Effects of Heavy Fields

Mark G. Jackson

Institute for Astrophysics-Paris,
University of Illinois Urbana-Champaign,
Paris Centre for Cosmological Physics

Collaborators: K. Schalm, B. Wandelt, F. Bouchet

Critical Tests of Inflation using nG
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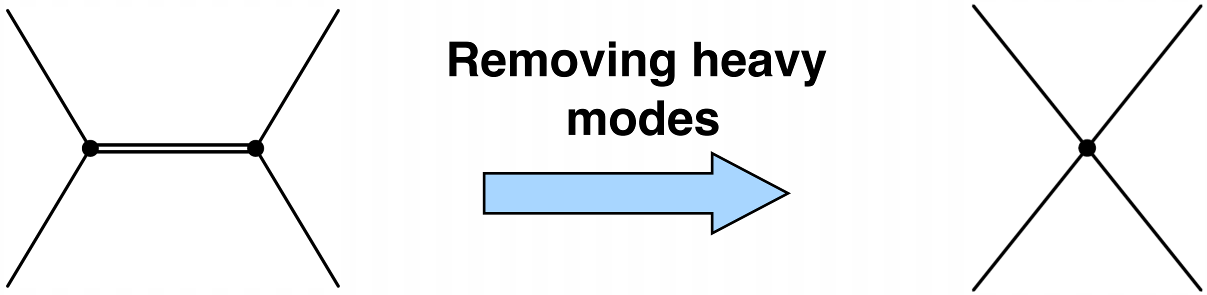
Heavy Fields (in Cosmology)

- **To calculate effects of heavy fields we usually rely upon Wilsonian effective actions:**



$$\mathcal{G}(x,y) = \int \frac{d^4p}{(2\pi)^4} \frac{e^{-ip\cdot(x-y)}}{p^2 - M^2} \approx -\left(\frac{1}{M^2} - \frac{\partial^2}{M^4} + \cdots\right) \delta^4(x-y).$$

- **Unfortunately, standard techniques rely upon energy conservation, which is absent during cosmological expansion**
- **How do we do this for inflationary theories?**

Power Spectrum Corrections from a Toy UV Model

- **We (MGJ, Schalm '10) recently developed the procedure to evaluate correlation functions from heavy fields**
- **Begin with inflating system,**

$$S_{\rm inf}[\phi] = -\int d^4x\sqrt{g}\left[\frac{1}{2}(\partial\phi)^2 - V(\phi)\right]$$

and add (for example) Yukawa interactions to a heavy field χ :

$$S_{\rm new}[\varphi,\chi] \;=\; -\int d^4x\sqrt{g}\left[\frac{1}{2}(\partial\chi)^2 + \frac{1}{2}M^2\chi^2 + \frac{g}{2}\varphi^2\chi\right]$$

- **The power spectrum can then be computed using the in-in formalism:**

$$P_\varphi(k) \;=\; \lim_{t\to\infty}\frac{k^3}{2\pi^2}\,\langle \mathbf{0}(t_0)|e^{i\int_{t_0}^t dt'\mathcal{H}(t')}|\varphi_{\mathbf{k}}(t)|^2 e^{-i\int_{t_0}^t dt''\mathcal{H}(t'')}|\mathbf{0}(t_0)\rangle$$

