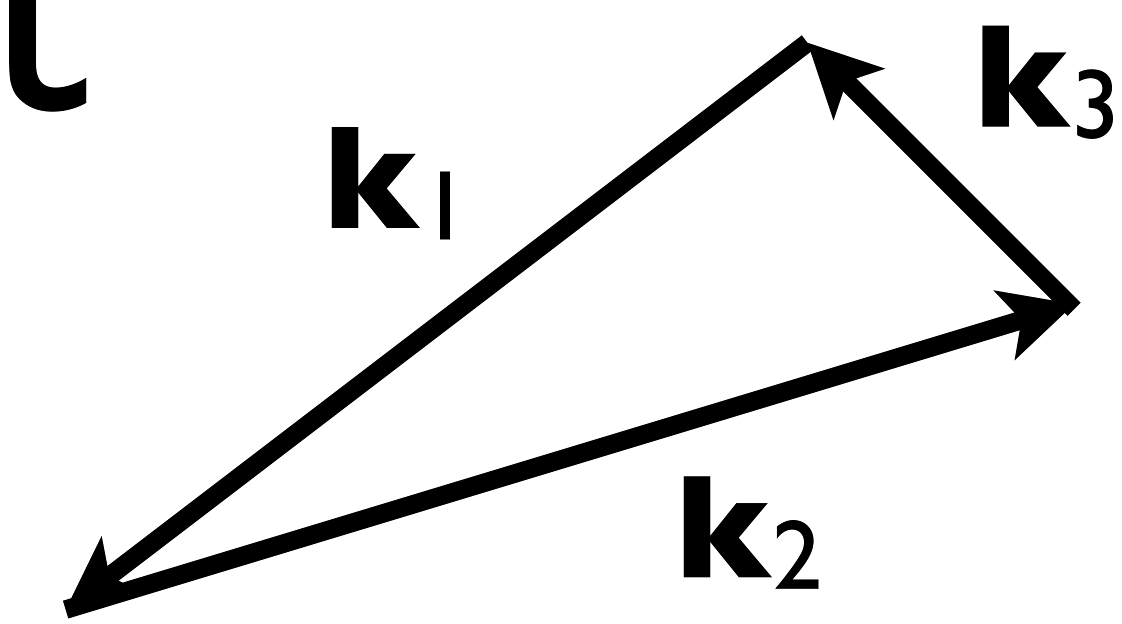
- Bunch-Davies vacuum, $a_{\mathbf{k}}|0\rangle = 0$ with

$$u_k(\eta) = \frac{H^2}{\dot{\phi}}\frac{1}{\sqrt{2k^3}}(1+ik\eta)e^{-ik\eta}$$

[η: conformal time]

Maldacena (2003)

Result



- $B_\zeta(\mathbf{k}_1,\mathbf{k}_2,\mathbf{k}_3)$
$= \langle \zeta_{\mathbf{k}1}\zeta_{\mathbf{k}2}\zeta_{\mathbf{k}3} \rangle$ = (amplitude) x $(2\pi)^3\delta(\mathbf{k}_1+\mathbf{k}_2+\mathbf{k}_3)b(k_1,k_2,k_3)$

- $b(k_1,k_2,k_3) = \dfrac{\dot\rho_*^4}{\dot\phi_*^4}\dfrac{H_*^4}{M_{pl}^4}\dfrac{1}{\prod_i(2k_i^3)}$

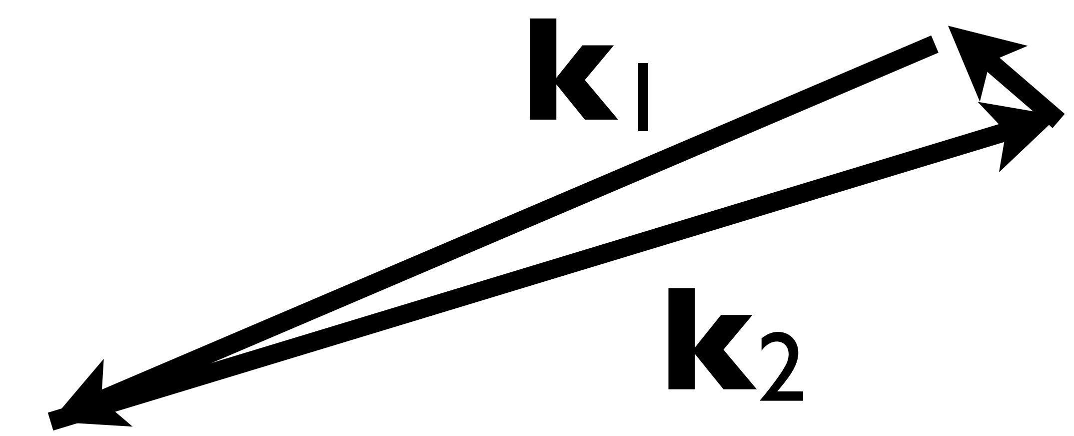
$$\boxed{H \equiv \dot\rho}$$

$$\times\left\{2\frac{\ddot\phi_*}{\dot\phi_*\dot\rho_*}\sum_i k_i^3 + \frac{\dot\phi_*^2}{\dot\rho_*^2}\left[\frac{1}{2}\sum_i k_i^3 + \frac{1}{2}\sum_{i\neq j} k_i k_j^2 + 4\frac{\sum_{i>j} k_i^2 k_j^2}{k_t}\right]\right\}$$

Complicated? But...

Maldacena (2003)

Taking the squeezed limit
($k_3 << k_1 \approx k_2$)



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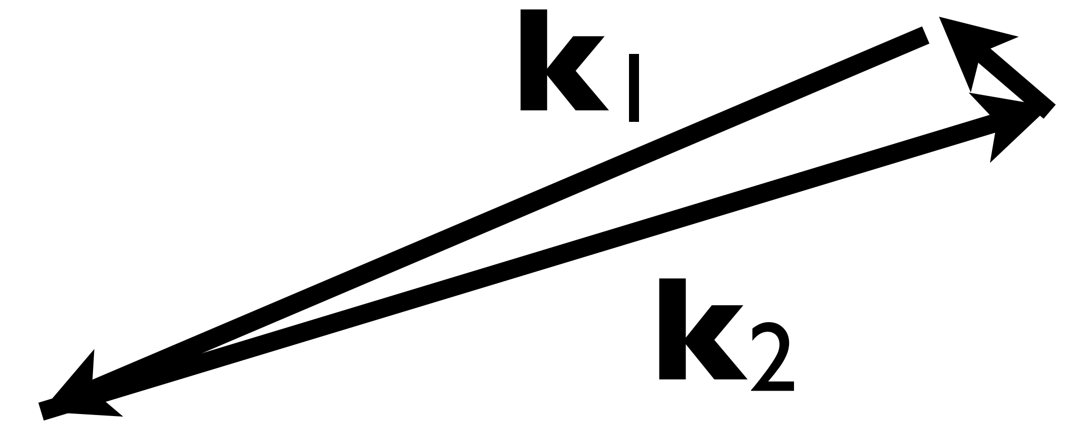
- $b(k_1,k_1,k_3\text{->}0)= \dfrac{\dot\rho_*^4}{\dot\phi_*^4}\dfrac{H_*^4}{M_{pl}^4}\dfrac{1}{\prod_i(2k_i^3)}$

$$\times\left\{2\frac{\ddot\phi_*}{\dot\phi_*\dot\rho_*}\underbrace{\sum_i k_i^3}_{2k_1^3}+\frac{\dot\phi_*^2}{\dot\rho_*^2}\left[\underbrace{\frac{1}{2}\sum_i k_i^3}_{k_1^3}+\underbrace{\frac{1}{2}\sum_{i\neq j}k_ik_j^2}_{k_1^3}+\underbrace{4\frac{\sum_{i>j}k_i^2k_j^2}{k_t}}_{2k_1^3}\right]\right\}$$

Taking the squeezed limit ($k_3 << k_1 \approx k_2$)



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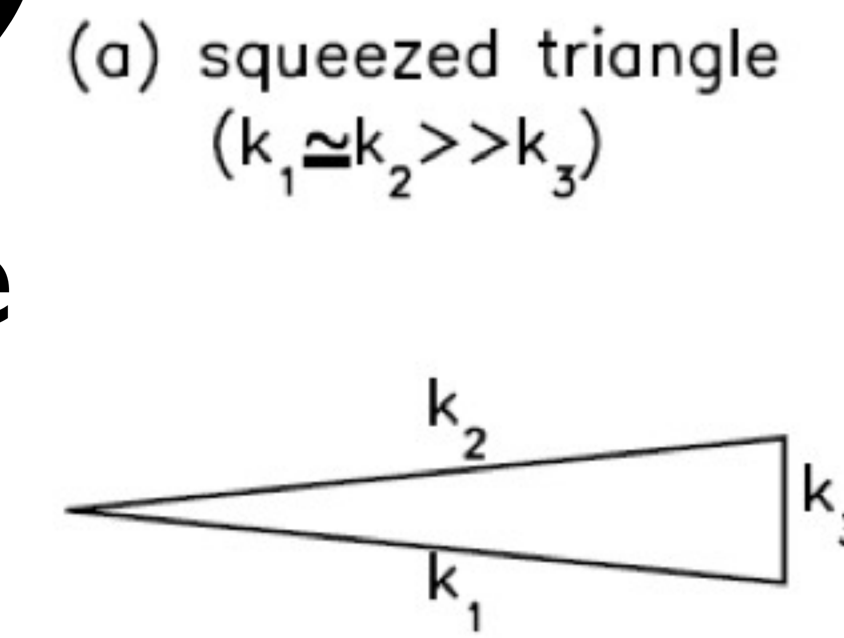
- $b(k_1,k_1,k_3\text{->}0) = \dfrac{\dot\rho_*^4}{\dot\phi_*^4}\dfrac{H_*^4}{M_{pl}^4}\underbrace{2\left[\dfrac{\ddot\phi_*}{\dot\phi_*\dot\rho_*}+\dfrac{\dot\phi_*^2}{\dot\rho_*^2}\right]}_{=1-n_s}\dfrac{1}{k_1{}^3k_3{}^3}$

$= (1-n_s)P_\zeta(k_1)P_\zeta(k_3)$

Maldacena (2003); Seery & Lidsey (2005); Creminelli & Zaldarriaga (2004)

Single-field Theorem (Consistency Relation)



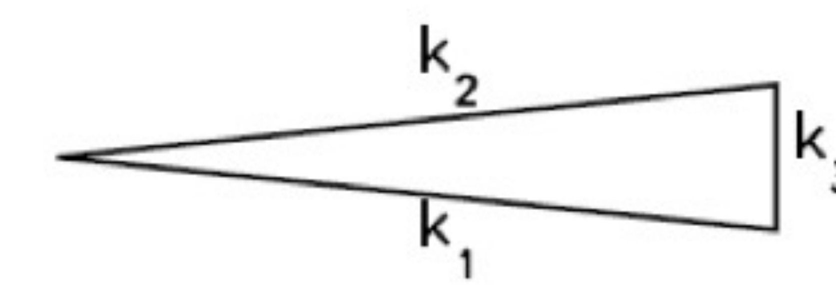
- For **<u>ANY</u>** single-field models*, the bispectrum in the squeezed limit ($k_3 << k_1 \approx k_2$) is given by
  - $B_\zeta(\mathbf{k}_1,\mathbf{k}_1,\mathbf{k}_3\text{->}0) = (1-n_s) \times (2\pi)^3\delta(\mathbf{k}_1+\mathbf{k}_2+\mathbf{k}_3) \times P_\zeta(k_1)P_\zeta(k_3)$

* for which the single field is solely responsible for driving inflation ***and*** generating observed fluctuations.

