Non-linearities vs primordial NG in the CMB

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A simple (obvious??) consideration……..

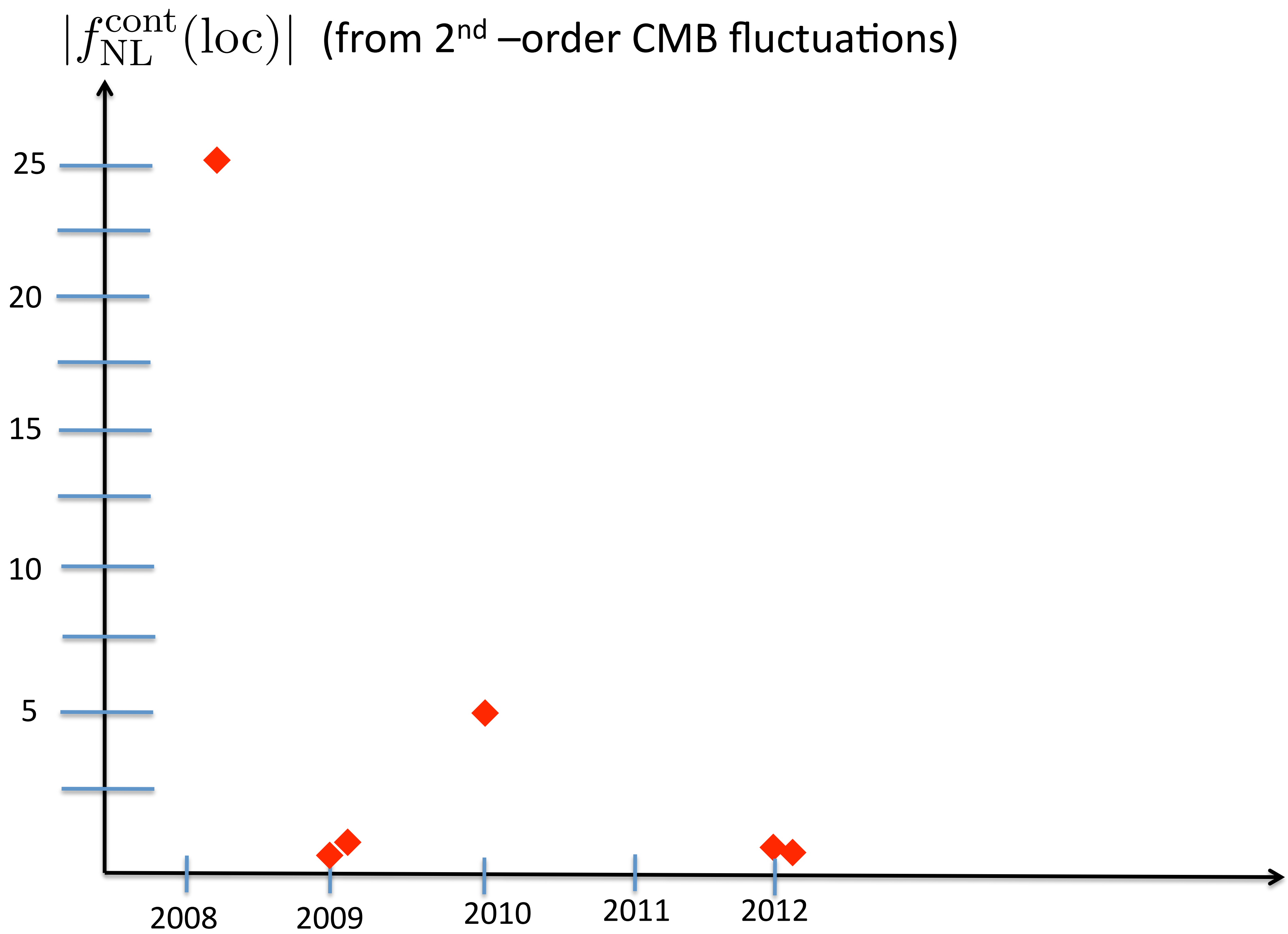
Gravity is nonlinear and it feeds non-linearitites into the cosmological perturbations;
expect CMB non-Gaussianity from 2nd-order effects of O(1) even with primordial f_NL=0;

Given the forecasted sensitivity of Planck the real question is ``what is the exact value'': is it 0.1 or 7?

And…….this is not ``abracadabra''……..

these effects are there because they are predicted by General Relativity with a fixed amplitude

They *must exist regardless of inflationary models, setting the minimum level of* non–Gaussianity in the cosmological perturbations


$|f_{\rm NL}^{\rm cont}(\rm loc)|$ (from 2nd –order CMB fluctuations)
25
20
15
10
5
2008
2009
2010
2011
2012

Numbers, numbers.......which one is correct?

Pitrou, Uzan, Bernardeau (Phys.Rev.D78, 2008) claim a contamination to local NG

$$f_{\rm NL}^{\rm cont}({\rm loc}) \simeq 25 \;!!$$

- N.B & Riotto (JCAP0903) show that this effect ***(small-scale evolution of 2nd-order gravitational potentials)*** mainly contaminates the equilateral

$$f_{\rm NL}^{\rm cont}({\rm loc}) \simeq 0.3 \qquad f_{\rm NL}^{\rm cont}({\rm equil}) \simeq 5$$

Pitrou, Uzan, Bernardeau (JCAP1007) ***(from squeezed configurations of*** δ_γ***)***

$$f_{\rm NL}^{\rm cont}({\rm loc}) \simeq 5$$

Given the forecasted sensitivity of Planck, might be non-negligible bias.
Do we believe this?

- N.B., Matarrese, Riotto (arXiv:1109.2043); Creminelli, Pitrou, Vernizzi (1109.1822) CMB bispectrum at recombination in the squeezed limit.

Both find $|f_{\rm NL}^{\rm cont}({\rm loc})| \leq \mathcal{O}(1)$

Other works on secondary sources of NG

- **NG from ISW-lensing (Rees-Sciama) correlation:** $f_{\rm NL}^{\rm cont}({\rm loc}) \simeq 10$

  Goldberg, Spergel 1999; Smith, Zaldarriaga 2006; Hanson, Smith, Challinor, Liguori, 2009; Mangilli, Verde 2009; Lewis, Challinor, Hanson 2011;Junk, Komatsu 2012; Lewis 2012.

