Predictions from multiple field inflation

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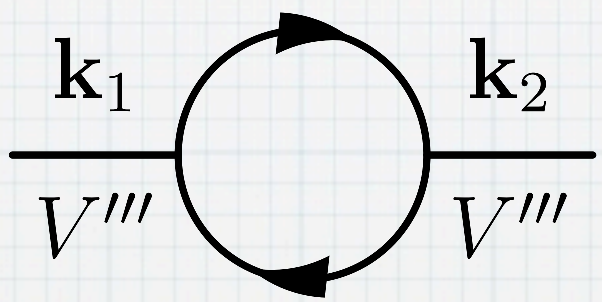
University of Sussex

Critical tests of inflation using non-gaussianity, 7 November 2012

Single-field inflation

On previous days we heard a lot about single-clock inflation.

This models are nice because the correlation functions have a simple structure.



Pimentel, Senatore & Zaldarriaga (2012); Senatore & Zaldarriaga (2012)
Assassi, Baumann & Green (2012)

In multiple field inflation this is no longer true.

(For me, that means inflation with multiple active, light fields.)

$$\langle \delta\phi_\alpha(\boldsymbol{k}_1)\delta\phi_\beta(\boldsymbol{k}_2)\rangle_\tau = (2\pi)^3\delta(\boldsymbol{k}_1+\boldsymbol{k}_2)\frac{H_*^2}{2k^3}$$

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$$\approx N$$

N measures the number of e-folds by which this *k*-mode is out of the horizon

We assumed these fields were light, so $\frac{m_{\alpha\beta}}{H^2} \ll 1$

By the end of inflation $N \approx 60$,
so you might think we can get a good estimate from
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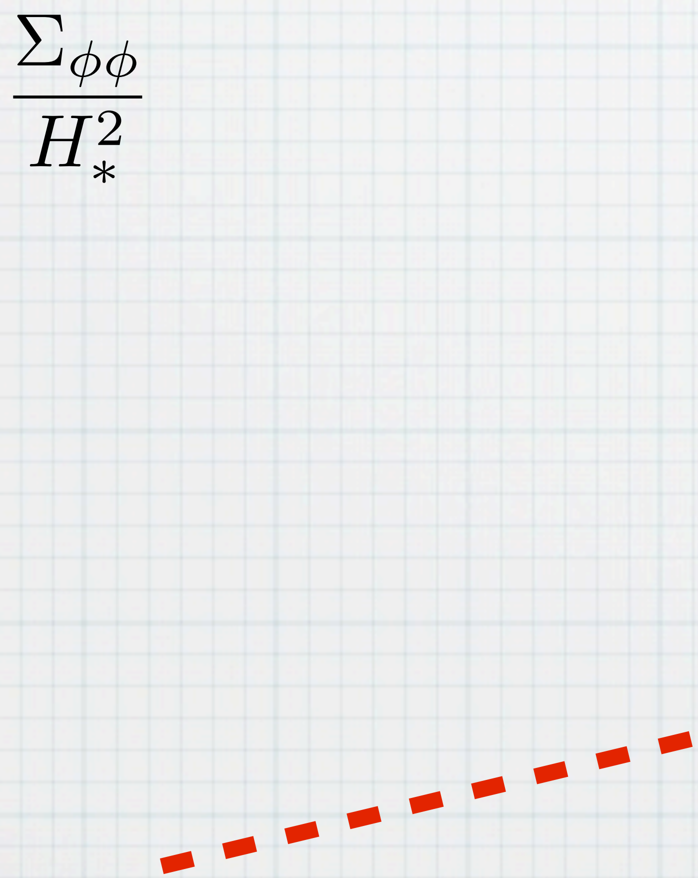

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The linear approximation is poor by N = 10

It is totally wrong, even qualitatively, for N $\gtrsim$ 10

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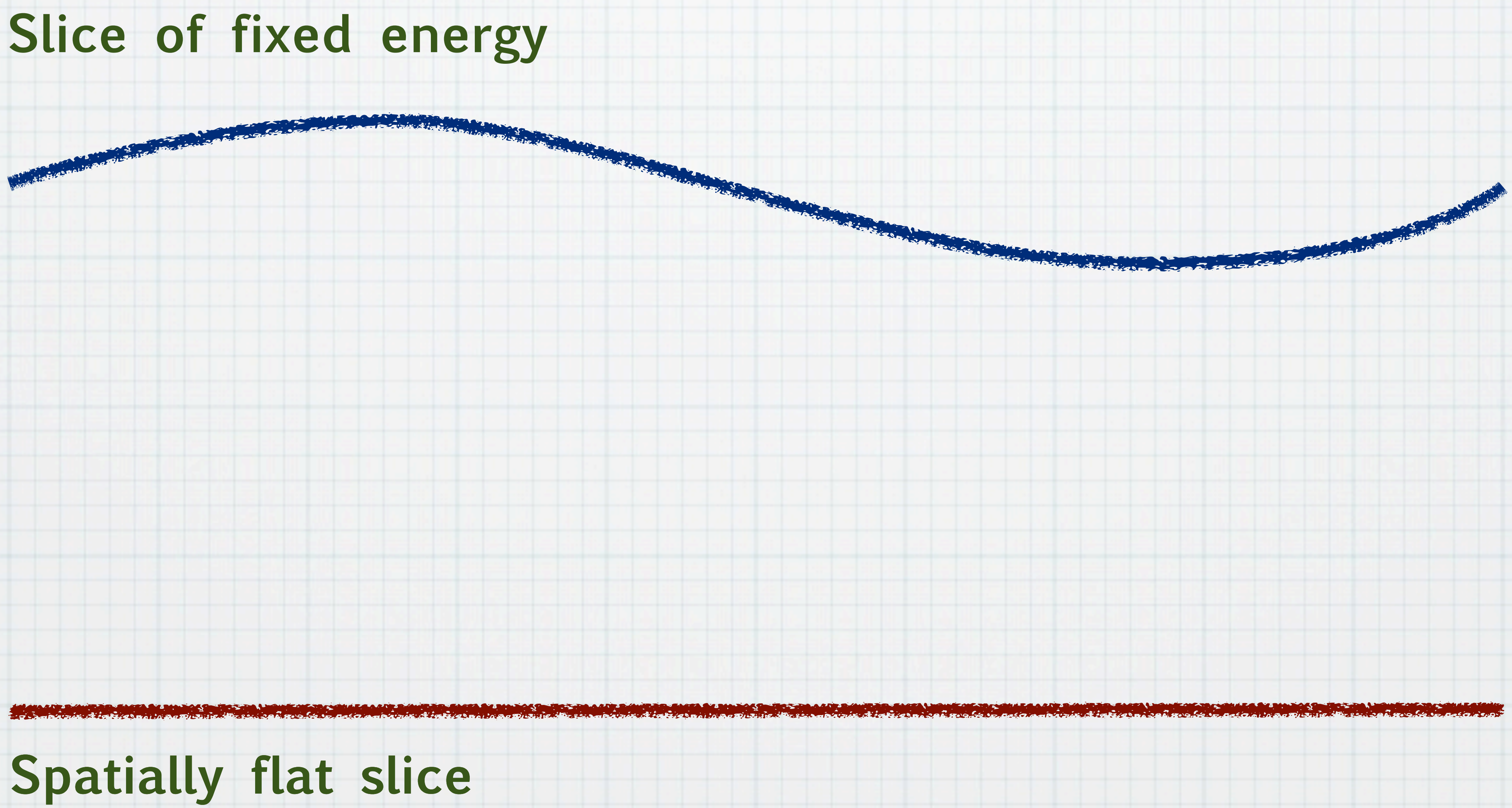