Maldacena (2003); Seery & Lidsey (2005); Creminelli & Zaldarriaga (2004)

Single-field Theorem (Consistency Relation)

(a) squeezed triangle ($k_1 \simeq k_2 >> k_3$)



- For **ANY** single-field models*, the bispectrum in the squeezed limit ($k_3 << k_1 \approx k_2$) is given by
  - $B_\zeta(\mathbf{k}_1,\mathbf{k}_1,\mathbf{k}_3 \to 0) = (1-n_s) \times (2\pi)^3\delta(\mathbf{k}_1+\mathbf{k}_2+\mathbf{k}_3) \times P_\zeta(k_1)P_\zeta(k_3)$

$$\boxed{\frac{6}{5} f_{NL} \equiv \frac{B_\zeta(k_1, k_2, k_3)}{P_\zeta(k_1)P_\zeta(k_2) + P_\zeta(k_2)P_\zeta(k_3) + P_\zeta(k_3)P_\zeta(k_1)}}$$

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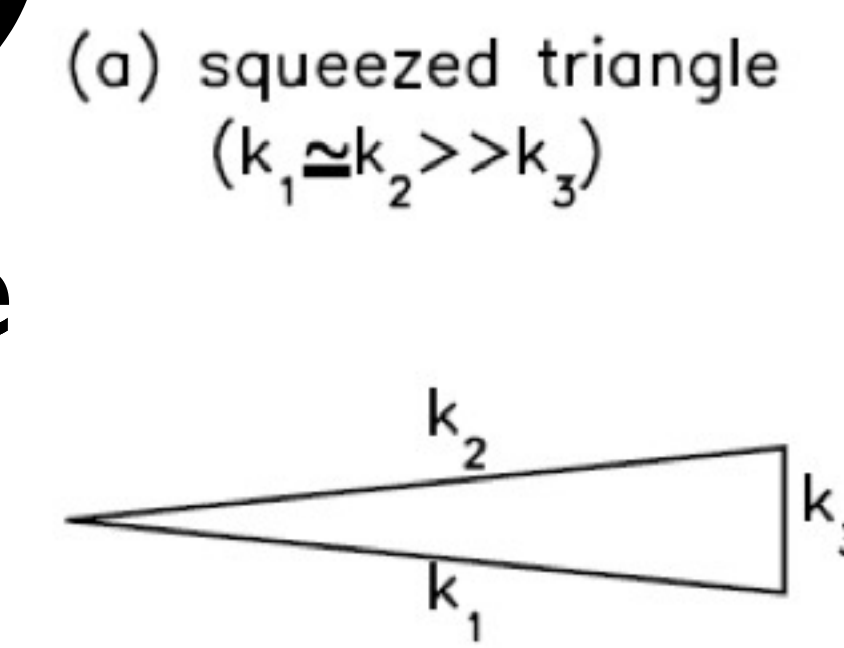


- For **ANY** single-field models*, the bispectrum in the squeezed limit ($k_3 << k_1 \approx k_2$) is given by
  - $B_\zeta(\mathbf{k}_1,\mathbf{k}_1,\mathbf{k}_3\to 0) = (1-n_s) \times (2\pi)^3\delta(\mathbf{k}_1+\mathbf{k}_2+\mathbf{k}_3) \times P_\zeta(k_1)P_\zeta(k_3)$
  - Therefore, all single-field models predict $f_{NL} \approx (5/12)(1-n_s)$.
  - With the current limit $n_s=0.96$, f_{NL} is predicted to be 0.017.

* for which the single field is solely responsible for driving inflation ***and*** generating observed fluctuations.

Limits on f_{NL}

$$\boxed{\frac{6}{5} f_{NL} \equiv \frac{B_\zeta(k_1, k_2, k_3)}{P_\zeta(k_1) P_\zeta(k_2) + P_\zeta(k_2) P_\zeta(k_3) + P_\zeta(k_3) P_\zeta(k_1)}}$$

When f_{NL} is independent of wavenumbers,
it is called the "**local type**."

Komatsu et al. (2011)

Limits on f_{NL}

$$\boxed{\frac{6}{5} f_{NL} \equiv \frac{B_\zeta(k_1, k_2, k_3)}{P_\zeta(k_1) P_\zeta(k_2) + P_\zeta(k_2) P_\zeta(k_3) + P_\zeta(k_3) P_\zeta(k_1)}}$$

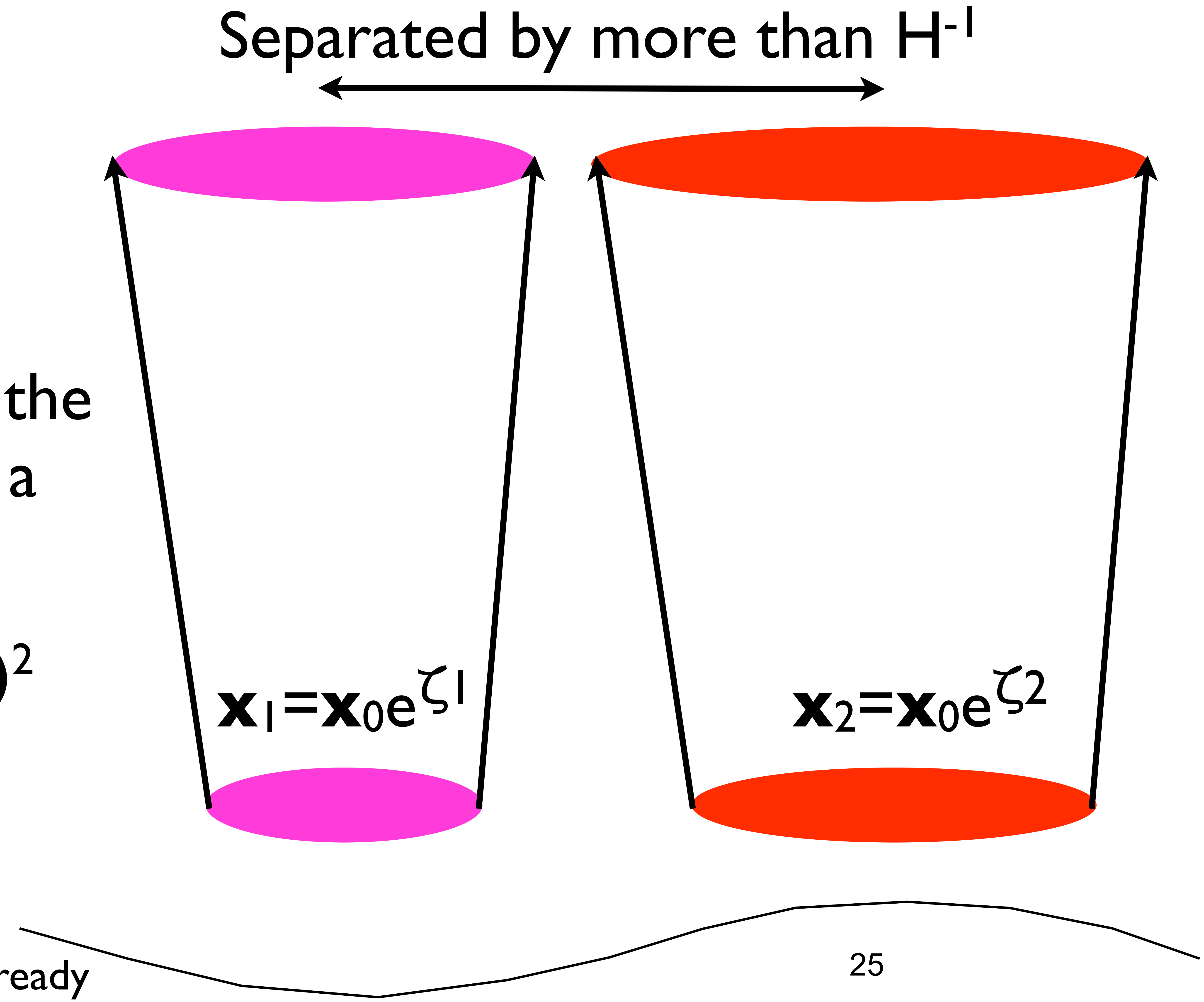
- **f_{NL} = 32 ± 21** (68%C.L.) from WMAP 7-year data
  - Planck's CMB data is expected to yield Δf_{NL}=5.
- **f_{NL} = 27 ± 16** (68%C.L.) from WMAP 7-year data combined with the limit from the large-scale structure (by Slosar et al. 2008)
  - Future large-scale structure data are expected to yield Δf_{NL}=1.

Understanding the Theorem

- First, the squeezed triangle correlates one very long-wavelength mode, k_L ($=k_3$), to two shorter wavelength modes, k_S ($=k_1 \approx k_2$):
    - $\langle \zeta_{\mathbf{k}1} \zeta_{\mathbf{k}2} \zeta_{\mathbf{k}3} \rangle \approx \langle (\zeta_{\mathbf{k}S})^2 \zeta_{\mathbf{k}L} \rangle$
- Then, the question is: "why should $(\zeta_{\mathbf{k}S})^2$ ever care about $\zeta_{\mathbf{k}L}$?"
    - The theorem says, "it doesn't care, if $\zeta_{\mathbf{k}}$ is exactly scale invariant."

$\zeta_{\mathbf{k}L}$ rescales coordinates

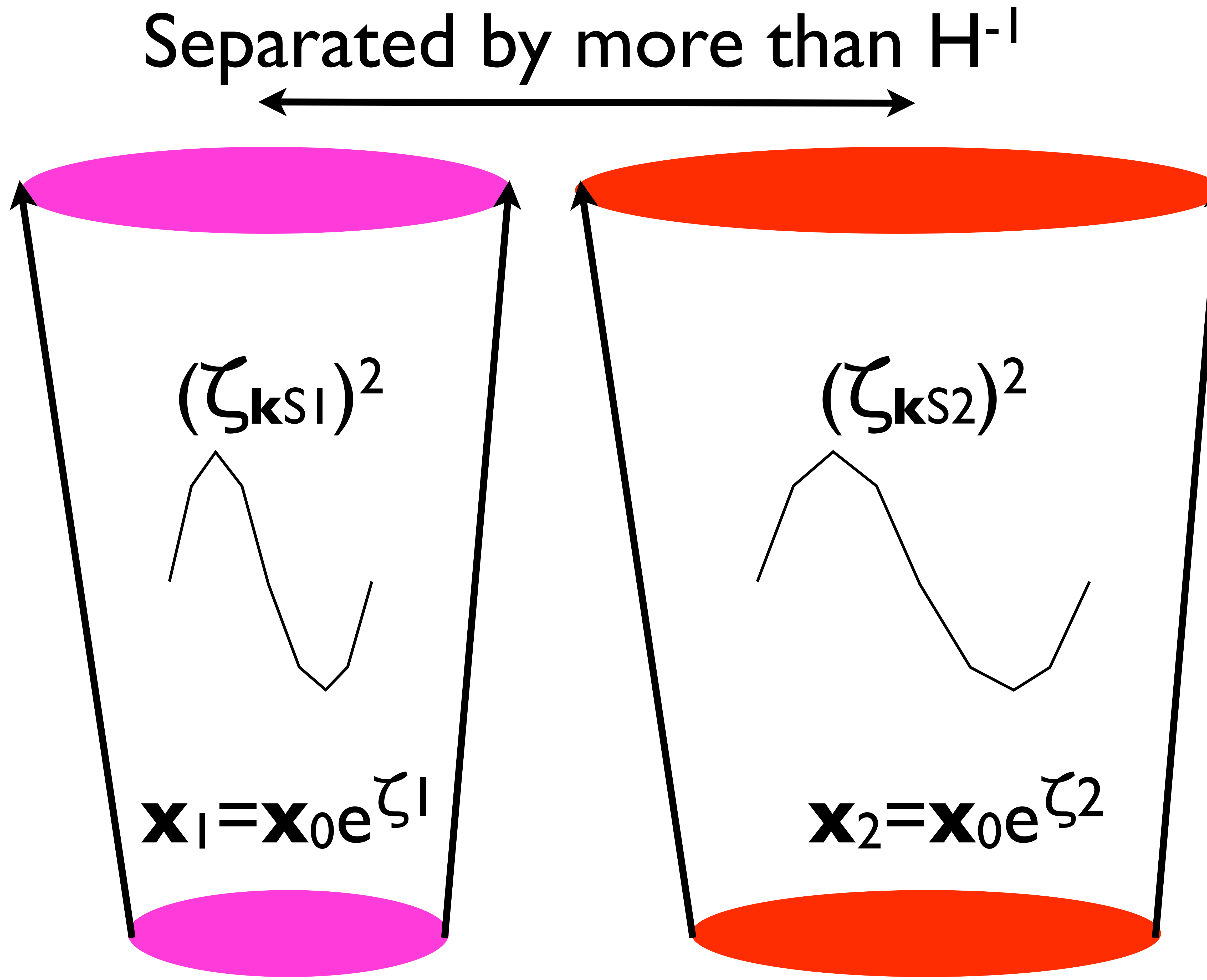
- The long-wavelength curvature perturbation rescales the spatial coordinates (or changes the expansion factor) within a given Hubble patch:
  - $ds^2 = -dt^2 + [a(t)]^2 e^{2\zeta} (d\mathbf{x})^2$

