(Still) Hunting for Primordial Non-Gaussianity: Current Status and Future Prospects

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Cosmology and Fundamental Physics: 6 Numbers

- Successful early-universe models **must** satisfy the following observational constraints:
  - The observable universe is nearly flat, **$|\Omega_K| < O(0.02)$**
  - The primordial fluctuations are
    - Nearly Gaussian, **$|f_{NL}| < O(100)$**
    - Nearly scale invariant, **$|n_s - 1| < O(0.05)$**, **$|dn_s/d\ln k| < O(0.05)$**
    - Nearly adiabatic, **$|S/R| < O(0.2)$**

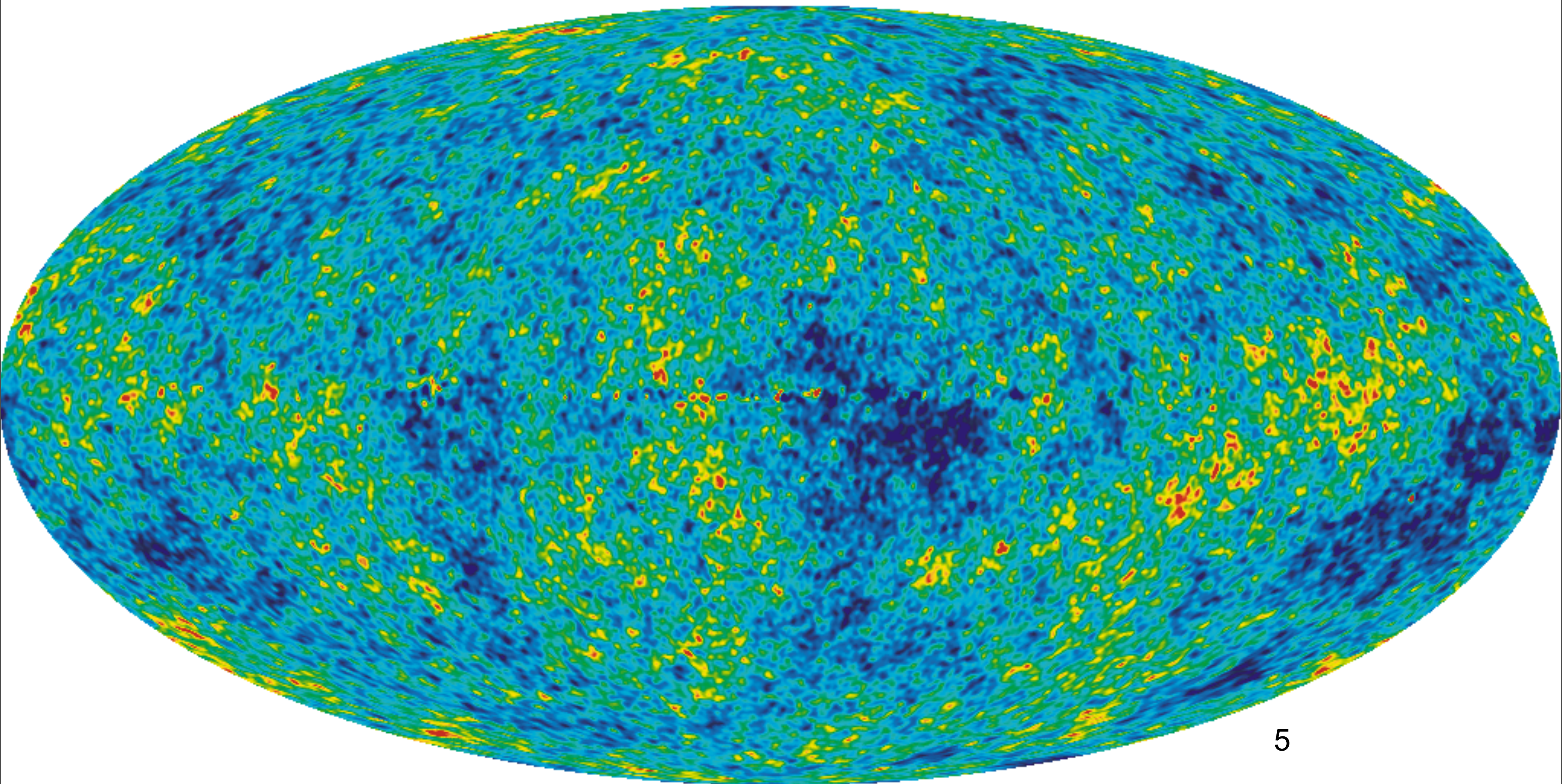
Cosmology and Fundamental Physics: 6 Numbers

- A "generous" theory would make cosmologists very happy by producing detectable primordial gravity waves ($r>0.01$)…
  - But, this is not a requirement yet.
  - Currently, **$r<O(0.5)$**

Why Study Non-Gaussianity?

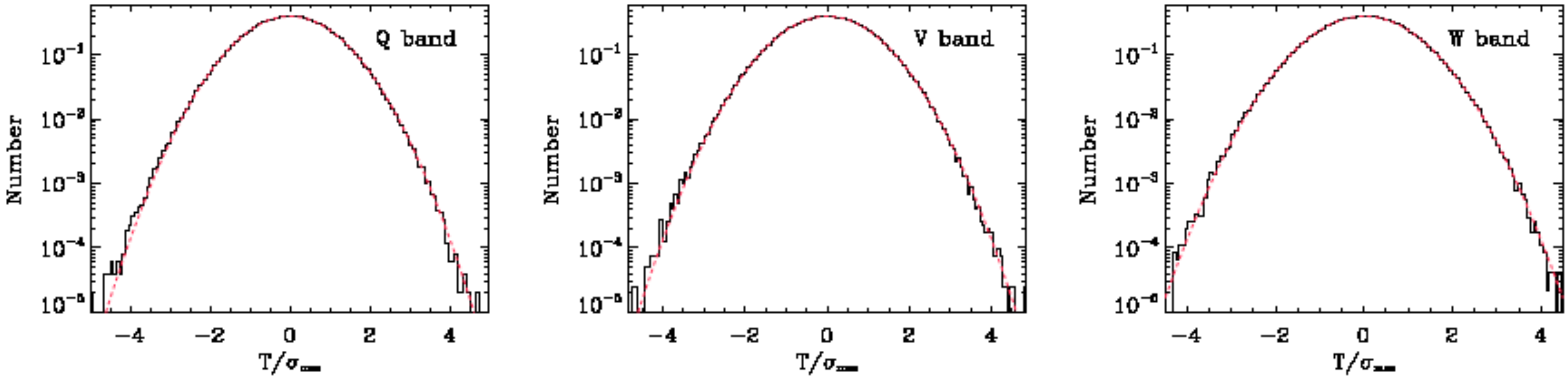
- **Who said that CMB must be Gaussian?**
  - **Don't let people take it for granted.**
  - It is rather remarkable that the distribution of the observed temperatures is so close to a Gaussian distribution.
  - The WMAP map, when smoothed to 1 degree, is entirely dominated by the CMB signal.
    - If it were still noise dominated, no one would be surprised that the map is Gaussian.
  - The WMAP data are telling us that primordial fluctuations are pretty close to a Gaussian distribution.
    - How common is it to have something so close to a Gaussian distribution in astronomy?
  - It is not so easy to explain why CMB is Gaussian, unless we have a compelling early universe model that predicts Gaussian primordial fluctuations: e.g., ***Inflation***.

