What every dynamicist should know about... Cosmology

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42nd Annual Meeting of AAS Division on Dynamical Astronomy
April 12, 2011

Cosmology: The Questions

- How much do we understand our Universe?
  - How old is it?
  - How big is it?
  - What shape does it take?
  - What is it made of?
  - How did it begin?

Dynamics of the Universe?

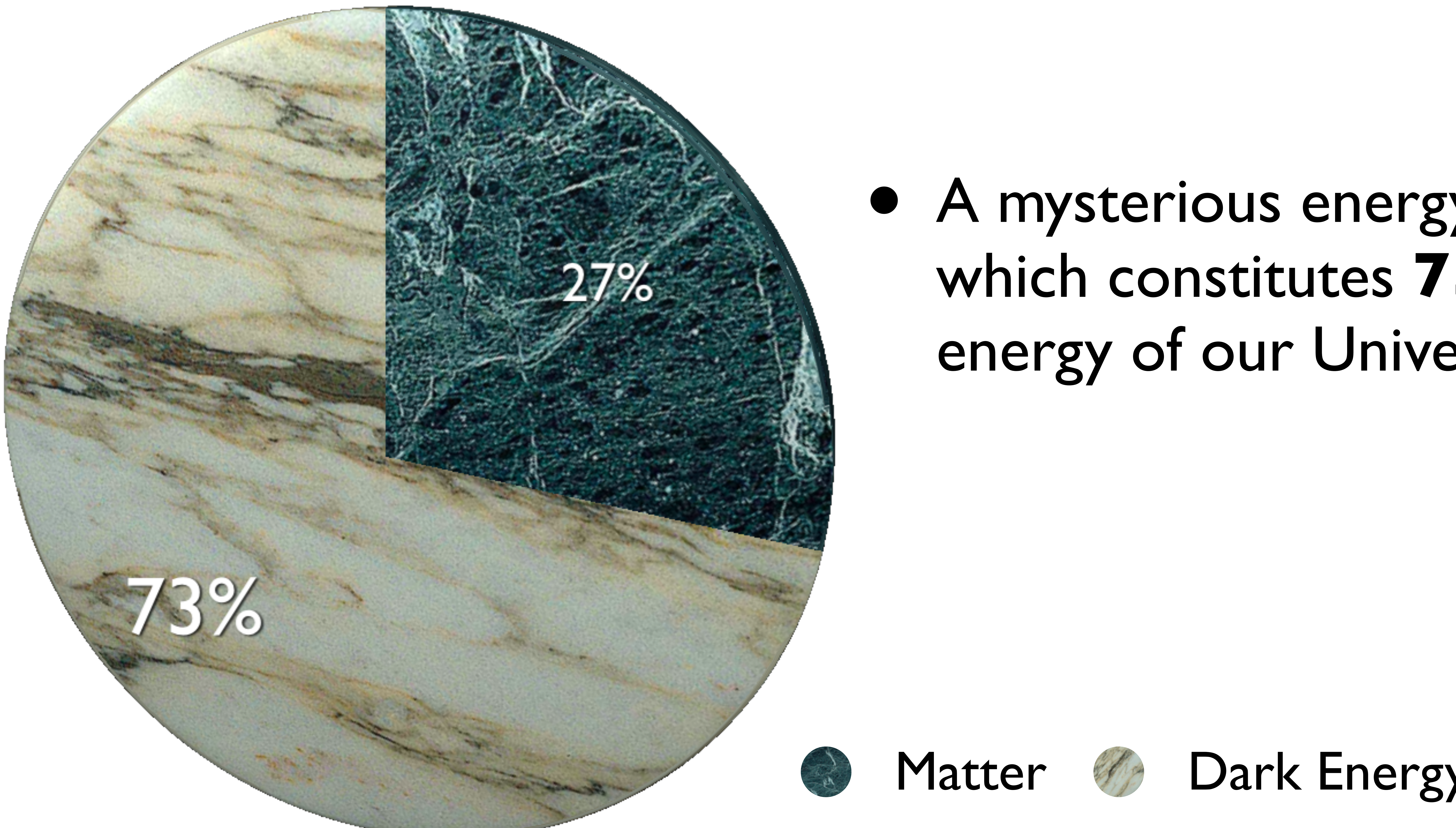
- The Universe expands, and how it expands depends on what is in it.
- As the Universe expands, the Universe cools. As the Universe cools, various things start to happen.
- We observe structures in the Universe! Where do they come from, and how were they formed?

From "Cosmic Voyage"

Strange things can happen

- In cosmology, it is not uncommon to see and think about something completely crazy.
- One good example is "dark energy."
- What does it do?

What is dark energy?



- A mysterious energy component, which constitutes **73%** of the energy of our Universe.

How is dark energy different from matter?

- Matter *slows down* the expansion of the Universe by gravity
- Dark Energy *accelerates* the expansion of the Universe by (what appears to be an) "anti-gravity"

Imagine you throw an apple to the above...


ZERO


ZERO
暗黒エネルギーが支配する場合

Newton thought about it (with the opposite sign)

- Everyone knows about Newton's formula for a gravitational acceleration:

$$a = -\frac{GM}{r^2}$$

- However, Newton also wrote down another term, which linear in distance (in Principia):

$$a = -\frac{GM}{r^2} + Br$$

Newton thought about it (with the opposite sign)

$$a = -\frac{GM}{r^2} + Br$$

- Newton was imagining an attractive force, so B was taken to be negative ($B_{Newton}<0$).
- What is special about these **two** particular terms?
  - These forces can have circular or elliptical orbits.
  - The force exerted by an extended body with mass *M* is the same as the force exerted by a point particle with the same mass *M*.

Newton thought about it (with the opposite sign)

$$a = -\frac{GM}{r^2} + Br$$

- So, if we take the opposite limit, B<0, then we can get an acceleration, similar to what we observe in cosmology!
- Another good example is Hooke's law (k>0):

$$F = -kr$$

However:

- These formulae are all non-relativistic. You must you General Relativity to describe a whole Universe.
- Let's see what you would get from

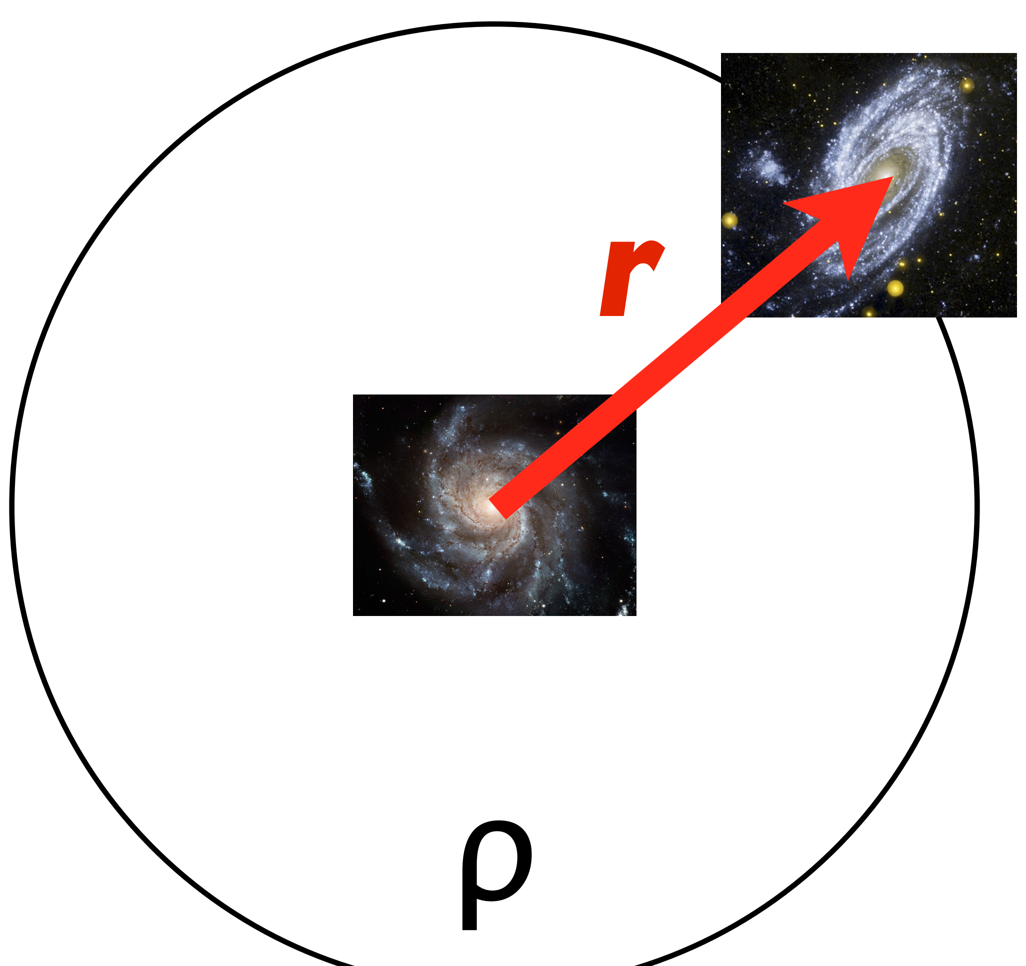
$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = \frac{8\pi G}{c^4} T_{\mu\nu}$$

Matter-dominated Universe

- For an expanding universe dominated by matter (where there is no dark energy), GR gives the acceleration between two galaxies is given by

$$a = -\frac{4\pi G\rho r}{3}$$

- where ρ is the mean mass density of the Universe.



Now, use $M = (4\pi/3)\,\rho\, r^3$

$$a = -\frac{GM}{r^2}$$

The same result as Newtonian!

General Relativity Adds One More Thing...

- *Pressure also contributes to the acceleration.*

$$a = -\frac{4\pi G(\rho + 3P)r}{3}$$

- From the current observations of the expansion of the universe, we have obtained:
  - $P_{\text{dark enrgy}} = (-1 \pm 0.1)\rho_{\text{dark energy}}$ **[<0; negative pressure!]**
  - $\rho_{\text{dark energy}} \sim$ constant
- Then, by defining "cosmological constant," $\Lambda = 8\pi G \rho_{\text{dark energy}}$, we obtain...