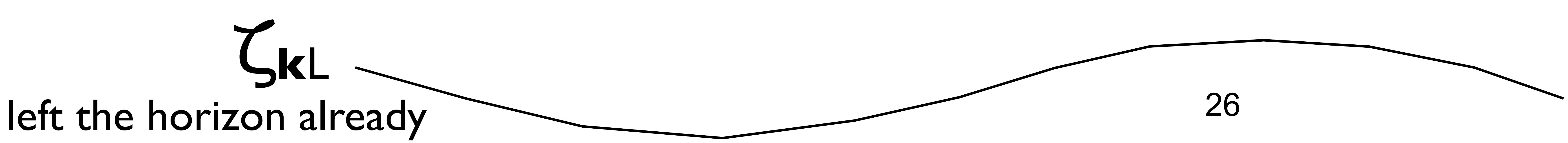


$\zeta_{\mathbf{k}L}$
left the horizon already

$\zeta_{\mathbf{k}L}$ rescales coordinates

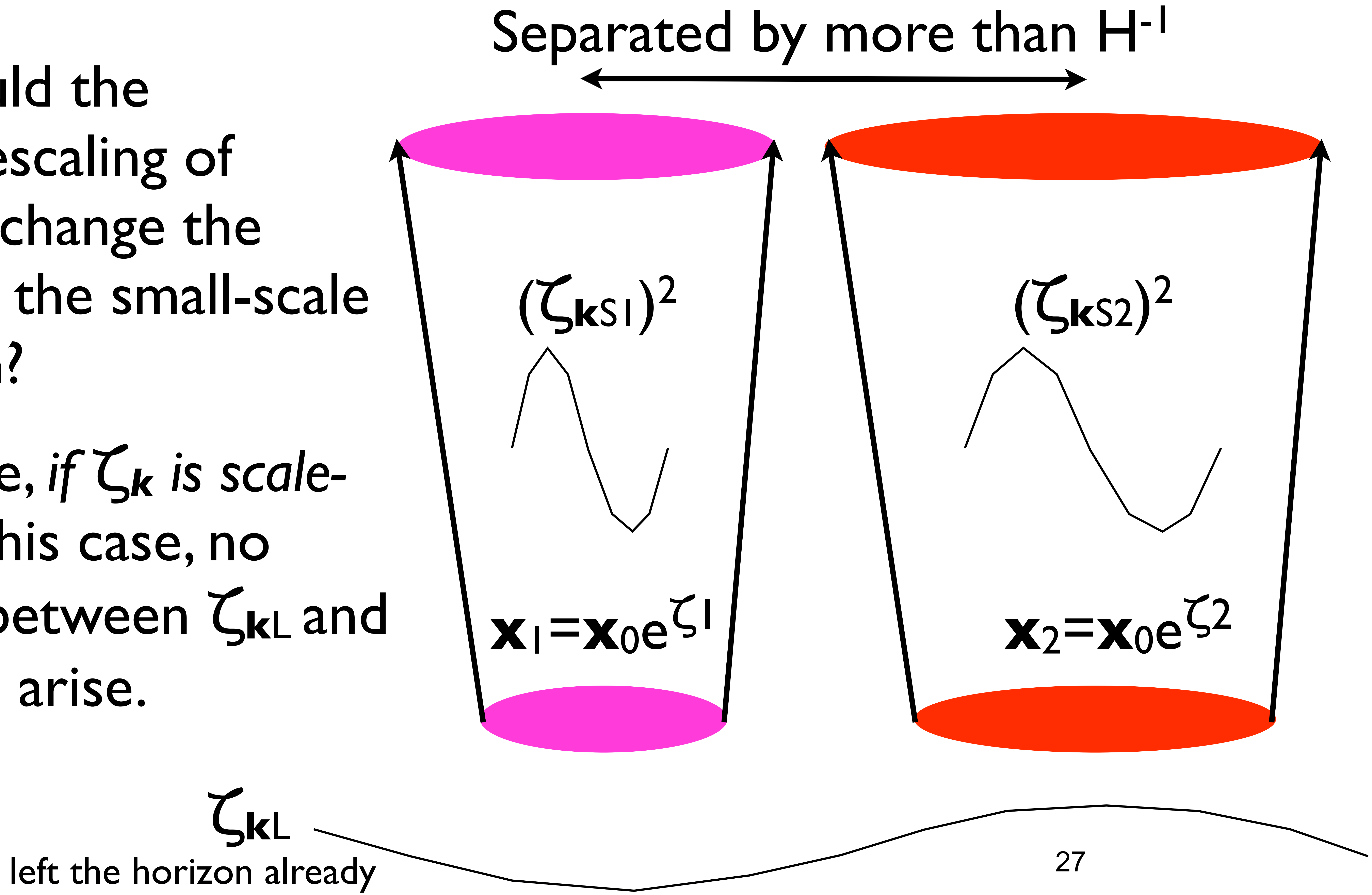
- Now, let's put small-scale perturbations in.
- Q. How would the conformal rescaling of coordinates change the amplitude of the small-scale perturbation?





$\zeta_{\mathbf{k}L}$ rescales coordinates

- Q. How would the conformal rescaling of coordinates change the amplitude of the small-scale perturbation?
- A. No change, *if $\zeta_{\mathbf{k}}$ is scale-invariant*. In this case, no correlation between $\zeta_{\mathbf{k}L}$ and $(\zeta_{\mathbf{k}S})^2$ would arise.

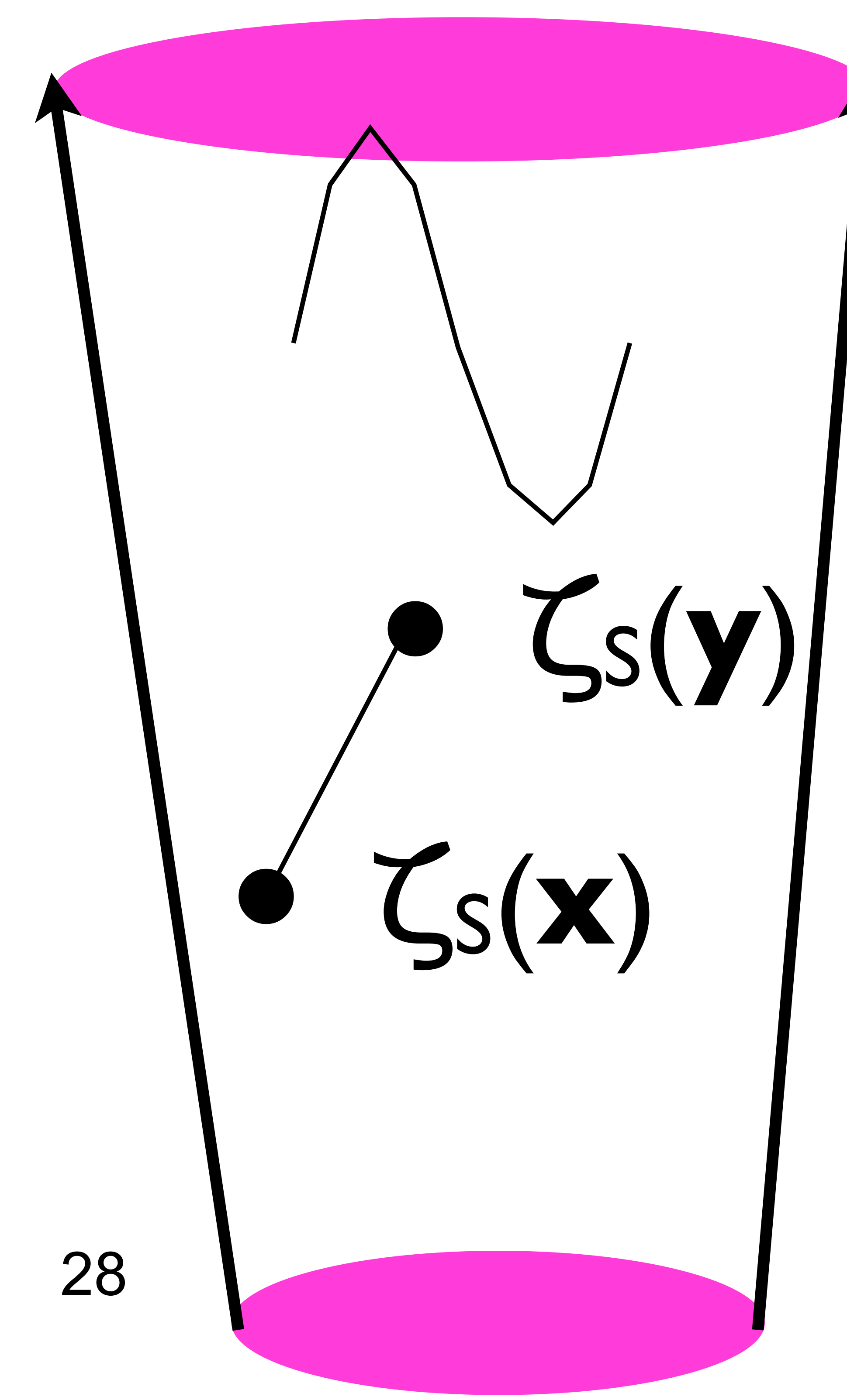


Creminelli & Zaldarriaga (2004); Cheung et al. (2008)

Real-space Proof

- The 2-point correlation function of short-wavelength modes, $\xi = \langle \zeta_S(\mathbf{x}) \zeta_S(\mathbf{y}) \rangle$, within a given Hubble patch can be written in terms of its vacuum expectation value (in the absence of ζ_L), ξ_0, as:

  - $\xi_{\zeta L} \approx \xi_0(|\mathbf{x}-\mathbf{y}|) + \zeta_L \, [d\xi_0(|\mathbf{x}-\mathbf{y}|)/d\zeta_L]$
  - $\xi_{\zeta L} \approx \xi_0(|\mathbf{x}-\mathbf{y}|) + \zeta_L \, [d\xi_0(|\mathbf{x}-\mathbf{y}|)/d\ln|\mathbf{x}-\mathbf{y}|]$
  - $\xi_{\zeta L} \approx \xi_0(|\mathbf{x}-\mathbf{y}|) + \zeta_L \, (1-n_s)\xi_0(|\mathbf{x}-\mathbf{y}|)$



$$3\text{-pt func.} = \langle (\zeta_S)^2 \zeta_L \rangle = \langle \xi_{\zeta L} \zeta_L \rangle$$
$$= (1-n_s)\xi_0(|\mathbf{x}-\mathbf{y}|) \langle \zeta_L^2 \rangle$$

This is great, but...

- The proof relies on the following Taylor expansion:
  - $<\zeta_S(\mathbf{x})\zeta_S(\mathbf{y})>_{\zeta_L} = <\zeta_S(\mathbf{x})\zeta_S(\mathbf{y})>_0 + \zeta_L\,[d<\zeta_S(\mathbf{x})\zeta_S(\mathbf{y})>_0/d\zeta_L]$
- Perhaps it is interesting to show this explicitly using the in-in formalism.
  - Such a calculation would shed light on the limitation of the above Taylor expansion.
  - Indeed it did - we found a non-trivial "counter-example" (more later)

An Idea

- How can we use the in-in formalism to compute the two-point function of short modes, given that there is a long mode, $\langle\zeta_S(\mathbf{x})\zeta_S(\mathbf{y})\rangle_{\zeta_L}$?
- Here it is!

$$\langle \zeta_S^2(\bar{t}) \rangle_{\zeta_L} = -i \int_{-(1-i\epsilon)\infty}^{\bar{t}} dt' \langle 0 | [\zeta_S^2(\bar{t}), H_I^{(3)}(t')] | 0 \rangle$$

Ganc & Komatsu, JCAP, 12, 009 (2010)

Long-short Split of H_I

$$\langle \zeta_S^2(\bar{t}) \rangle_{\zeta_L} = -i \int_{-(1-i\epsilon)\infty}^{\bar{t}} dt' \langle 0 | [\zeta_S^2(\bar{t}), H_I^{(3)}(t')] | 0 \rangle$$

- Inserting $\zeta = \zeta_L + \zeta_S$ into the cubic action of a scalar field, and retain terms that have one ζ_L and two ζ_S's.

$$S_{\rm int}^{(3)} = \int d^4x \left[\left(\frac{1}{4}\frac{\dot\phi_0^4}{H^4} - \frac{1}{16}\frac{\dot\phi_0^6}{H^6} \right) a^3 \zeta_L \dot\zeta_S^2 + \frac{1}{4}\frac{\dot\phi_0^4}{H^4} a \zeta_L (\partial\zeta_S)^2 - \frac{\dot\phi_0^4}{2H^4} a^3 \dot\zeta_S \partial_i \zeta_S \partial_i \partial^{-2} \dot\zeta_L + \right.$$
$$+ \frac{1}{16}\frac{\dot\phi_0^6}{H^6} a^3 \partial_i\partial_j\partial^{-2}\dot\zeta_S\, \partial_i\partial_j\partial^{-2}\dot\zeta_S\, \zeta_L + 2\frac{\dot\phi_0^2}{H^2} a^3 \zeta_L \frac{d}{dt}\left[\frac{1}{2}\frac{\ddot\phi_0}{\dot\phi_0 H} + \frac{1}{4}\frac{\dot\phi_0^2}{H^2} \right] \dot\zeta_S \zeta_S$$
$$\left. - f(\zeta)\frac{\delta L_0}{\delta\zeta_S} \right],$$

Ganc & Komatsu, JCAP, 12, 009 (2010)

Result

$$\langle \zeta_{S,\mathbf{k}_1} \zeta_{S,\mathbf{k}_2} \rangle_{\zeta_{\mathbf{k}_3}} = \zeta_{L,\mathbf{k}_1+\mathbf{k}_2} \left[K + \left(\frac{\ddot{\phi}_0}{\dot{\phi}_0 H} + \frac{1}{2}\frac{\dot{\phi}_0^2}{H^2} \right) P(k_1) \right]$$

- where

$$K \equiv i u_{k_1}^2(\bar{\eta}) \int_{-\infty(1-i\epsilon)}^{\bar{\eta}} d\eta \left[\frac{1}{2}\frac{\dot{\phi}_0^4}{H^4} a^2 u_{k_1}^{\prime *2}(\eta) + \frac{1}{2}\frac{\dot{\phi}_0^4}{H^4} a^2 k_1^2 u_{k_1}^{*2}(\eta) + \right.$$
$$\left. + 2\frac{\dot{\phi}_0^2}{H^2} a^3 \frac{d}{dt}\left(\frac{\ddot{\phi}_0}{\dot{\phi}_0 H} + \frac{1}{2}\frac{\dot{\phi}_0^2}{H^2} \right) u_{k_1}^{\prime *}(\eta) u_{k_1}^{*}(\eta) \right] + \text{c.c.}$$

Result

- Although this expression looks nothing like $(1-n_S)P(k_1)\zeta_{kL}$, we have verified that it leads to the known consistency relation for (i) slow-roll inflation, and (ii) power-law inflation.
- But, there was a curious case – Alexei Starobinsky's exact $n_S=1$ model.
  - If the theorem holds, we should get a vanishing bispectrum in the squeezed limit.