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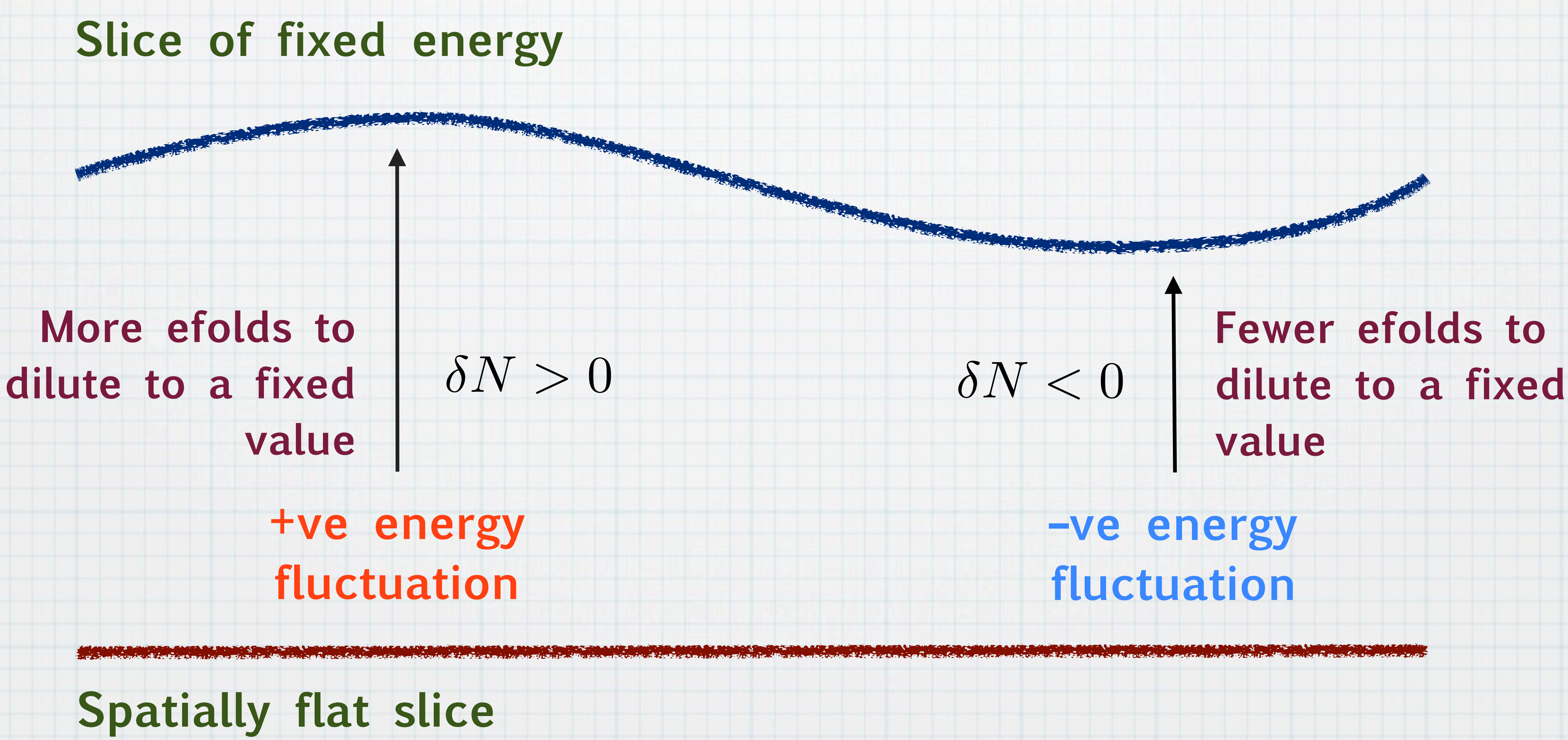
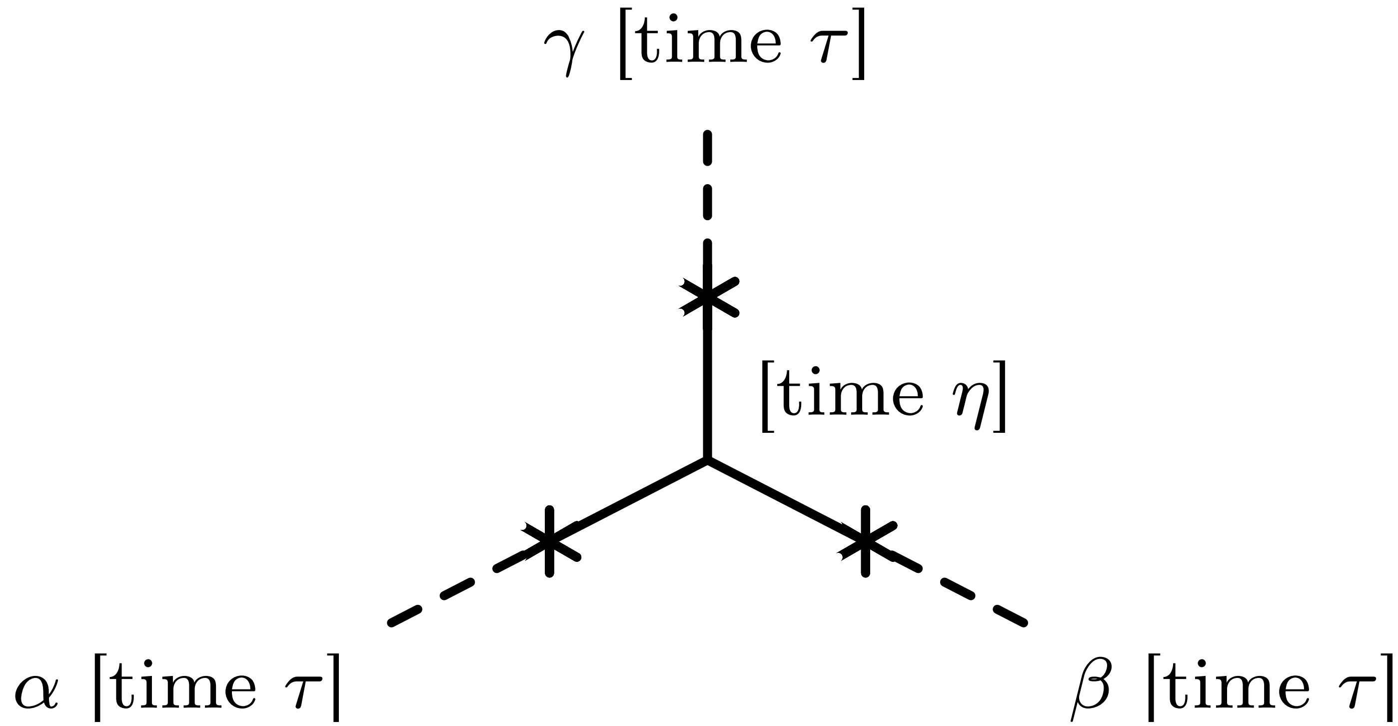


The δN method tells us how to handle this time dependence

$$\delta N = \delta[N(\phi, \rho, \cdots)] = \frac{\partial N}{\partial \phi^*_\alpha}\delta\phi^*_\alpha + \frac{1}{2}\frac{\partial^2 N}{\partial \phi^*_\alpha \partial \phi^*_\beta}\delta\phi^*_\alpha \delta\phi^*_\beta + \cdots$$

Lyth & Rodríguez (2005)

- Although no-one doubts this formula, it has never been demonstrated to be correct.
  So, what do I do if I am dealing with a different model?
- ... maybe I want to include loop corrections?
- ... interesting models may have nontrivial kinetic sector, for which the δN formula may not apply. Maybe I'm interested in these?