- **Note that this can be interpreted as an in-out correlation using**

$$\mathcal{S} \equiv S[\varphi_+,\chi_+] - S[\varphi_-,\chi_-]$$

Feynman Rules in Keldysh Basis

- **The correlations can now be evaluated using these:**

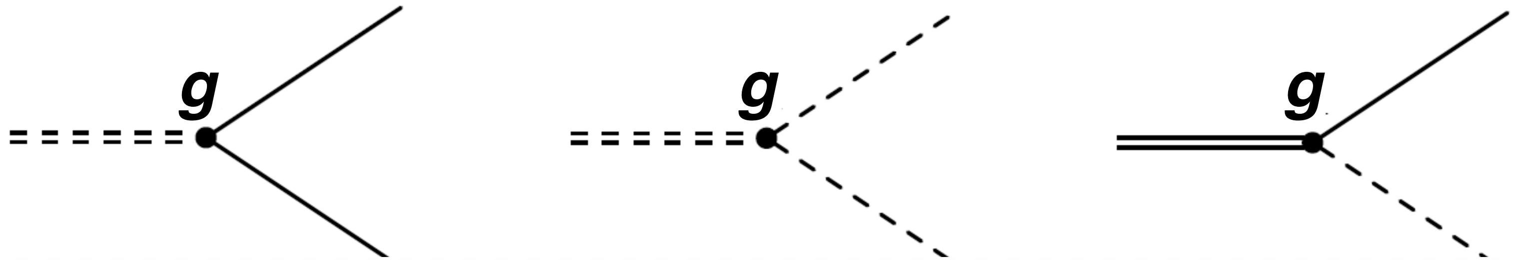
$$G^R_{\mathbf{k}}(\tau_1,\tau_2) \equiv i\langle\bar{\varphi}_{\mathbf{k}}(\tau_1)\Phi_{-\mathbf{k}}(\tau_2)\rangle = -2\theta(\tau_1-\tau_2)\mathrm{Im}\left[U_{\mathbf{k}}(\tau_1)U^*_{\mathbf{k}}(\tau_2)\right],$$

$$F_{\mathbf{k}}(\tau_1,\tau_2) \equiv \langle\bar{\varphi}_{\mathbf{k}}(\tau_1)\bar{\varphi}_{-\mathbf{k}}(\tau_2)\rangle = \mathrm{Re}\left[U_{\mathbf{k}}(\tau_1)U^*_{\mathbf{k}}(\tau_2)\right],$$

$$\mathcal{G}^R_{\mathbf{k}}(\tau_1,\tau_2) \equiv i\langle\bar{\chi}^{(0)}_{\mathbf{k}}(\tau_1)\mathrm{X}^{(0)}_{-\mathbf{k}}(\tau_2)\rangle = -2\theta(\tau_1-\tau_2)\mathrm{Im}\left[V_{\mathbf{k}}(\tau_1)V^*_{\mathbf{k}}(\tau_2)\right],$$

$$\mathcal{F}_{\mathbf{k}}(\tau_1,\tau_2) = \langle\bar{\chi}^{(0)}_{\mathbf{k}}(\tau_1)\bar{\chi}^{(0)}_{-\mathbf{k}}(\tau_2)\rangle = \mathrm{Re}\left[V_{\mathbf{k}}(\tau_1)V^*_{\mathbf{k}}(\tau_2)\right],$$

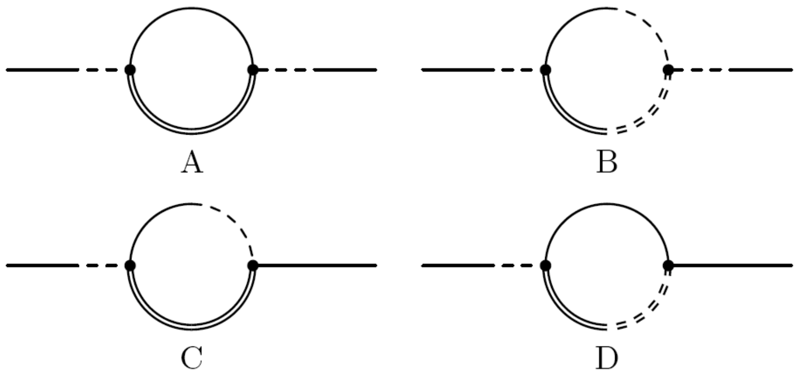
- **The interactions are given by:**



(MGJ, Schalm '10)

Power Spectrum Corrections

- **2-pt correlation can then be computed using normal methods, producing four Feynman diagrams:**



$$P_\varphi^{(\mathrm{A})}(k) = \frac{k^3}{2\pi^2}(-ig)^2 \int_{\tau_{\mathrm{in}}}^0 d\tau_1 \; a(\tau_1)^4 \int_{\tau_{\mathrm{in}}}^0 d\tau_2 \; a(\tau_2)^4 \times$$

$$\int \frac{d^3\mathbf{q}}{(2\pi)^3} \left[-iG_{\mathbf{k}}^R(0,\tau_1)\right] \mathcal{F}_{\mathbf{q}+\mathbf{k}}(\tau_1,\tau_2) F_{\mathbf{q}}(\tau_1,\tau_2) \left[-iG_{\mathbf{k}}^A(\tau_2,0)\right].$$