- **Study of second-order Boltzmann equations:**

  N.B. Matarrese, Riotto 2006; 2007; 2009
  Pitrou 2007, 2009; Pitrou, Bernardeau, Uzan 2008
  Khatri, Wandelt 2009,2010
  Senatore, Tassev, Zaldarriaga 2009
  Beneke, Fidler 2010
  see poster by Guido Pettinari

  In particular:

- Contamination from (first-order)^2 terms: $f_{\rm NL}^{\rm cont}({\rm loc}) \simeq 0.5$
  Nitta, Komatsu, N.B., Matarrese, Riotto 2009

- ``Inhomogeneous'' recombination: $|f_{\rm NL}^{\rm cont}({\rm loc})| \simeq 0.7$
  Khatri, Wandelt 2009; Senatore, Tassev, Zaldarriaga 2009

CMB Non-Gaussianity from non-linear effects at Recombination in the squeezed limit

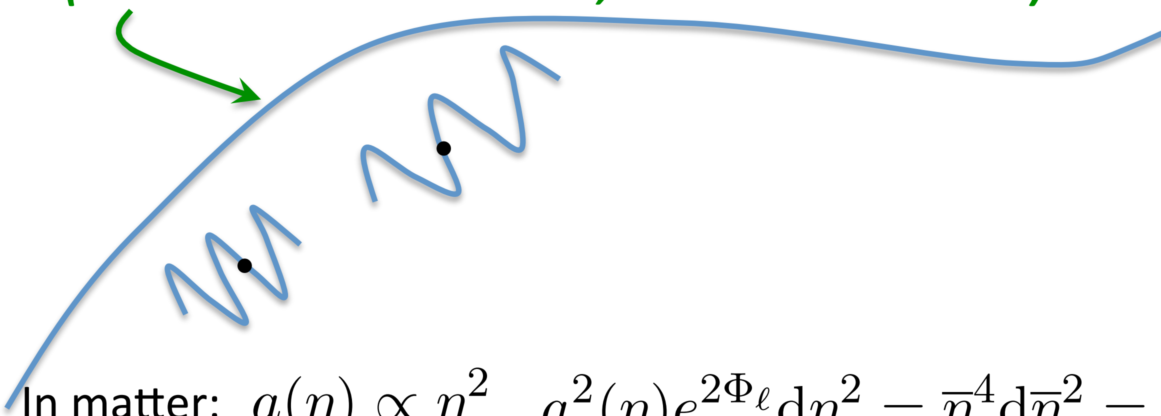
N.B., Matarrese, Riotto (arXiv:1109.2043) and see talk by Filippo Vernizzi
(see also Lewis 2012)

Squeezed bispectrum: a coordinate rescaling story....

✓ Squeezed limit: $\mathbf{k_1} + \mathbf{k_2} + \mathbf{k_3} = 0\,, \quad k_1 \ll k_2 \simeq k_3$

Origin of squeezed non-Gaussian signal: short-wavelength fluctuations modulated by long-wave

✓ ***Long-background mode k_1***
(outside at recombination, but observable now)



$$\mathrm{d}s^2 = a^2(\eta)\left[-e^{2\Phi}\mathrm{d}\eta^2 + e^{-2\Psi}\mathrm{d}\mathbf{x}^2\right]$$

In matter: $a(\eta) \propto \eta^2 \quad a^2(\eta)e^{2\Phi_\ell}\mathrm{d}\eta^2 = \overline{\eta}^4\mathrm{d}\overline{\eta}^2 = a^2(\overline{\eta})\mathrm{d}\overline{\eta}^2$

$$a^2(\eta)e^{-2\Psi_\ell}\mathrm{d}\mathbf{x}^2 = a^2(\overline{\eta})\mathrm{d}\overline{\mathbf{x}}^2$$

$$\Rightarrow \begin{cases} \overline{\eta} = e^{\frac{1}{3}\Phi_\ell}\eta \\ \overline{\mathbf{x}} = e^{-\frac{2}{3}\Phi_\ell}e^{-\Psi_\ell}\mathbf{x} \end{cases}$$

$$\overline{k}\overline{\eta} = e^{\Phi_\ell + \Psi_\ell}\, k\eta$$

The effect of the background-wave is a coordinate rescaling

(Creminelli, Zaldarriaga 04; N.B. Matarrese, Riotto 05;
similar to what happens for the squeezed limit of single-field inflation, Maldacena 03)

Coordinate rescaling: it works!

✓***An example:*** consider the first moment of Boltzmann equation for CMB photons

Linear equation

$$4\Theta_{00}^{(1)'} + \frac{4}{3}\partial_i v_\gamma^{(1)i} - 4\Psi^{(1)'} = 0$$

Coordinate rescaling →

$$4\Theta'_{00} + \frac{4}{3}e^{\Phi_\ell+\Psi_\ell}\partial_i v_\gamma^i - 4\Psi' = 0$$

↓ Expand at second-order

$$4\Theta_{00}^{(2)'} + \frac{4}{3}\partial_i v_\gamma^{i(2)} - 4\Psi'^{(2)} = -2(\Phi_\ell^{(1)} + \Psi_\ell^{(1)})(4\Psi^{(1)'} - 4\Theta_{00}^{(1)'})$$

This coincides in the squeezed limit with the 2nd-order Boltzmann equations obtained in N.B., Matarrese, Riotto (06,07,2010)

$$4\Theta_{00}^{(2)'} + \frac{4}{3}\partial_i v_\gamma^{(2)i} - 4\Psi^{(2)'} = \mathcal{S}_\Delta$$

$$\mathcal{S}_\Delta = -2(\Phi_\ell^{(1)} + \Psi_\ell^{(1)})(4\Psi^{(1)\prime} - \Theta_{00}^{(1)\prime}) - \frac{8}{3}v_\gamma^{(1)i}(\Theta_{00}^{(1)} + 4\Phi^{(1)})_{,i} + \frac{16}{3}(\Phi^{(1)} + \Psi^{(1)})^{,i}v_i - \frac{8}{3}R\left(\frac{\mathcal{H}}{1+R}v_\gamma^{(1)2} - \frac{1}{4}\frac{v_{\gamma i}^{(1)}\Theta_{00}^{(1),i}}{1+R}\right)$$

✓ Similar conclusions for photon velocity continuity eq. and 2nd-order evolution of gravitational potentials (for a numerical check Creminelli, Pitrou Vernizzi 2011)

Bispectrum from recombination in the squeezed limit

$$\langle a(\vec{\ell}_1)a(\vec{\ell}_2)a(\vec{\ell}_3)\rangle = (2\pi)^2\delta^{(2)}(\vec{\ell}_1+\vec{\ell}_2+\vec{\ell}_3)B(\ell_1,\ell_2,\ell_3)$$

$$T = \overline{T}e^{\Theta} \longrightarrow \frac{\Delta T}{\overline{T}} = \Theta^{(1)} + \frac{1}{2}\Theta^{(2)} + \frac{1}{2}\left(\Theta^{(1)}\right)^2$$

1 2

Squeezed limit: $\ell_1 \ll \ell_2 \sim \ell_3$

- 2 $$B_{\frac{1}{2}\left(\Theta^{(1)}\right)^2}(\ell_1,\ell_2,\ell_3) = 2\;C(\ell_1)C(\ell_2)$$

Recall that a primordial local NG in the squeezed limit $B_{\rm loc}(\ell_1,\ell_2,\ell_3) \simeq -12\, f_{\rm NL}^{\rm loc} C(\ell_1)C(\ell_2)$

the contamination to local primordial NG is $$f_{\rm NL}^{\rm cont} = -\frac{1}{6}$$