γ [time τ]
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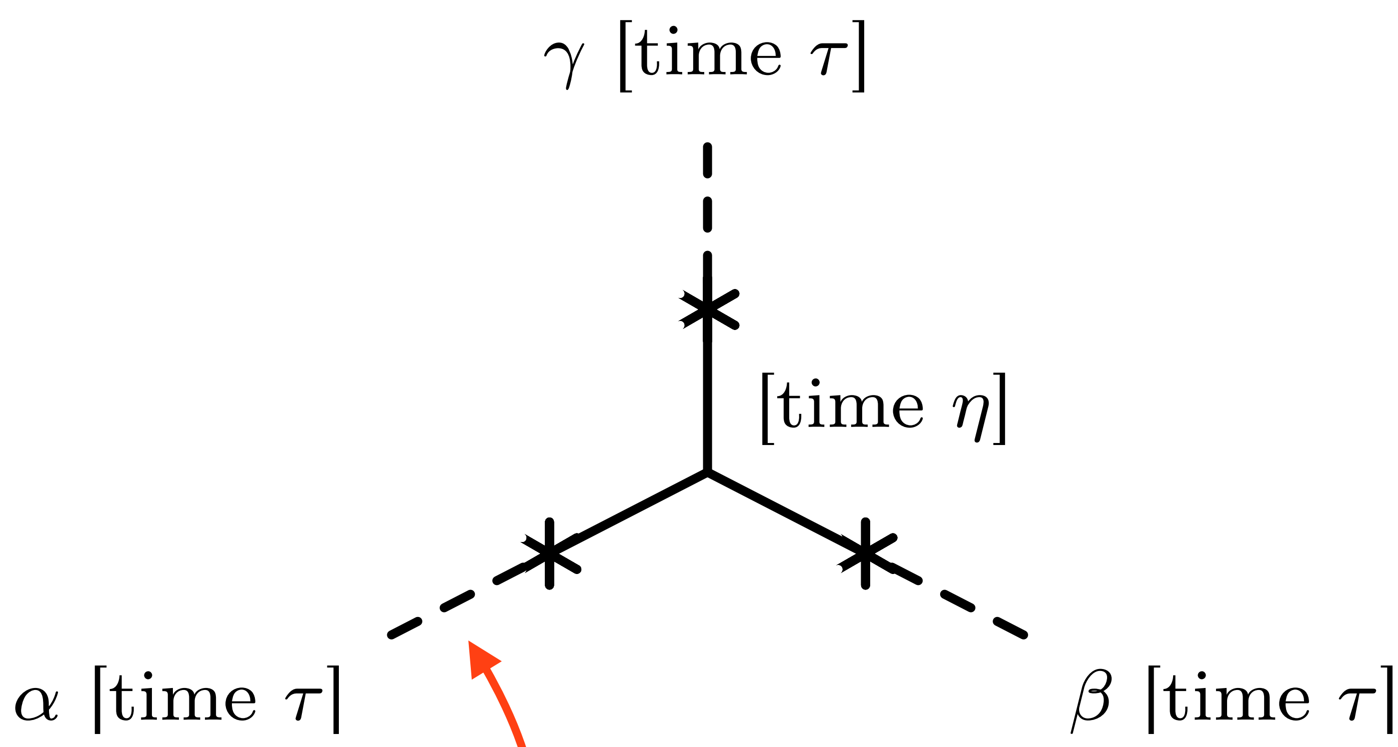
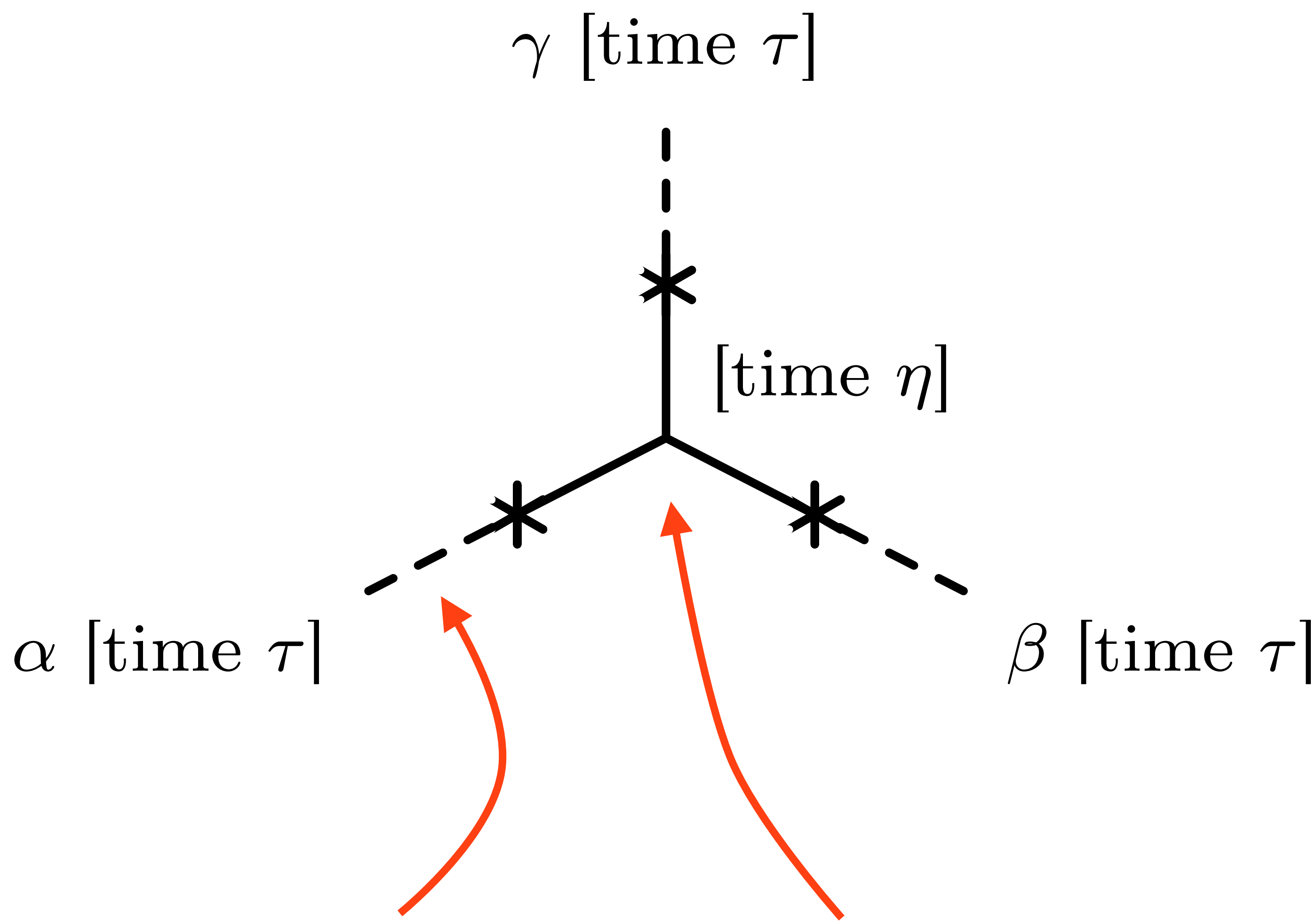


Time dependence from external wavefunctions

$$\psi(k,\tau) = (1 + \mathrm{i}k\tau)\mathrm{e}^{-\mathrm{i}k\tau}$$



Time dependence from external wavefunctions

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3-body “collision integral”

$$\int \mathrm{d}t \; a(t)^3 \psi(k_1, \tau)\psi(k_2, \tau)\psi(k_3, \tau)$$

Two-step strategy, borrowed from QCD

Dias, Ribeiro & DS arXiv:1210.7800

1. Including masses perturbatively, argue that logarithmic divergences can only be produced in combination with certain functions of the external momenta

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2. Write renormalization-group equations for the unknown coefficients

We require a guarantee that we only need a finite number of unknown functions to do this — the analogue of renormalizability.
Here it is the statement that correlation functions factorize.

In conventional models you can show this reproduces the usual δN formula to leading-logarithm order.