- **Which are significant? We need some good approximations!**

(MGJ, Schalm '10)

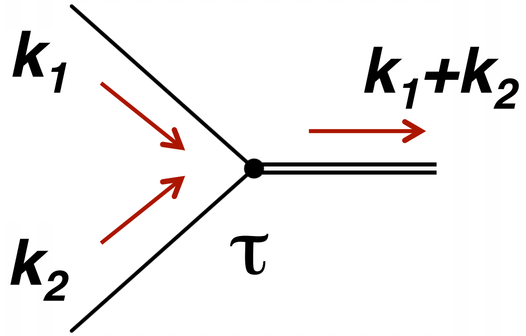
Power Spectrum Corrections

- **Each vertex is an integral over the time of interaction, and has the following form:**

$$\mathcal{A}_1(\mathbf{k}_1,\mathbf{k}_2) \equiv \int_{\tau_0}^{0} d\tau\ a^4(\tau) U_{\mathbf{k}_1}(\tau) U_{\mathbf{k}_2}(\tau) V^*_{-(\mathbf{k}_1+\mathbf{k}_2)}(\tau)$$

$$\approx -\frac{1}{2\sqrt{2k_1^3k_2^3}H}\int_{\tau_0}^{0}\frac{d\tau}{\tau^3}\frac{(1-ik_1\tau)(1-ik_2\tau)}{\left(|\mathbf{k}_1+\mathbf{k}_2|^2+\frac{M^2}{H^2\tau^2}\right)^{1/4}}$$

$$\times \exp\left[-i(k_1+k_2)\tau + i\int^{\tau} d\tau'\sqrt{|\mathbf{k}_1+\mathbf{k}_2|^2+\frac{M^2}{H^2\tau'^2}}\right].$$



- **This admits a stationary phase approximation near the moment of energy-conservation,**

$$\tau_*^{-1} = -\frac{H}{M}\sqrt{2k_1k_2(1-\cos\theta)}, \qquad \cos\theta = \frac{\mathbf{k}_1\cdot\mathbf{k}_2}{k_1k_2}.$$

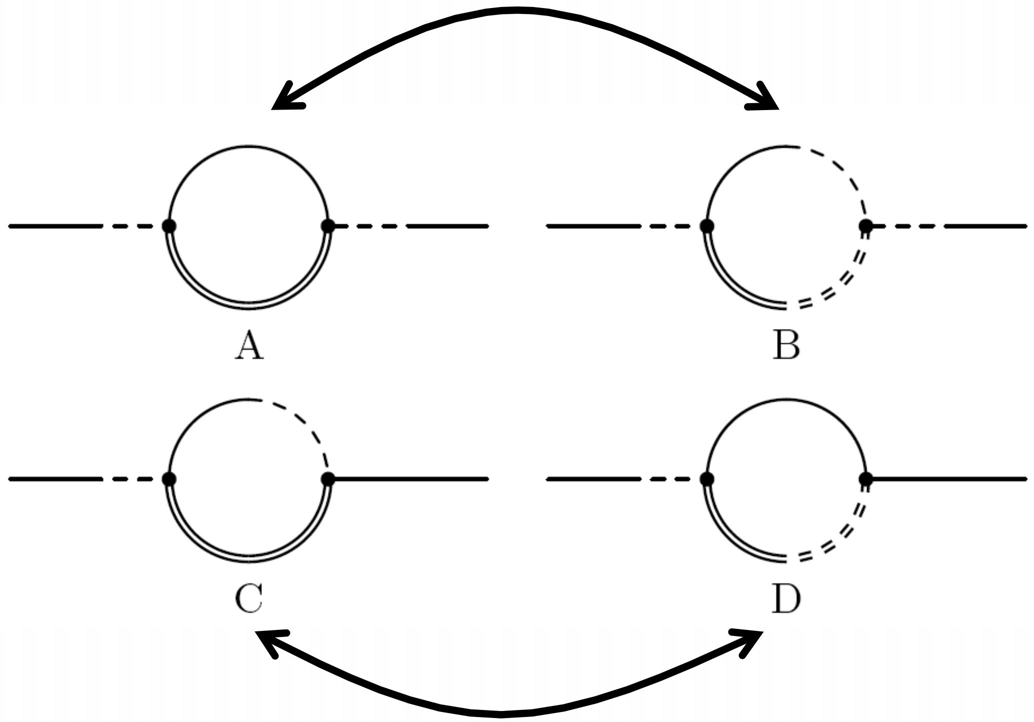
- **The vertex (to leading order in *H/M*) is then simply**