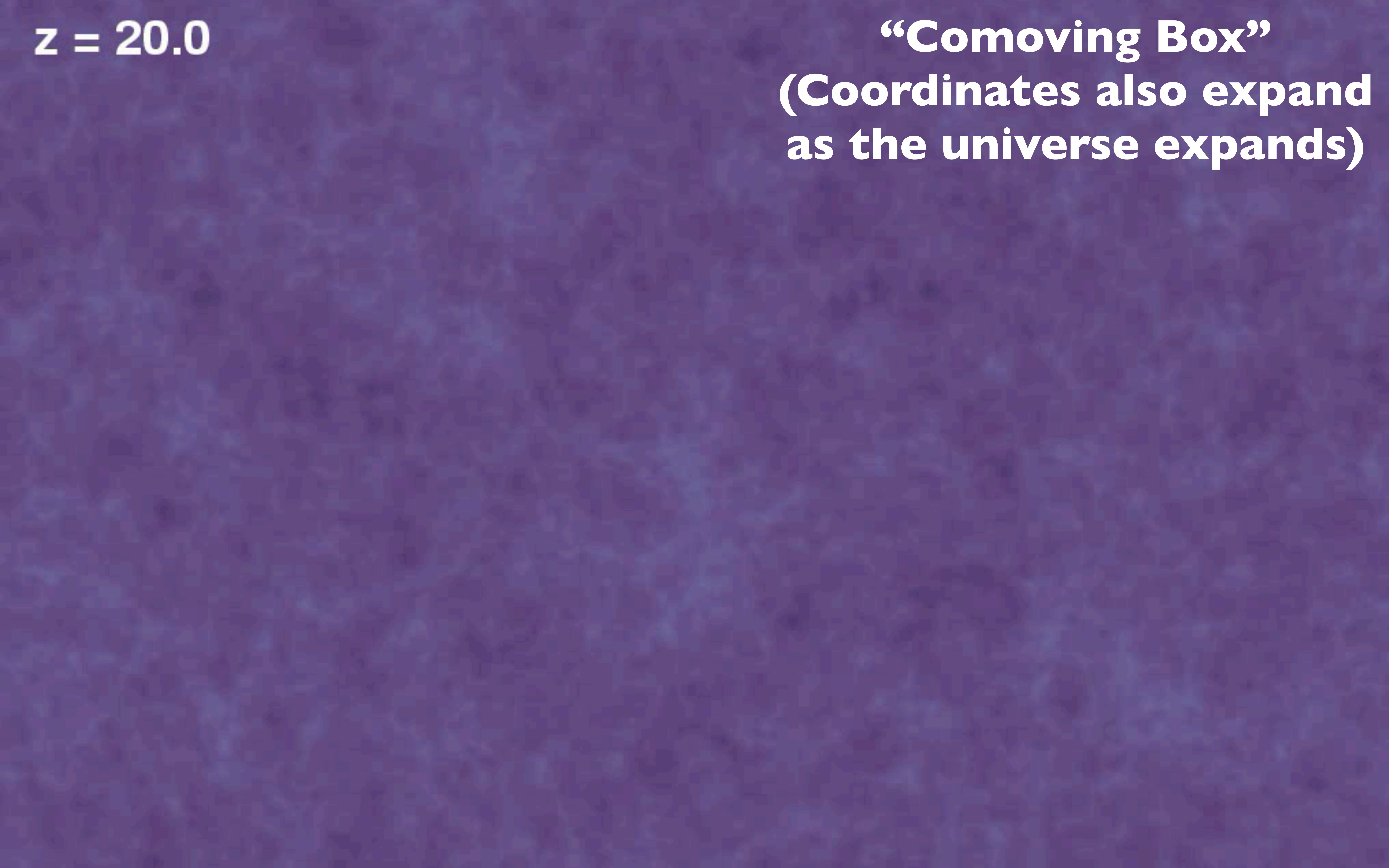
General Relativistic Acceleration Equation

$$a = -\frac{4\pi G\rho r}{3} + \frac{\Lambda}{3} r$$

- which is identical to the formula that Newton conceived:

$$a = -\frac{GM}{r^2} + Br \qquad (M = (4\pi/3)\,\rho\, r^3)$$

With, of course, the "wrong sign" - Λ>0 leads to an acceleration of the Universe!



“Comoving Box”
(Coordinates also expand as the universe expands)

How do particles move in an expanding universe?

Velocity = [Expansion Velocity (Hubble Flow)] + [**Peculiar Velocity**]

- A surprise again! The equation of motion for peculiar velocity is the same as the usual Euler equation, except for the cosmological redshift effect.
  - Namely, in the absence of external forces, the peculiar velocity decays as $V_{peculiar} \sim 1/a(t)$ where $a(t)$ is the expansion factor.

Euler Equation in an Expanding Universe

*for non-relativistic particles

$$\dot{\vec{V}} = -\frac{\dot{a}}{a}\vec{V} - \frac{1}{a}\vec{\nabla}\phi - \frac{1}{a\rho}\vec{\nabla}P$$

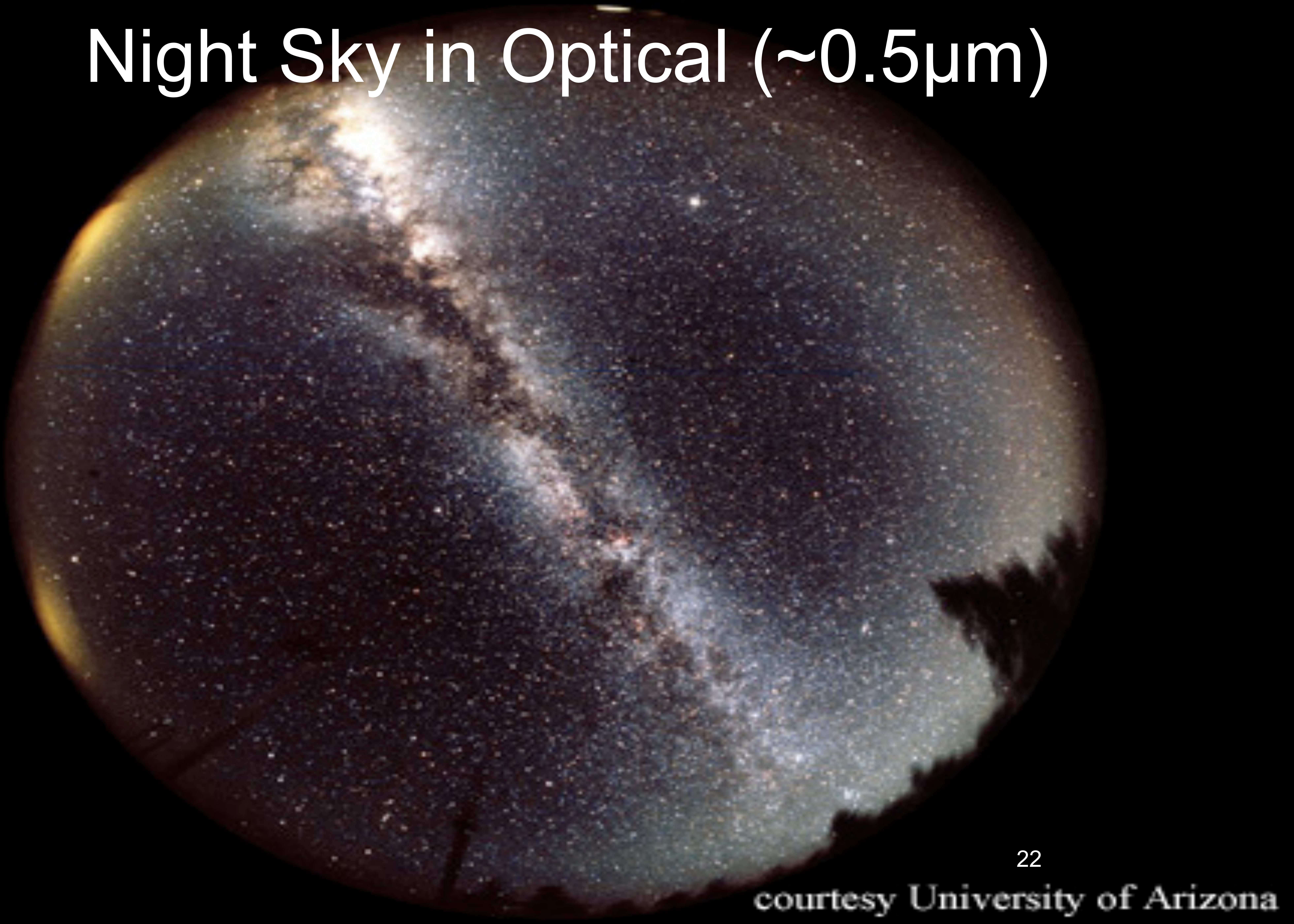
- The usual story!
  - 1st term: cosmological redshift
  - 2nd term: gravitational force
  - 3rd term: pressure gradient

Yet, this is a fully General Relativistic result (for linear perturbations)

Cosmological Hydrodynamics

- Very successful application to a redshift of z=1100 (when the Universe was 380,000 years old)
  - Cosmic Microwave Background

Night Sky in Optical (~0.5μm)

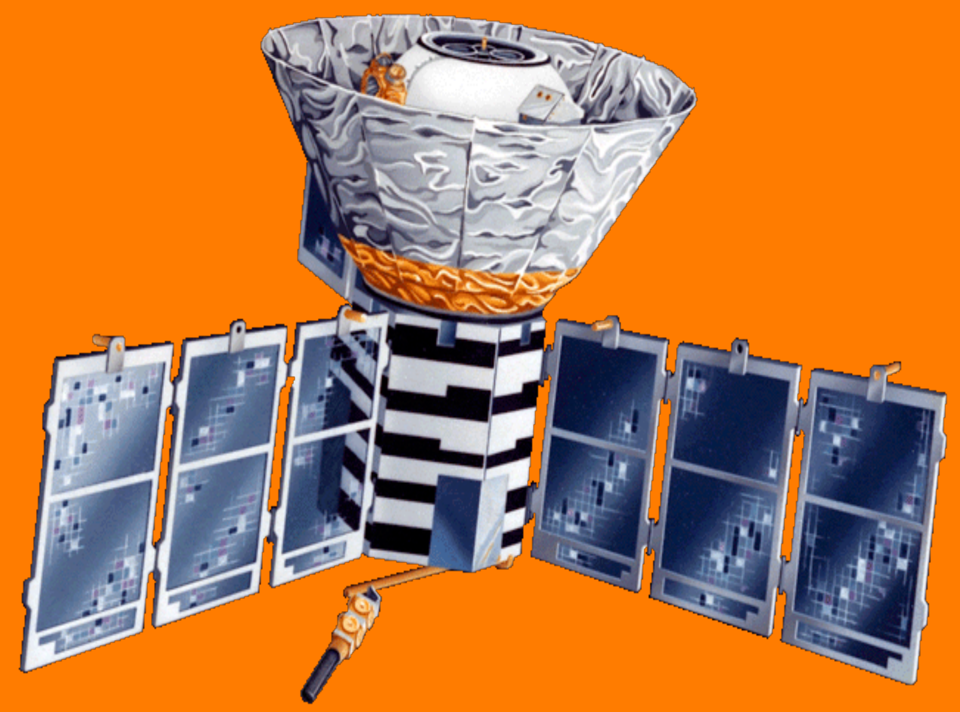


Night Sky in Microwave (~1mm)