Starobinsky's Model

- The famous Mukhanov-Sasaki equation for the mode function is

$$\frac{d^2 u_k}{d\eta^2} + \left(k^2 - \frac{1}{z}\frac{d^2 z}{d\eta^2}\right) u_k = 0$$

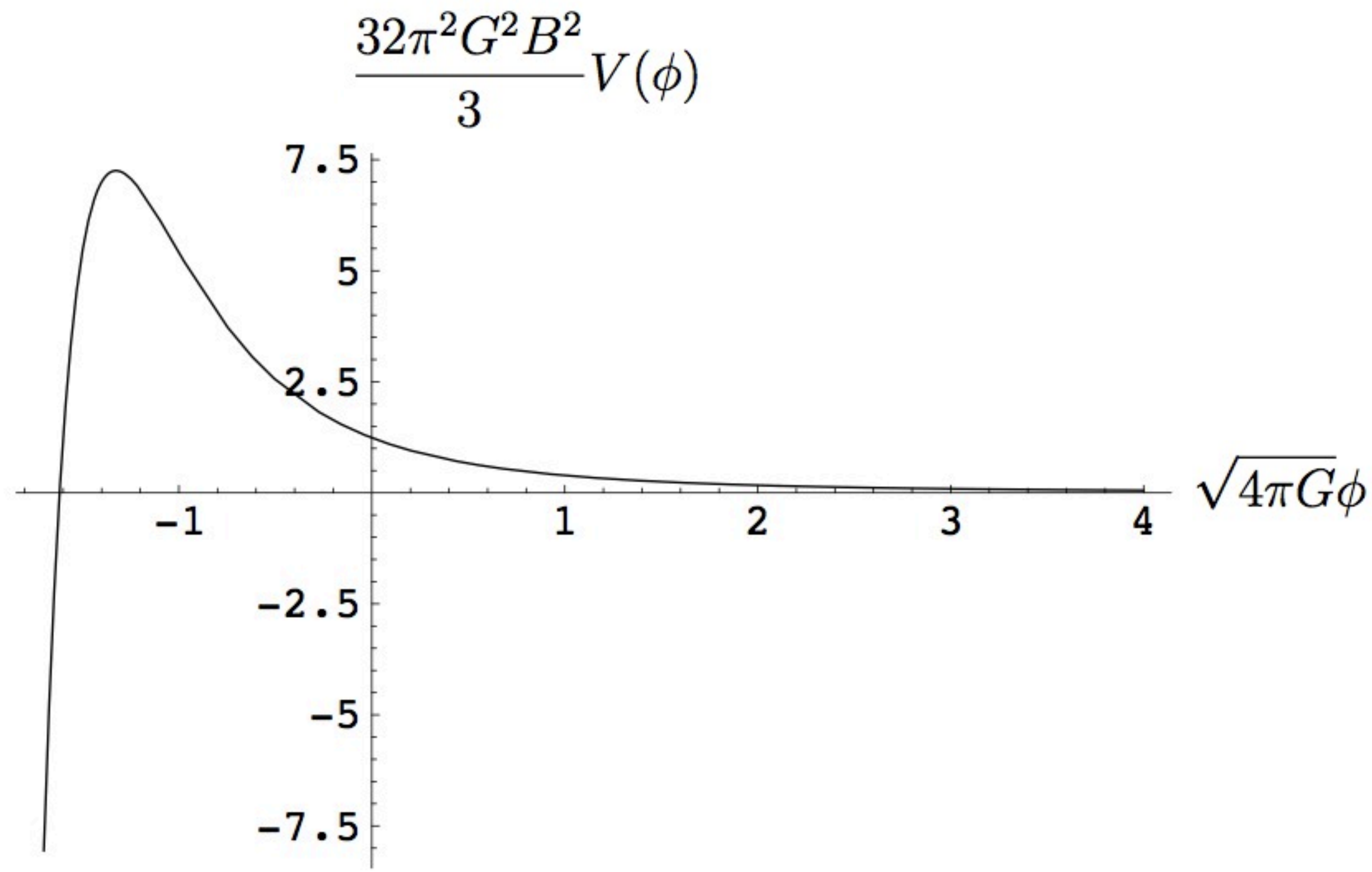
where

$$z = \frac{a\dot{\phi}}{H}$$

•The scale-invariance results when $\frac{1}{z}\frac{d^2 z}{d\eta^2} = \frac{2}{\eta^2}$

So, let's write **z=B/η**

Starobinsky's Potential



- This potential is a one-parameter family; this particular example shows the case where inflation lasts very long: $\varphi_{end} \to \infty$

Result

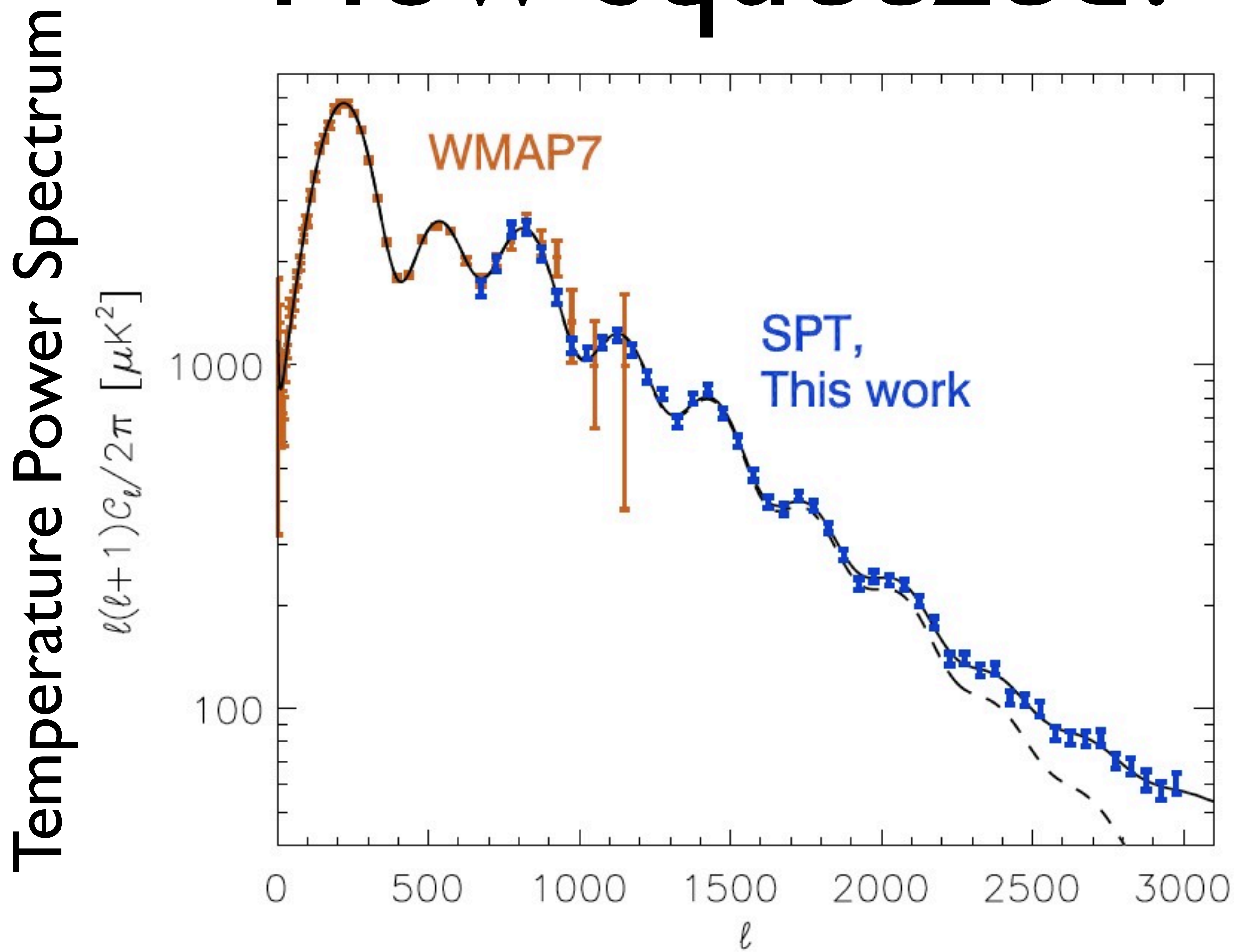
$$\langle \zeta_{S,\mathbf{k}_1} \zeta_{S,\mathbf{k}_2} \rangle_{\zeta_{\mathbf{k}_3}} = \zeta_{L,\mathbf{k}_1+\mathbf{k}_2} 4P(k_1)(k_1\eta_{\rm start})^2 e^{-\frac{1}{2}\phi_{\rm end}^2}$$

- **It does not vanish!**
- But, it approaches zero when $\Phi_{\rm end}$ is large, meaning the duration of inflation is very long.
  - In other words, this is a condition that **the longest wavelength that we observe, k_3, is far outside the horizon.**
  - In this limit, the bispectrum approaches zero.

Initial Vacuum State?

- What we learned so far:
  - The squeezed-limit bispectrum is proportional to $(1-n_S)P(k_1)P(k_3)$, provided that ζ_{k3} is far outside the horizon when k_1 crosses the horizon.
- What if the state that $\zeta_{\mathbf{k}3}$ sees is not a Bunch-Davies vacuum, but something else?
  - The exact squeezed limit (k_3->0) should still obey the consistency relation, but perhaps something happens when **k_3/k_1 is small but finite**.