How Do We Test *Gaussianity of CMB*?



Spergel et al. (2007)

One-point PDF from WMAP



- The one-point distribution of CMB temperature anisotropy looks pretty Gaussian.
  - Left to right: Q (41GHz), V (61GHz), W (94GHz).
- We are therefore talking about quite a subtle effect.

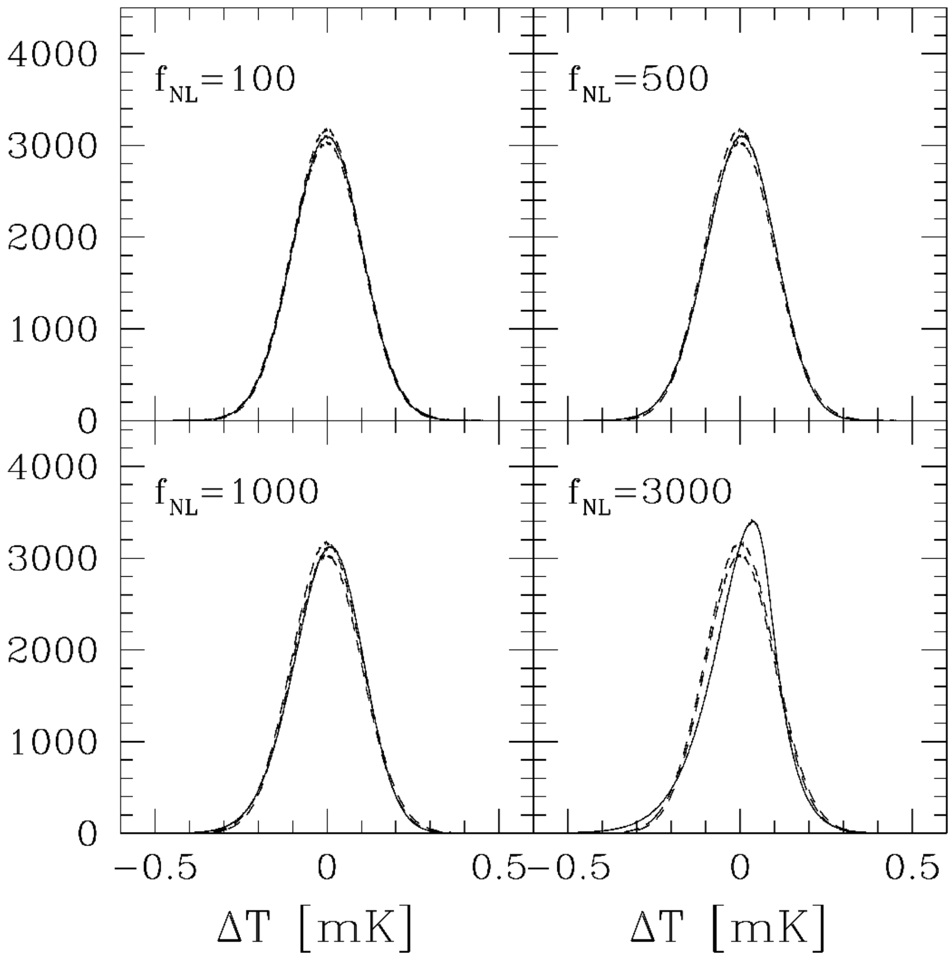
Gaussianity vs Flatness

- We are generally happy that geometry of our observable Universe is flat.
  - Geometry of our Universe is consistent with a flat geometry to **~2%** accuracy at 95% CL. (Spergel et al., WMAP 3yr)

- What do we know about Gaussianity?
  - Parameterize non-Gaussianity: $\Phi=\Phi_L+f_{NL}\Phi_L^2$
    - $\Phi_L \sim 10^{-5}$ is a Gaussian, linear curvature perturbation in the matter era
  - Therefore, $f_{NL}<100$ means that the distribution of Φ is consistent with a Gaussian distribution to $\sim 100\times(10^{-5})^2/(10^{-5})=$**0.1%** accuracy at 95% CL.

- **Remember this fact: "Inflation is supported more by Gaussianity than by flatness."**

How Would f_{NL} Modify PDF?



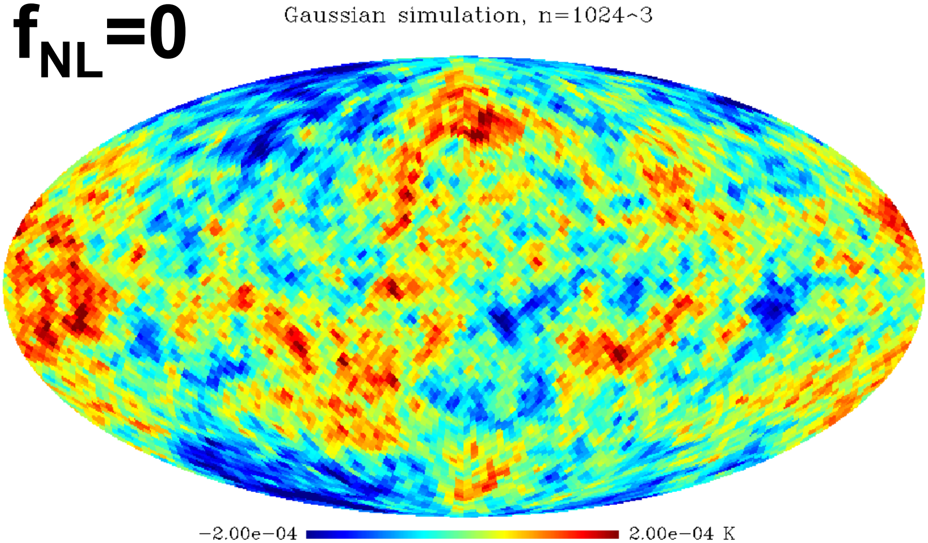
One-point PDF is not useful for measuring primordial NG. We need something better:

- Three-point Function
  - Bispectrum
- Four-point Function
  - Trispectrum
- Morphological Test
  - Minkowski Functionals