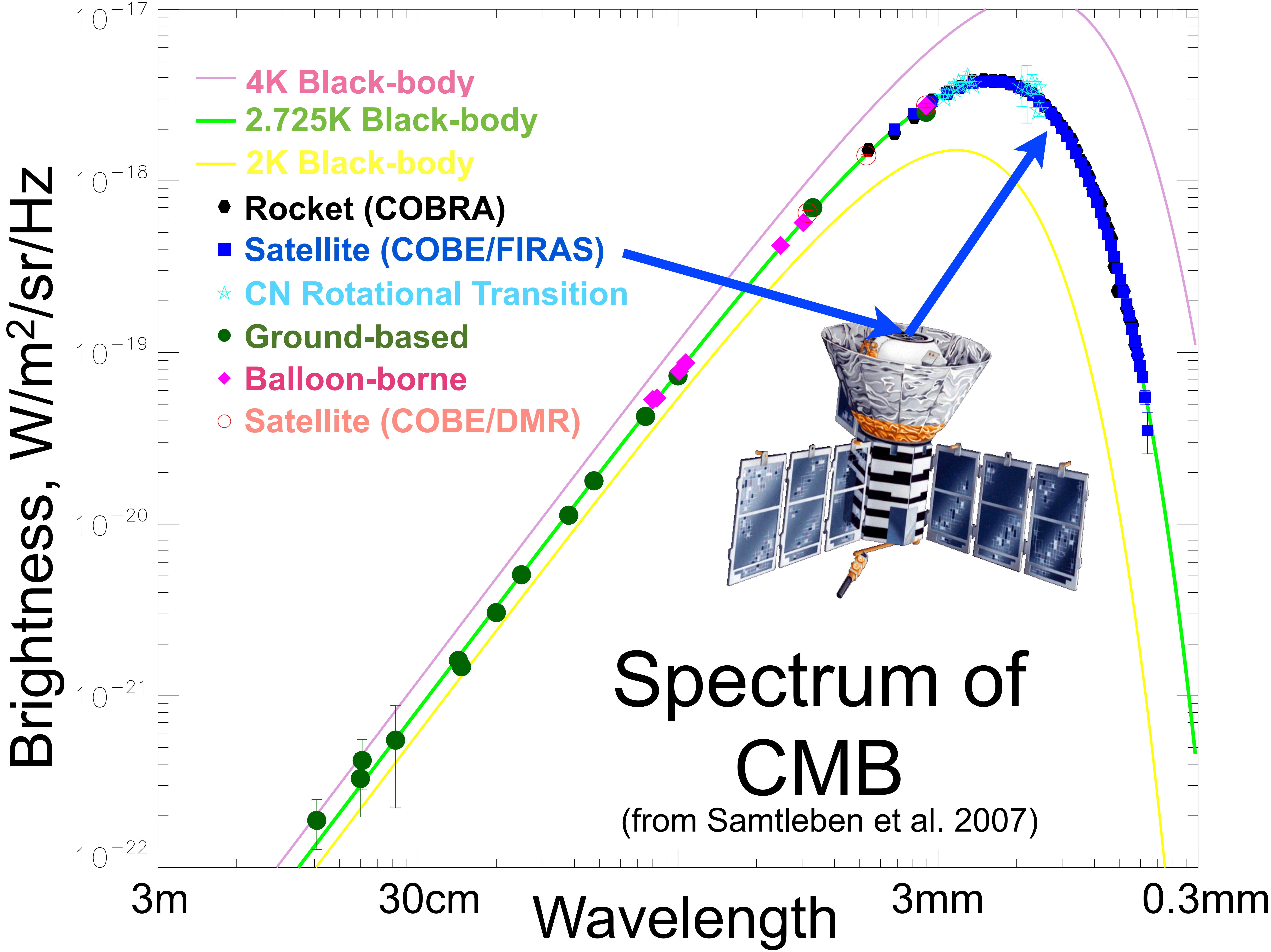
courtesy University of Arizona

Night Sky in Microwave (~1mm)

T_{today}=2.725K



COBE Satellite, 1989-1993

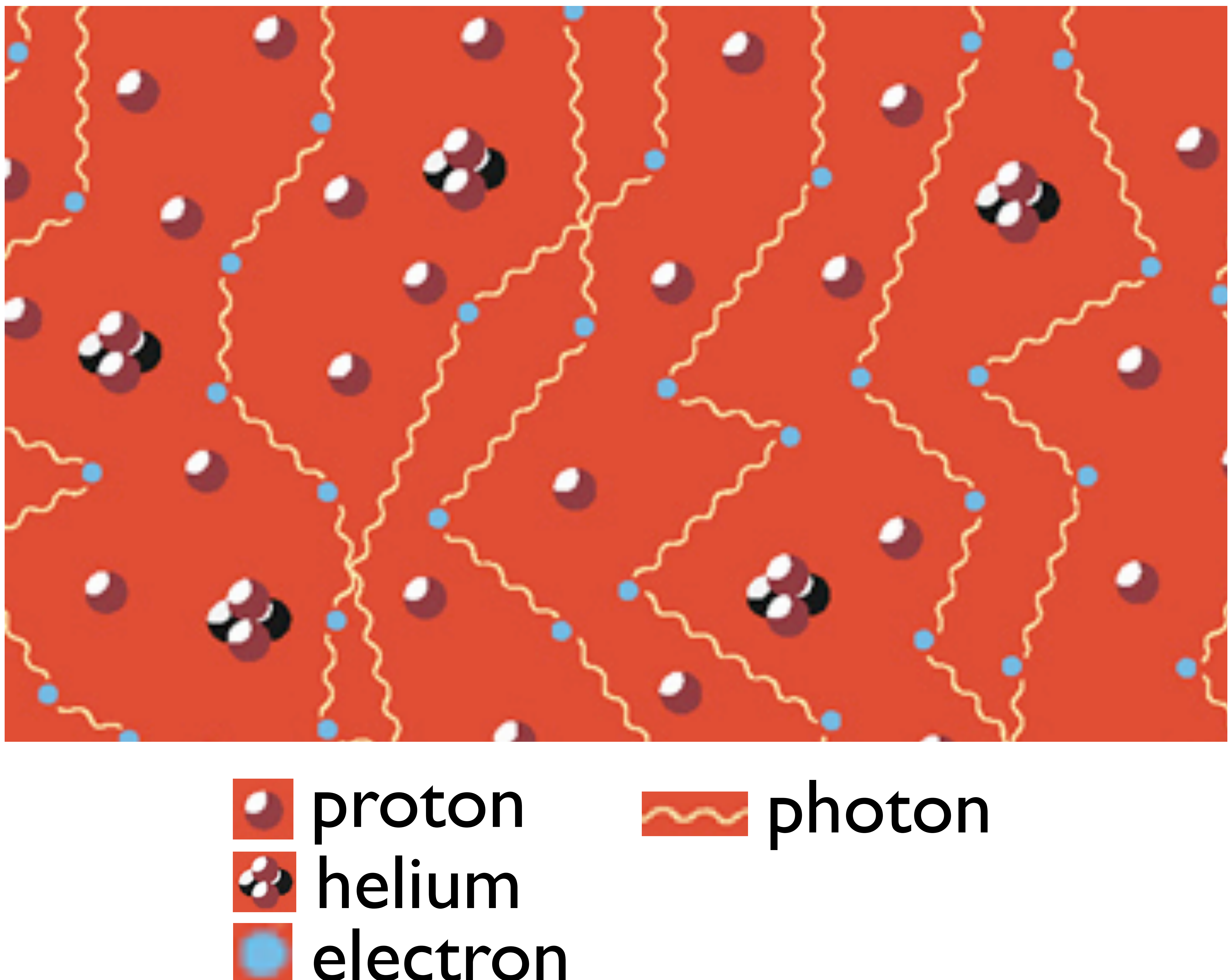


4K Black-body
2.725K Black-body
2K Black-body
Rocket (COBRA)
Satellite (COBE/FIRAS)
CN Rotational Transition
Ground-based
Balloon-borne
Satellite (COBE/DMR)
Spectrum of CMB
(from Samtleben et al. 2007)
Brightness, W/m²/sr/Hz
10^{-17}
10^{-18}
10^{-19}
10^{-20}
10^{-21}
10^{-22}
3m
30cm
3mm
0.3mm
Wavelength

How was CMB created?

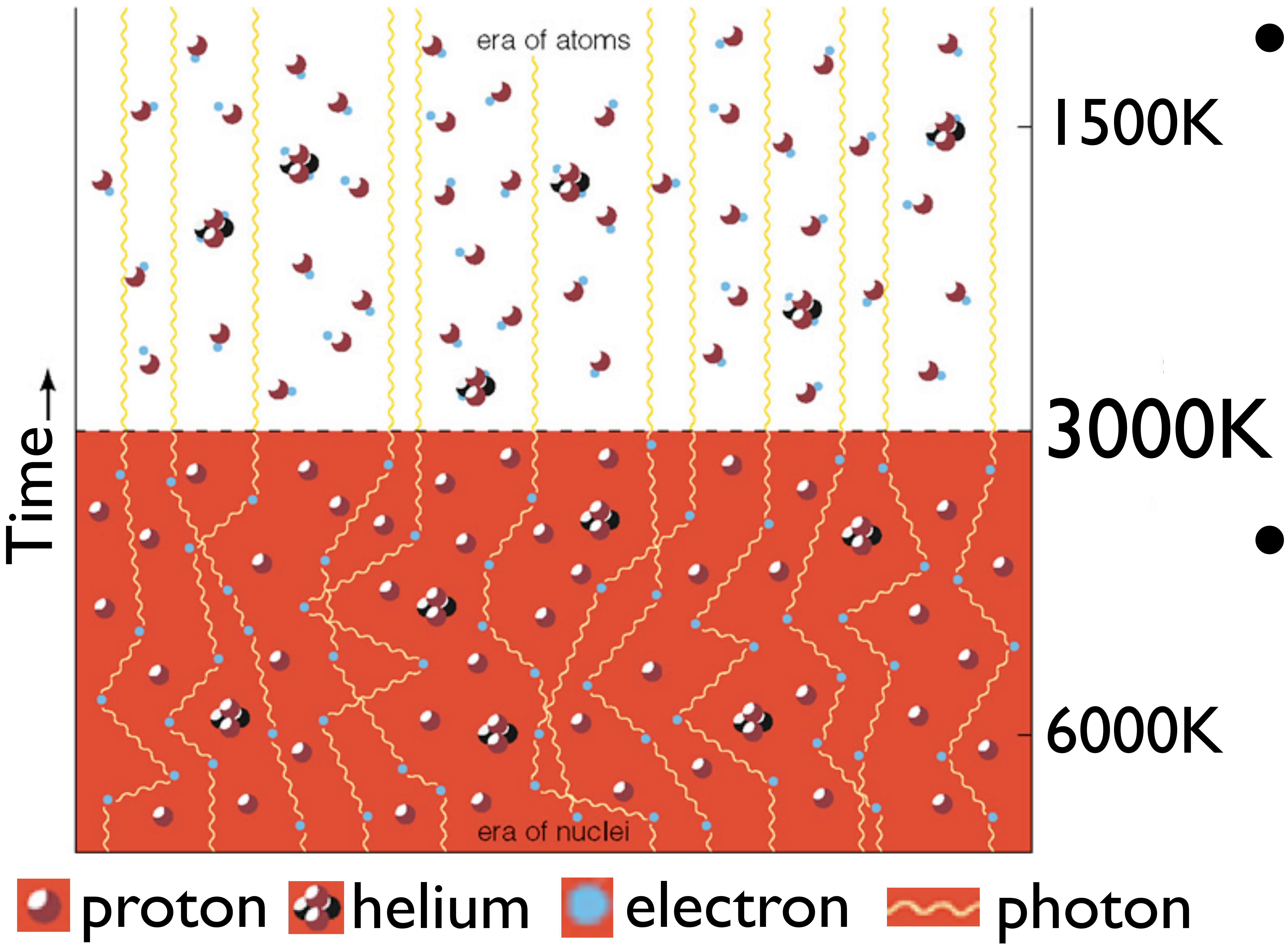
- When the Universe was hot, it was a hot soup made of:
  - Protons, electrons, and helium nuclei
  - Photons and neutrinos
  - Dark matter (DM)
    - DM does not do much, except for providing a a gravitational potential because $\rho_{DM}/\rho_{H,He}$~5)