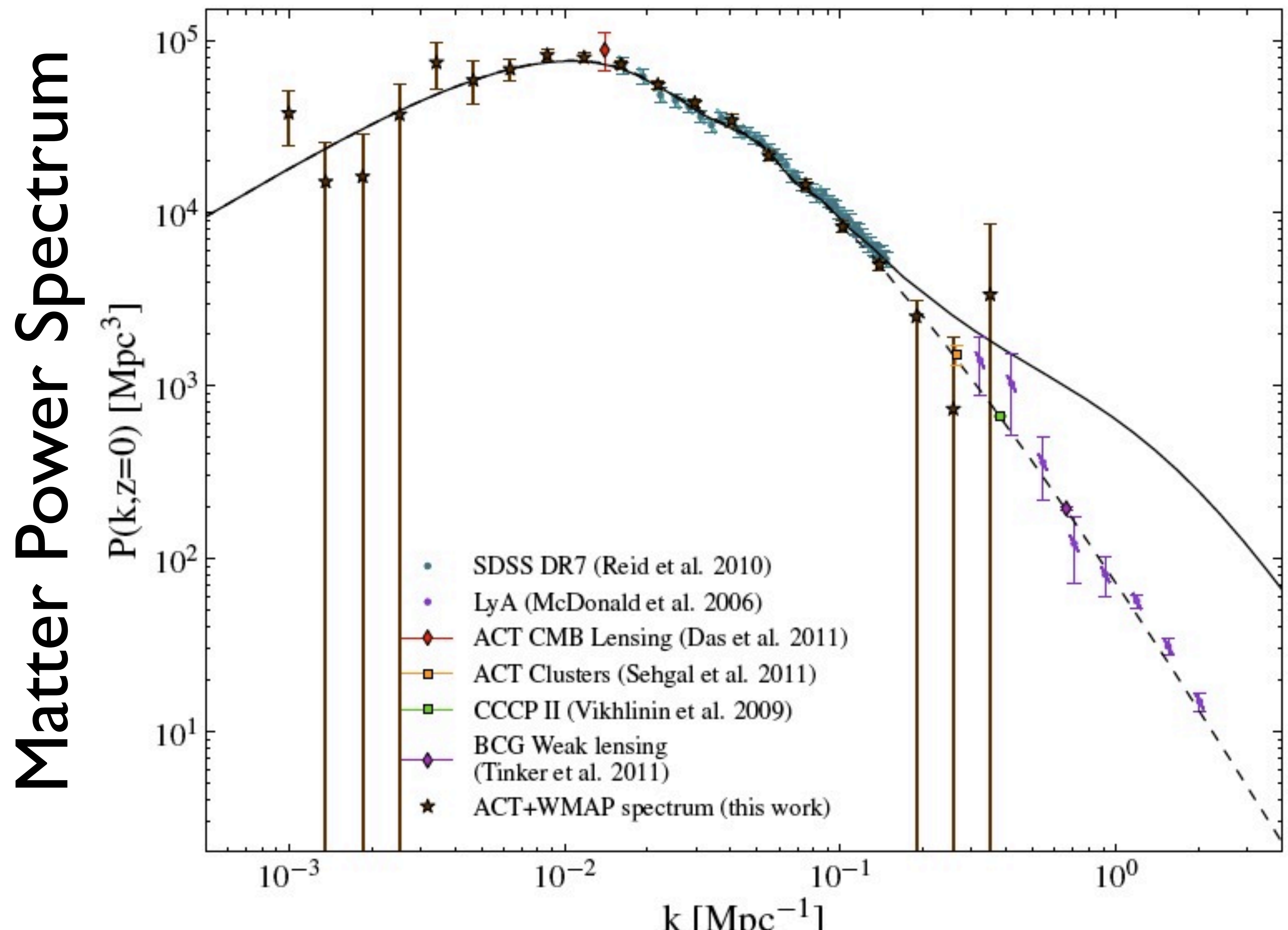
How squeezed?



Keisler et al. (2011)

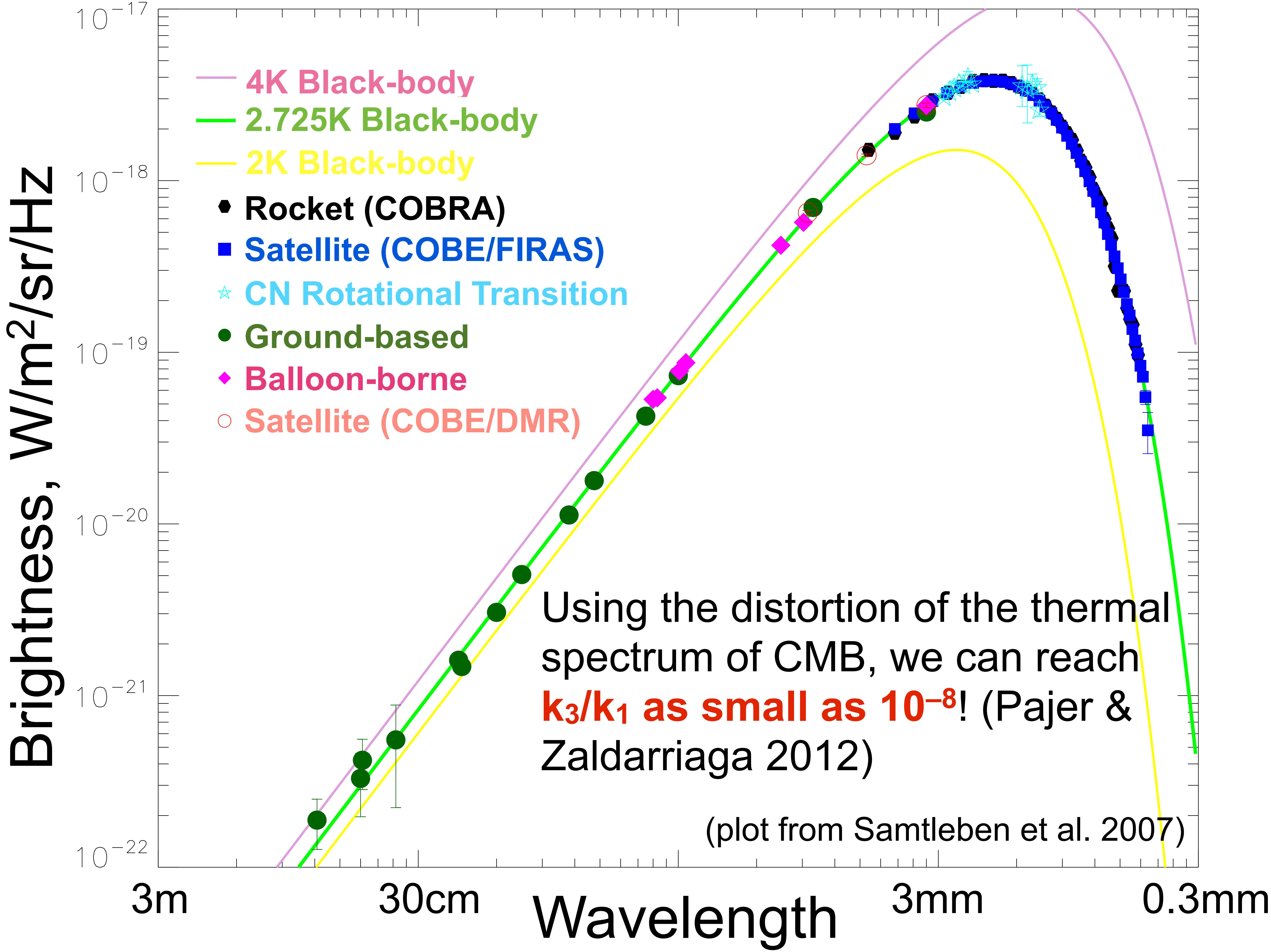
- With CMB, we can measure primordial modes in l=2–3000. Therefore, **k_3/k_1 can be as small as 1/1500**.

How squeezed?



Hlozek et al. (2011)

- With large-scale structure, we can measure primordial modes in $k=10^{-3}$–1 Mpc^{-1}. Therefore, **k_3/k_1 can be as small as 1/1000**.



4K Black-body
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Rocket (COBRA)
Satellite (COBE/FIRAS)
CN Rotational Transition
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Using the distortion of the thermal spectrum of CMB, we can reach k_3/k_1 as small as 10^{-8}! (Pajer & Zaldarriaga 2012)
(plot from Samtleben et al. 2007)
Brightness, W/m²/sr/Hz
10^{-17}
10^{-18}
10^{-19}
10^{-20}
10^{-21}
10^{-22}
3m
30cm
3mm
0.3mm
Wavelength

Back to in-in

$$\langle \zeta^3(t^*) \rangle = -i \int_{t_0}^{t^*} dt' \langle 0 | [\zeta^3(t^*), H_I(t')] | 0 \rangle$$

$$B_\zeta(k_1, k_2, k_3) = 2i \frac{\dot{\phi}^4}{H^6} \sum_i \left(\frac{1}{k_i^2} \right) \tilde{u}_{k_1}(\bar{\eta}) \tilde{u}_{k_2}(\bar{\eta}) \tilde{u}_{k_3}(\bar{\eta}) \int_{\eta_0}^{\bar{\eta}} d\eta \frac{1}{\eta^3} u'^*_{k_1} u'^*_{k_2} u'^*_{k_3} + \text{c.c.}$$

- The Bunch-Davies vacuum: $u_k' \sim \eta e^{-ik\eta}$ (positive frequency mode)
  - The integral yields $1/(k_1+k_2+k_3) \to 1/(2k_1)$ in the squeezed limit

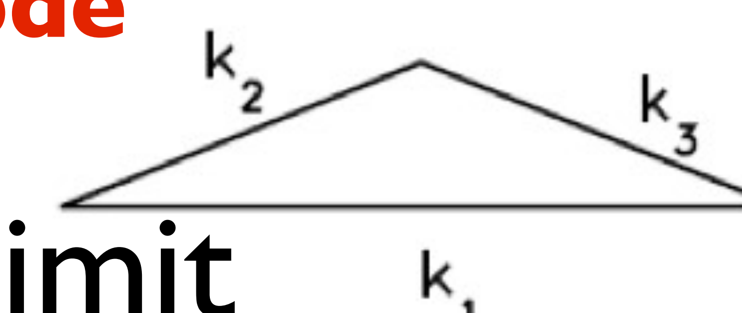
Back to in-in

$$\langle \zeta^3(t^*) \rangle = -i \int_{t_0}^{t^*} dt' \langle 0 | [\zeta^3(t^*), H_I(t')] | 0 \rangle$$

$$B_\zeta(k_1, k_2, k_3) = 2i \frac{\dot{\phi}^4}{H^6} \sum_i \left(\frac{1}{k_i^2} \right) \tilde{u}_{k_1}(\bar{\eta}) \tilde{u}_{k_2}(\bar{\eta}) \tilde{u}_{k_3}(\bar{\eta}) \int_{\eta_0}^{\bar{\eta}} d\eta \frac{1}{\eta^3} u'^{*}_{k_1} u'^{*}_{k_2} u'^{*}_{k_3} + \text{c.c.}$$

negative frequency

- Non-Bunch-Davies vacuum: $u_k' \sim \eta(A_k e^{-ik\eta} + B_k e^{+ik\eta})$ **mode**



- The integral yields $1/(k_1-k_2+k_3)$, peaking in the folded limit

Chen et al. (2007); Holman & Tolley (2008)

- The integral yields $1/(k_1-k_2+k_3)$ -> $1/(2k_3)$ in the squeezed limit

Enhanced by k_1/k_3: this can be a big factor!

Agullo & Parker (2011)

Agullo & Parker (2011)

How about the consistency relation?

$$B_\zeta(k_1,k_2,k_3) \xrightarrow[k_3/k_1 \ll 1]{} P_\zeta(k_1)P_\zeta(k_3)\left\{(1-n_s) + 4\frac{\dot\phi^2}{H^2}\frac{k_1}{k_3}[1-\cos(k_3\eta_0)]\right\}$$

- When k_3 is far outside the horizon at the onset of inflation, η_0 (whatever that means), $k_3\eta_0 \to 0$, and thus the above additional term vanishes.
  - The consistency relation is restored.

An interesting possibility:

- What if $k_3\eta_0 = O(1)$?
- The squeezed bispectrum receives an enhancement of order $\varepsilon k_1/k_3$, which can be sizable.
- Most importantly, **the bispectrum grows faster than the local-form toward k_3/k_1 -> 0!**
  - $B_\zeta(k_1,k_2,k_3) \sim 1/k_3^3$ [Local Form]
  - $B_\zeta(k_1,k_2,k_3) \sim 1/k_3^{\mathbf{4}}$ [non-Bunch-Davies]
- This has an observational consequence – particularly a scale-dependent bias and distortion of CMB spectrum.

Power Spectrum of Galaxies

- Galaxies do not trace the underlying matter density fluctuations perfectly. They are **biased tracers**.
- "Bias" is operationally defined as
  - $\mathbf{b_{galaxy}^2(k) = \langle|\delta_{galaxy,k}|^2\rangle / \langle|\delta_{matter,k}|^2\rangle}$

Density-ζ Relation

- It is given by the Poisson equation:

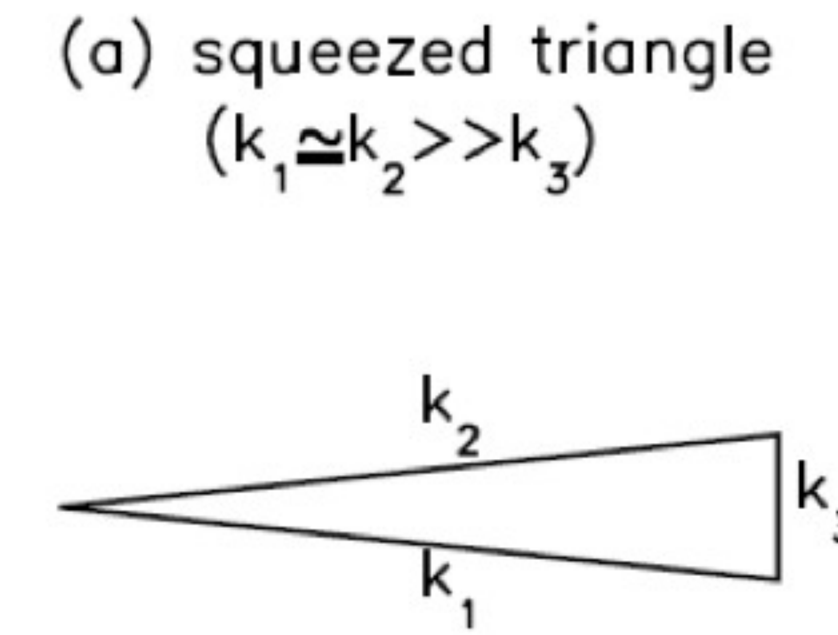
$$\delta_{m,\mathbf{k}}(z) = \frac{2k^2}{5H_0^2\Omega_m}\zeta_{\mathbf{k}} T(k) D(k,z)$$

$T(k) \to 1$ for $k \ll 10^{-2}$ Mpc^{-1}
$T(k) \to (\ln k)^2/k^4$ for $k \gg 10^{-2}$ Mpc^{-1}

$D(k,z)=1/(1+z)$ during the matter-dominated era

Positive ζ_k -> positive $\delta_{m,k}$!