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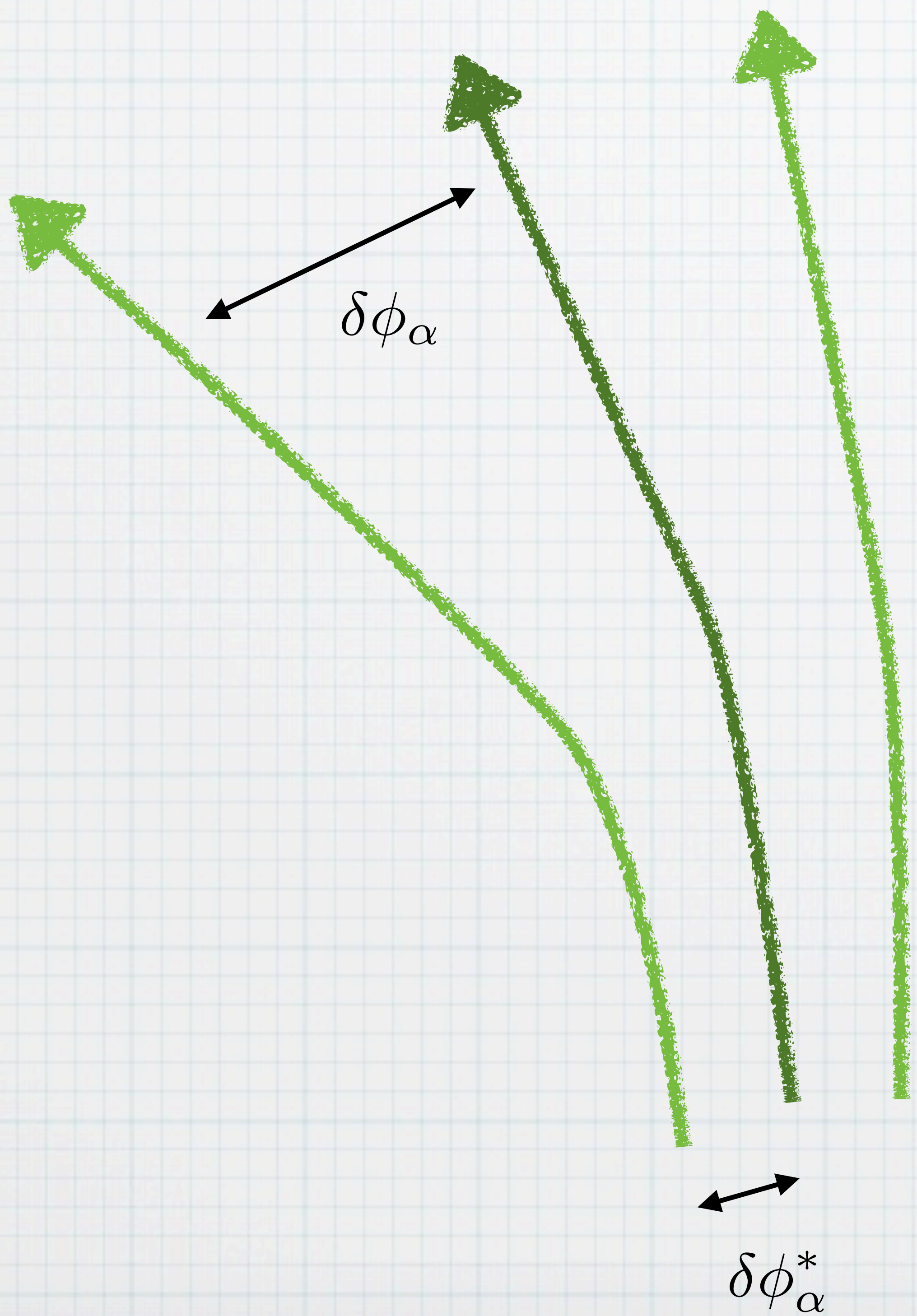

The RGE can be interpreted as an evolution equation for each Jacobi field of the flow.

Then we have to track the correlation functions along the flow, à la Callan-Symanzik equation, critical phenomena, ...

García-Bellido & Wands (1996)
Bernardeau & Uzan (2002)

Yokoyama, Suyama & Tanaka (2007)
DS, Mulryne, Frazer & Ribeiro (2012)

Example: nontrivial kinetic term

$$S = -\frac{1}{2}\int \mathrm{d}^4x\,\sqrt{-g}\,\left(M_{\mathrm{P}}^2 R + \boldsymbol{G}_{\alpha\beta}\partial_a\phi^\alpha\partial_b\phi^\beta + 2V\right)$$

Nakamura & Stewart (1996)
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Tegmark & Peterson arXiv:1111.0927
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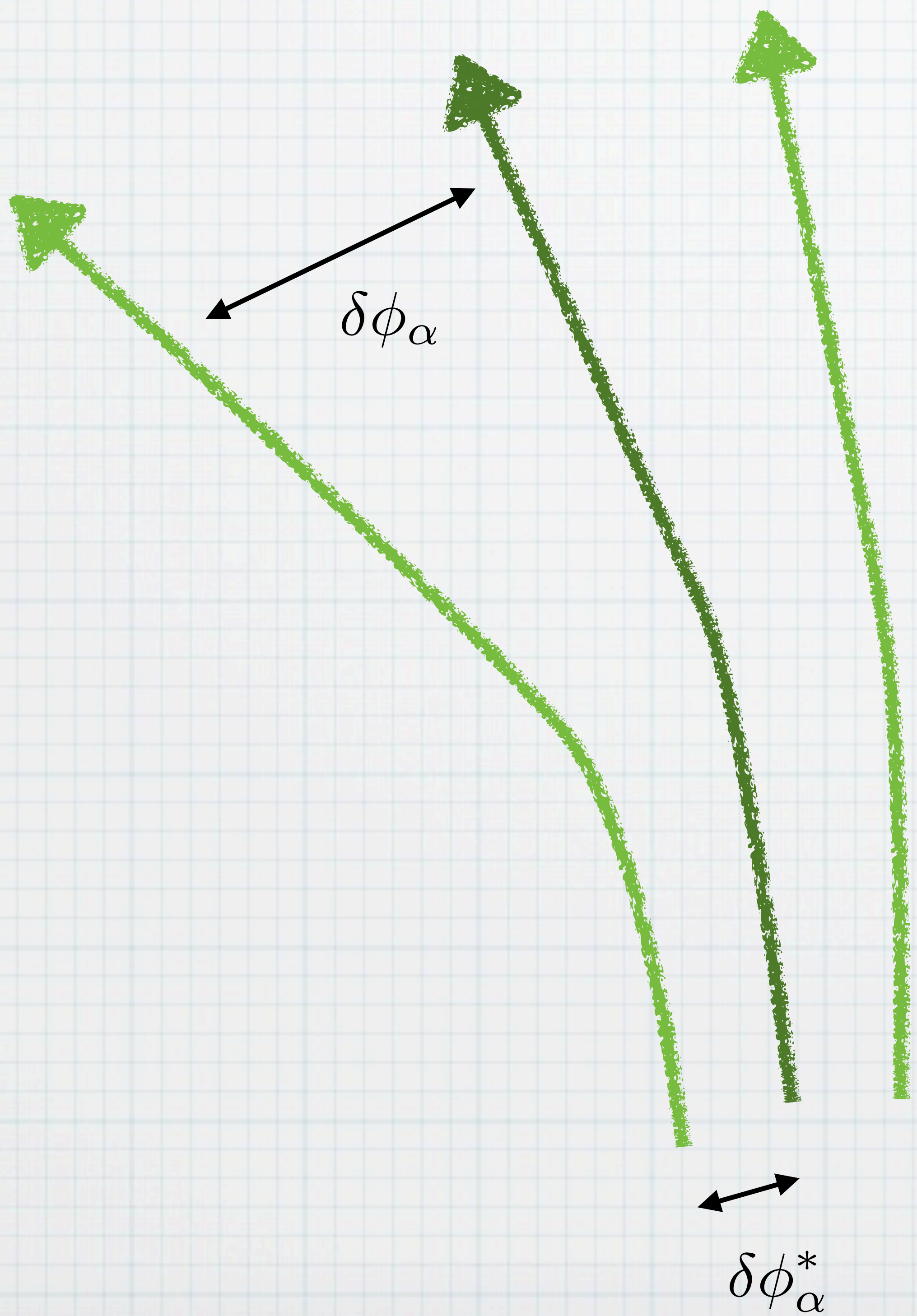