Universe as a hot soup

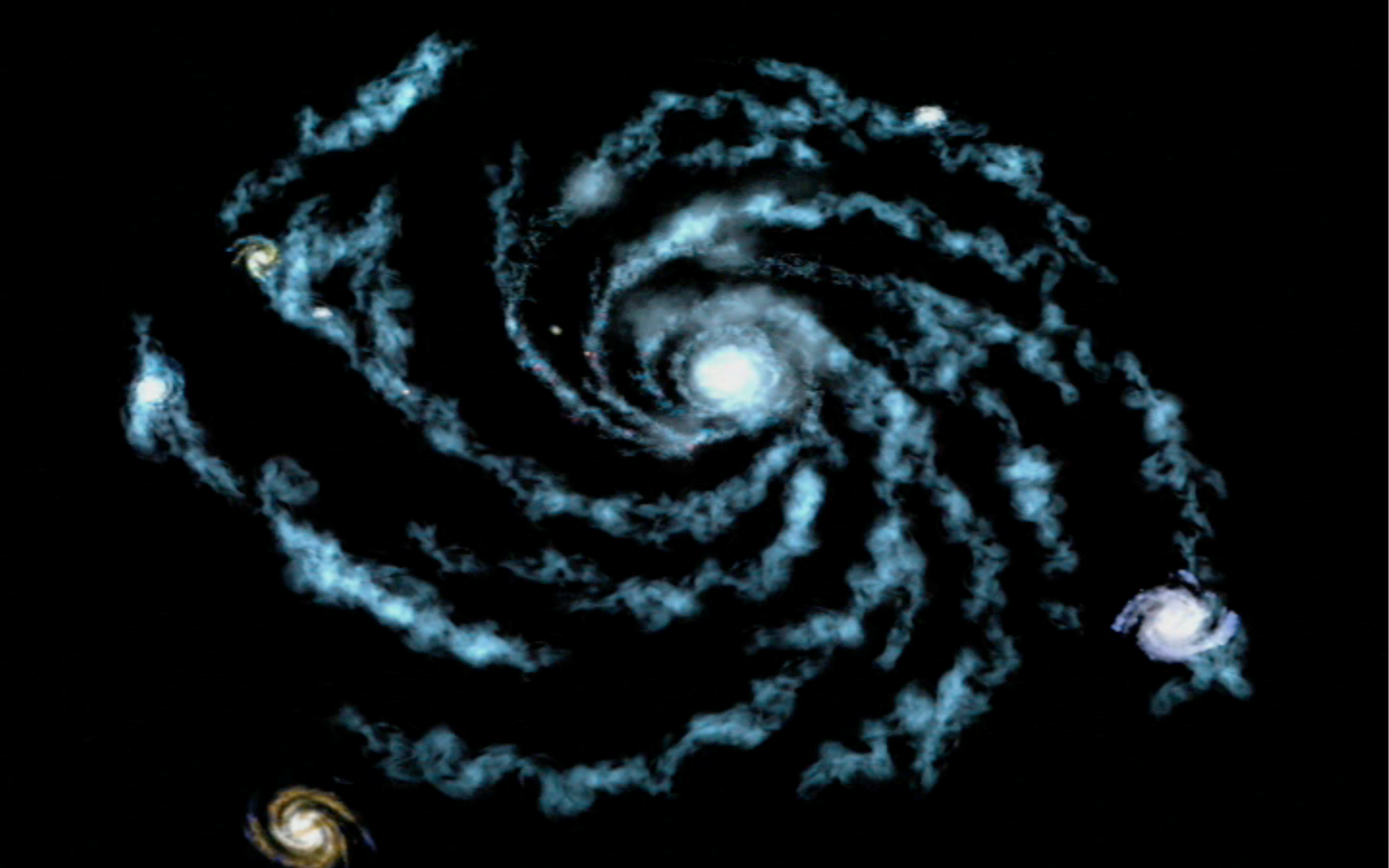


- Free electrons can scatter photons efficiently.
- Photons cannot go very far.

Recombination and Decoupling

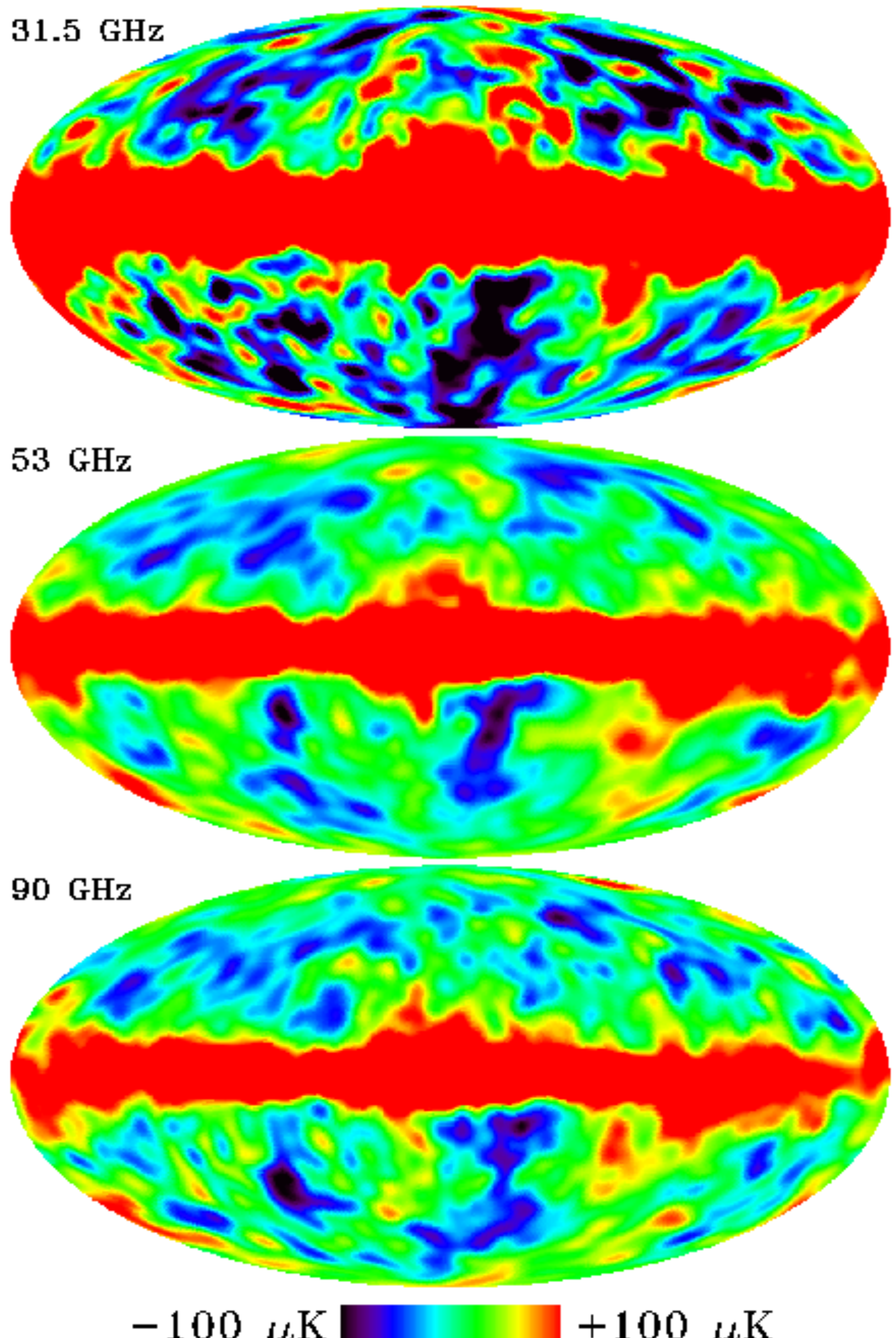


- [***recombination***] When the temperature falls below 3000 K, almost all electrons are captured by protons and helium nuclei.
- [***decoupling***] Photons are no longer scattered. I.e., photons and electrons are no longer coupled.



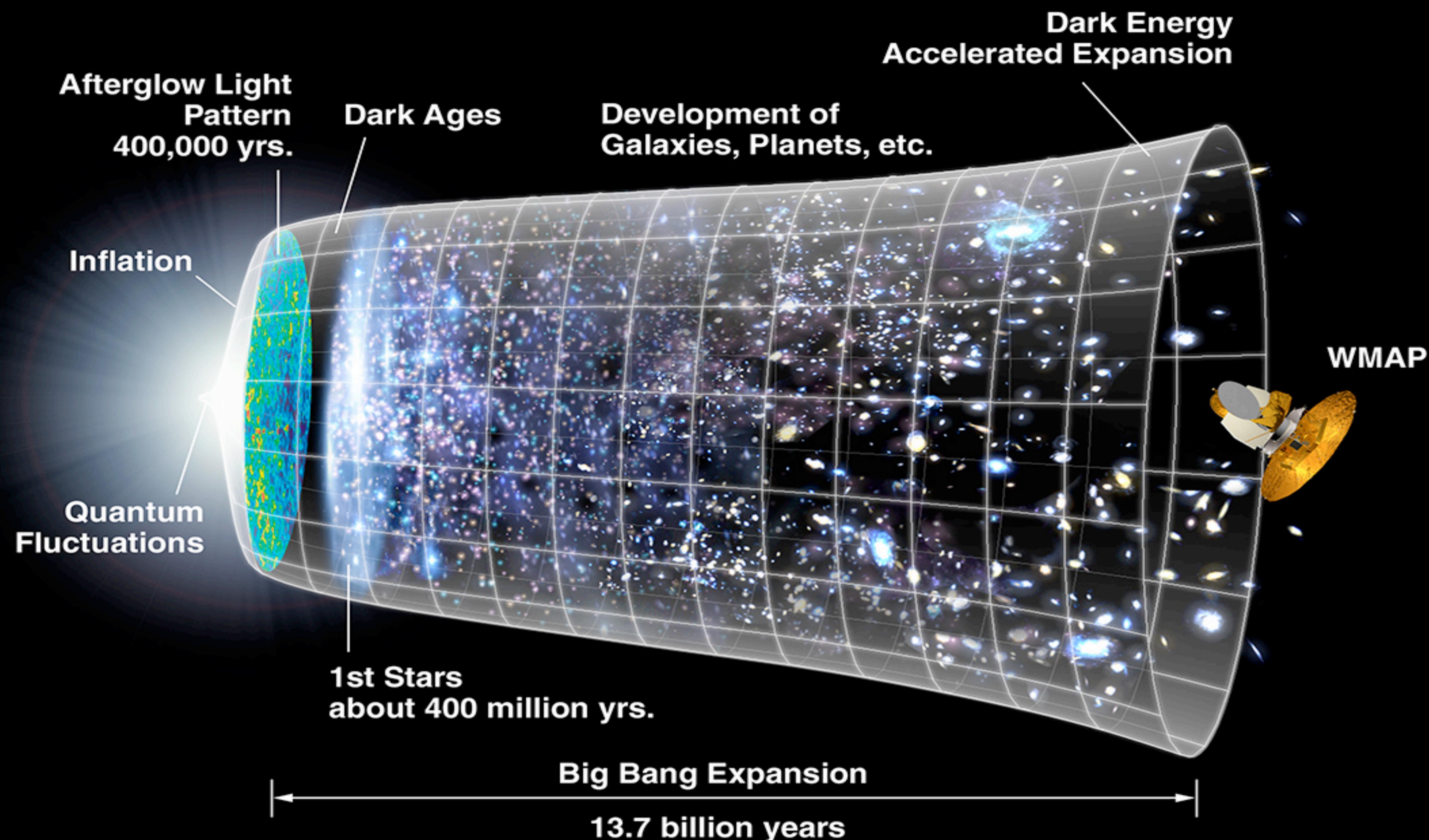
COBE/DMR, 1992

Smoot et al. (1992)