Galaxy clustering modified by the squeezed limit



- The existence of long-wavelength ζ changes the small-scale power of δ_m.
- **A positive long-wavelength ζ -> more power on small scales**.
- More power on small scales -> more galaxies formed.

Dalal et al. (2008); Matarrese & Verde (2008); Desjacques et al. (2011)

Scale-dependent Bias

$$\Delta b(k, R) = 2\frac{\mathcal{F}_R(k)}{\mathcal{M}_R(k)}\left[(b_1 - 1)\delta_c\right],$$

$M_R(k) \sim k^2$ for $k \ll 1/R$ and small for $k \gg 1/R$

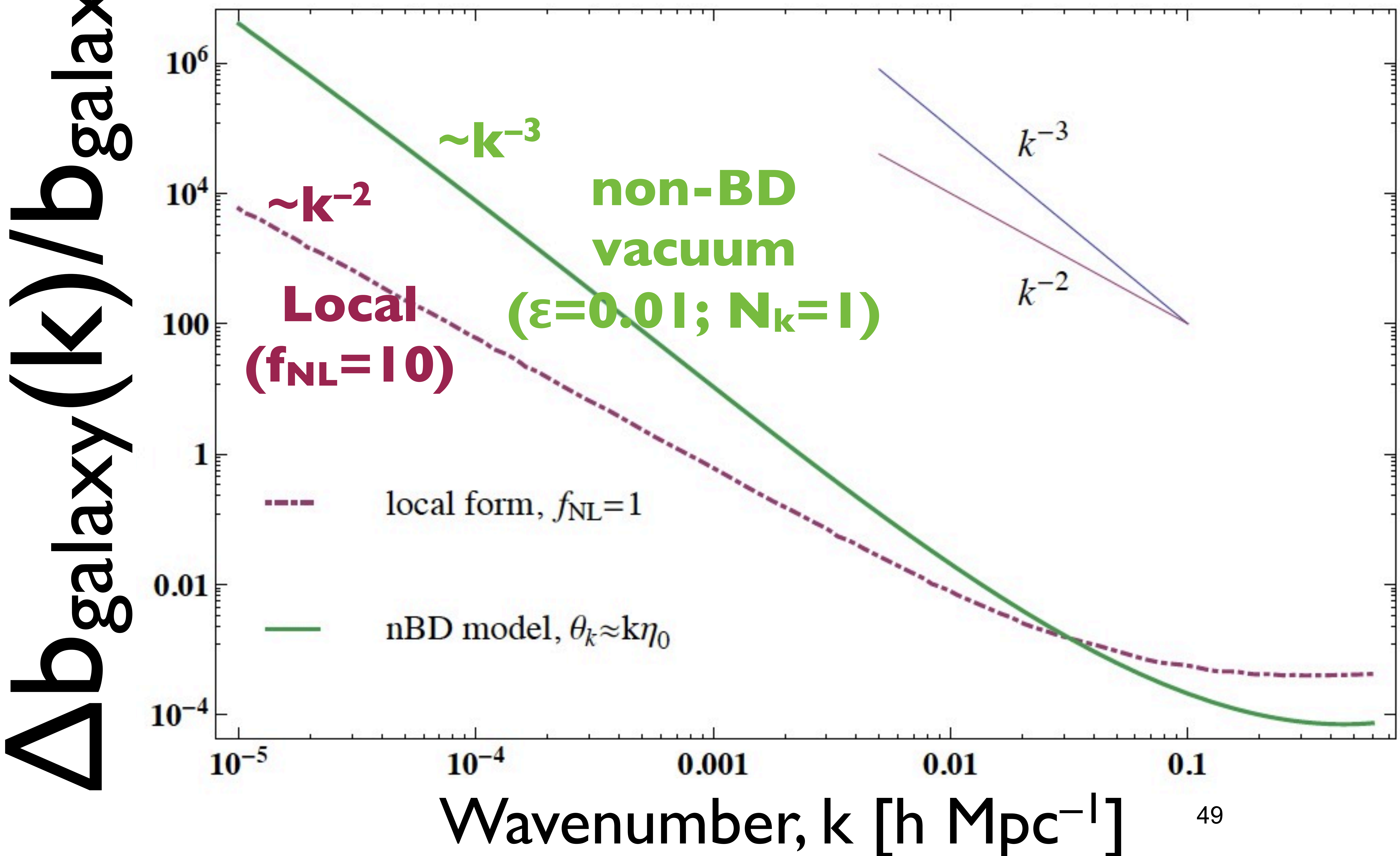
$$\mathcal{F}_R(k) \approx \frac{1}{4\sigma_R^2 P_\zeta(k)} \int \frac{d^3k_1}{(2\pi)^3} \mathcal{M}_R^2(k_1) B_\zeta(k_1, k_1, k)$$

R is the linear size of dark matter halos

- A rule-of-thumb: $$\sigma_R^2 \equiv \int \frac{d^3k}{(2\pi)^3} P_\zeta(k)\mathcal{M}_R^2(k)$$
  - For $B(k_1,k_2,k_3) \sim 1/k_3^{\mathbf{p}}$, the scale-dependence of the halo bias is given by $b(k) \sim 1/k^{\mathbf{p-1}}$
  - For a local-form (p=3), it goes like $b(k) \sim 1/k^2$
  - For a non-Bunch-Davies vacuum (p=4), would it go like $b(k) \sim 1/k^3$?

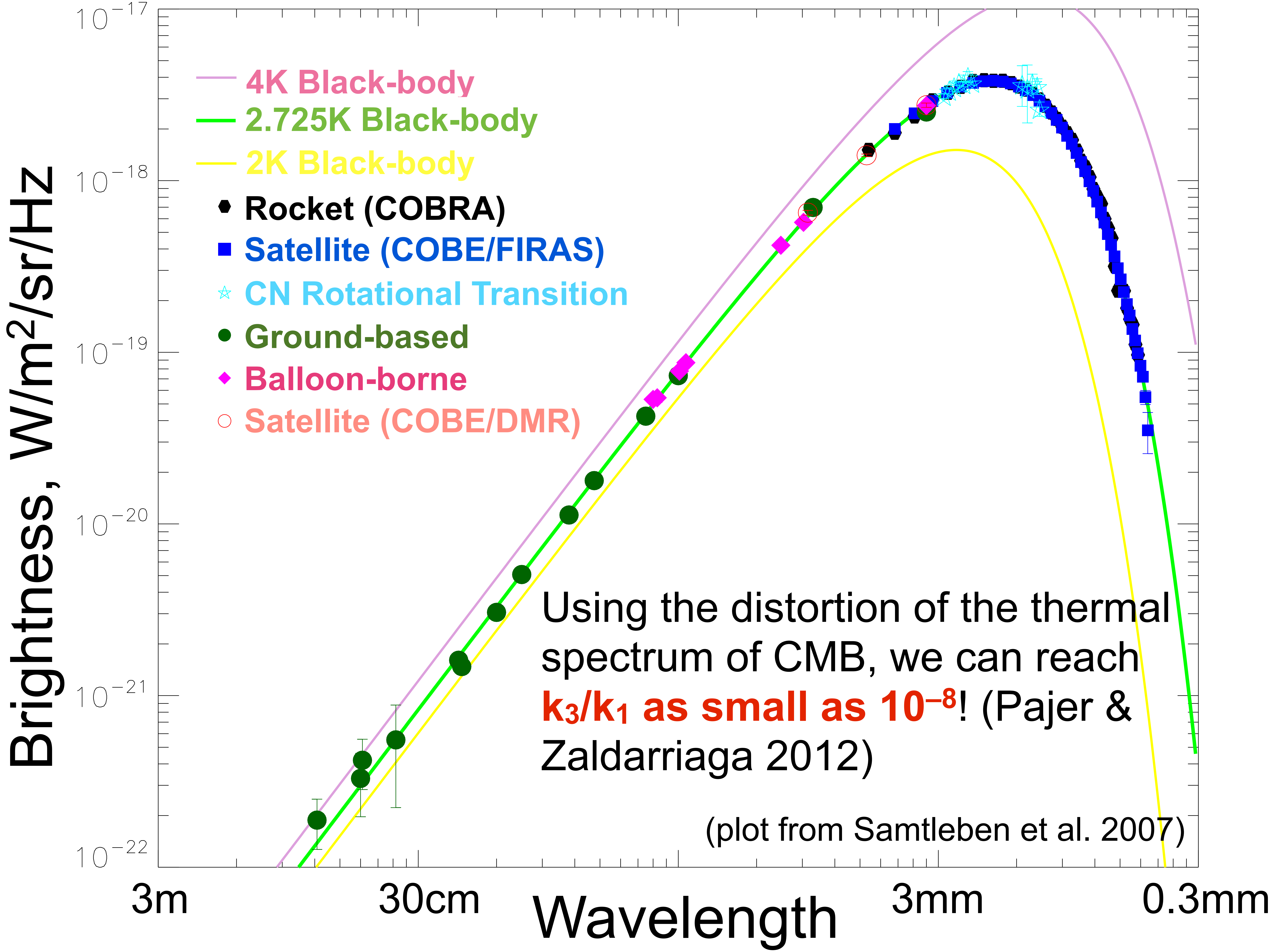
It does!

Ganc & Komatsu (2012)



CMB Bispectrum

- The expected contribution to f_{NL} as measured by the CMB bispectrum is typically $f_{NL} \approx 8(\varepsilon/0.01)$.
  - A lot bigger than $(5/12)(1-n_S)$, and could be detectable with Planck.
- Note that this does not mean a violation of the single-field consistency condition, which is valid in the exact squeezed limit, $k_3 \to 0$.
- We have an enhanced bispectrum in the squeezed configuration where **k_3/k_1 is small but finite**.