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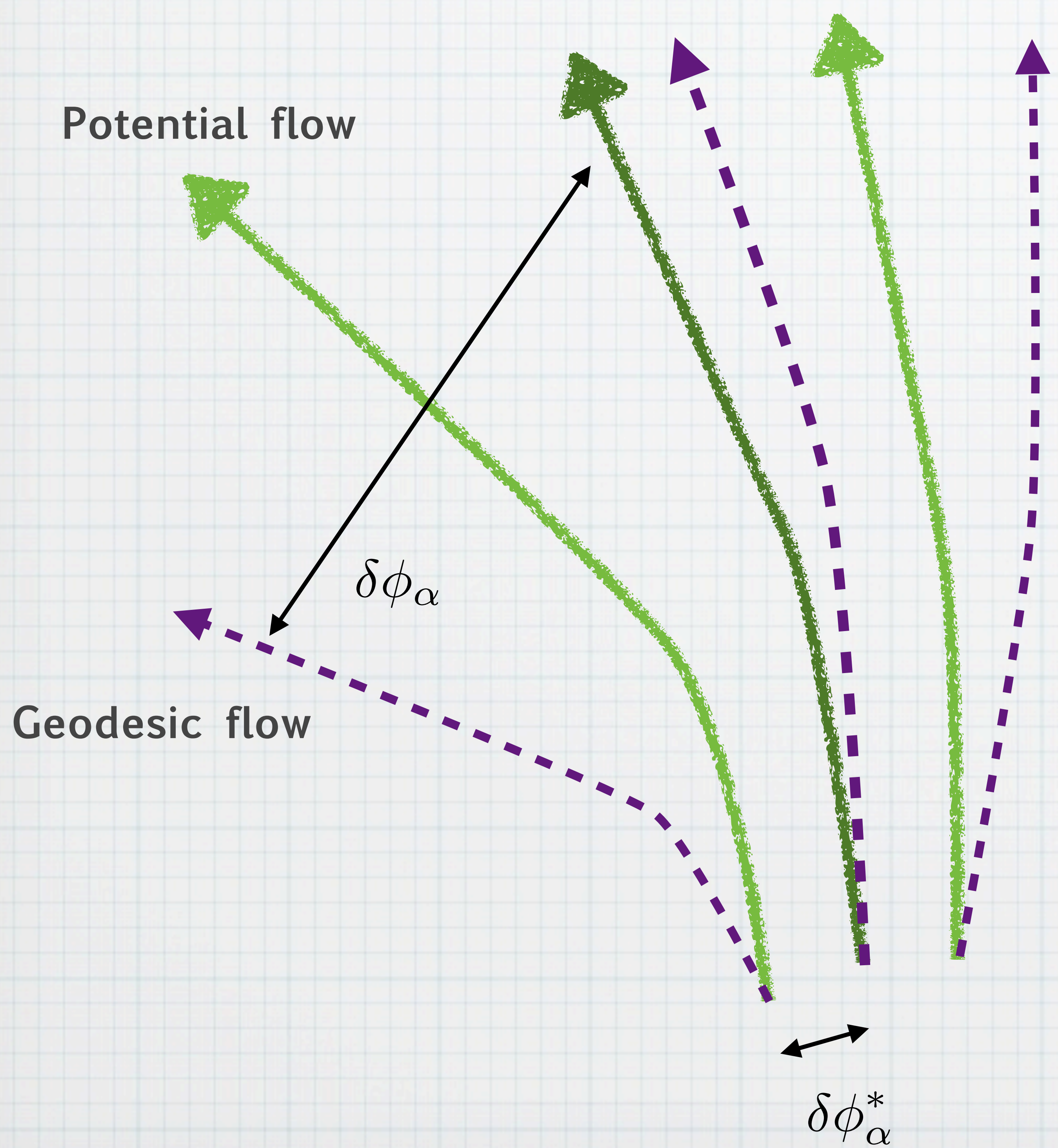

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Now there are contributions from what would be geodesic deviation in spacetime

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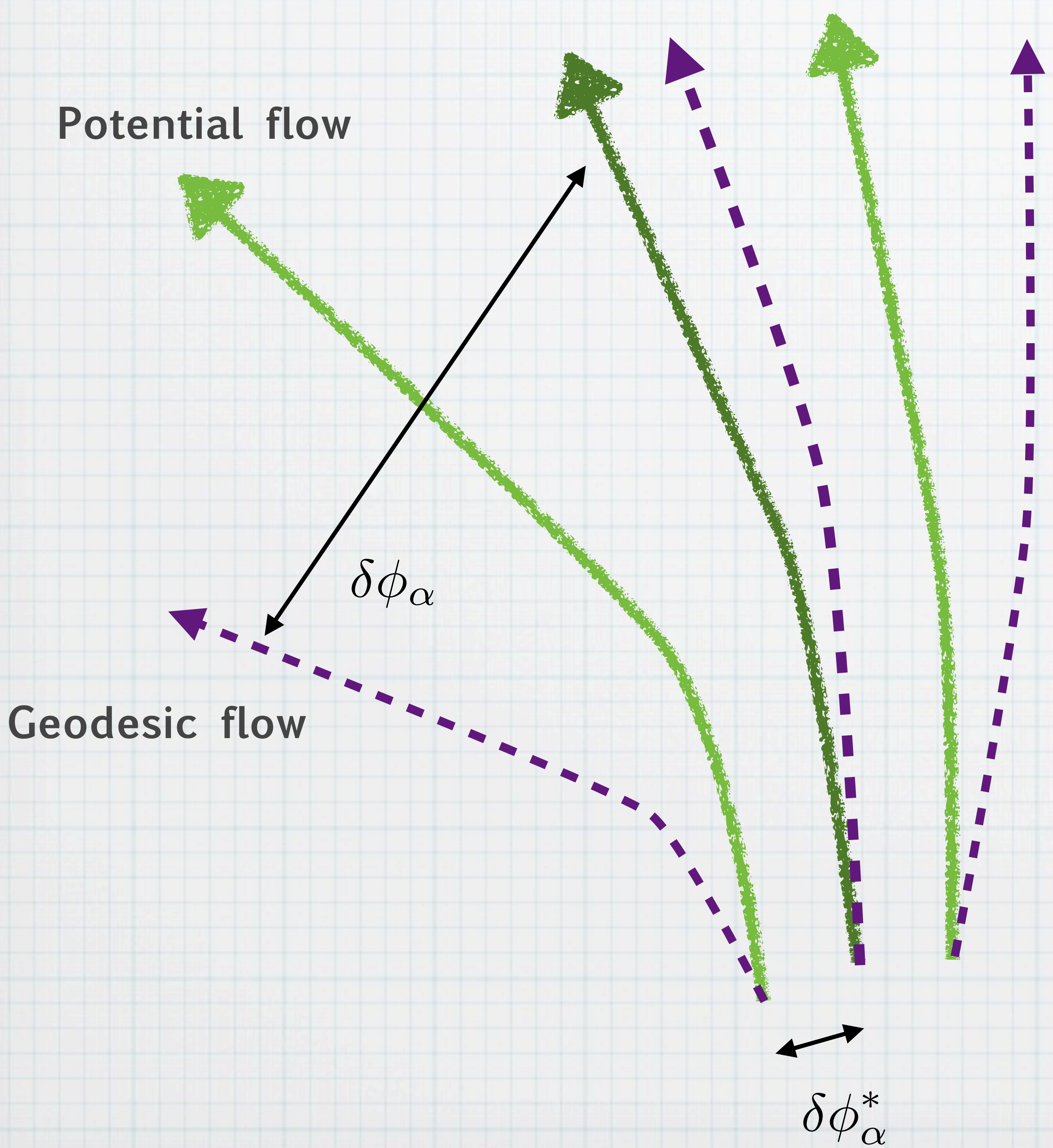

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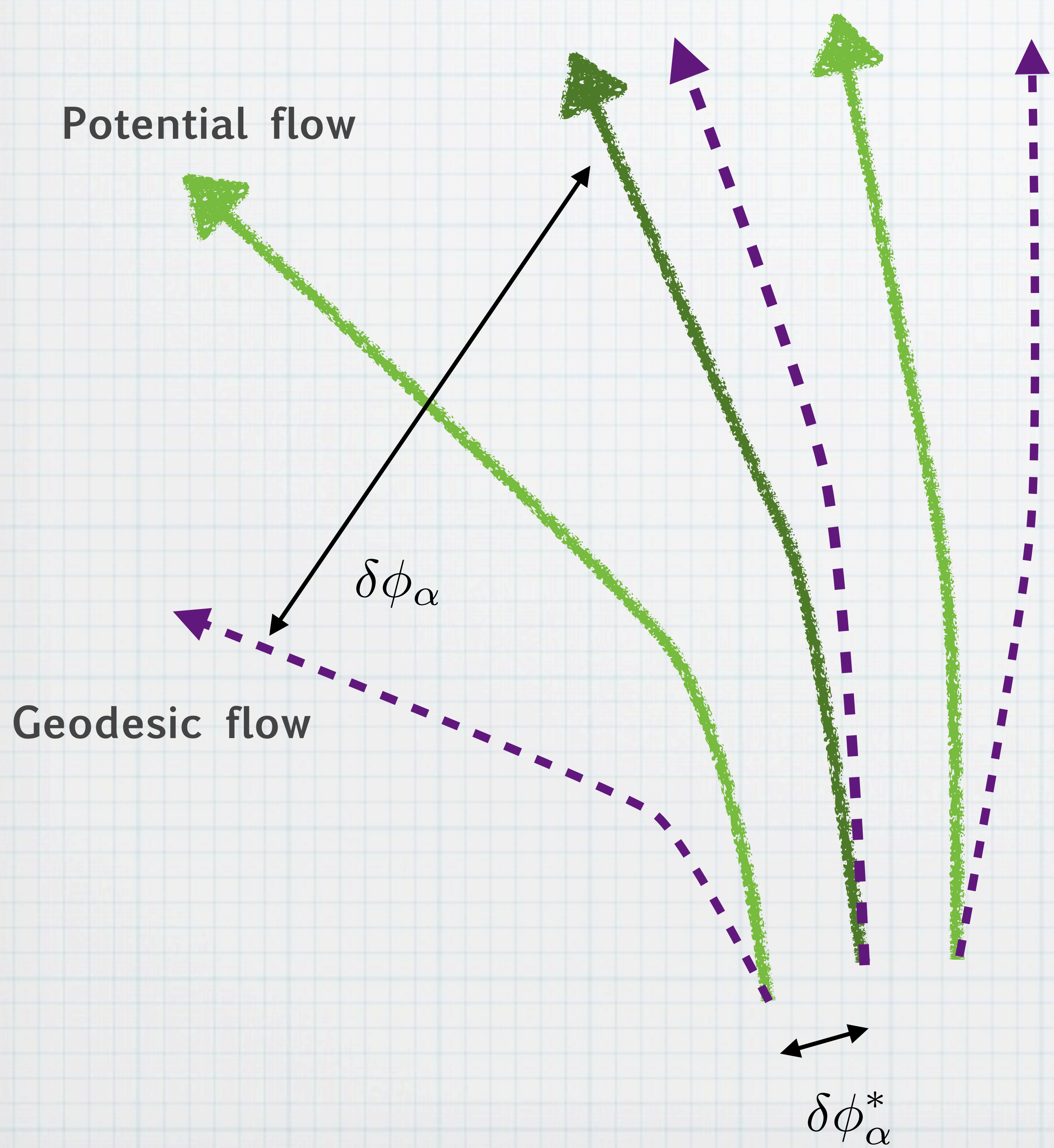