(first found in N.B., Matarrese, Riotto, PRL 04)

➢ 1 Bispectrum from intrinsically second-order temperature $\Theta^{(2)}$: coordinate rescaling

- First compute the short-scale 2-point function ***in the background*** of $\Phi^{(1)}_{\mathbf{k}_1}$ (or $\zeta^{(1)}_{\mathbf{k}_1} = -\frac{5}{3}\Phi^{(1)}_{\mathbf{k}_1}$)
  $k_1 << k_2 \sim k_3$

$$\langle a(\vec{\ell}_2)a(\vec{\ell}_3)\rangle_{\Phi^{(1)}_{\mathbf{k}_1}} = \langle a(\vec{\ell}_2)a(\vec{\ell}_3)\rangle_0 + 5\ a(-\vec{\ell}_1)\ C(\ell_2)\frac{\mathrm{d}\ln\left[\ell_2^2 C(\ell_2)\right]}{\mathrm{d}\ln\ell_2}$$

coordinate rescaling: trade the derivative w.r.t. $\zeta_{\mathbf{k}_1}$ with derivative w.r.t. spatial coordinates

- correlate this result with the long-wavelength temperature

$$B_{\Theta^{(2)}}(\ell_1,\ell_2,\ell_3) = \left\langle a(\vec{\ell}_1)\langle a(\vec{\ell}_2)a(\vec{\ell}_3)\rangle\right\rangle = 5\ C(\ell_1)C(\ell_2)\ \frac{\mathrm{d}\ln\left[\ell_2^2 C(\ell_2)\right]}{\mathrm{d}\ln\ell_2}$$

➡ Bispectrum

$$B_{\rm rec}(\ell_1,\ell_2,\ell_3) = C(\ell_1)C(\ell_2)\left[2 + 5\,\frac{\mathrm{d}\ln\left[\ell_2^2 C(\ell_2)\right]}{\mathrm{d}\ln\ell_2}\right]$$

Bispectrum from recombination in the squeezed limit: coord. rescaling

1. $\langle a(\vec{\ell}_2)a(\vec{\ell}_3)\rangle = \langle a(\vec{\ell}_2)a(\vec{\ell}_3)\rangle_{\Phi^{(1)}_{\mathbf{k}_1}}$

Coordinate rescaling

$$\langle a(\vec{\ell}_2)a(\vec{\ell}_3)\rangle_{\Phi^{(1)}_{\mathbf{k}_1}} = \langle a(\vec{\ell}_2)a(\vec{\ell}_3)\rangle_0 + \frac{\partial\langle a(\vec{\ell}_2)a(\vec{\ell}_3)\rangle}{\partial\zeta_{\rm L}}\,\zeta_L \sim \langle a(\vec{\ell}_2)a(\vec{\ell}_3)\rangle_0 - \frac{\partial\langle a(\vec{\ell}_2)a(\vec{\ell}_3)\rangle}{\partial\ln\ell}\,\zeta_L$$

On large scales $\zeta_{\rm L} = -\frac{5}{3}\Phi_{\rm L}$

2. $B_{\Theta^{(2)}}(\ell_1,\ell_2,\ell_3) = \left\langle a(\vec{\ell}_1)\langle a(\vec{\ell}_2)a(\vec{\ell}_3)\rangle\right\rangle \sim \left\langle a(\vec{\ell}_1)\frac{5}{3}\Phi_{\rm L}\right\rangle\frac{\partial\langle a(\vec{\ell}_2)a(\vec{\ell}_3)\rangle}{\partial\ln\ell} \sim 5C_{\ell_1}\frac{\partial\langle a(\vec{\ell}_2)a(\vec{\ell}_3)\rangle}{\partial\ln\ell}$

Contamination to the primordial local NG

Fisher matrix $$F_{ij} = \int d^2\ell_1 d^2\ell_2 d^2\ell_3\, \delta^{(2)}(\vec{\ell}_1 + \vec{\ell}_2 + \vec{\ell}_3)\, \frac{B_i(\ell_1,\ell_2,\ell_3)\, B_j(\ell_1,\ell_2,\ell_3)}{6\, C(\ell_1)\, C(\ell_2)\, C(\ell_3)}$$

$B_i(\ell_1,\ell_2,\ell_3)$: primordial or secondary bispectra

Fit the primordial bispectrum template to the recombination bispectrum to find the best-fitting contamination $f_{\rm NL}^{\rm con}$ by minimizing

$$\chi^2 = \int d^2\ell_1 d^2\ell_2 d^2\ell_3\, \delta^{(2)}(\vec{\ell}_1 + \vec{\ell}_2 + \vec{\ell}_3)\, \frac{\left[f_{\rm NL}^{\rm con} B_{\rm loc}(\ell_1,\ell_2,\ell_3; f_{\rm NL}^{\rm loc}=1) - B_{\rm rec}(\ell_1,\ell_2,\ell_3)\right]^2}{6\, C(\ell_1)\, C(\ell_2)\, C(\ell_3)}$$

$$f_{\rm NL}^{\rm con} = \left.\frac{F_{\rm rec,loc}}{F_{\rm loc,loc}}\right|_{f_{\rm NL}^{\rm loc}=1}$$

Contamination to the primordial local NG

- bispectrum from recombination $B_{\rm rec}(\ell_1,\ell_2,\ell_3) = C(\ell_1)C(\ell_2)\left[2+5\,\frac{{\rm d}\ln\left[\ell_2^2 C(\ell_2)\right]}{{\rm d}\ln \ell_2}\right]$

  primordial local NG $B_{\rm loc}(\ell_1,\ell_2,\ell_3) \simeq -12\, f_{\rm NL}^{\rm loc} C(\ell_1)C(\ell_2)$

  transfer function on small scales $C(\ell_2) \simeq a^2 \frac{A}{\pi}\frac{\ell_*}{\ell_2^3}\, e^{-(\ell_2/\ell_*)^{0.85}}$
  ($\ell_1 \ll \ell_2 \sim \ell_3$)

$$f_{\rm NL}^{\rm con} \simeq -\frac{1}{6} + \frac{5}{12}\left[1 + 0.6\frac{\left(\frac{\ell_{\rm max}}{\ell_*}\right)^{0.85} - \left(\frac{\ell_{\rm min}}{\ell_{\rm max}}\right)^2\left(\frac{\ell_{\rm min}}{\ell_*}\right)^{0.85}}{1-\left(\frac{\ell_{\rm min}}{\ell_{\rm max}}\right)^2}\right] \simeq 0.94$$

Below Planck forecasted sensitivity and smaller than 5 recently claimed

- Main conclusion unchanged even including:
  - lensing term correlated to the CMB anisotropies at recombination
  (Creminelli, Vernizzi, Pitrou, 2011)
  where overlap our results perfectly agree with them

The bispectrum computed with the coordinate rescailing accounts for ***exact squeezed NG*** from non-linearities

What about beyond squeezed?