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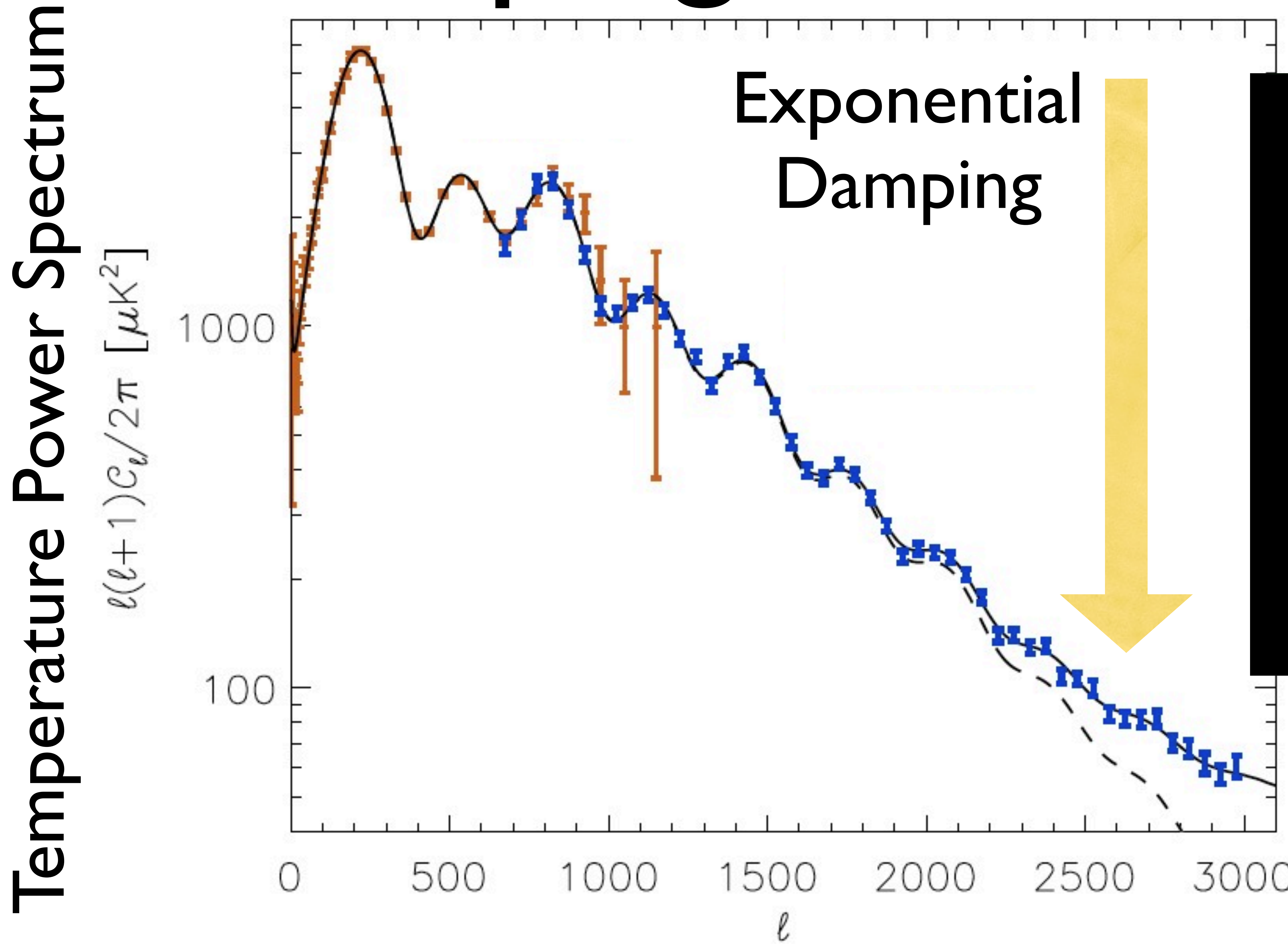
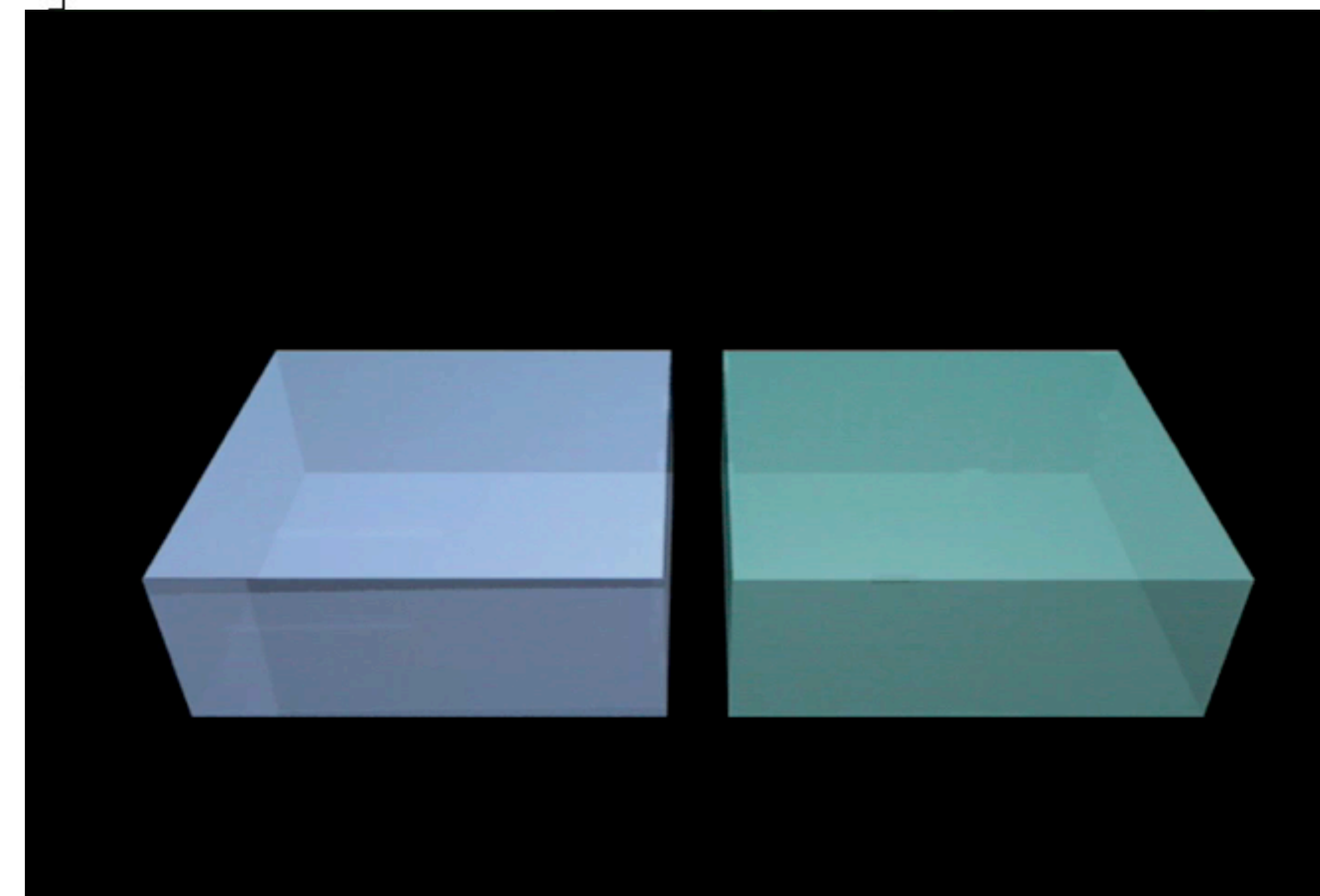
Using the distortion of the thermal spectrum of CMB, we can reach **k_3/k_1 as small as 10^{-8}**! (Pajer & Zaldarriaga 2012)
(plot from Samtleben et al. 2007)

Damping of Acoustic Waves





- Energy stored in the acoustic waves must go somewhere -> heating of CMB photons -> distortion of the thermal spectrum

Chemical potential from energy injection

- Suppose that some energy, ΔE, is injected into the cosmic plasma during the radiation dominated era.

- What happens? The thermal spectrum of CMB should be distorted!

Chemical potential from energy injection

- For $z>\mathbf{z_i=2x10^6}$, double Compton scattering, $e^-+\gamma \to e^- + 2\gamma$, is effective, erasing the distortion of the thermal spectrum of CMB.

- Black-body spectrum is restored.

Chemical potential from energy injection

- For $z<z_i=2\times10^6$, double Compton scattering, $e^-+\gamma \to e^-+2\gamma$, freezes out.
- However, the elastic scattering, $e^-+\gamma \to e^-+\gamma$, remains effective [until $\mathbf{z_f=5\times10^4}$]
- Black-body spectrum is not restored, but the spectrum relaxes to a Bose-Einstein spectrum with a non-zero chemical potential, μ, **for $z_f<z<z_i$**:

$$n(\nu)=\frac{1}{e^{h\nu/(k_BT)}-1} \to \frac{1}{e^{h\nu/(k_BT)+\mu}-1}$$

Chemical potential from energy injection

$$n(\nu) = \frac{1}{e^{h\nu/(k_B T)} - 1} \rightarrow \frac{1}{e^{h\nu/(k_B T)+\mu} - 1}$$

- Energy density is added to the plasma ($\mu \ll 1$):
  - $aT^4 + \Delta E/V = a(T')^4(1-1.11\mu)$
- Number density is conserved ($\mu \ll 1$):
  - $bT^3 = b(T')^3(1-1.37\mu)$
- Solving for μ gives
  - $\mu = 1.4[\Delta E/(aT^4V)]$ **$= 1.4(\Delta E/E)$**

How much energy?

- Only **1/3** of the total energy stored in the acoustic wave during radiation era is used to *heat* CMB (thus distorting the CMB spectrum) (papers by Jens Chluba):
  - $Q = (1/3)(9/4)c_s^2\rho_\gamma(\delta_\gamma)^2 = (1/4)\rho_\gamma(\delta_\gamma)^2$
- $\mu \approx 1.4 \int dz[(dQ/dz)/\rho_\gamma]$

  $=(1.4/4)[(\delta_\gamma)^2(z_i)-(\delta_\gamma)^2(z_f)]$
  - where $z_i=2\mathrm{x}10^6$ and $z_f=5\mathrm{x}10^4$

Bottom Line

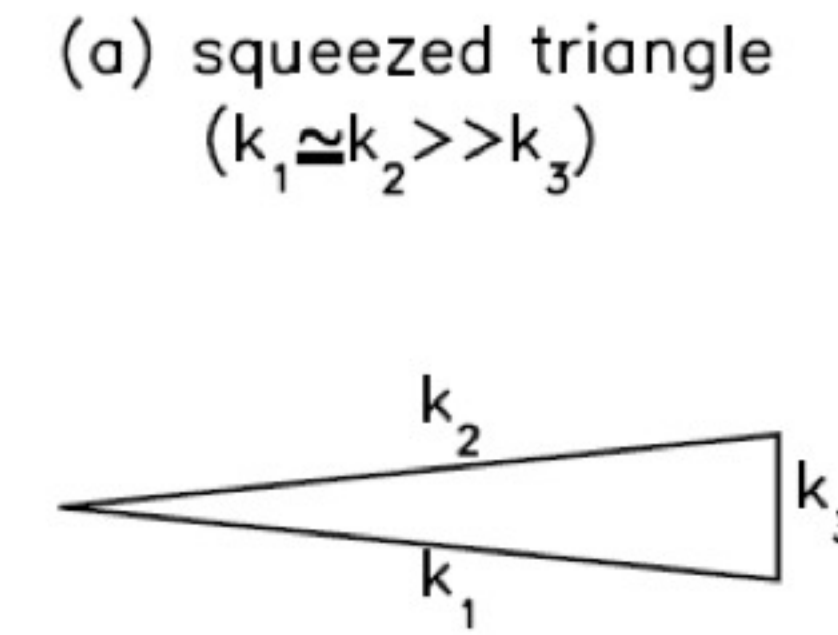
- Therefore, the chemical potential is generated by the photon density perturbation **squared**.
- At what scale? The diffusion damping occurs at the mean free path of photons. In terms of the wavenumber, it is given by:

$$k_D \approx 130\ [(1+z)/10^5]^{3/2}\ \mathrm{Mpc}^{-1}$$

$$k_D(z_i) \approx 12000\ \mathrm{Mpc}^{-1}\ \ ;\ \ k_D(z_f) \approx 46\ \mathrm{Mpc}^{-1}$$

It's a very small scale!
(compared to the large-scale structure, k~1 Mpc^{-1})

μ-distortion modified by the squeezed limit



- The existence of long-wavelength ζ changes the small-scale power of δ_γ.
- **A positive long-wavelength ζ -> more power on small scales**.
- More power on small scales -> more μ-distortion.
- **μ-distortion becomes anisotropic on the sky!** (Pajer & Zaldarriaga 2012)

μ-T cross-correlation

- In real space:
  - $\mu = (1.4/4)[(\delta_\gamma)^2(z_i)-(\delta_\gamma)^2(z_f)]$ at $k_1 \sim O(10^2)-O(10^4)$
  - $\Delta T/T = -(1/5)\zeta$ at $k_3 \sim O(10^{-4})$ [in the Sachs-Wolfe limit]
- Correlating these will probe the bispectrum in the squeezed configuration with **$k_3/k_1 = O(10^{-6})-O(10^{-8})$!!**

More exact treatment

- Going to harmonic space:
  - $\Delta T/T(\mathbf{n})=\sum a_{lm}{}^{T}Y_{lm}(\mathbf{n})$; $\mu(\mathbf{n})=\sum a_{lm}{}^{\mu}Y_{lm}(\mathbf{n})$
- $$a^T_{lm} = \frac{12\pi}{5}(-i)^l \int \frac{d^3k}{(2\pi)^3} \underline{\zeta(\mathbf{k})} g_{Tl}(k) Y^*_{lm}(\hat{k})$$ [$g_{Tl}(k)$ contains info about the acoustic oscillation]
- $$a^\mu_{lm} = 18\pi(-i)^l \int \frac{d^3k_1 d^3k_2}{(2\pi)^6} Y^*_{lm}(\hat{k}) \underline{\zeta(\mathbf{k}_1)\zeta(\mathbf{k}_2)} W\left(\frac{k}{k_s}\right) \times$$
  $$\times\, j_l(kr_L)\langle \cos(k_1 r)\cos(k_2 r)\rangle_p \left[e^{-(k_1^2+k_2^2)/k_D^2(z)}\right]^{z_i}_{z_f}$$