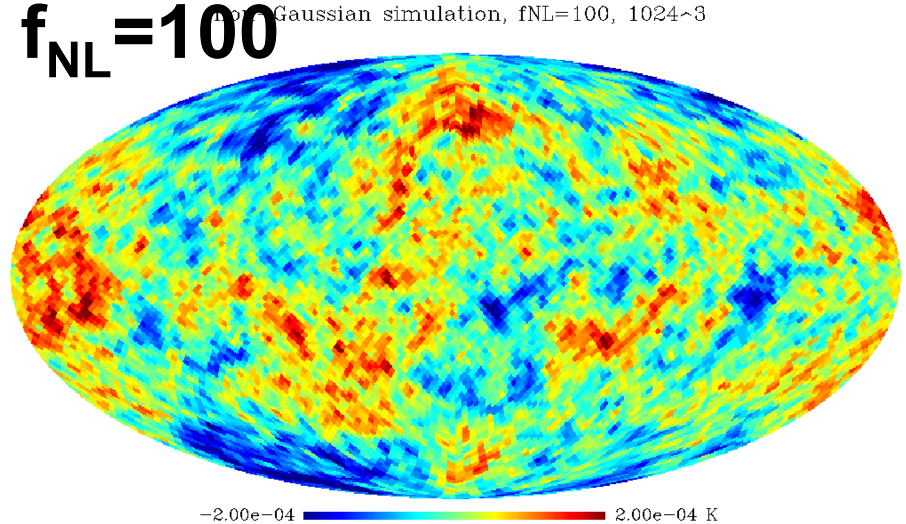
Positive f_{NL} = More Cold Spots

Simulated temperature maps from $\Phi(x) = \Phi_G(x) + f_{NL}\Phi_G^2(x)$

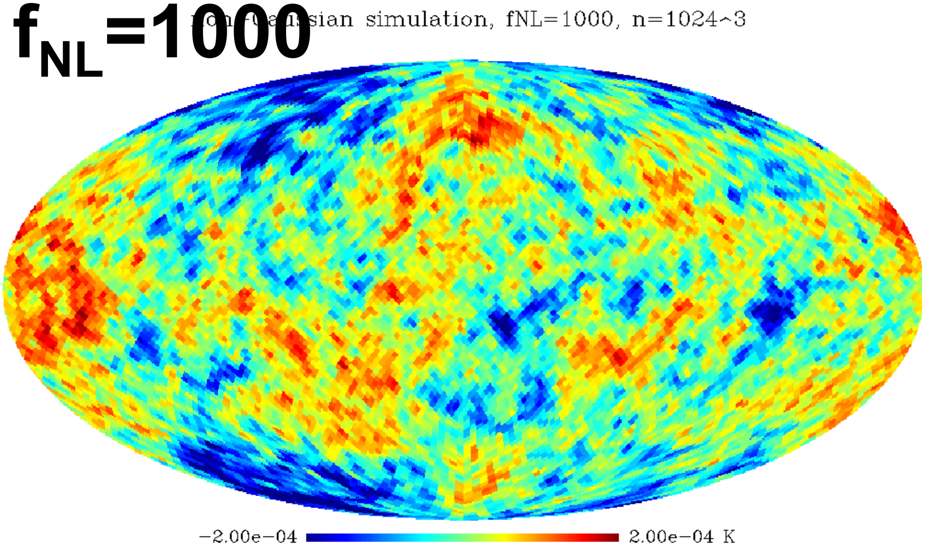
f_{NL}=0



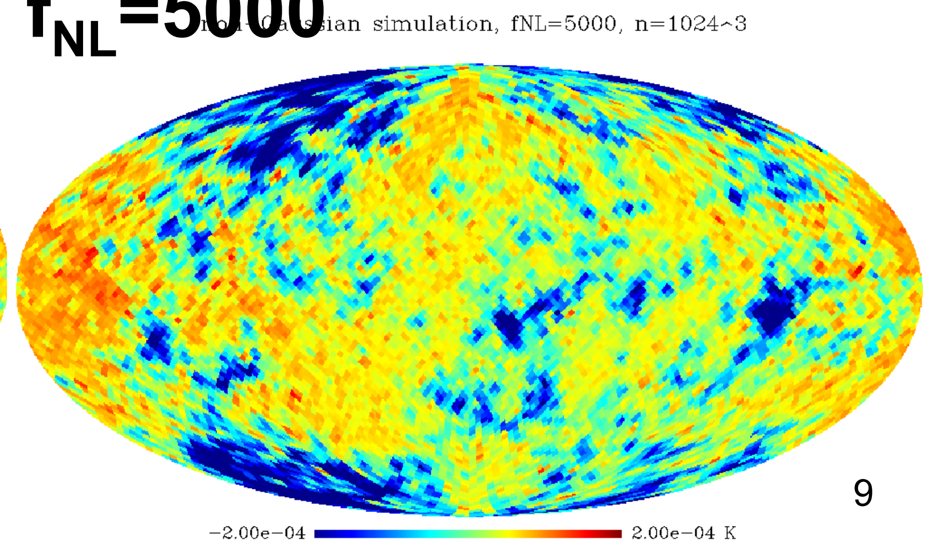
f_{NL}=100



f_{NL}=1000

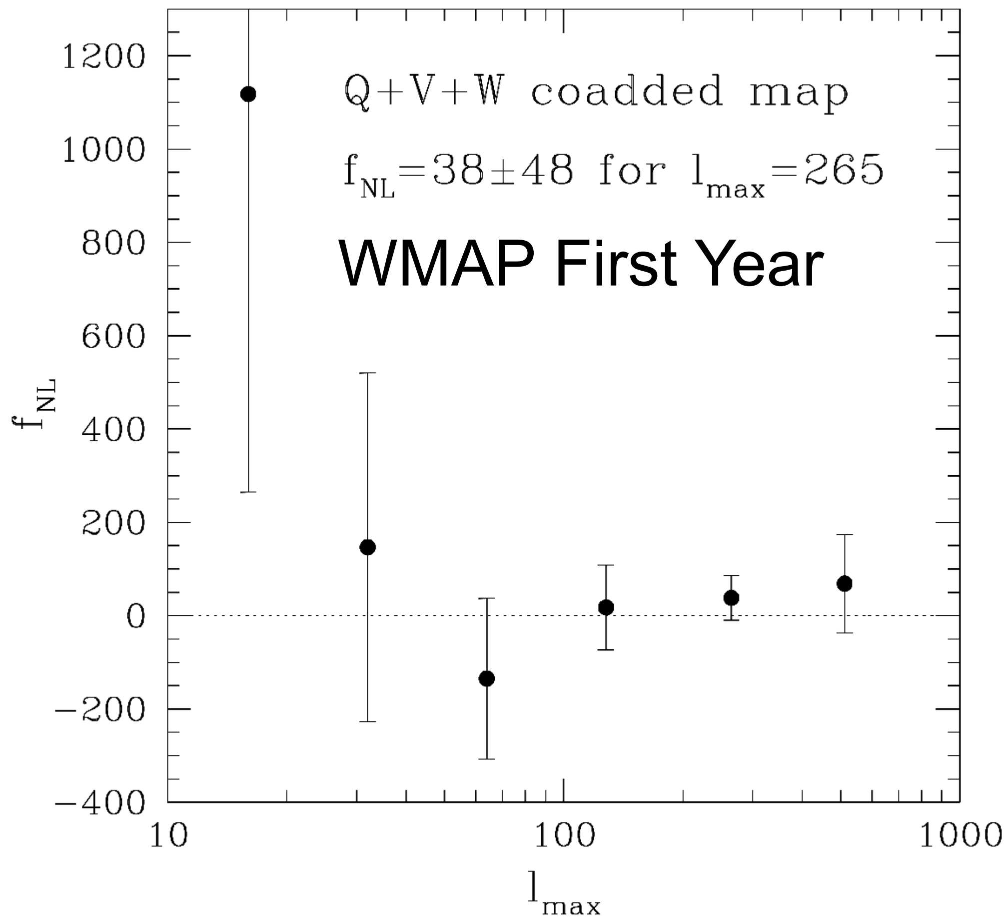


f_{NL}=5000



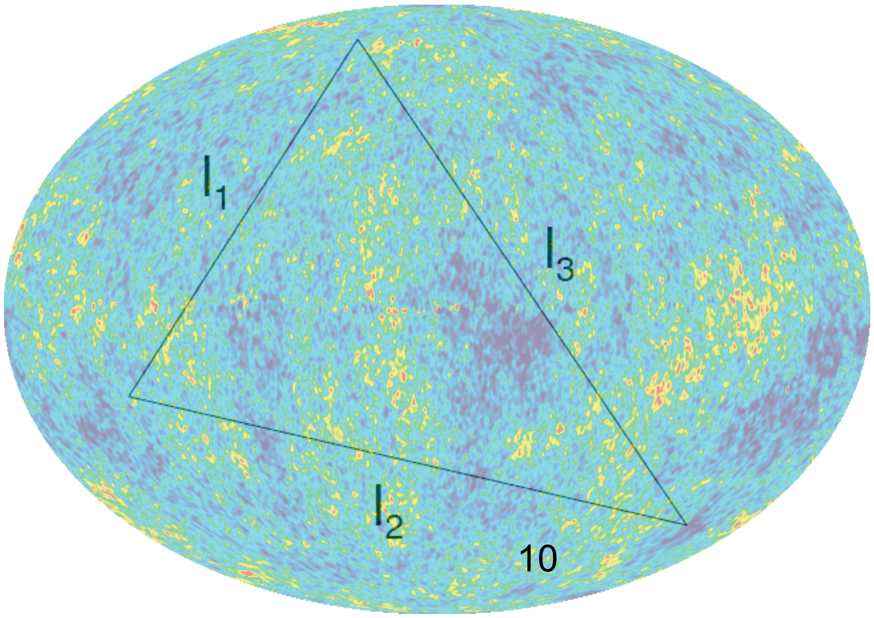
Komatsu et al. (2003); Spergel et al. (2007)

Bispectrum Constraints



$-58 < f_{NL} < +134$ (95% CL) (1yr)

$-54 < f_{NL} < +114$ (95% CL) (3yr)



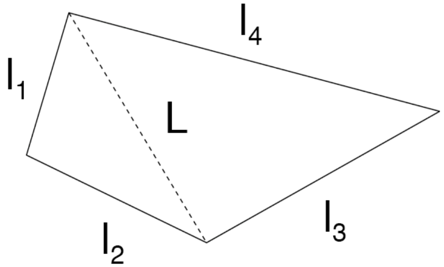
Trispectrum of Primordial Perturbations

- Trispectrum is the Fourier transform of four-point correlation function.
- Trispectrum(k_1,k_2,k_3,k_4)

  $$=\langle\Phi(k_1)\Phi(k_2)\Phi(k_3)\Phi(k_4)\rangle$$

  which can be sensitive to the higher-order terms:

$$\Phi(\boldsymbol{x}) = \Phi_{\rm L}(\boldsymbol{x}) + f_{\rm NL}\left[\Phi_{\rm L}^2(\boldsymbol{x}) - \langle\Phi_{\rm L}^2(\boldsymbol{x})\rangle\right] + f_2\Phi_{\rm L}^3(\boldsymbol{x})$$



Okamoto & Hu (2002); Kogo & Komatsu (2006)