- **Isotropic?**
- **CMB is anisotropic! (at the 1/100,000 level)**

CMB: The Farthest and Oldest Light That We Can Ever Hope To Observe Directly

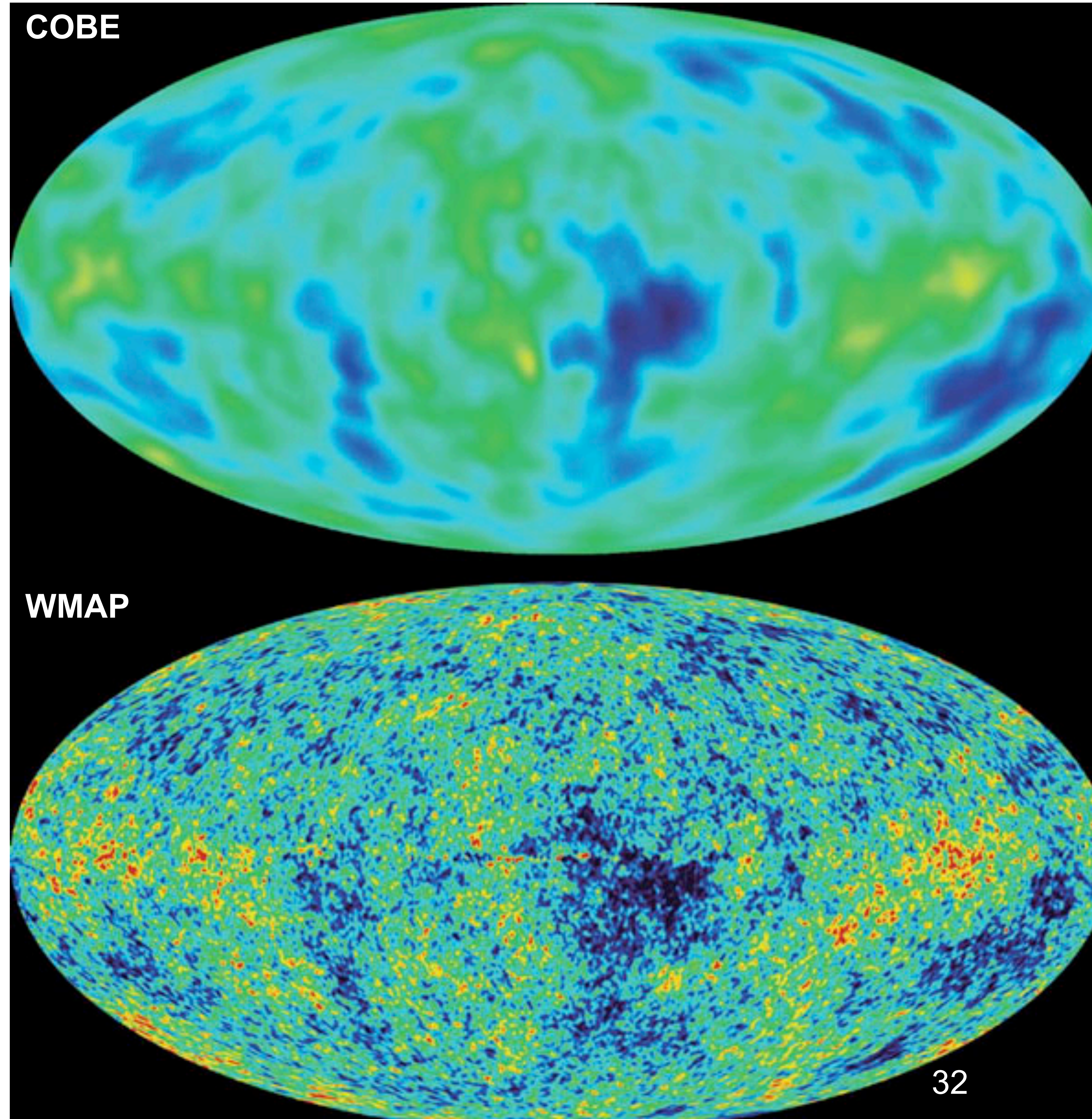


- When the Universe was 3000K (~380,000 years after the Big Bang), electrons and protons were combined to form neutral hydrogen.

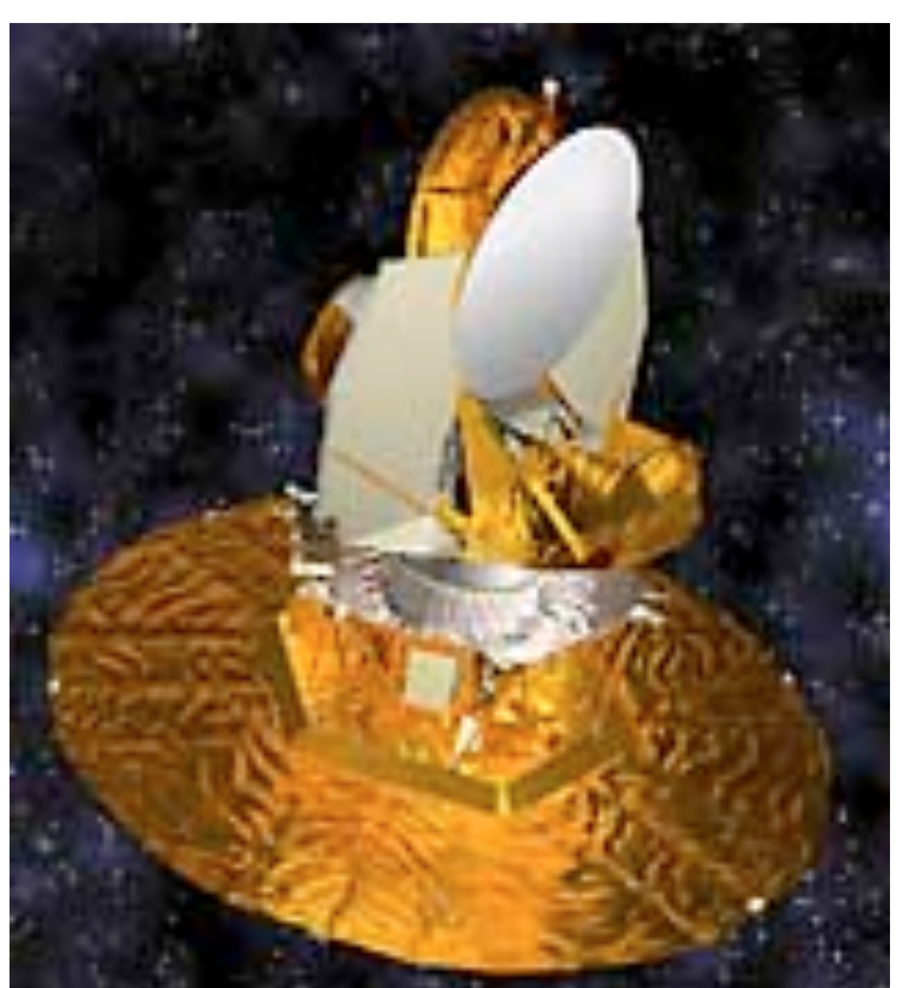
NASA/WMAP Science Team

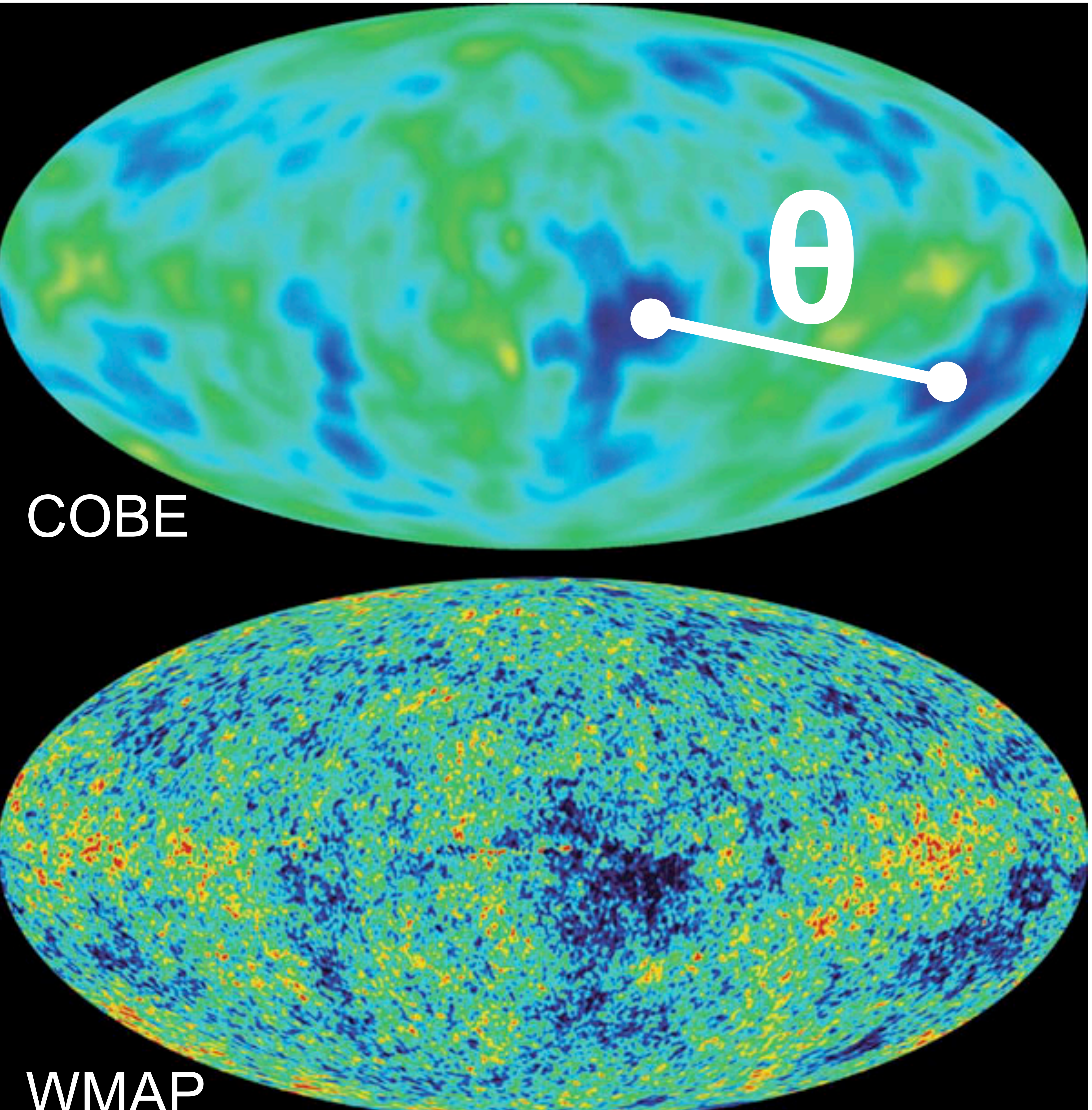
COBE to WMAP (x35 better resolution)