$$\hat{\mathcal{A}}_1(\mathbf{k}_1,\mathbf{k}_2) \approx -\frac{\sqrt{\pi i}}{2\sqrt{k_1k_2}\,[2k_1k_2(1-\cos\theta)]^{1/4}\,H}\sqrt{\frac{H}{M}}\left[\frac{2M}{H}\left(k_1+k_2+\sqrt{2k_1k_2(1-\cos\theta)}\right)\right]^{-i\frac{M}{H}}$$

Power Spectrum Corrections

- **Now things simplify immensely:**

$P^{(A)} = -2\, P^{(B)}$

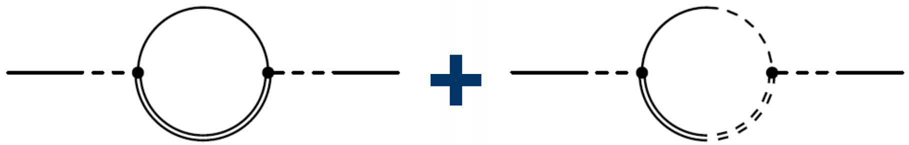


These cancel!

(MGJ, Schalm '10)

Power Spectrum Corrections

$$\Delta P_\varphi \approx$$



$$\approx \frac{g^2 H \Lambda^3}{192\pi^3 M^4}\left[1 - \frac{8\epsilon_1}{3} - (2\epsilon_1+\epsilon_2)C - 4\epsilon_1 \ln\left(\frac{4\Lambda H_*}{M^2}\right) + (6\epsilon_1+\epsilon_2)\ln\left(\frac{k}{k_*}\right)\right]$$

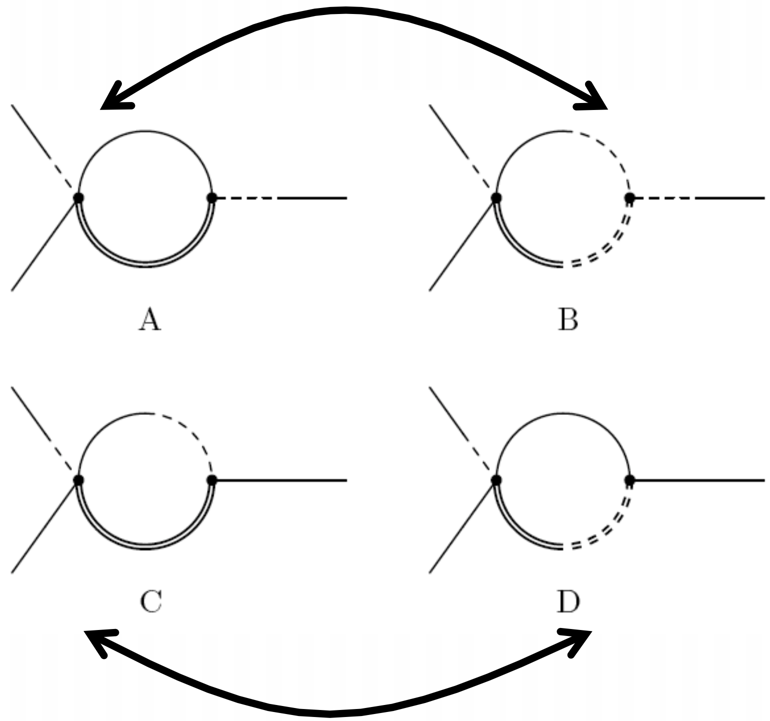
- **Thus high-energy physics produces a nearly scale-invariant shift in the power spectrum.**
- **The integral over loop momentum smears out oscillations into an overall enhancement**
- **It is of magnitude ~ $H\Lambda^3/M^4$, which is ~ H/M**