Trispectrum of CMB

$$\langle a_{l_1m_1}a_{l_2m_2}a_{l_3m_3}a_{l_4m_4}\rangle = \sum_{LM}(-1)^M \begin{pmatrix} l_1 & l_2 & L \\ m_1 & m_2 & -M \end{pmatrix}\begin{pmatrix} l_3 & l_4 & L \\ m_3 & m_4 & M \end{pmatrix} T^{l_1l_2}_{l_3l_4}(L)$$

$$T_{c\,l_3l_4}^{\,l_1l_2}(L) = P^{l_1l_2}_{l_3l_4}(L) + (2L+1)\sum_{L'}\left[(-1)^{l_2+l_3}\begin{Bmatrix} l_1 & l_2 & L \\ l_4 & l_3 & L'\end{Bmatrix}P^{l_1l_3}_{l_2l_4}(L') + (-1)^{L+L'}\begin{Bmatrix} l_1 & l_2 & L \\ l_3 & l_4 & L'\end{Bmatrix}P^{l_1l_4}_{l_3l_2}(L')\right],$$

where

$$P^{l_1l_2}_{l_3l_4}(L) = t^{l_1l_2}_{l_3l_4}(L) + (-1)^{2L+l_1+l_2+l_3+l_4}t^{l_2l_1}_{l_4l_3}(L) + (-1)^{L+l_3+l_4}t^{l_1l_2}_{l_4l_3}(L) + (-1)^{L+l_1+l_2}t^{l_2l_1}_{l_3l_4}(L).$$

$$\begin{aligned} t^{l_1l_2}_{l_3l_4}(L) \;=\; & \int r_1^2dr_1r_2^2dr_2\; F_L(r_1,r_2)\alpha_{l_1}(r_1)\beta_{l_2}(r_1)\alpha_{l_3}(r_2)\beta_{l_4}(r_2)h_{l_1Ll_2}h_{l_3Ll_4} \\ & + \int r^2dr\;\beta_{l_2}(r)\beta_{l_4}(r)\left[\mu_{l_1}(r)\beta_{l_3}(r)+\beta_{l_1}(r)\mu_{l_3}(r)\right]h_{l_1Ll_2}h_{l_3Ll_4}, \end{aligned}$$