Turn to the full second-order Boltzmann equations

Contamination to local NG from recombination: beyond squeezed

➢ Main conclusion unchanged even including contributions beyond the ***exact*** squeezed limit

- corrections $\mathcal{O}(\Phi^{(1)}_{\mathbf{k}_{\rm SHORT}} \nabla \Phi^{(1)}_{\mathbf{k}_{\rm LONG}})$ vanishing for exact squeezed limit bring

$|f_{\rm NL}^{\rm cont}({\rm loc})| \simeq 0.1$ to be added (Bartolo, Matarrese, Riotto 2011)

- ``Inhomogeneous'' recombination:
  Khatri, Wandelt 2009; Senatore, Tassev, Zaldarriaga 2009

$|f_{\rm NL}^{\rm cont}({\rm loc})| \simeq 0.7$

- small-scale (k_i η_{rec} >>1) evolution of 2nd-order gravitational potentials at recomb. brings

$f_{\rm NL}^{\rm cont}({\rm loc}) \simeq 0.3$ to be added (Bartolo, Riotto 2009)

$$|f_{\rm NL}^{\rm cont}({\rm loc})| \leq \mathcal{O}(0.8)$$

What about beyond squeezed?

Hence what about contamination to primordial NG other than local ?

Turn to the full second-order Boltzmann equations

Second-order CMB Anisotropies

$$\frac{df}{d\eta} = a\,C[f]$$

Collision term

$$\frac{df}{d\eta} = \frac{\partial f}{\partial \eta} + \frac{\partial f}{\partial x^i}\frac{dx^i}{d\eta} + \frac{\partial f}{\partial p}\frac{dp}{d\eta} + \frac{\partial f}{\partial n^i}\frac{dn^i}{d\eta}$$

Gravity effects

Metric perturbations: Poisson gauge

$$ds^2 = a^2(\eta)\left[-e^{2\Phi}d\eta^2 + 2\omega_i dx^i d\eta + (e^{-2\Psi}\delta_{ij} + \chi_{ij})dx^i dx^j\right]$$

$$\Phi = \Phi^{(1)} + \frac{1}{2}\Phi^{(2)}, \qquad \psi = \psi^{(1)} + \frac{1}{2}\psi^{(2)}$$

Example: using the geodesic equation for the photons

$$\frac{1}{p}\frac{dp}{d\eta} = -\mathcal{H} + \Psi' - \Phi_{,i}\, n^i e^{\Phi+\Psi} - \omega_i' n^i - \frac{1}{2}\chi_{ij}' n^i n^j$$

Redshift of the photon (***Sachs-Wolfe and ISW effects***)

PS: Here the photon momentum is **p=** pn^i with $p^2 = g_{ij} P^i P^j$
($P^\mu = dx^\mu(\lambda)/d\lambda$ quadri-momentum vector)

The 2nd-order photon Boltzmann equation

$$\Delta^{(2)\prime} + n^i \frac{\partial \Delta^{(2)}}{\partial x^i} - \tau' \Delta^{(2)} = S$$

N.B: for a derivation of the Boltzmann equations see also
C. Pitrou CQG 09 (includes polarization);
Senatore, Tassev, Zaldarriaga, JCAP 09

$$\Delta = \Delta^{(1)} + \Delta^{(2)}/2 \qquad \Delta^{(2)}(x^i, n^i, \eta) = \frac{\int dp p^3 f^{(2)}}{\int dp p^3 f^{(0)}}$$

with $\tau' = -n_e \sigma_T a$
optical depth

Source term $S = S^{(2)} + S^{(I \times I)}$

Second-order baryon velocity

Sachs-Wolfe effect

$$S^{(2)} = -\tau'(\Delta_{00}^{(2)} + 4\Phi^{(2)}) + 4(\Phi^{(2)} + \Psi^{(2)})' - 8\omega_i' n^i - 4\chi_{ij}' n^i n^j - \tau' \left[4\mathbf{v}^{(2)} \cdot \mathbf{n} - \frac{1}{2} \sum_{m=-2}^{2} \frac{\sqrt{4\pi}}{5^{3/2}} \Delta_{2m}^{(2)} Y_{2m}(\mathbf{n}) \right]$$

$$\begin{aligned} S^{(I\times I)} &= 8\Delta^{(1)}(\Psi^{(1)\prime} - n^i \Phi^{(1)}_{,i}) - 2n^i(\Phi^{(1)} + \Psi^{(1)})(\Delta^{(1)} + 4\Phi^{(1)})_{,i} \\ &- 2\left[(\Phi^{(1)} + \Psi^{(1)})_{,j} n^i n^j - (\Phi^{(1)} + \Psi^{(1)})^{,i} \right] \frac{\partial \Delta^{(1)}}{\partial n^i} \\ &- \tau' \left[2\delta_e^{(1)} \left(\Delta_0^{(1)} - \Delta^{(1)} + 4\mathbf{v} \cdot \mathbf{n} + \frac{1}{2} \Delta_2^{(1)} P_2(\hat{\mathbf{v}} \cdot \mathbf{n}) \right) \right. \\ &+ \left. 2(\mathbf{v} \cdot \mathbf{n}) \left[\Delta^{(1)} + 3\Delta_0^{(1)} - \Delta_2^{(1)} \left(1 - \frac{5}{2} P_2(\hat{\mathbf{v}} \cdot \mathbf{n}) \right) \right] - v\Delta_1^{(1)}(4 + 2P_2(\hat{\mathbf{v}} \cdot \mathbf{n})) + 14(\mathbf{v} \cdot \mathbf{n})^2 - 2v^2 \right] \end{aligned}$$

Gravitational lensing

Quadratic-Doppler effect

Coupling velocity and linear photon anisotropies

CMB angular bispectrum

$$B_{l_1 l_2 l_3}^{m_1 m_2 m_3} \equiv \langle a_{l_1 m_1} a_{l_2 m_2} a_{l_3 m_3} \rangle \, ,$$

$$B_{l_1 l_2 l_3} = \sum_{\text{all } m} \begin{pmatrix} l_1 & l_2 & l_3 \\ m_1 & m_2 & m_3 \end{pmatrix} B_{l_1 l_2 l_3}^{m_1 m_2 m_3} ,$$

$$\Delta^{(1)} + ik\mu\Delta^{(1)} - \tau'\Delta^{(1)} = S^{(1)}(\mathbf{k}, \hat{\mathbf{n}}, \eta)$$
$$\Delta^{(2)} + ik\mu\Delta^{(2)} - \tau'\Delta^{(2)} = S^{(2)}(\mathbf{k}, \hat{\mathbf{n}}, \eta)$$

$$S_{lm}^{(2)}(\mathbf{k}) = \int \frac{d^3k'}{(2\pi)^3} \int d^3k'' \delta^3(\mathbf{k}' + \mathbf{k}'' - \mathbf{k}) \times \mathcal{S}_{lm}^{(2)}(\mathbf{k}', \mathbf{k}'', \mathbf{k}) \zeta(\mathbf{k}') \zeta(\mathbf{k}'')$$