Each trajectory is a solution of

$$\frac{1}{3}\frac{\mathrm{D}^2\phi^\alpha}{\mathrm{d}N^2} + \frac{\mathrm{D}\phi^\alpha}{\mathrm{d}N} + \frac{\boldsymbol{G}^{\alpha\beta}V_{,\beta}}{3H^2} = 0$$

Each infinitesimal connecting vector is a solution of

$$\delta\left\{\frac{1}{3}\frac{\mathrm{D}^2\phi^\alpha}{\mathrm{d}N^2} + \frac{\mathrm{D}\phi^\alpha}{\mathrm{d}N} + \frac{\boldsymbol{G}^{\alpha\beta}V_{,\beta}}{3H^2}\right\} = 0$$

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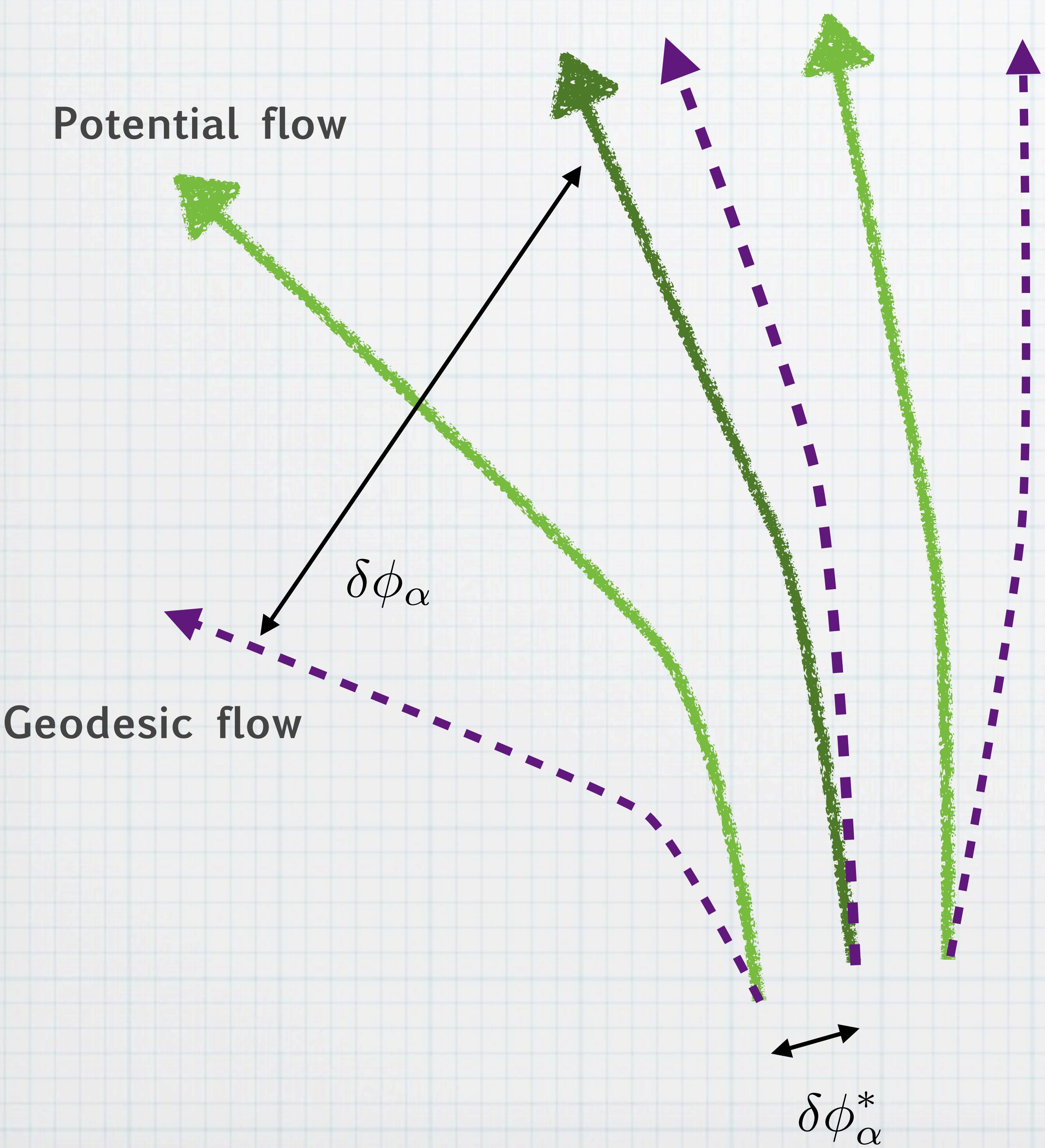

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$$\frac{1}{3}\mathbf{R}^\alpha{}_{\lambda\mu\nu}\frac{\mathrm{d}\phi^\lambda}{\mathrm{d}N}\frac{\mathrm{d}\phi^\mu}{\mathrm{d}N}\delta\phi^\nu$$

We get relatively simple evolution equations which account for the geodesic deviation effect

$$\frac{\mathrm{D}\Sigma^{\alpha\beta}}{\mathrm{d}N} = \boldsymbol{w}^{\alpha}{}_{\gamma}\Sigma^{\gamma\beta} + \boldsymbol{w}^{\beta}{}_{\gamma}\Sigma^{\gamma\alpha}$$

$$\frac{\mathrm{D}\alpha_{\alpha|\beta}}{\mathrm{d}N} = \boldsymbol{w}_{\alpha}{}^{\lambda}a_{\lambda|\beta\gamma} + \boldsymbol{w}_{\beta}{}^{\lambda}a_{\alpha|\lambda\gamma} + \boldsymbol{w}_{\gamma}{}^{\lambda}a_{\alpha|\beta\lambda} + \boldsymbol{w}_{\alpha}{}^{\lambda\mu}\Sigma_{\lambda\beta}\Sigma_{\mu\gamma}$$