(MGJ, Schalm ‘11)

Heavy Field Corrections to the Bispectrum

- **Nearly identical to power spectrum evaluation:**

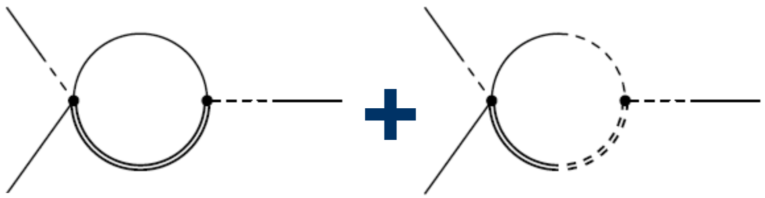
$B^{(A)} = -2\, B^{(B)}$



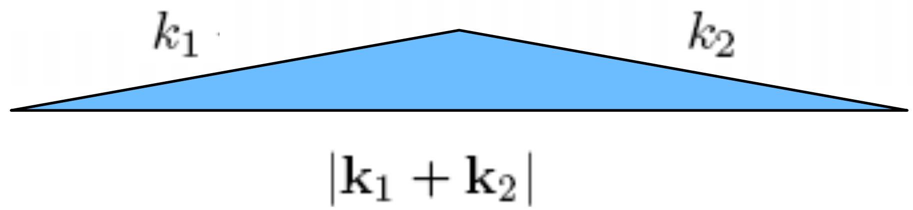
These cancel!

(MGJ, Schalm '12)

Heavy Field Corrections to the Bispectrum

$B_\varphi \approx$ 

$\approx \Delta P_\varphi \, \delta(\epsilon)$



where $\epsilon \equiv k_1 + k_2 - |\mathbf{k}_1 + \mathbf{k}_2|$

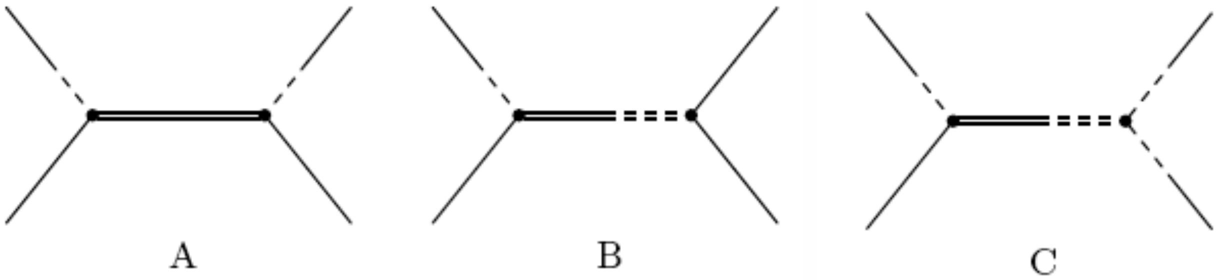
- **Thus high-energy physics produces a nearly scale-invariant enfolded-triangle bispectrum**
- **This enfolded shape was precisely the shape predicted by**

  R. Holman and A. Tolley ‘07; D. Meerburg, P. Corasaniti, MGJ, J. P. van der Schaar ‘09
- **The loop momentum again smears any oscillations into an overall enhancement.**

(MGJ, Schalm ‘12)

Heavy Field Corrections to the Bispectrum

- **There are three tree-level trispectrum corrections:**



$$T_\varphi = \frac{g_1^2 H^3}{2^{10}\pi^2 (k_1 k_2 k_3 k_4)^2 M} \left[(k_1+k_2)^2 - |\mathbf{k}_1+\mathbf{k}_2|^2\right]^{-1/4} \left[(k_3+k_4)^2 - |\mathbf{k}_3+\mathbf{k}_4|^2\right]^{-1/4} \times$$

$$\left(\frac{k_1+k_2+\sqrt{(k_1+k_2)^2 - |\mathbf{k}_1+\mathbf{k}_2|^2}}{k_3+k_4+\sqrt{(k_3+k_4)^2 - |\mathbf{k}_3+\mathbf{k}_4|^2}}\right)^{-iM/H} + \text{c.c.}$$

Important: phase oscillates as ~ *M/H*

- **No loops, no cutoffs: completely clean calculation**

(MGJ, Schalm '12)

Observability? (!)