Harmonic components of the CMB source function

CMB angular bispectrum (II)

$$a_{lm}^{(1)} = 4\pi(-i)^l \int \frac{d^3k}{(2\pi)^3} g_l(k) Y_{lm}^{*} \zeta(\mathbf{k})$$

$$\Phi(\mathbf{x}) = \Phi_L(\mathbf{x}) + f_{NL}\left(\Phi_L^2(\mathbf{x}) - \langle \Phi_L^2(\mathbf{x}) \rangle\right)$$

$$a_{lm}^{(2)} = \frac{4\pi}{8}(-i)^l \int \frac{d^3k}{(2\pi)^3} \int \frac{d^3k'}{(2\pi)^3} \int d^3k'' \delta^3(\mathbf{k}' + \mathbf{k}'' - \mathbf{k}) \times \sum_{l'm'} F_{lm}^{l'm'}(\mathbf{k}', \mathbf{k}'', \mathbf{k}) Y_{l'm'}^{*}(\hat{\mathbf{k}}) \zeta(\mathbf{k}') \zeta(\mathbf{k}'')$$

Second-order radiation transfer function

Primordial curvature perturbation

$$F_{lm}^{l'm'}(\mathbf{k}', \mathbf{k}'', \mathbf{k}) = i^l \sum_{\lambda\mu} (-1)^m (-i)^{\lambda - l'} \mathcal{G}_{ll'\lambda}^{-mm'\mu} \times \sqrt{\frac{4\pi}{2\lambda + 1}} \int_0^{\eta_0} d\eta e^{-\tau} \mathcal{S}_{\lambda\mu}^{(2)}(\mathbf{k}', \mathbf{k}'', \mathbf{k}) j_{l'}[k(\eta - \eta_0)].$$

Nitta, Komatsu, N.B, Matarrese, Riotto 09

CMB source function

A closer look at the CMB source function

Two types of contributions:

1) (first-order)^2-terms (oscillating/constant in time)

→ Contamination to local NG
(but detailed shape diff. w.r.t. local)

2) Intrinsically second-order term

$$\Theta^{(2)} = (\Delta_{00}^{(2)}/4 + \Phi^{(2)})$$

on large scales it gives rise to the Sachs-Wolfe effect;
on small scales it grows as η^2

(pointed out by Pitrou, Uzan, Bernadeau 08)

→ Contamination to equilateral NG

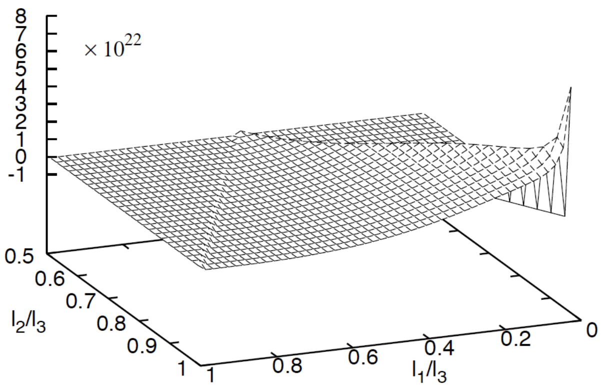
Second-order bispectrum from products of 1st-order terms

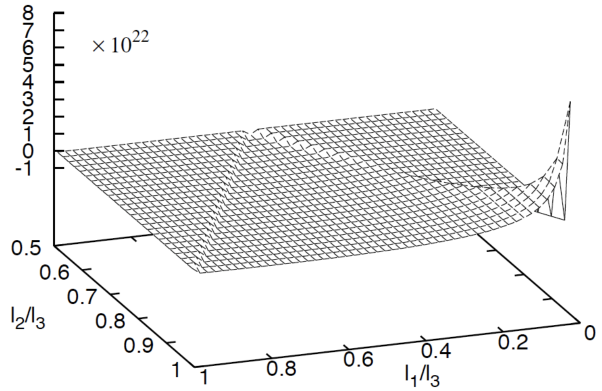
$$B_{l_1 l_2 l_3} \equiv \sum_{\text{all}\, m} \begin{pmatrix} l_1 & l_2 & l_3 \\ m_1 & m_2 & m_3 \end{pmatrix} \langle a_{l_1 m_1} a_{l_2 m_2} a_{l_3 m_3} \rangle$$

Maximum signal in the squeezed triangles, $l_1 \ll l_2 \sim l_3$, similar to the local primordial bispectrum

$$l_1 l_2 \langle a^{(1)}_{l_1 m_1} a^{(1)}_{l_2 m_2} a^{(2)}_{l_3 m_3} \rangle (\mathcal{G}^{m_1 m_2 m_3}_{l_1 l_2 l_3})^{-1} / (2\pi)^2 \times 10^{22}$$

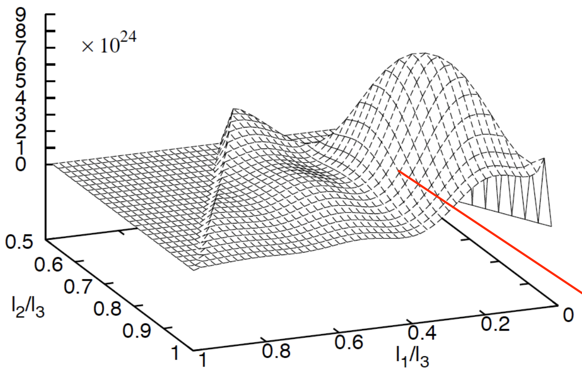
Local primordial

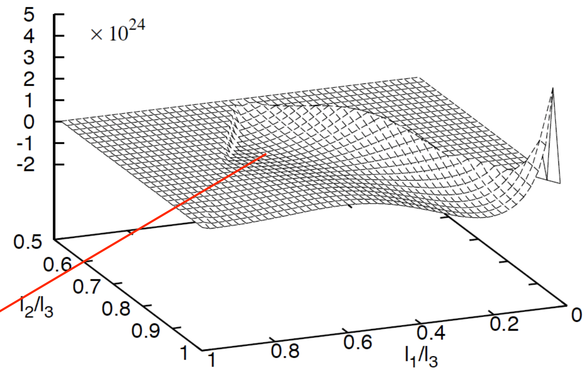




l_3=200

Local primordial





Acoustic oscillations

l_3=1000

Shape of the second-order bispectrum from products of 1st-order terms

✓ The bispectrum has the maximum signal in the squeezed triangles $l_1 << l_2 \sim l_3$, as the local-type primordial bispectrum: both generate non-linearities via products of first-order terms in position space