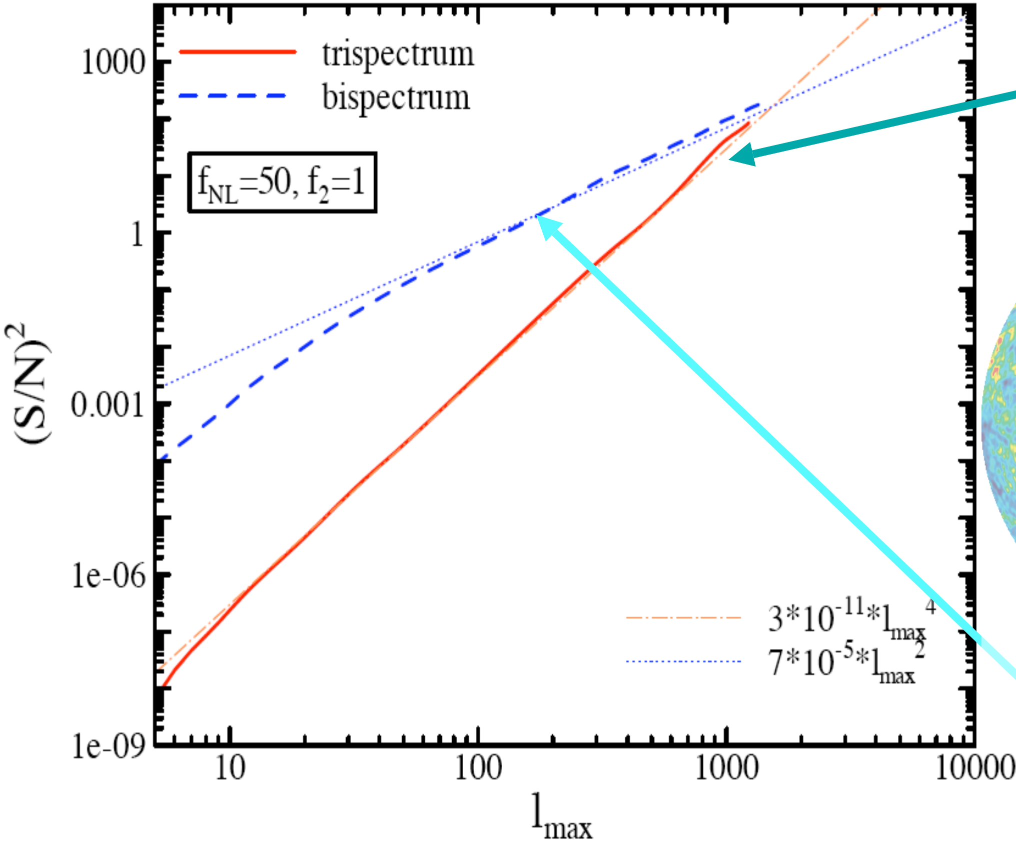
$\text{alpha}_l(r)=2b_l^{NL}(r)$; $\text{beta}_l(r)=b_l^{L}(r)$; $\mu_l(r) \equiv \frac{2}{\pi}\int k^2dk\; f_2 g_{Tl}(k)j_l(kr)$

Measuring Trispectrum

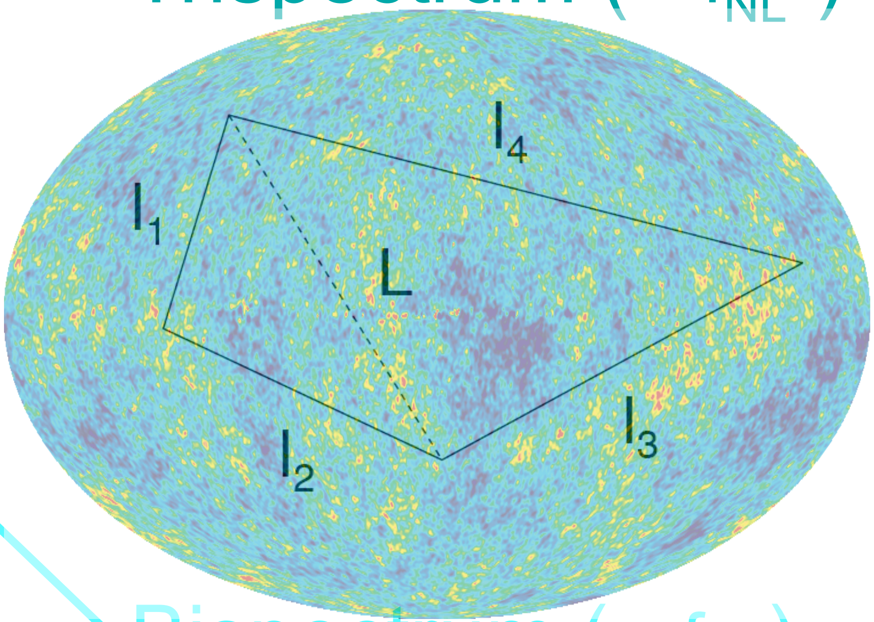
- It's pretty painful to measure all the quadrilateral configurations.
  - Measurements from the COBE 4-year data (Komatsu 2001; Kunz et al. 2001)
- Only limited configurations measured from the WMAP 3-year data
  - Spergel et al. (2007)
- No evidence for non-Gaussianity, but f_{NL} has not been constrained by the trispectrum yet. (Work to do.)

Kogo & Komatsu (2006)

Trispectrum: Not useful for WMAP, but maybe useful for Planck, if f_{NL} is greater than ~50