- **We see that integrating out high energy physics produces low energy interactions**
- **These appear in the spectrum and bispectrum as scale-invariant corrections, and in the trispectrum as oscillations**
- **But are these observable?**
- **We can see about four decades of comoving *k* in the CMB,**

$$k_{\rm min} \le k_{\rm obs} \le 10^4 \; k_{\rm min}$$

- **If** $H/M_{\rm string} \sim 10^{-2}$ **then we should see about 10^2 oscillations, just at the threshold of *Planck*'s sensitivity.**

k- versus position-space Trispectra

- **Now instead consider the position-space trispectrum:**

$$\langle \frac{\Delta T(\hat{\mathbf{n}}_1)}{T}\frac{\Delta T(\hat{\mathbf{n}}_2)}{T}\frac{\Delta T(\hat{\mathbf{n}}_3)}{T}\frac{\Delta T(\hat{\mathbf{n}}_4}{T} = \int \prod_{i=1}^{4} \frac{d^3\mathbf{k}_i}{(2\pi)^3} \langle \varphi_{\mathbf{k}_1}\varphi_{\mathbf{k}_2}\varphi_{\mathbf{k}_3}\varphi_{\mathbf{k}_4}\rangle \exp -i\eta \left(\sum_{i=1}^{4} \mathbf{k}_i \cdot \hat{\mathbf{n}}_i\right)$$

where

$$\cdot\langle \varphi_{\mathbf{k}_1}\varphi_{\mathbf{k}_2}\varphi_{\mathbf{k}_3}\varphi_{\mathbf{k}_4}\rangle \sim \delta^3\left(\sum_{i=1}^{4}\mathbf{k}_i\right)\left(\frac{k_1+k_2+\sqrt{(k_1+k_2)^2-|\mathbf{k}_1+\mathbf{k}_2|^2}}{k_3+k_4+\sqrt{(k_3+k_4)^2-|\mathbf{k}_3+\mathbf{k}_4|^2}}\right)^{-iM/H}$$

- **Using the Fourier representation of the δ-function**

$$(2\pi)^3\delta^3\left(\sum_{i=1}^{4}\mathbf{k}_i\right) = \left(\frac{M}{H}\right)^3 \int d^3\mathbf{w} \exp -i\frac{M}{H}\left(\sum_{i=1}^{4}\mathbf{k}_i\right)\cdot\mathbf{w}$$

allows us to put it in the form

$$\left(\frac{\Delta T}{T}\right)^4 \sim \int \prod_{i=1}^{4} d^3\mathbf{k}_i \; d^3\mathbf{w} \; G(\mathbf{k}_i) e^{-i\frac{M}{H}F(\mathbf{k}_i,\mathbf{w})}$$

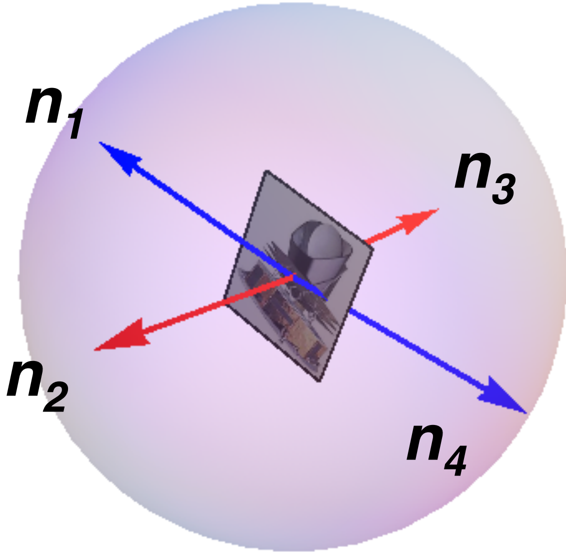
k- versus position-space Trispectra

- **There will be a solution satisfying**

$$0 = \frac{\partial F}{\partial k_i^a}, \qquad 0 = \frac{\partial F}{\partial w^a} = \sum_i k_i^a.$$