✓ However the shapes are sufficiently different

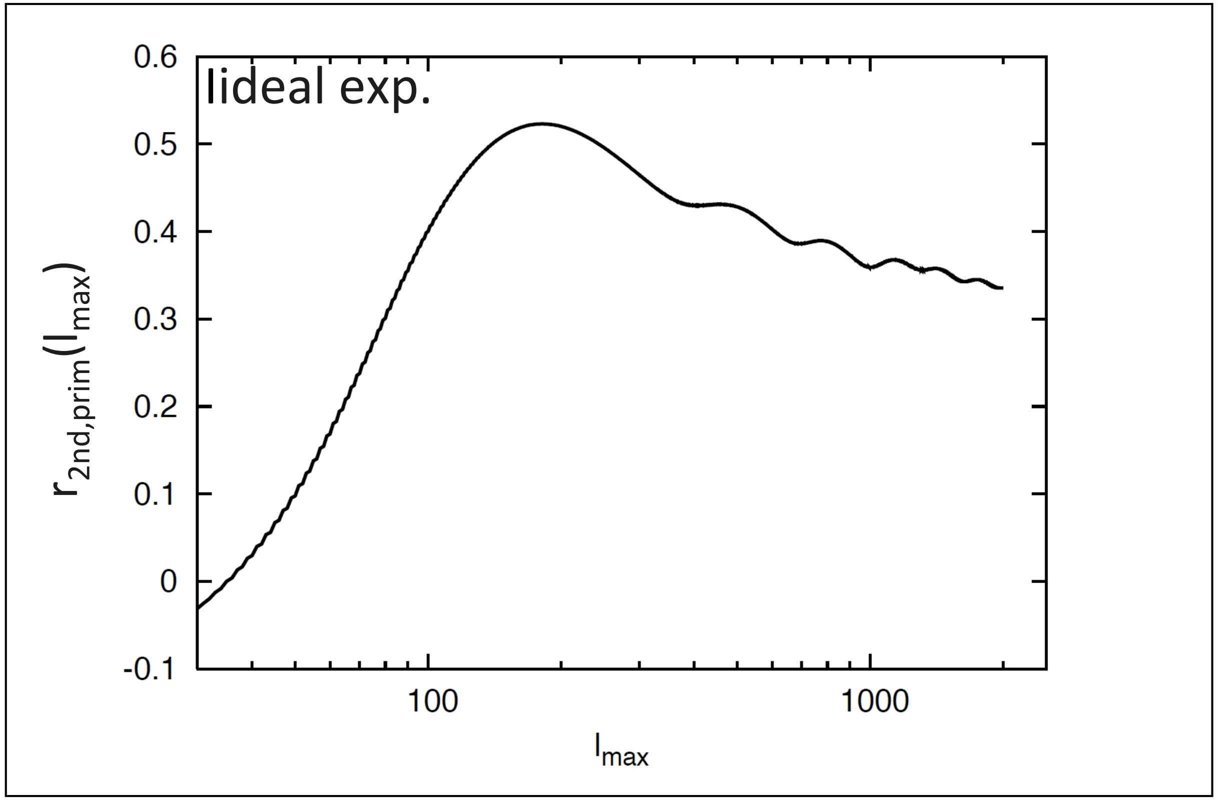
- different dependence on the transfer functions (acoustic oscillations);
  the primordial bispectrum contain $[g_l(k)]^3$
  the 2nd-order one goes like $[g_l(k)]^\alpha$, with $2 \leq \alpha \leq 4$

- second-order effects are not scale-invariant because of extra powers of k (e.g. from velocity terms)
  $B(k_1,k_2,k_3) \sim (k_1)^m (k_2)^n / (k_1)^3 (k_2)^3$+cycl. still peaks in the squeezed configuration with $m,n \leq 3$

Cross-correlation

$$r_{ij} = \frac{F_{ij}^{-1}}{\sqrt{F_{ii} F_{ij}}}$$

→ How similar are 2 bispectra?