COBE
1989



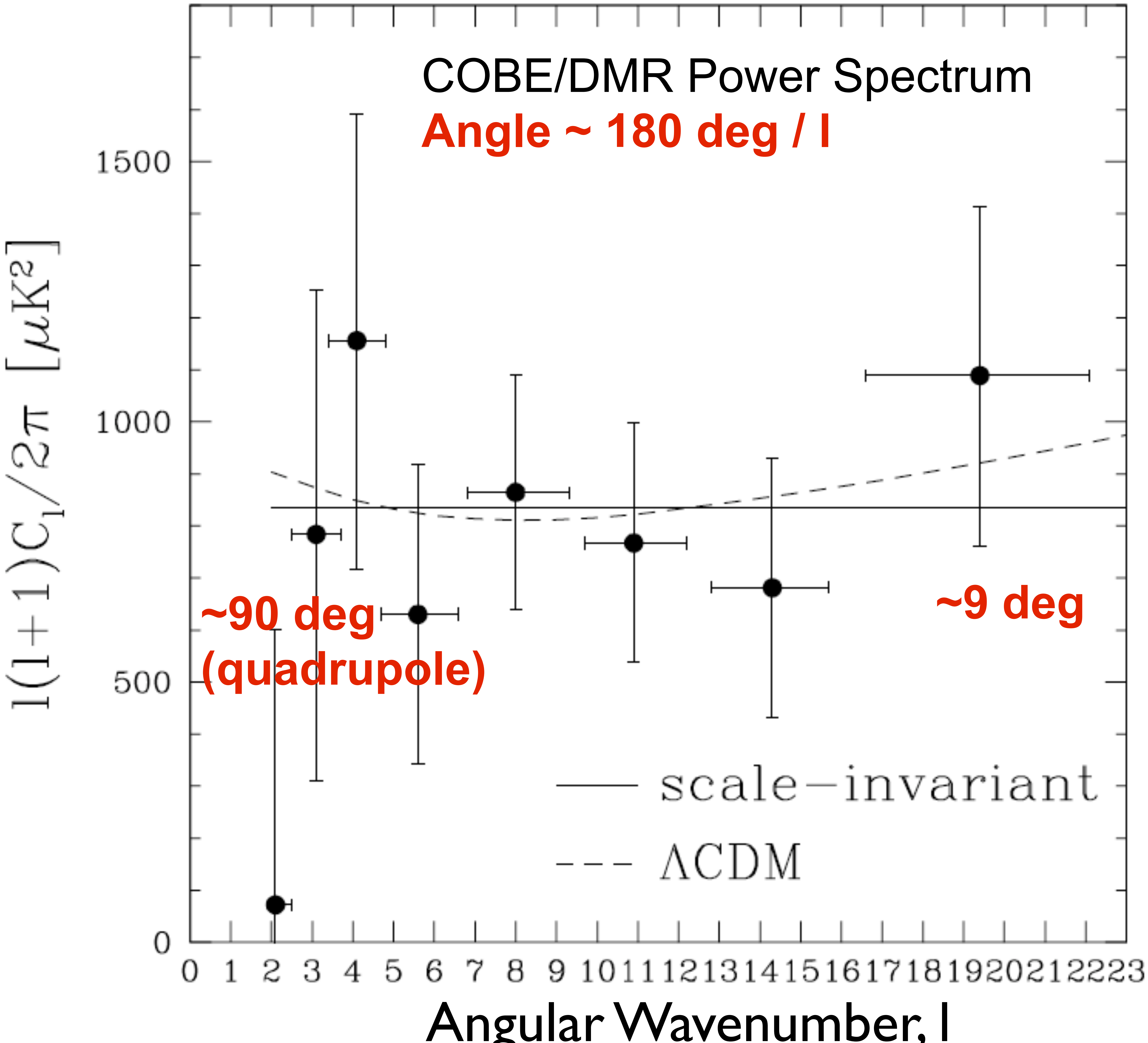
WMAP
2001

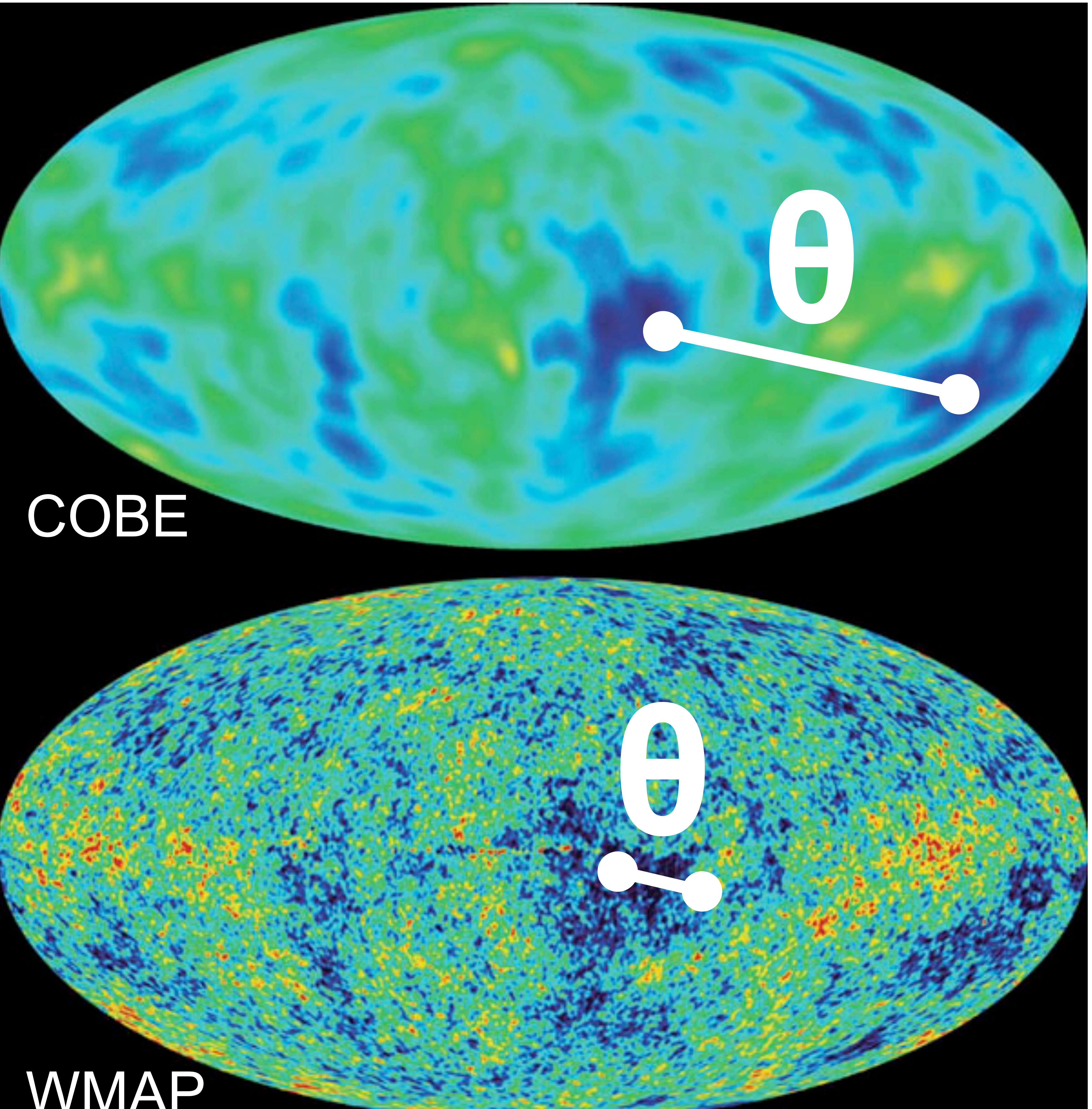




Analysis: 2-point Correlation

- $C(\theta)=(1/4\pi)\sum(2l+1)C_l P_l(\cos\theta)$
- How are temperatures on two points on the sky, separated by θ, are correlated?
- **"Power Spectrum," C_l**
  - How much fluctuation power do we have at a given angular scale?
  - **$l \sim 180$ degrees / θ**