where

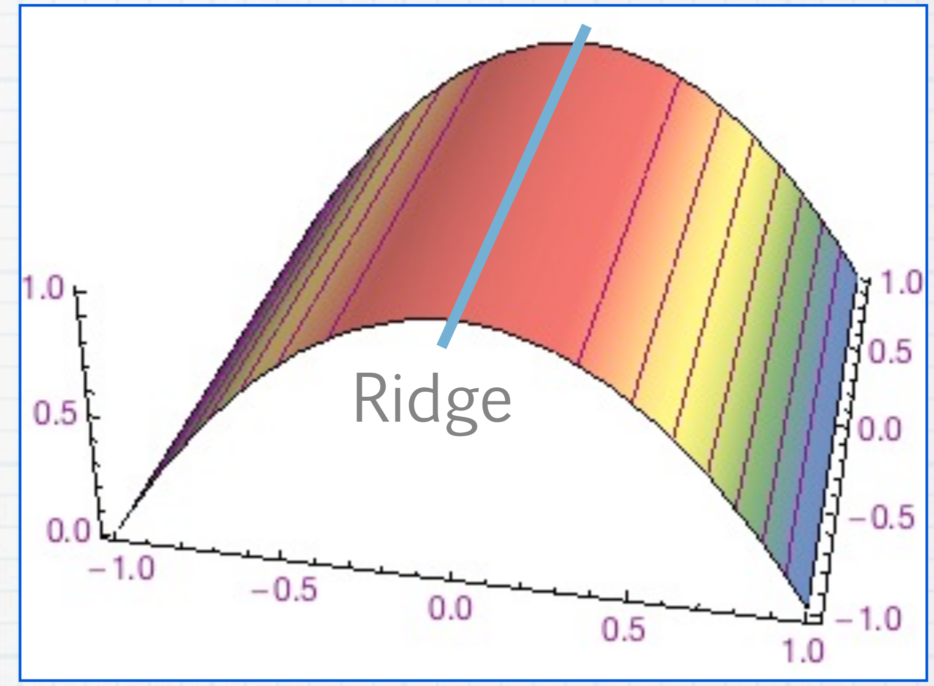
$$\langle\delta\phi_{\alpha}(\boldsymbol{k}_1)\delta\phi_{\beta}(\boldsymbol{k}_2)\delta\phi_{\gamma}(\boldsymbol{k}_3)\rangle \sim \frac{a_{\alpha|\beta\gamma}}{k_2^3k_3^3} + \frac{a_{\beta|\alpha\gamma}}{k_1^3k_3^3} + \frac{a_{\gamma|\alpha\beta}}{k_1^3k_2^3}$$

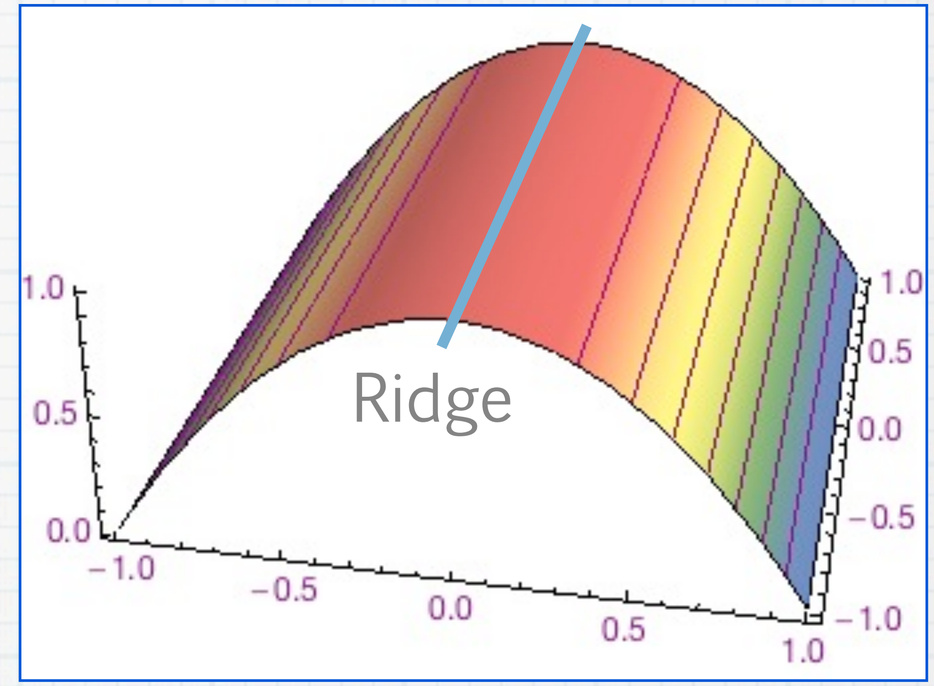
$$\boldsymbol{w}_{\alpha\beta} = -\frac{V_{\alpha\beta}}{3H^2} + \frac{1}{3H^2}\frac{1}{a^3}\frac{\mathrm{D}}{\mathrm{d}t}\left(\frac{a^3}{H}\dot{\phi}_{\alpha}\dot{\phi}_{\beta}\right) + \frac{1}{3}\mathbf{R}_{\alpha\lambda\mu\beta}\frac{\dot{\phi}^{\lambda}}{H}\frac{\dot{\phi}^{\mu}}{H}$$

$$\boldsymbol{w}_{\alpha\beta\gamma} = \nabla_{(\alpha}\boldsymbol{w}_{\beta\gamma)} + \frac{1}{3}\left(\nabla_{(\alpha}\mathbf{R}_{\beta|\lambda\mu|\gamma)}\frac{\dot{\phi}^{\lambda}}{H}\frac{\dot{\phi}^{\mu}}{H} - 4\mathbf{R}_{\alpha(\beta\gamma)\lambda}\frac{\dot{\phi}^{\lambda}}{H}\right)$$