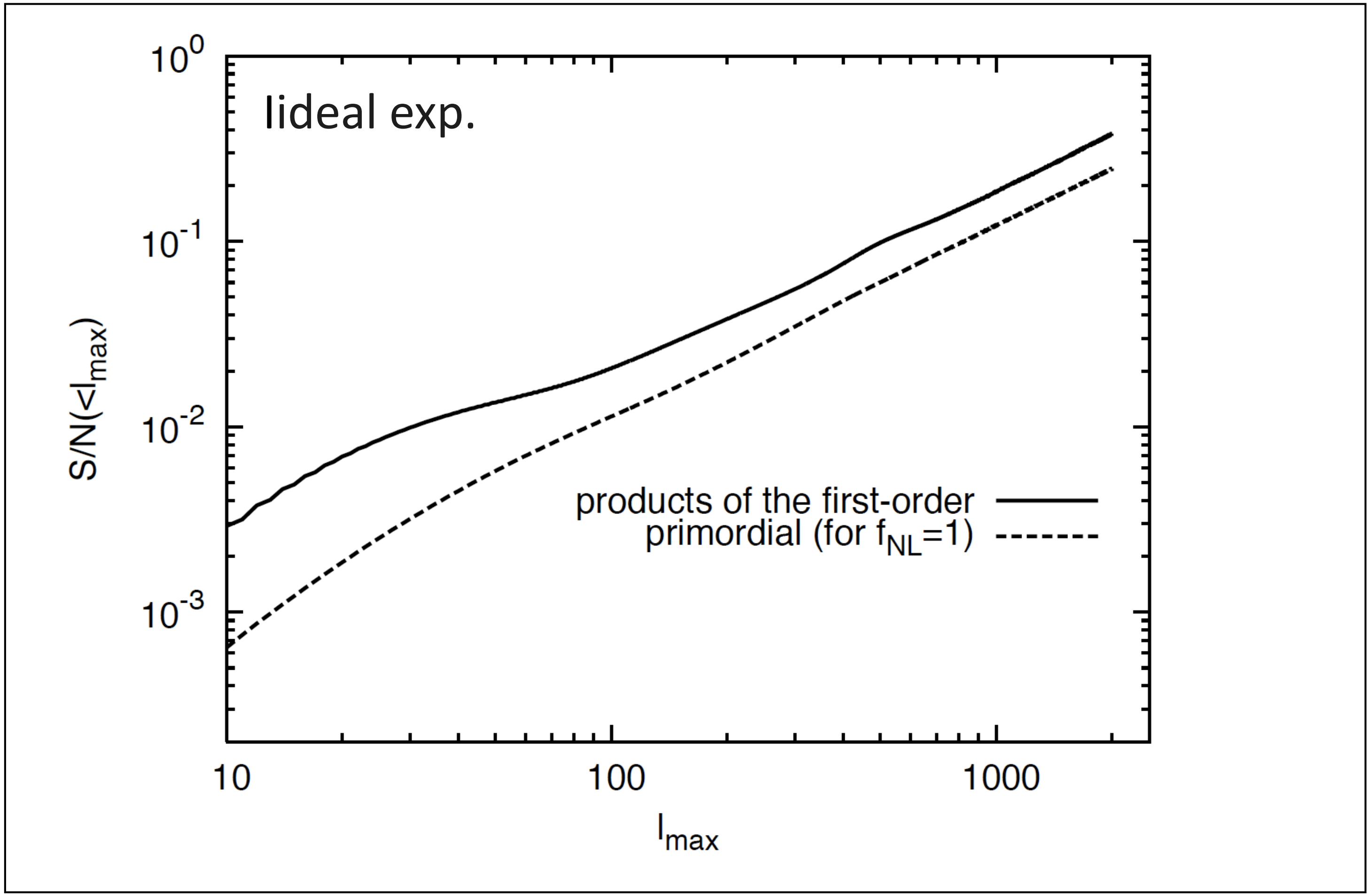


2^{nd}-order bispectrum
and local primordial
are fairly similar

$r_{2nd,prim}$~ 0.5 at lmax~200

$r_{2nd,prim}$~ 0.3 at l_max~2000

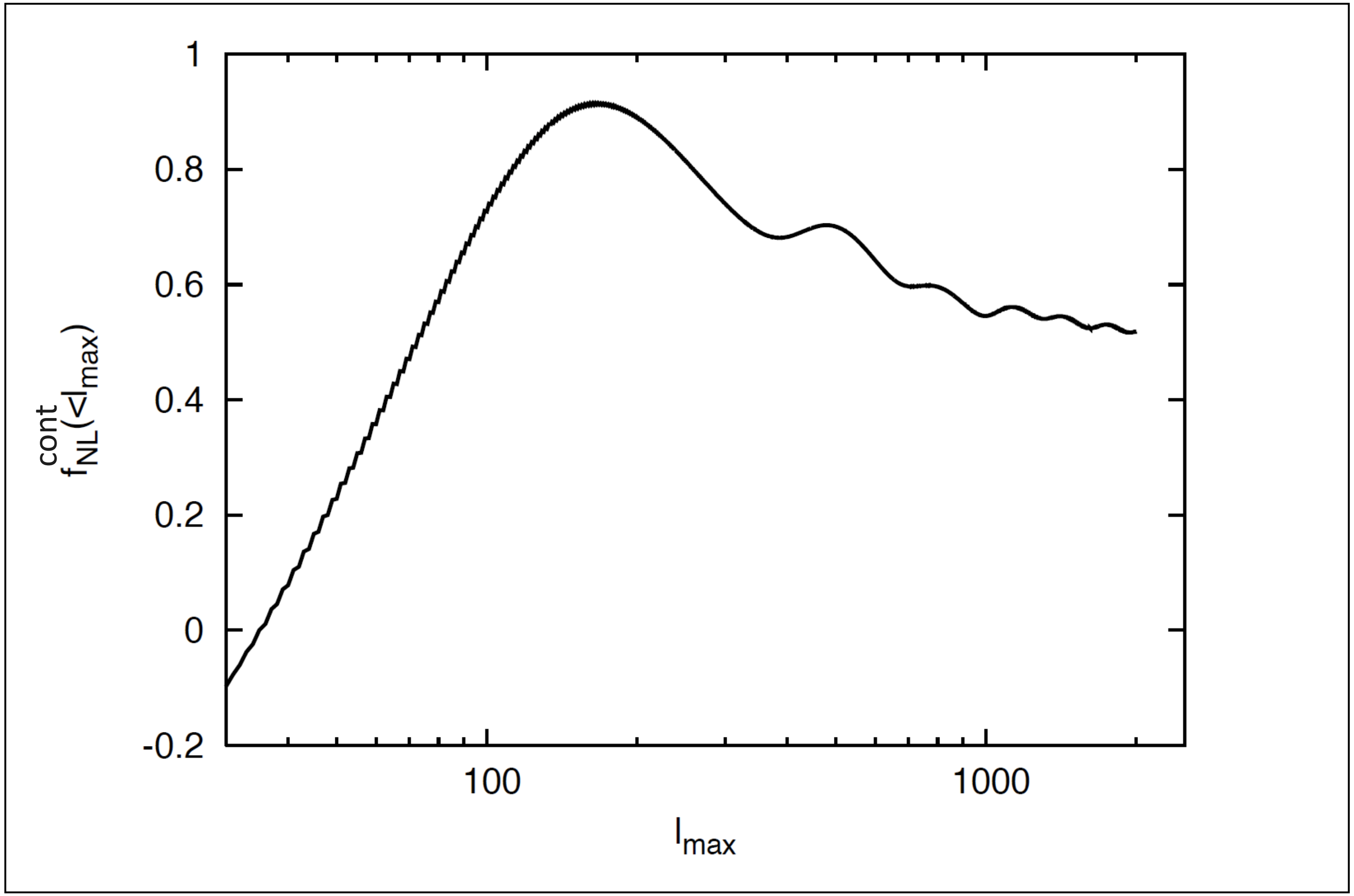
Signal to Noise ratio: numerical results for (first-order)^2-terms



(Nitta, Komatsu, N.B., Matarrese, Riotto, JCAP 09)

(S/N) from the (first-order×first-order) terms is about 0.4 at l_{max}≈2000 for an ideal full-sky experiment

Contamination to primordial f_NL of the local type from (first-order)^2-terms



(Nitta, Komatsu, N.B., Matarrese, Riotto, JCAP 09)

Contamination is 0.9 at l_{max}≈200 and 0.5 at l_{max}≈2000
vs 5 which is the minimum detectable value forcasted for Planck

Non-linear dynamics at recombination

On small scales, i.e. modes k >> k_{eq}, the second-order anisotropies at recombination are dominated by the 2nd-oder gravitational potential sourced by dark matter perturbations

$$\Phi^{(2)} \simeq \Psi^{(2)} = \Psi^{(2)}(0) - \frac{1}{14}\left(\partial_k \Phi^{(1)} \partial^k \Phi^{(1)} - \frac{10}{3}\frac{\partial_i \partial^j}{\nabla^2}\left(\partial_i \Phi^{(1)} \partial_j \Phi^{(1)}\right)\right)\eta^2$$

Initial conditions that contain the primordial NG

in Fourier space gives the convolution kernel

$$G(\mathbf{k}_1,\mathbf{k}_2,\mathbf{k})\eta^2 = \left[\mathbf{k}_1 \bullet \mathbf{k}_2 - \frac{10}{3}\frac{(\mathbf{k}\bullet\mathbf{k}_2)(\mathbf{k}\bullet\mathbf{k}_1)}{k^2}\right]\eta^2$$

This is a generalization to the well known expression at linear-order

$$\Theta^{(2)} = \frac{1}{4}\Delta^{(2)}_{00} + \Phi^{(2)} \sim A\cos[kc_s\eta]\, e^{-(k/k_D)^2} - R\Phi^{(2)} + S$$

(For details see Pitrou et al. 08; see also N.B, Matarrese, Riotto 07)

On small scales the combination of the damping effects AND the growth of the potential as η^2 make dominant the term