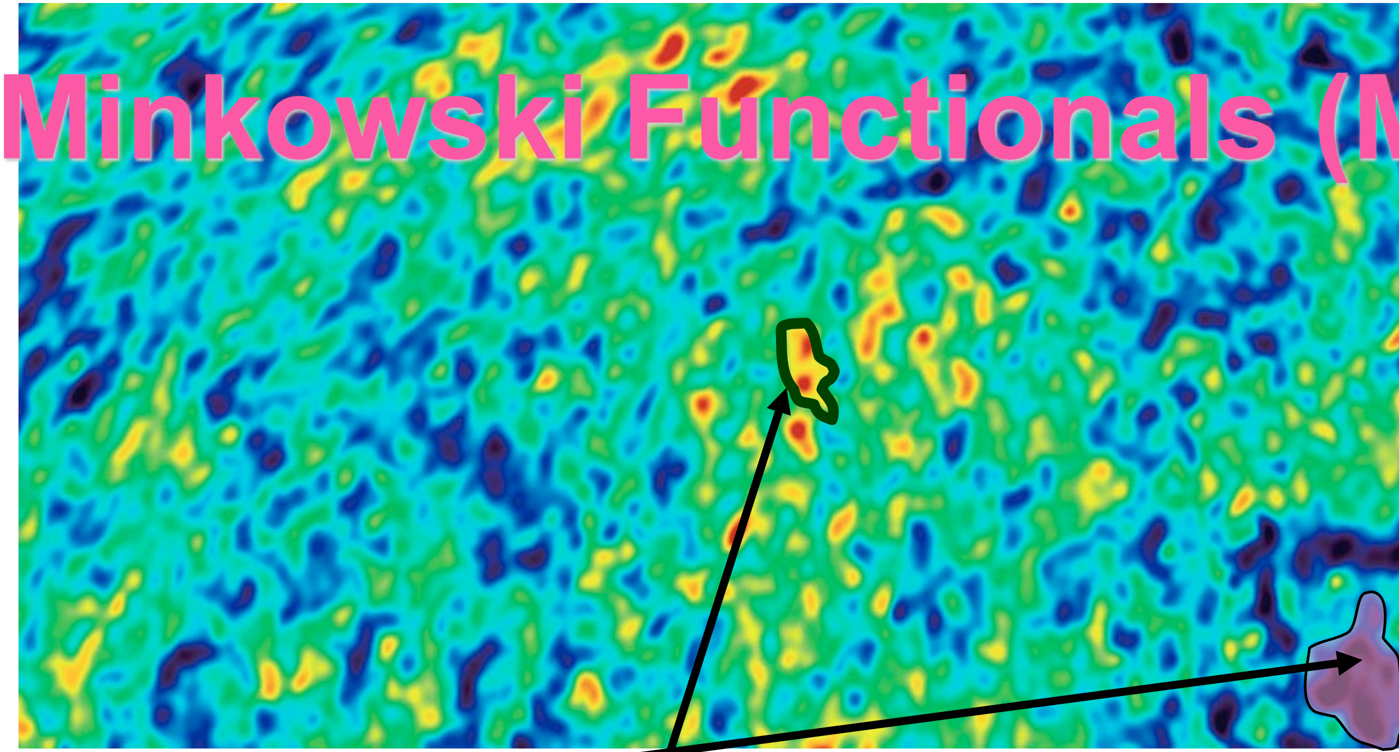


- Trispectrum (~ f_{NL}^2)



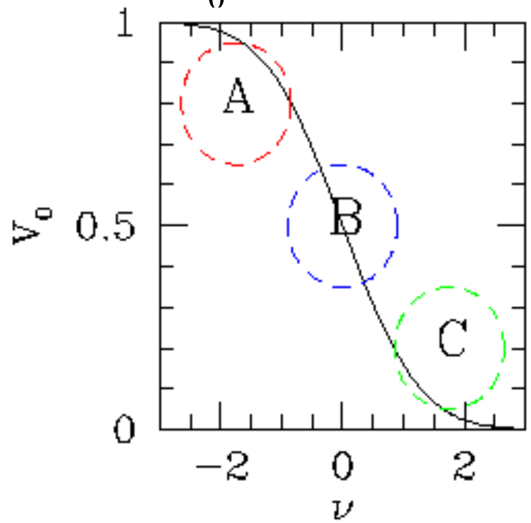
- Bispectrum (~ f_{NL})

Minkowski Functionals (MFs)

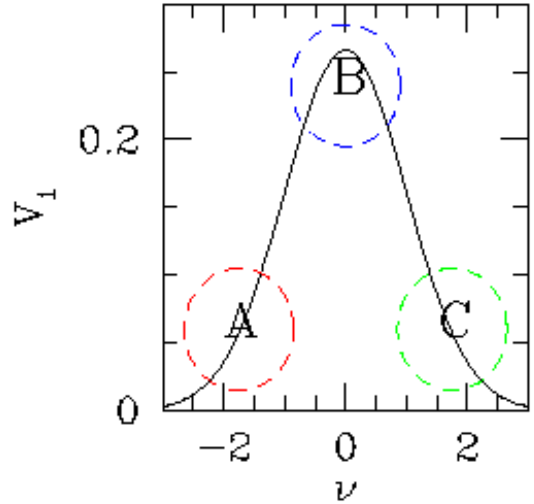


The number of hot spots minus cold spots.

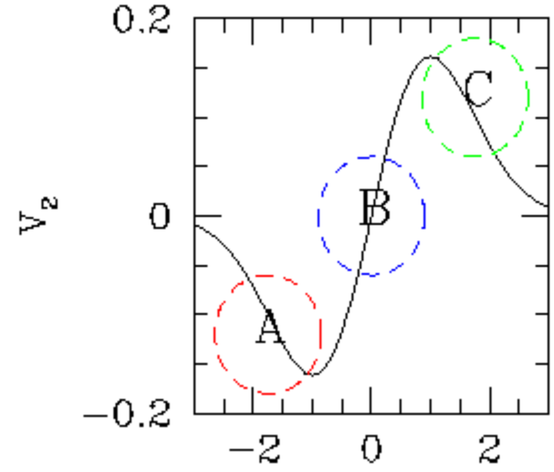
V$_0$:surface area



V$_1$: Contour Length



V$_2$: Euler Characteristic



Hikage, Komatsu & Matsubara (2006)

Analytical formulae of MFs

Perturbative formulae of MFs (Matsubara 2003)

$$V_k(\nu) = \frac{1}{(2\pi)^{(k+1)/2}} \frac{\omega_2}{\omega_{2-k}\omega_k} \left(\frac{\sigma_1}{\sqrt{2}\sigma_0}\right)^k e^{-\nu^2/2} \{ H_{k-1}(\nu)$$

Gaussian term

$(k = 0,1,2)$

$$+ \left[\frac{1}{6} S^{(0)} H_{k+2}(\nu) + \frac{k}{3} S^{(1)} H_k(\nu) + \frac{k(k-1)}{6} S^{(2)} H_{k-2}(\nu)\right]\sigma_0 + O(\sigma_0^2)\}$$