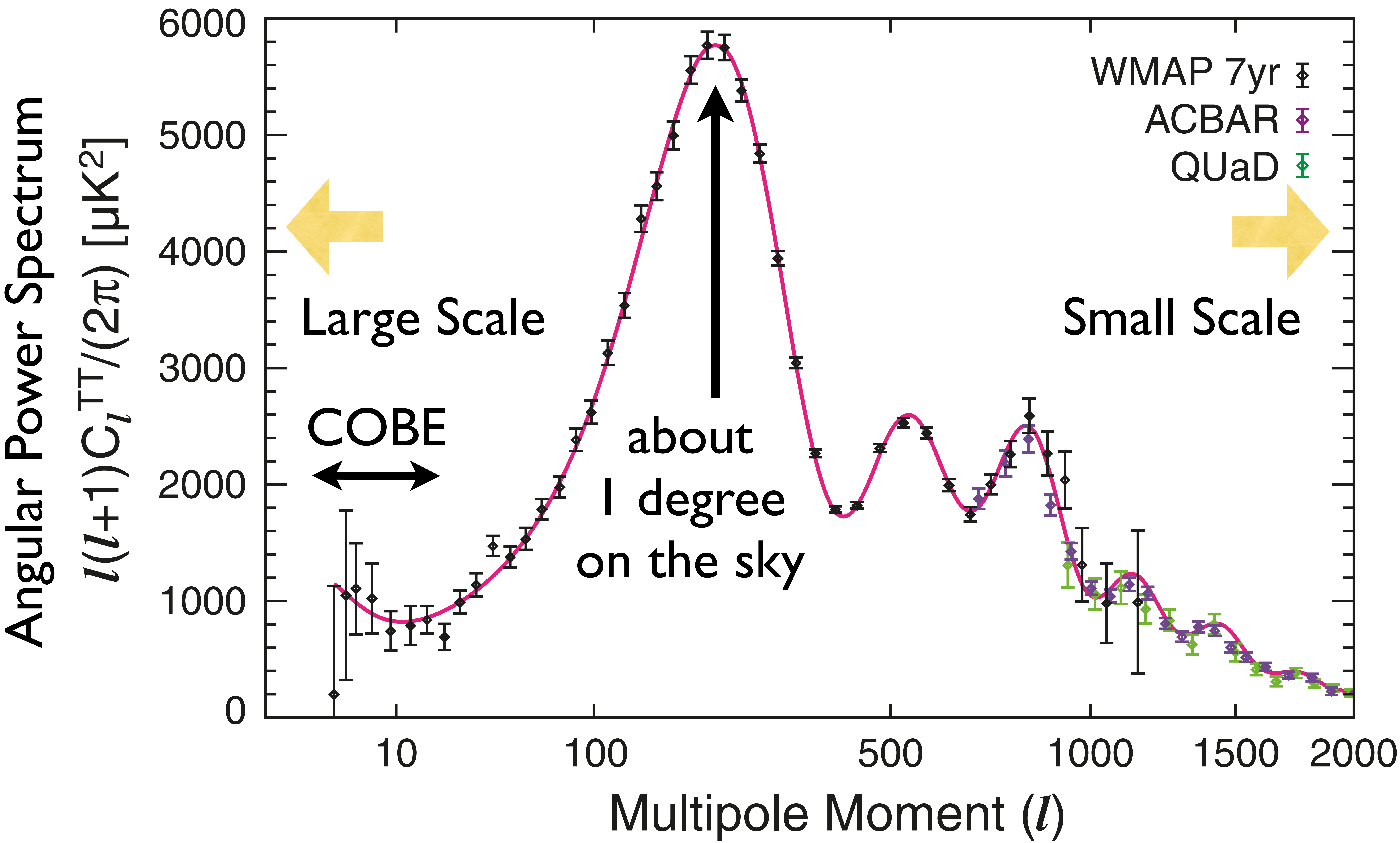

COBE/DMR Power Spectrum
Angle ~ 180 deg / l
~90 deg
(quadrupole)
~9 deg
scale−invariant
ΛCDM
$l(l+1)C_l/2\pi$ [μK^2]
0
500
1000
1500
0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23
Angular Wavenumber, l



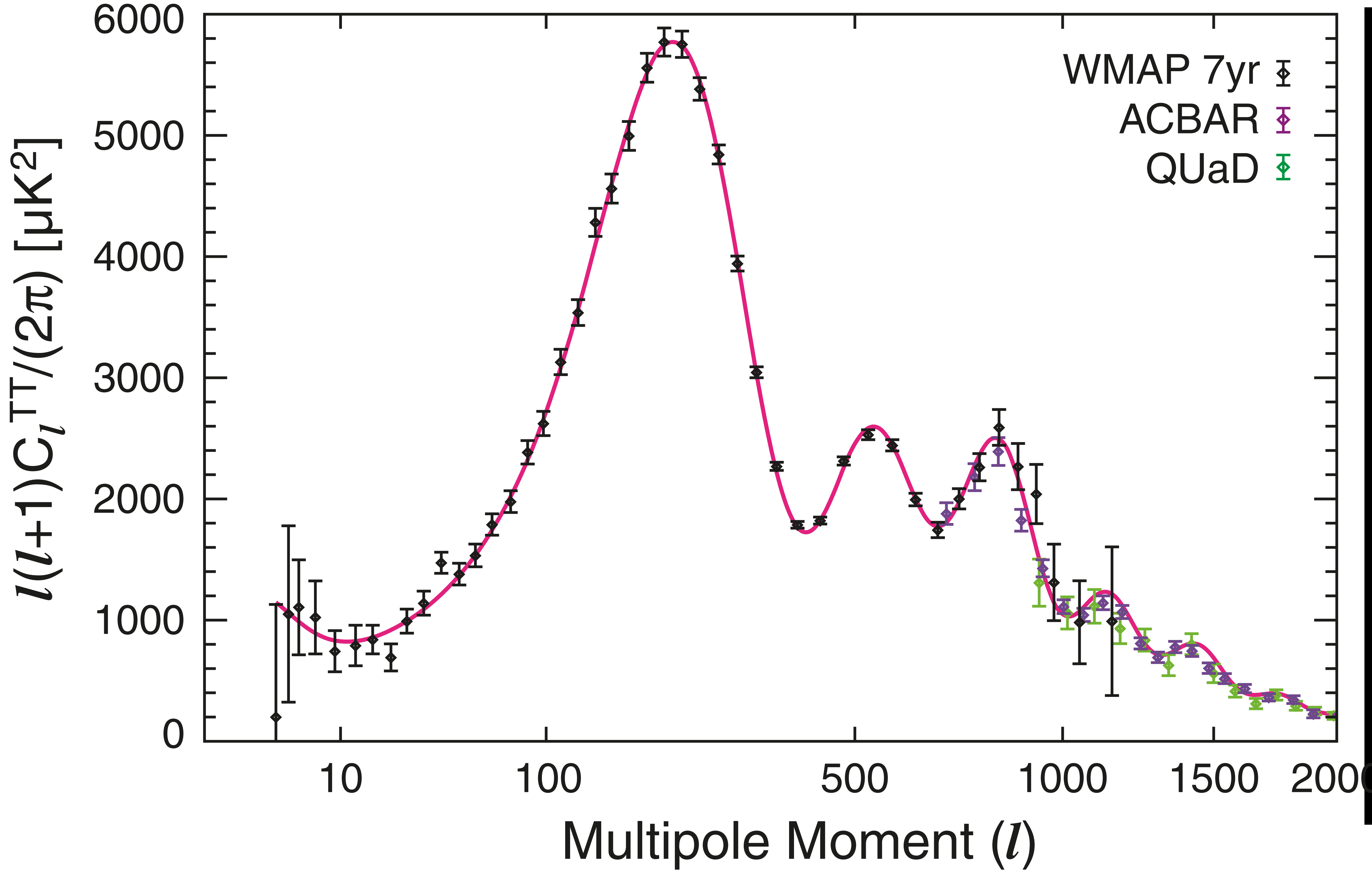
COBE To WMAP

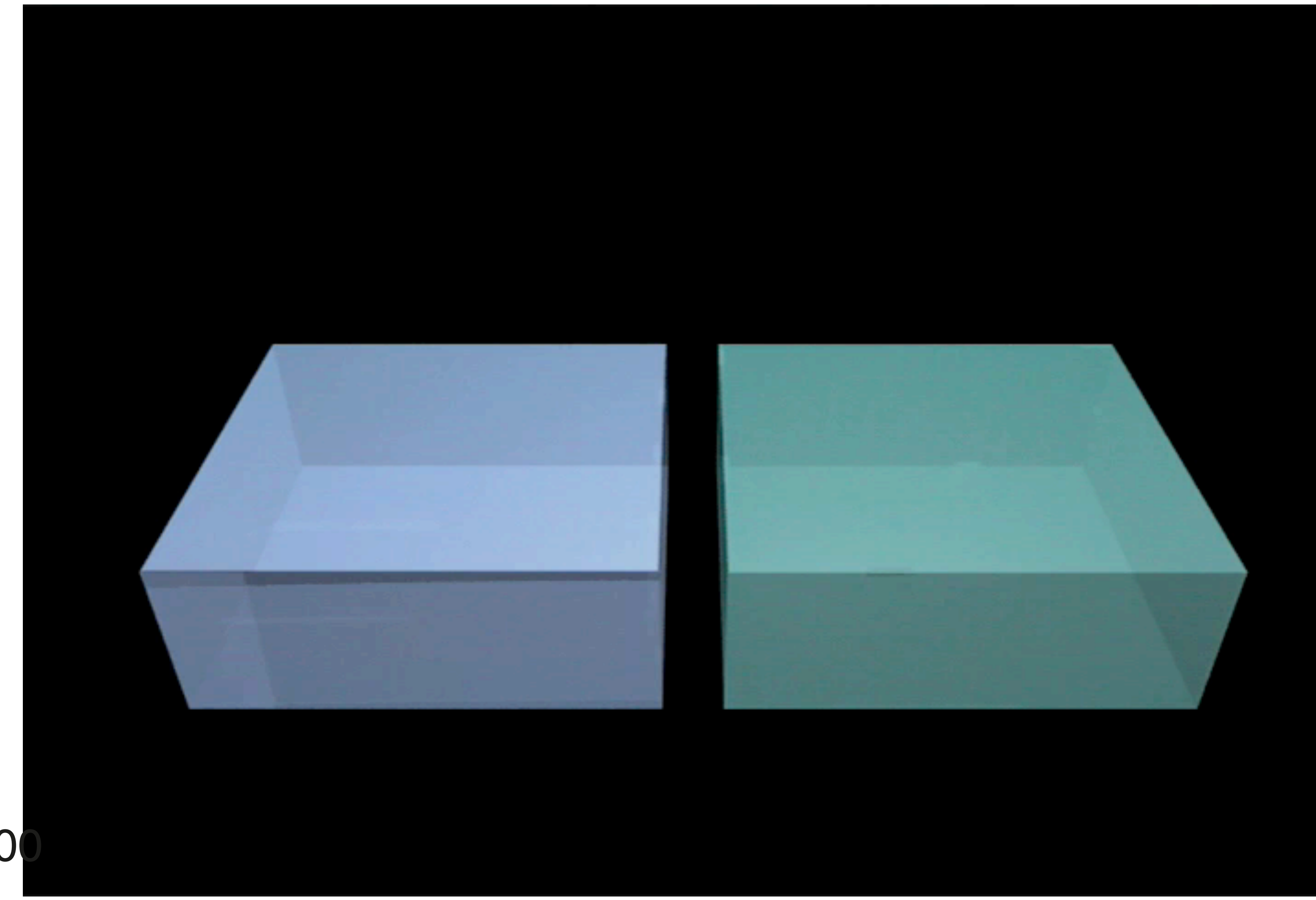
- COBE is unable to resolve the structures below ~7 degrees
- WMAP's resolving power is 35 times better than COBE.
- What did WMAP see?

Acoustic Wave in the Universe!