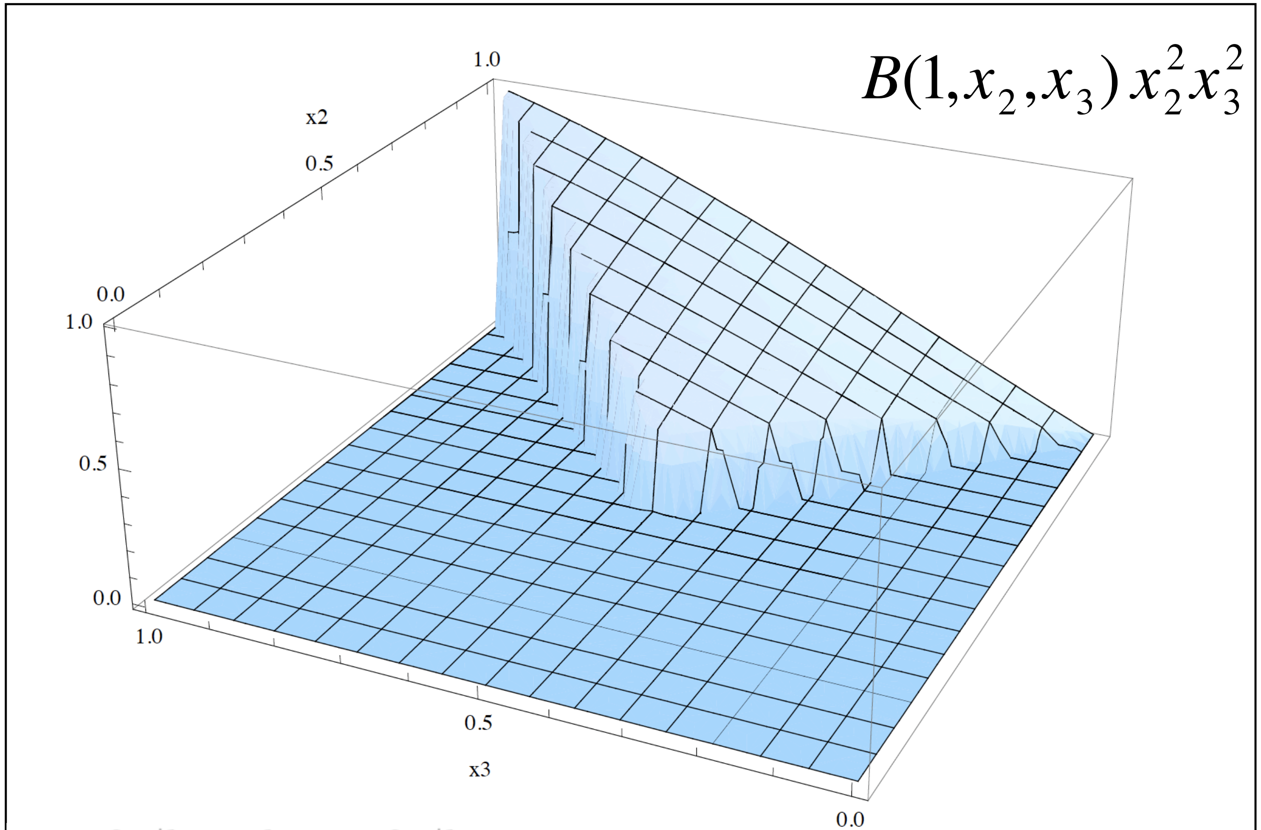
$$-R\Phi^{(2)} = -\frac{R}{14}G(\mathrm{k}_1,\mathrm{k}_2,\mathrm{k})\eta^2\Phi^{(1)}(\mathrm{k}_1)\Phi^{(1)}(\mathrm{k}_2)$$

Non-Gaussianity from 2nd-order gravitational potential

$$\Theta^{(2)} \cong -R\Phi^{(2)} \cong -\frac{R}{14} G(\mathrm{k}_1,\mathrm{k}_2,\mathrm{k})\eta^2\Phi^{(1)}(\mathrm{k}_1)\Phi^{(1)}(\mathrm{k}_2)$$

This effect is a causal one, i.e. developing on small scales; its origin is gravitational (due to the non-linear growth sourced by dark matter perturbations)

We expect the corresponding CMB bispectrum will be of the equilateral type.



$x_3 \equiv k_3/k_1$ and $x_2 \equiv k_2/k_1$

so, even if the NG at recombination is dominated by this effect, it will have a minimal contamination to the local primordial NG

Correlation to primordial f_{NL} of the intrinsically second-order term $\theta^{(2)}$=-R$\Phi^{(2)}$

Fisher matrix

$$F_{ij} = \frac{f_{\rm sky}}{\pi}\frac{1}{(2\pi)^2}\int d^2\ell_1 d^2\ell_2 d^2\ell_3\, \delta^{(2)}(\vec{\ell}_{123})\,\frac{B^i(\ell_1,\ell_2,\ell_3)\,B^j(\ell_1,\ell_2,\ell_3)}{6\,C(\ell_1)\,C(\ell_2)\,C(\ell_3)}$$

For EQUILATERAL primordial f^{eq}_{NL}

$$\left(\frac{S}{N}\right)_{\rm equil} = \frac{1}{\sqrt{F^{-1}_{\rm equil,equil}}} \simeq 12.6\times 10^{-3} f_{\rm NL}^{\rm equil}$$

$$\left(\frac{S}{N}\right)_{\rm rec} = \frac{1}{\sqrt{F^{-1}_{\rm rec,rec}}} \simeq 0.1$$

$$r_{\rm rec,equil} = \frac{F^{-1}_{\rm rec,equil}}{\sqrt{F^{-1}_{\rm equil,equil}F^{-1}_{\rm rec,rec}}} \simeq -0.53$$

$$d_{\rm rec} = F_{\rm rec,rec}F^{-1}_{\rm rec,rec} \simeq 1.4$$

$$d_{\rm equil} = F_{\rm equil,equil}F^{-1}_{\rm equil,equil} \simeq 1.4$$

As a confirmation of our expectations the NG from recombination (governed by the non-linear evolution of the 2nd-order gravitational potential) shows a quite high correlation with an equilateral primordial bispectrum

N.B, Riotto JCAP 09

Contamination to primordial f_{NL}

Contamination to equilateral $f_{NL}^{cont} = 5$

Contamination to local $f_{NL}^{cont} = 0.3$

Given the 1-σ uncertainty $f^{loc}_{NL} \sim 5$ for local, and $f^{eq}_{NL} \sim 67$ for equilateral, not relevant.

So the contamination from the intrinsically second-order term $\theta^{(2)}=-R\Phi^{(2)}$
to a primordial local NG is minimal
(definitely smaller than $f_{NL}^{cont}(loc) \approx 25$ claimed in Pitrou, Uzan, Bernardeau (PRD78, 08)

Conclusions

- ✓ Reassuring that results on NG from second-order fluctuations in the Boltzmann equations for photon-baryon and DM fluids converge
- ✓ Reassuring various groups worked on this (equations and computations quite messy)
- ✓ no significant contamination to primordial NG from non-linearities at recombination
- ✓ Still some work (and fun???) to do