leading order of Non-Gaussian term

$$\sigma_j^2 = \frac{1}{4}\sum_l (2l+1)[l(l+1)]^j C_l W_l^2$$

W_l: smoothing kernel

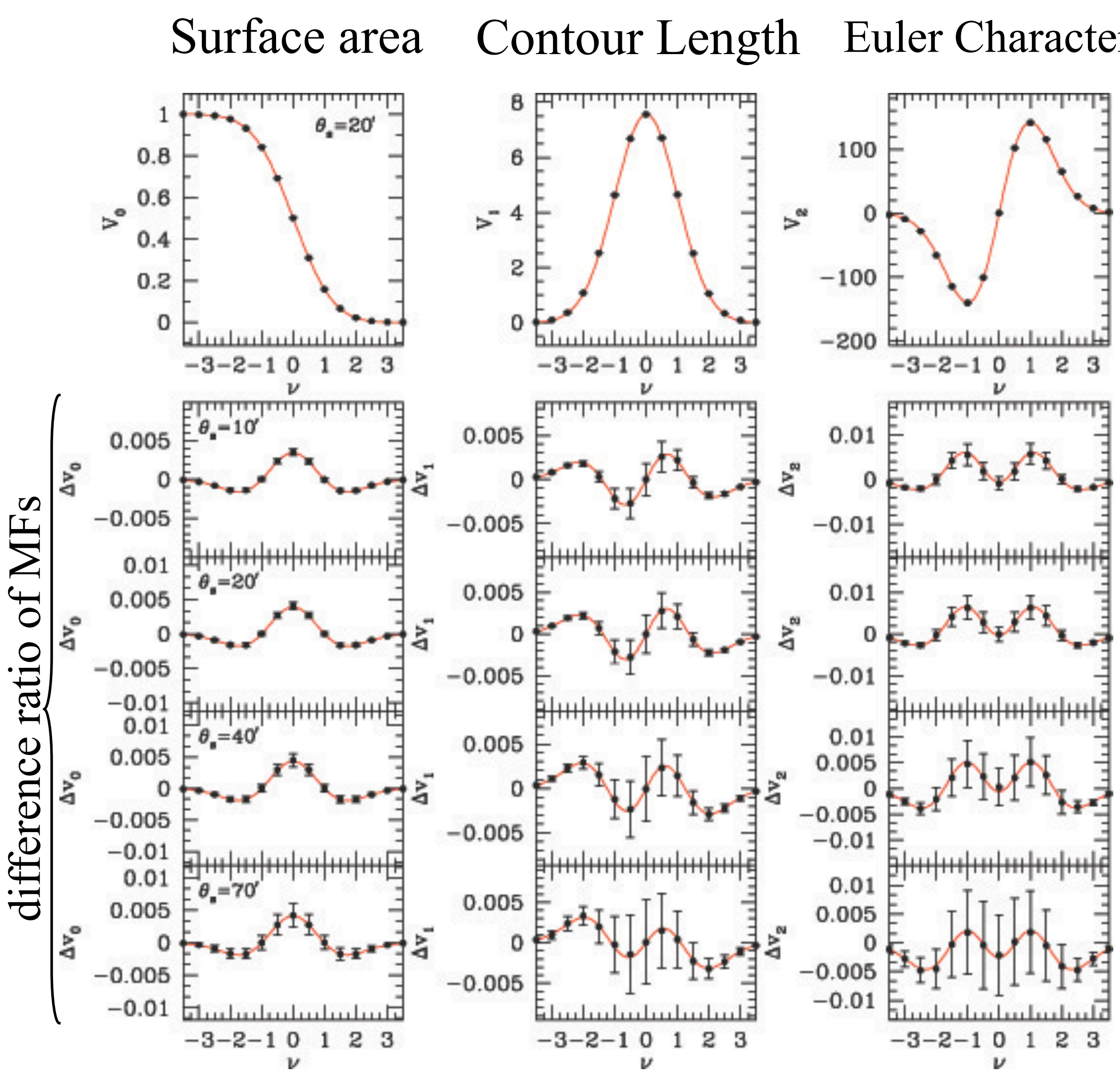
$\omega_0 = 1, \omega_1 = 1, \omega_2 = \pi, \omega_3 = 4\pi/3$ H_k : k- th Hermite polynomial

$S^{(a)}$: skewness parameters (a = 0,1,2)

In weakly non-Gaussian fields (σ_0<<1) , the non-Gaussianity in MFs is characterized by three skewness parameters $S^{(a)}$.

Hikage et al. (2007)

Comparison of analytical formulae with Non-Gaussian simulations



Comparison of MFs between analytical predictions and non-Gaussian simulations with f_{NL}=100 at different Gaussian smoothing scales, $\boldsymbol{\theta_s}$

Simulations are done for WMAP.

Analytical formulae agree with non-Gaussian simulations very well.

Komatsu et al. (2003); Spergel et al. (2007); Hikage et al. (2007)

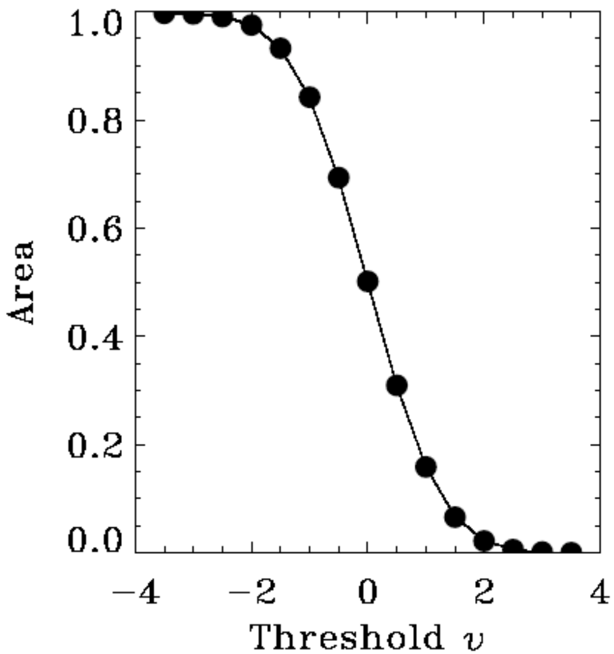
MFs from *WMAP*

(1yr)

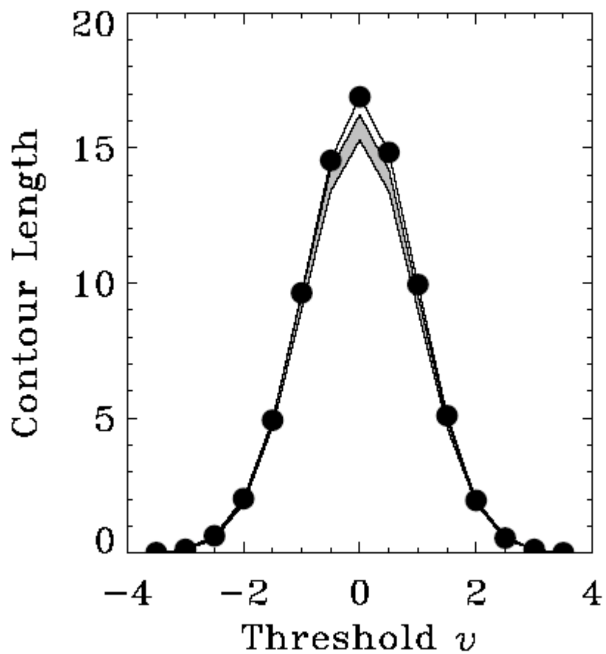
f_{NL} < +117 (95% CL)

(3yr)