This will determine the ideal direction vectors n_i to correlate

- **The trispectrum is then approximately**



$$\left(\frac{\Delta T}{T}\right)^4 \sim G(\mathbf{k}_{i(0)}) e^{-i\frac{M}{H}F(\mathbf{k}_{i(0)}\mathbf{w}_{(0)})}$$

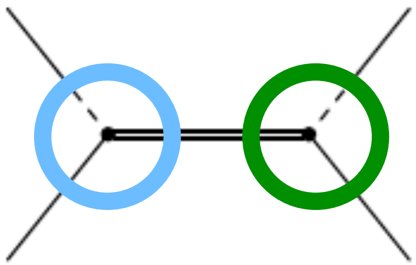
$$\times \int \prod_{i=1}^{4} d^3\mathbf{k}_i \; d^3\mathbf{w} \; e^{-i\frac{M}{H}\left[\frac{1}{2}\mathcal{M}_{ab}^{ij}\delta k_i^a \delta k_j^b + \delta\mathbf{k}_i\cdot\delta\mathbf{w}\right]}$$

$$\sim G(\mathbf{k}_{i(0)}) \left(\frac{2\pi i H}{M}\right)^{15/2} \left[\det\left(\mathcal{M}_{ab}^{ij} + 2\delta_{ab}^{\mathbf{k}^i,\mathbf{w}}\right)\right]^{-1/2} \qquad \mathcal{M}_{ab}^{ij} \equiv \left.\frac{\partial^2 F}{\partial k_i^a \partial k_j^b}\right|_{\mathbf{k}_{i(0)},\mathbf{w}_{(0)}}$$

Solving the Constraints, 1

- **A huge simplification occurs by noticing that the problem factorizes into two 'halves' coupled only via *w*:**

$$\begin{aligned} F(\mathbf{k}_i, \mathbf{w}) &= F_L(\mathbf{k}_1, \mathbf{k}_2, \mathbf{w}) + F_R(\mathbf{k}_3, \mathbf{k}_4, \mathbf{w}), \\ G(\mathbf{k}_i) &= G_L(\mathbf{k}_1, \mathbf{k}_2) G_R(\mathbf{k}_3, \mathbf{k}_4). \end{aligned}$$



- **We can thus solve each half separately, then combine later via *w*-constraint, or conservation of momentum. The left-half consists of k_1, k_2 - constraints:**

$$-\frac{\hat{\mathbf{k}}_1 + \frac{k_2(\hat{\mathbf{k}}_1 - \hat{\mathbf{k}}_2)}{\sqrt{2k_1k_2(1-\hat{\mathbf{k}}_1\cdot\hat{\mathbf{k}}_2)}}}{k_1 + k_2 + \sqrt{2k_1k_2\left(1 - \hat{\mathbf{k}}_1\cdot\hat{\mathbf{k}}_2\right)}} = \gamma\hat{\mathbf{n}}_1 + \mathbf{w},$$

$$-\frac{\hat{\mathbf{k}}_2 + \frac{k_1(\hat{\mathbf{k}}_2 - \hat{\mathbf{k}}_1)}{\sqrt{2k_1k_2(1-\hat{\mathbf{k}}_1\cdot\hat{\mathbf{k}}_2)}}}{k_1 + k_2 + \sqrt{2k_1k_2\left(1 - \hat{\mathbf{k}}_1\cdot\hat{\mathbf{k}}_2\right)}} = \gamma\hat{\mathbf{n}}_2 + \mathbf{w}.$$

Solving the Constraints, 2

- **An exact solution exists:**

$$\mathbf{N} \equiv \hat{\mathbf{n}}_1 - \hat{\mathbf{n}}_2, \quad \mathbf{v} \equiv \frac{2}{\gamma}\mathbf{w} + \hat{\mathbf{n}}_1 + \hat{\mathbf{n}}_2$$