μ-T cross-power spectrum

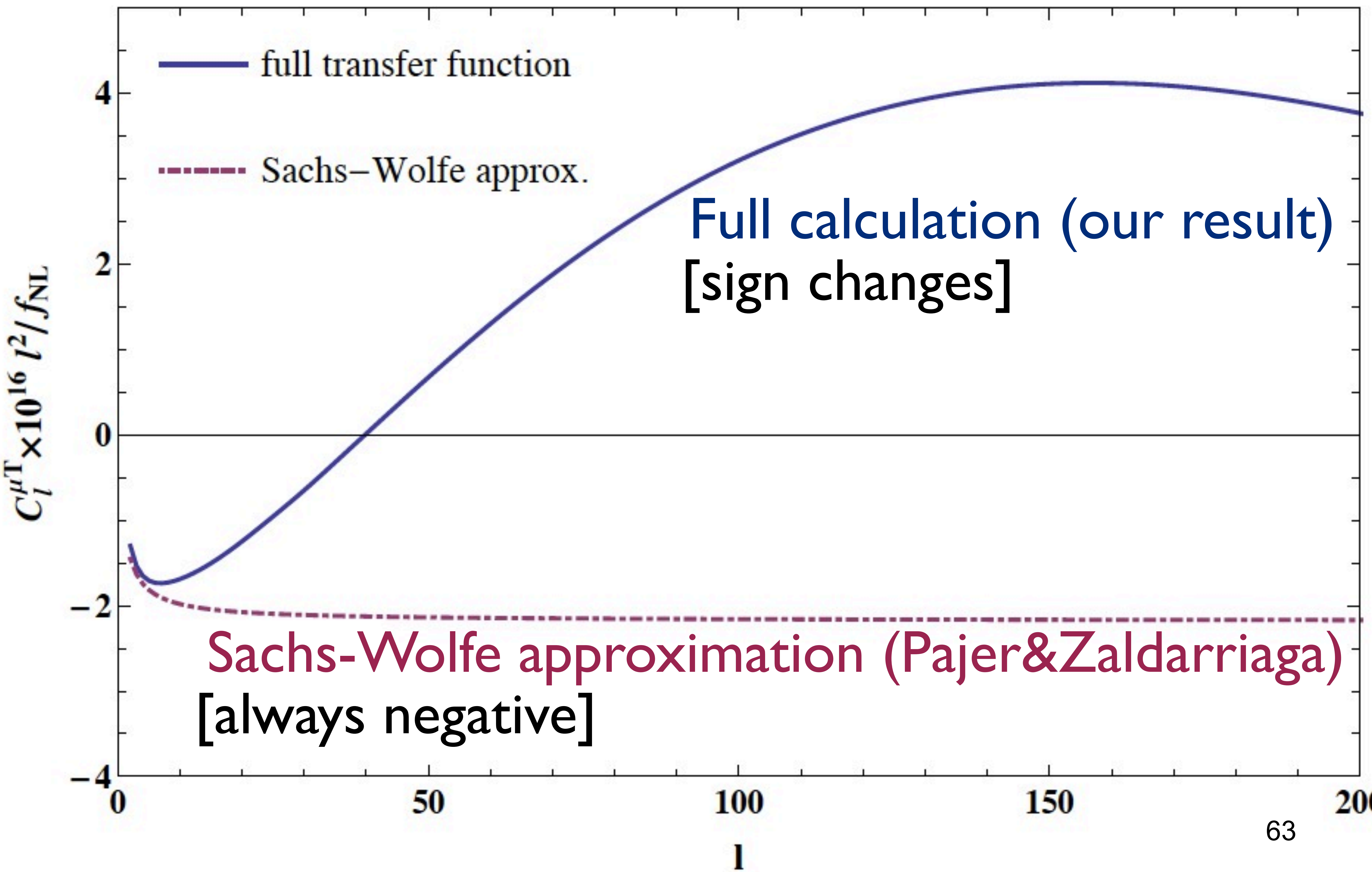
$$C_l^{\mu T} = \frac{27}{20\pi^3} \int_0^\infty k_1^2 dk_1 \left[e^{-2k_1^2/k_D^2(z)} \right]_{z_f}^{z_i} \times \int_0^\infty k^2 dk \, W\left(\frac{k}{k_s}\right) B_\zeta(k_1, k_2, k) j_l(kr_L) g_{Tl}(k)$$

- Here, the integral is dominated by $k_1 \approx k_2 \approx k_D$ (which is big) and $k \approx l/r_L$ (which is small because r_L=14000 Mpc)
- Very squeezed limit bispectrum

Local-form Result

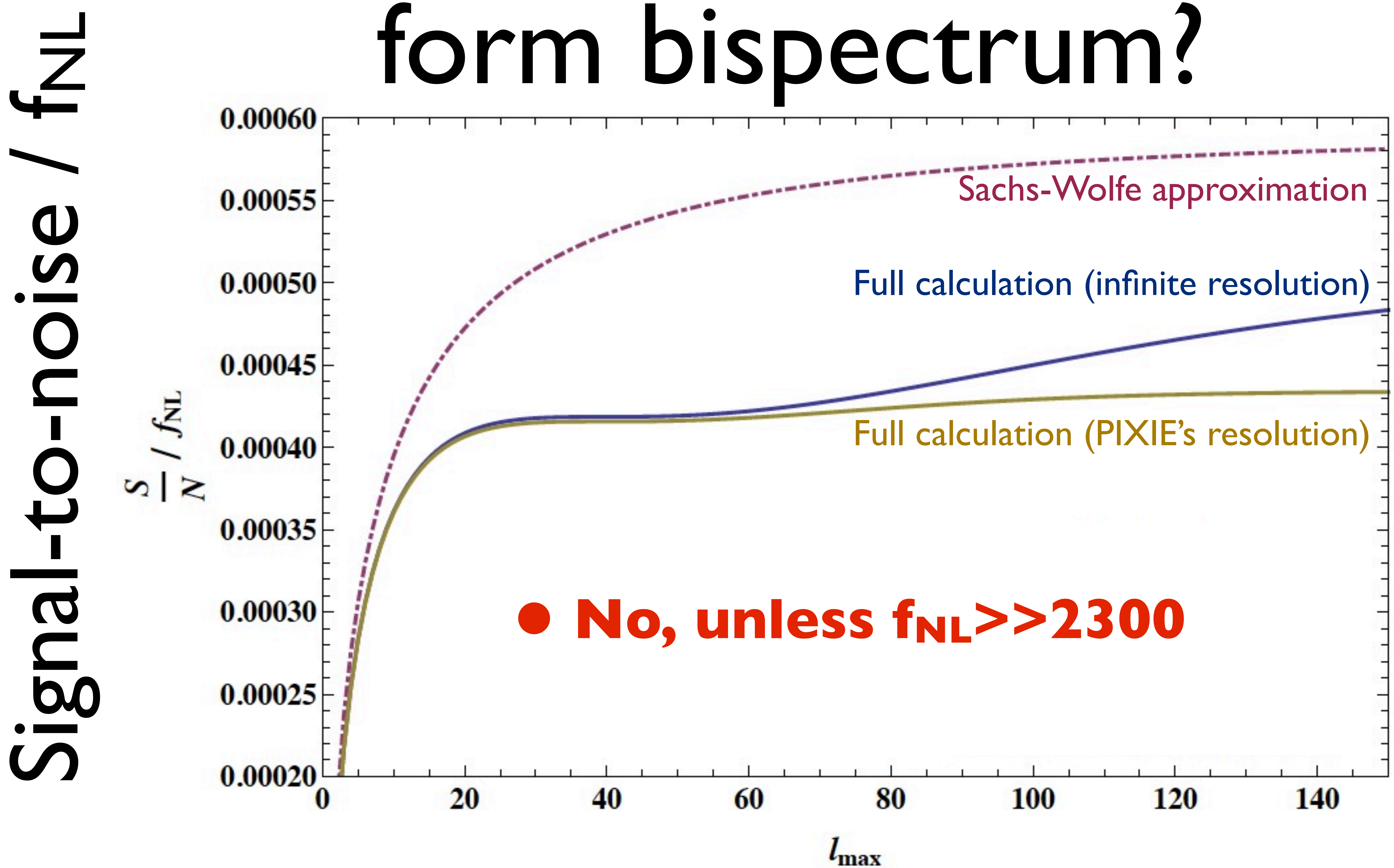
Ganc & Komatsu (2012)

μ-T cross-correlation



Ganc & Komatsu (2012)

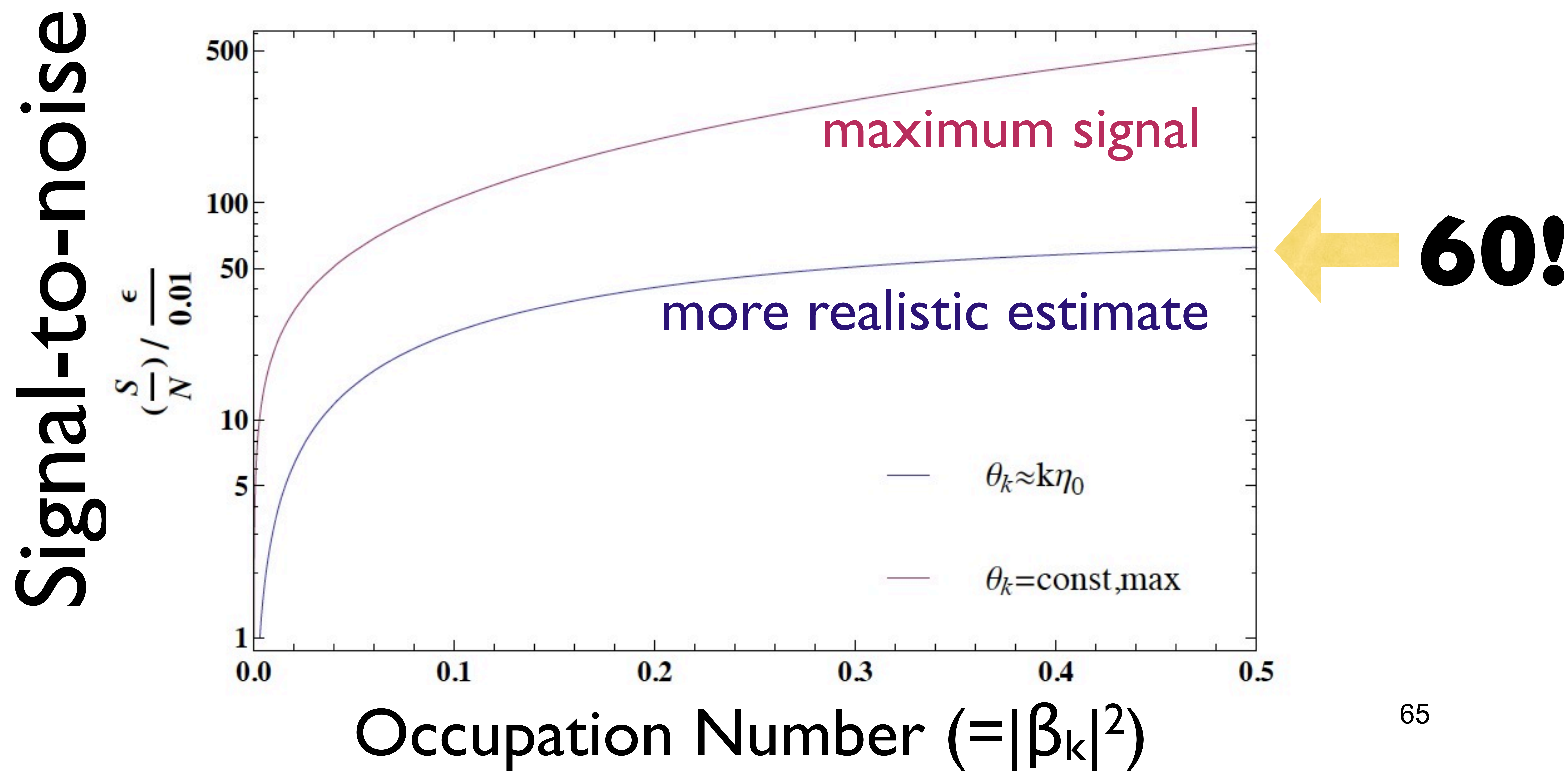
Can we detect the local-form bispectrum?



- **No, unless f_{NL}>>2300**

Ganc & Komatsu (2012)

But, a modified initial state enhances the signal



Future Work

- All we did was to impose the following mode function at a finite past:
- $u_k = \frac{H^2}{\dot{\phi}} \frac{1}{\sqrt{2k^3}} [\alpha_k(1+ik\eta)e^{-ik\eta} + \beta_k(1-ik\eta)e^{ik\eta}]$
  - with the condition: $\beta_k \to 0$ for $k \to \infty$
- However, it is desirable to construct an explicit model which will give explicit forms of α_k and β_k, so that we do not need to put an arbitrary model function at an arbitrary time by hand.

Summary

New probes of initial state of quantum fluctuations!

- A more insight into the single-field consistency relation for the squeezed-limit bispectrum using in-in formalism.
- Non-Bunch-Davies vacuum can give an enhanced bispectrum in the $k_3/k_1 \ll 1$ limit, yielding a distinct form of the scale-dependent bias.
- The μ-type distortion of the CMB spectrum becomes anisotropic, and it can be detected by correlating μ on the sky with the temperature anisotropy.

Squeezed-limit bispectrum
= Test of single-field inflation
& initial state of quantum fluctuations