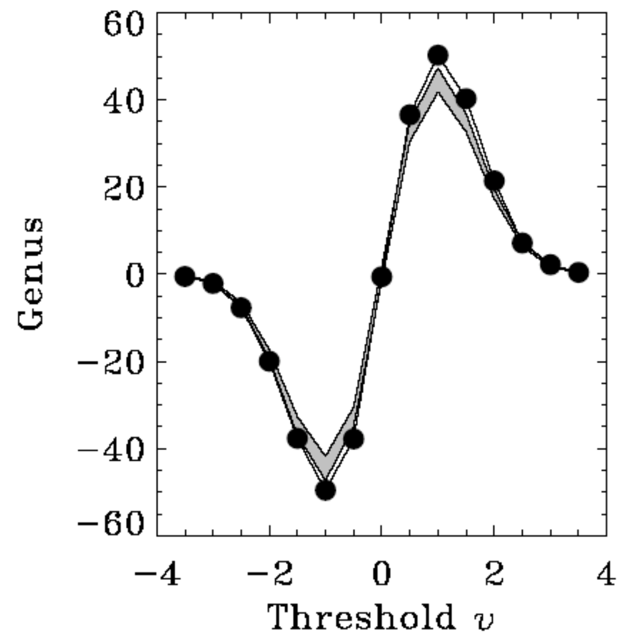
-70 < f_{NL} < +90 (95% CL)



Area



Contour Length



Euler
Characteristic

Gaussianity vs Flatness: Future

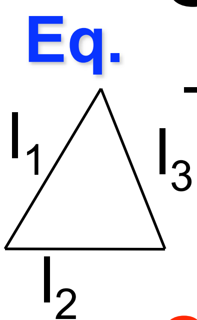
- **Flatness will never beat Gaussianity.**
  - In 5-10 years, we will know **flatness** to **0.1%** level.
  - In 5-10 years, we will know **Gaussianity** to **0.01%** level (f_{NL}~10), or even to **0.005%** level (f_{NL}~5), at 95% CL.
- However, a real potential of Gaussianity test is that **we might detect something at this level** (multi-field, curvaton, DBI, ghost cond., new ekpyrotic…)
  - Or, we might detect curvature first?
  - Is 0.1% curvature interesting/motivated?

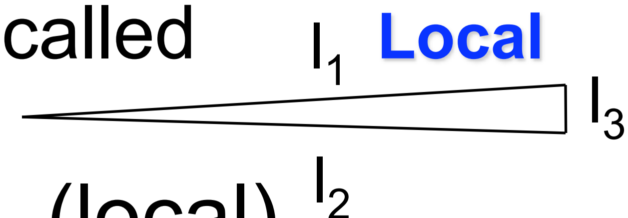
Journey For Measuring f_{NL}

- **2001**: Bispectrum method proposed and developed for f_{NL} (*Komatsu & Spergel*)
- **2002**: First observational constraint on f_{NL} from the COBE 4-yr data (*Komatsu, Wandelt, Spergel, Banday & Gorski*)
  - **$-3500 < f_{NL} < +2000$** (95%CL; **l_{max}=20**)
- **2003**: First numerical simulation of CMB with f_{NL} (*Komatsu*)
- **2003**: WMAP 1-year (*Komatsu, WMAP team*)
  - **$-58 < f_{NL} < +134$** (95% CL; **l_{max}=265**)

Journey For Measuring f_{NL}

- **2004**: Classification scheme of triangle dependence proposed (Babich, Creminelli & Zaldarriaga)
  - There are two "f_{NL}": the original f_{NL} is called "local," and the new one is called "equilateral."





- **2005**: Fast estimator for f_{NL}(local) developed ("KSW" estimator; *Komatsu, Spergel & Wandelt*)

Journey For Measuring f_{NL}

- **2006**: Improvement made to the KSW method, and applied to WMAP 1-year data by Harvard group (*Creminelli, et al.)*
  - **$-27 < f_{NL}$(local) $< +121$** (95% CL; **lmax=335**)
- **2006**: Fast estimator for f_{NL}(equilateral) developed, and applied to WMAP 1-year data by Harvard group (*Creminelli, et al.)*
  - **$-366 < f_{NL}$(equilateral) $< +238$** (95% CL; **lmax=405**)

Journey For Measuring f_{NL}

- **2007**: WMAP 3-year constraints
  - **-54 < f_{NL}(local) < +114** (95% CL; **lmax=350**) (*Spergel, WMAP team*)
  - **-36 < f_{NL}(local) < +100** (95% CL; **lmax=370**) (*Creminelli, et al.*)
  - **-256 < f_{NL}(equilateral) < +332** (95% CL; **lmax=475**) (*Creminelli, et al.*)
- **2007**: We've made further improvement to Harvard group's extension of the KSW method; **now, the estimator is very close to optimal** (*Yadav, Komatsu, Wandelt*)

Latest News on f_{NL}

- **2007**: Latest constraint from the WMAP 3-year data using the new YKW estimator
  - **+27 < f_{NL}(local) < +147** (95% CL; **lmax=750**) (Yadav & Wandelt, arXiv:0712.1148)
  - Note a significant jump in lmax.
  - A "hint" of f_{NL}(local)>0 at more than two σ?
- **Our independent analysis showed a similar level of f_{NL}(local), but no evidence for f_{NL}(equilateral).**

There have been many claims of non-Gaussianity at the 2-3 σ.
This is the best physically motivated one, and will be testable with more data.

WMAP: Future Prospects

- Could more years of data from WMAP yield a definitive answer?
  - 3-year latest [Y&W]: f_{NL}(local) = 87 +/- 60 (95%)
- Projected 95% uncertainty from WMAP
  - 5yr: Error[f_{NL}(local)] ~ 50
  - 8yr: Error[f_{NL}(local)] ~ 42
  - 12yr: Error[f_{NL}(local)] ~ 38

An unambiguous (>4σ) detection of f_{NL}(local) at this level with the future (e.g., 8yr) WMAP data could be a truly remarkable discovery.

More On Future Prospects

- CMB: Planck (temperature + polarization): **f_{NL}(local)<6 (95%)**
  - Yadav, Komatsu & Wandelt (2007)
- Large-scale Structure: e.g., ADEPT, CIP: **f_{NL}(local)<7 (95%); f_{NL}(equilateral)<90 (95%)**
  - Sefusatti & Komatsu (2007)
- CMB and LSS are independent. By combining these two constraints, we get **f_{NL}(local)<4.5**. This is currently the best constraint that we can possibly achieve in the foreseeable future (~10 years)

Classifying Non-Gaussianities in the Literature

- Local Form
  - Ekpyrotic models
  - Curvaton models
- Equilateral Form
  - Ghost condensation, DBI, low speed of sound models
- Other Forms
  - Features in potential, which produce large non-Gaussianity within narrow region in l

•Is any of these a winner?
•Non-Gaussianity may tell us soon. We will find out!

Summary

- Since the introduction of f_{NL}, the research on non-Gaussianity as a probe of the physics of early universe has evolved tremendously.
- I hope I convinced you that f_{NL} is as important a tool as Ω_K, n_s, $dn_s/d\ln k$, and r, for constraining inflation models.
- In fact, it has the best chance of ruling out the largest population of models...

Concluding Remarks

- Stay tuned: WMAP continues to observe, and Planck will soon be launched.
- Non-Gaussianity has provided cosmologists and string theorists with a unique opportunity to work together.
- For me, this is one of the most important contributions that f_{NL} has made to the community.