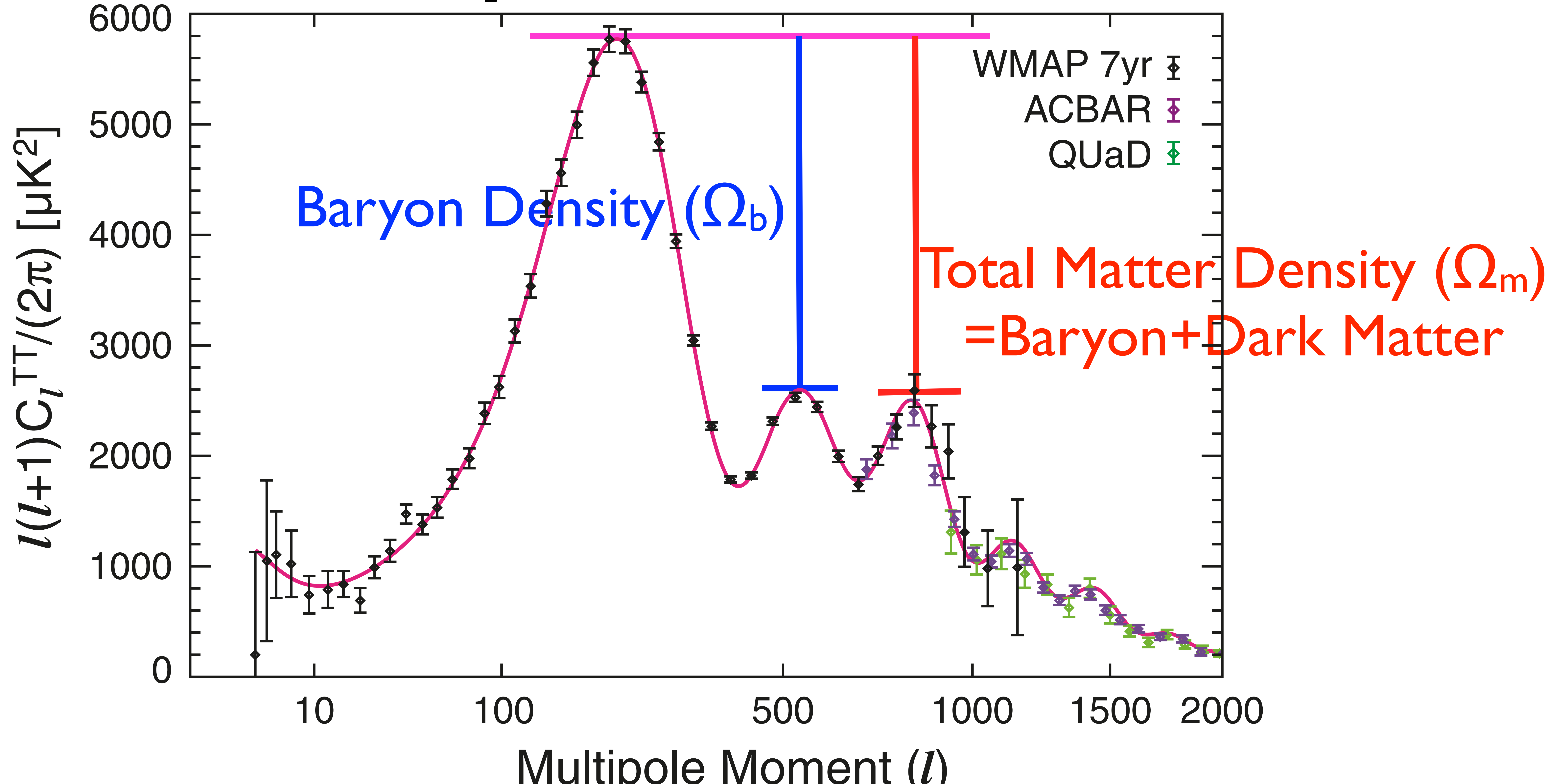
The Cosmic Sound Wave





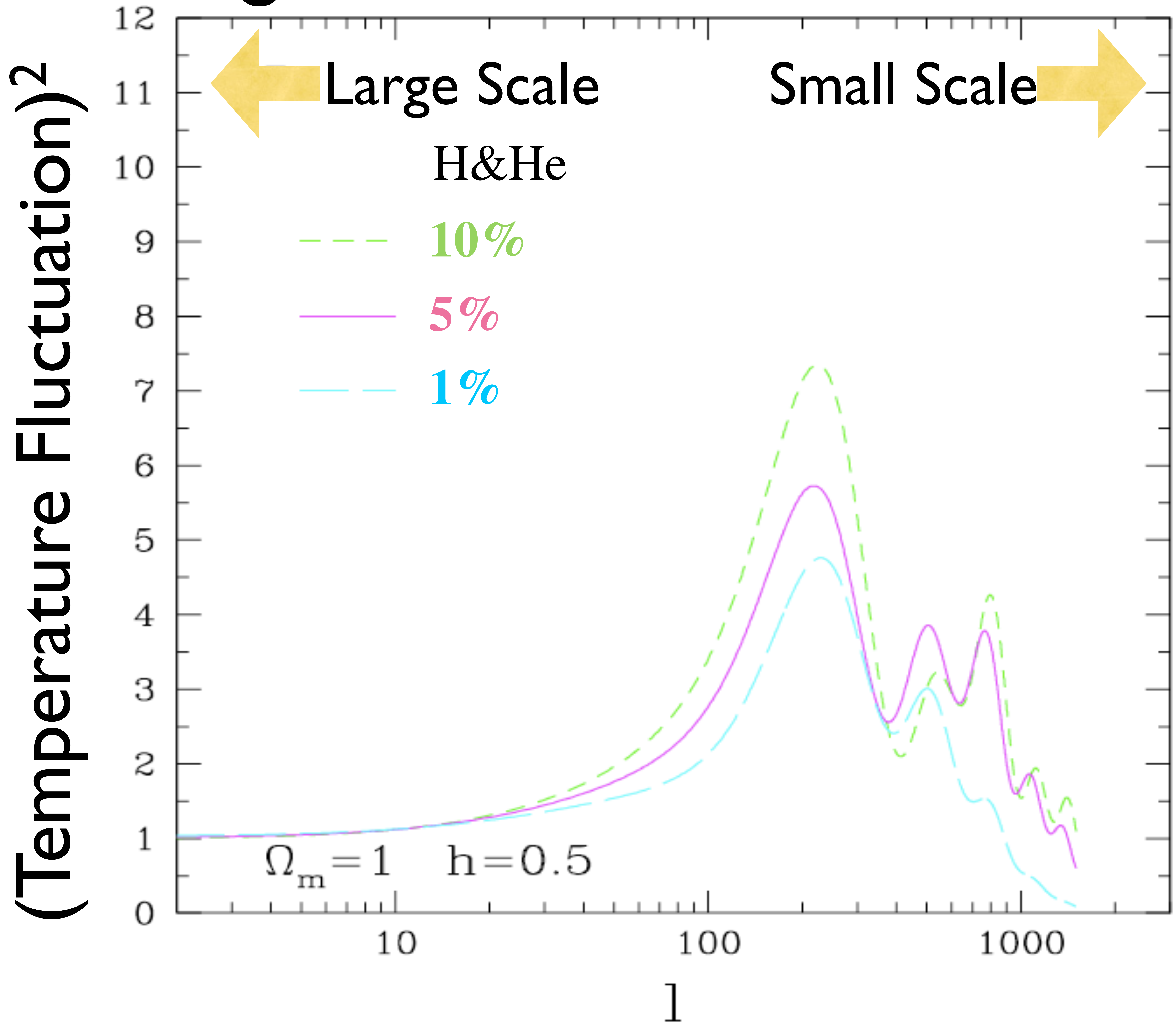
- *"The Universe as a Miso soup"*
  - *Main Ingredients: protons, helium nuclei, electrons, photons*
- We measure the composition of the Universe by analyzing the wave form of the cosmic sound waves.

CMB to Baryon & Dark Matter



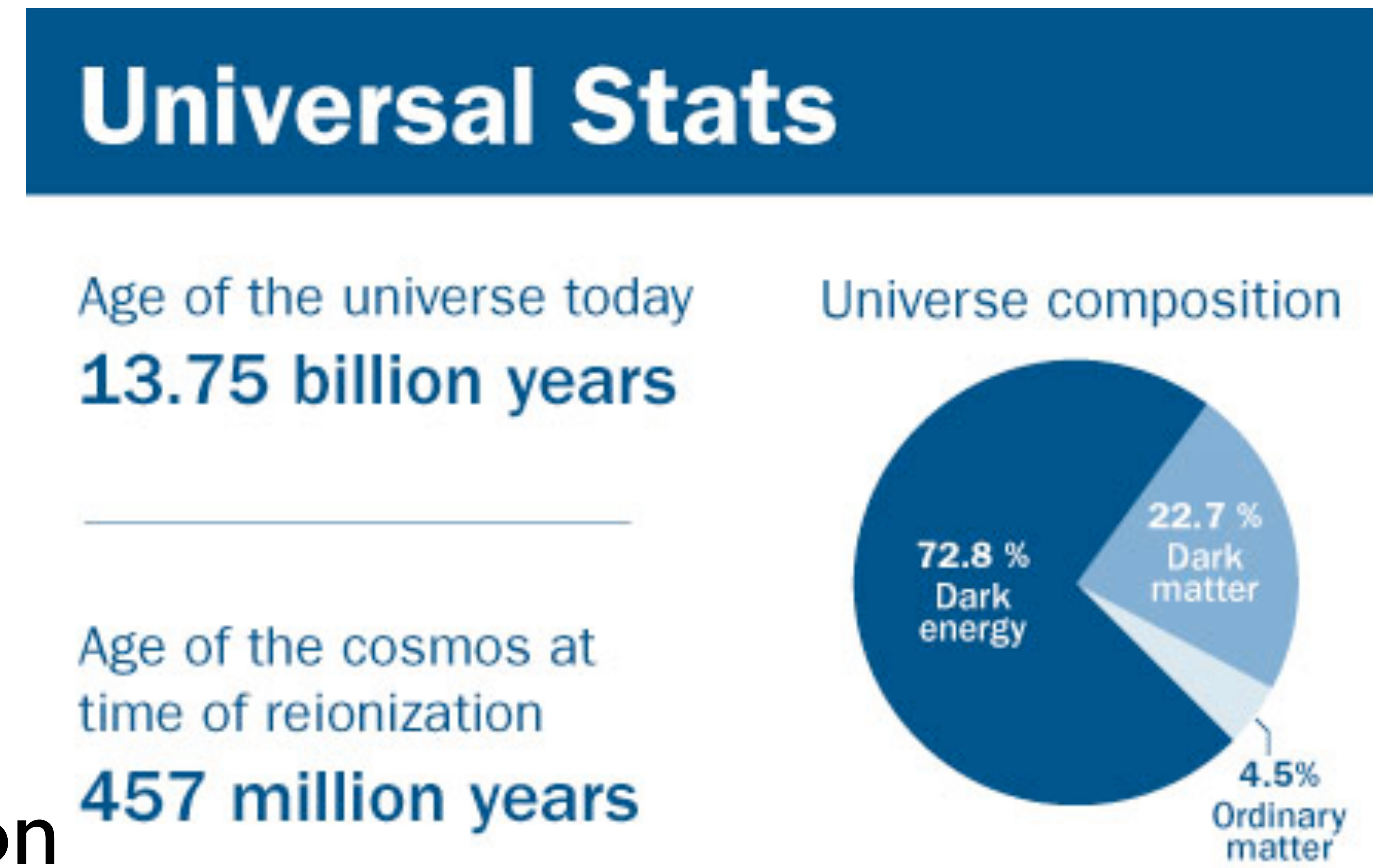
- 1-to-2: baryon-to-photon ratio
- 1-to-3: matter-to-radiation ratio (z_{EQ}: equality redshift)

Using the Wave Form: H&He



Results: Cosmic Pie Chart

- **Standard Model**
  - H&He = **4.5**% (±0.16%)
  - Dark Matter = **22.7**% (±1.5%)
  - Dark Energy = **72.8**% (±1.6%)
  - H_0=70.2±1.4 km/s/Mpc
  - Age of the Universe = 13.75 billion years (±0.11 billion years)



"ScienceNews" article on the WMAP 7-year results

Summary: Cosmology is Simple

- In principle, dynamics of the Universe cannot be studied without using General Relativity. However, in many important applications, the familiar non-relativistic formulae yield the same results.
  - Even including dark energy!
- Equation of motion of non-relativistic particles in an expanding universe is analogous to the usual Euler equation - this allows us to use simpler, non-relativistic codes to simulate large-scale structure of the Universe.
- Finally, we ***see*** hydrodynamics of a cosmic fluid at work at z=1100, and use it to determine the basic cosmological parameters.