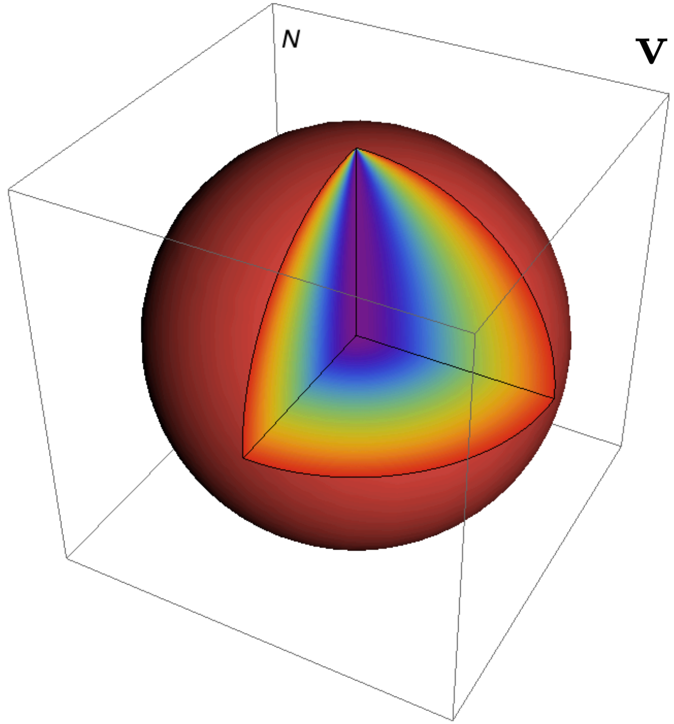
$$K(\rho) = \frac{2\rho}{\sqrt{1+\rho^2}}$$

$$k_1 = \frac{1}{2\gamma}\left[\frac{2KN}{N^2 - v^2}\left(1 - \frac{n}{N}\right) - \frac{K}{N}\right],$$

$$k_2 = \frac{1}{2\gamma}\left[\frac{2KN}{N^2 - v^2}\left(1 + \frac{n}{N}\right) - \frac{K}{N}\right],$$

$$\hat{\mathbf{k}}_1 = -\frac{1}{2}\left(\sqrt{4 - K(\rho)^2}\hat{\mathbf{M}} + K(\rho)\hat{\mathbf{N}}\right),$$

$$\hat{\mathbf{k}}_2 = -\frac{1}{2}\left(\sqrt{4 - K(\rho)^2}\hat{\mathbf{M}} - K(\rho)\hat{\mathbf{N}}\right).$$



Color indicates time of interaction

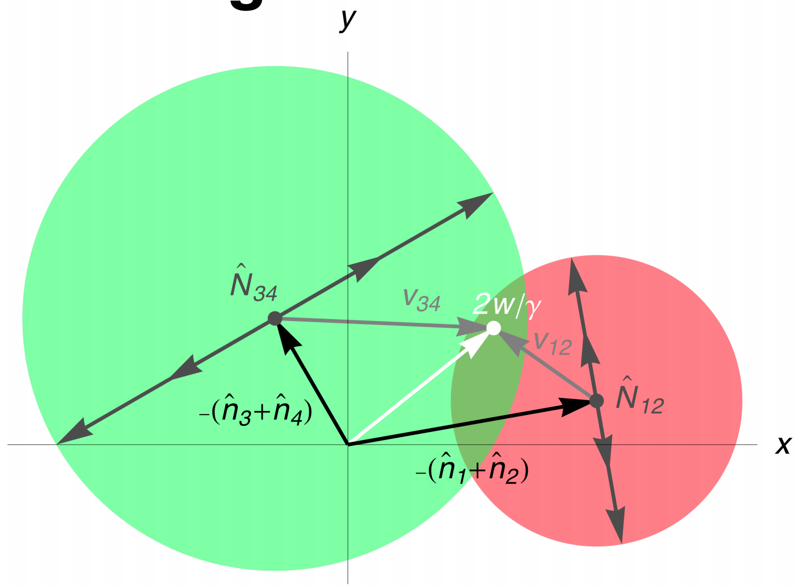
- **The identical result applies for the right half**

Solving the Constraints, 3

- **Next step: solving the combined system via momentum conservation,**

$$\mathbf{k}_1 + \mathbf{k}_2 = -(\mathbf{k}_3 + \mathbf{k}_4)$$

- **This amounts to finding a mutual solution to the two halves,**



- **Work in progress….**

Conclusion

- **We can now calculate the effect of heavy fields to the spectrum, bispectrum, and trispectrum, as well as construct effective actions**
- **Currently working with the Planck team for the trispectrum search templates.**