Elliston, DS & Tavakol arXiv:1208.6011

Ridge

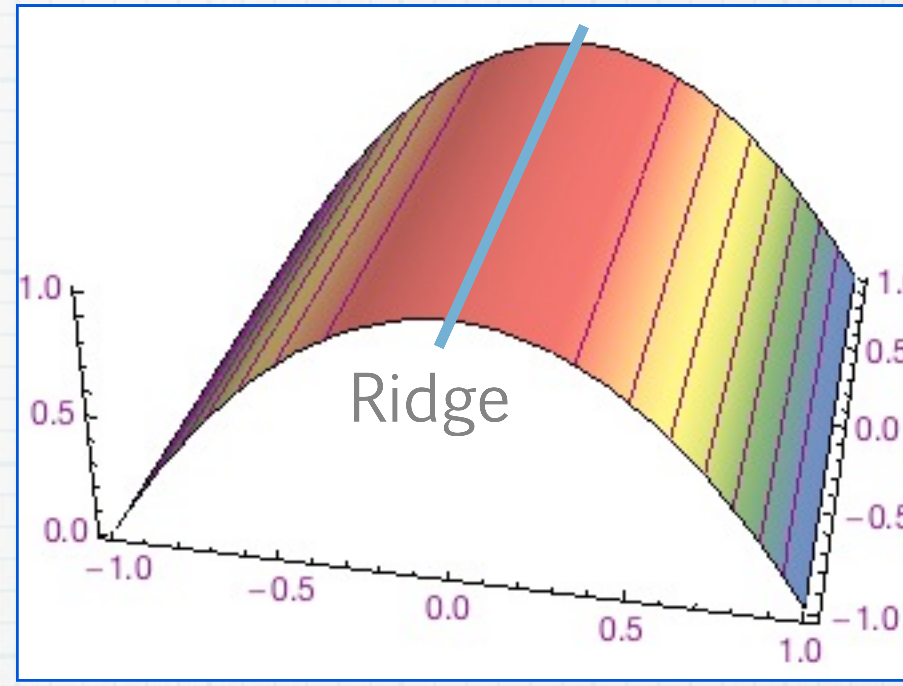


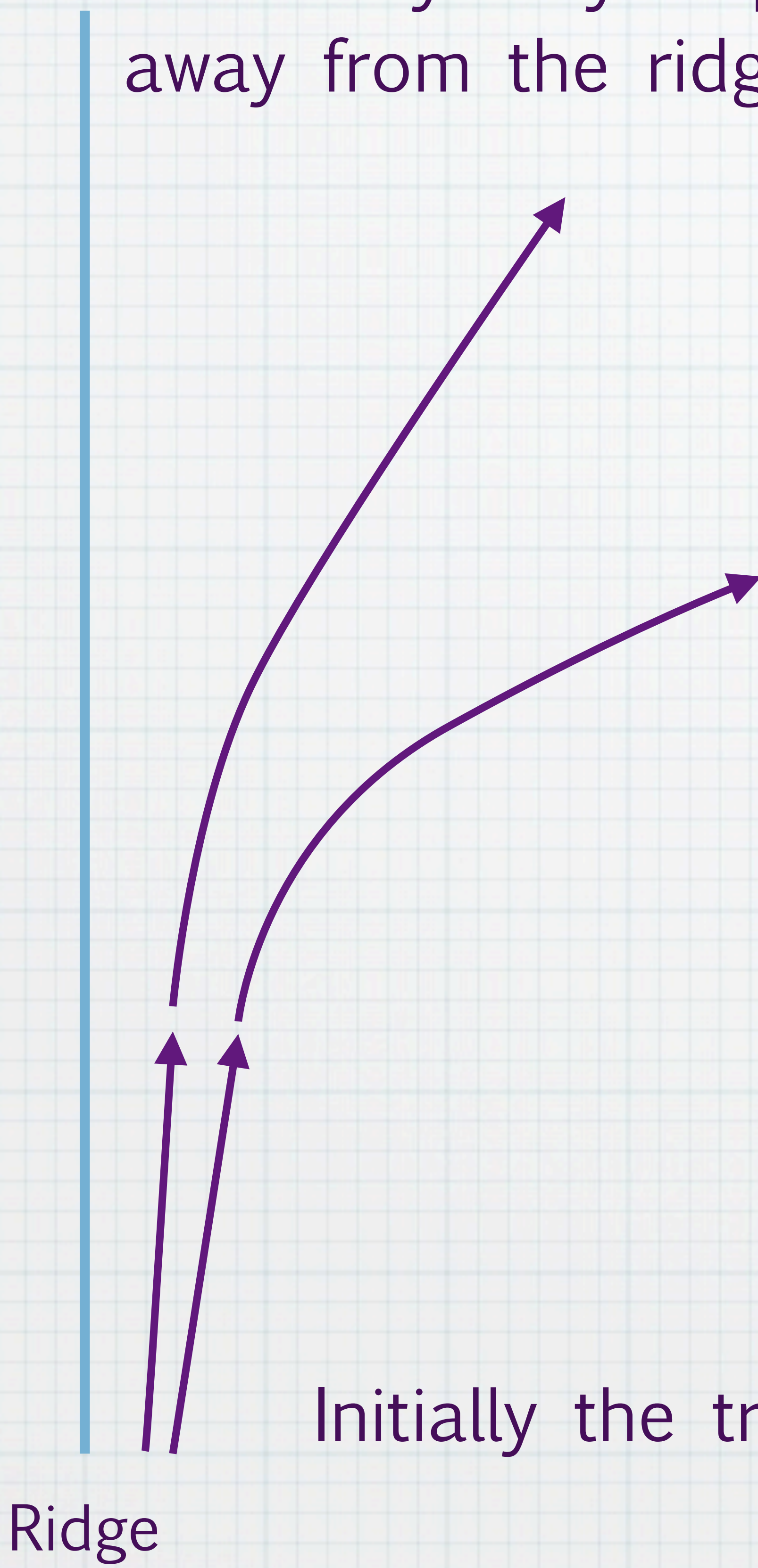


Initially the trajectories keep close to each other

Ridge

Eventually they disperse *nonlinearly* away from the ridge





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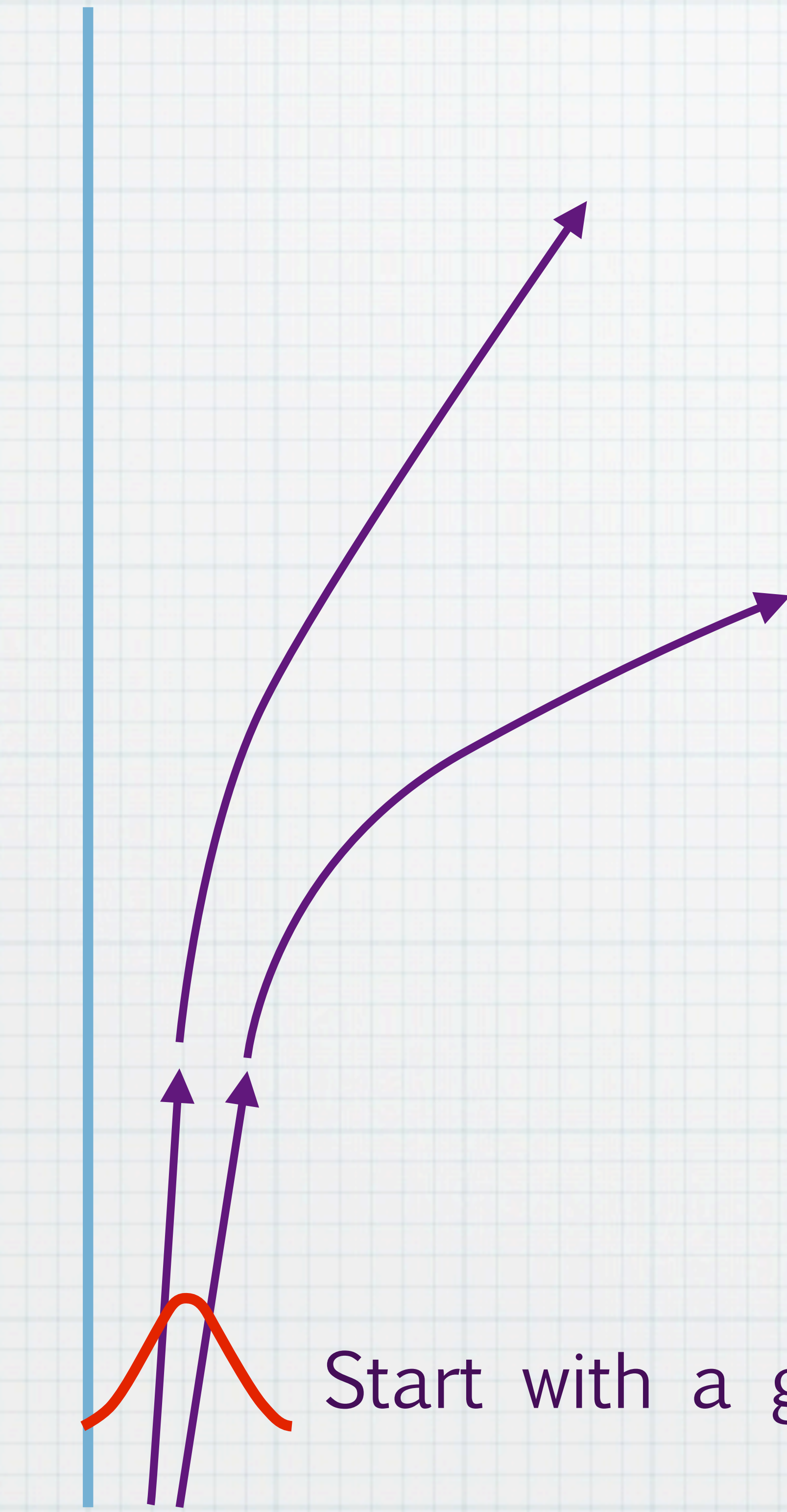
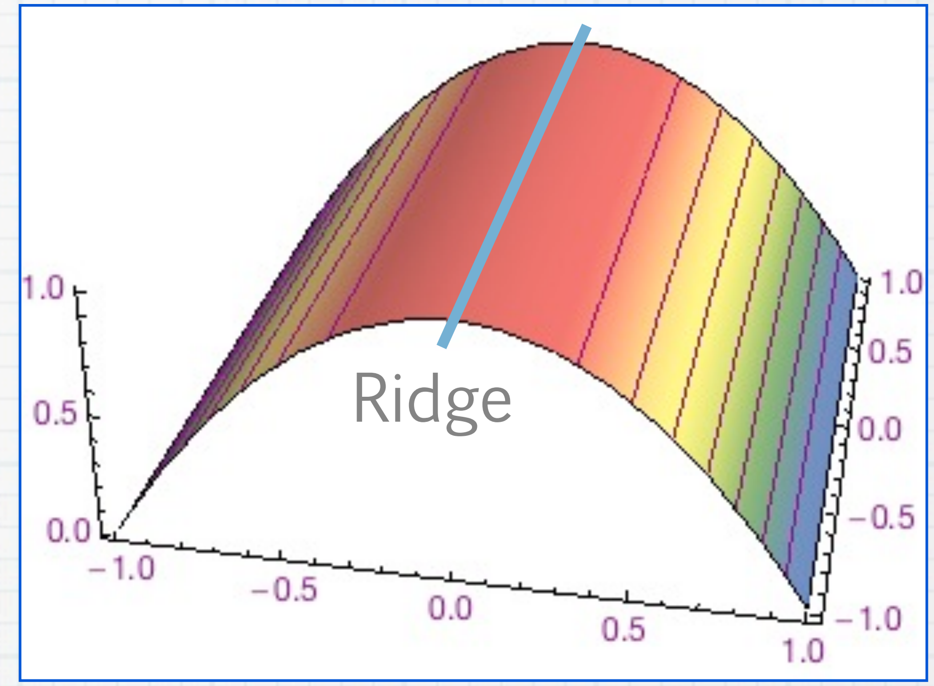
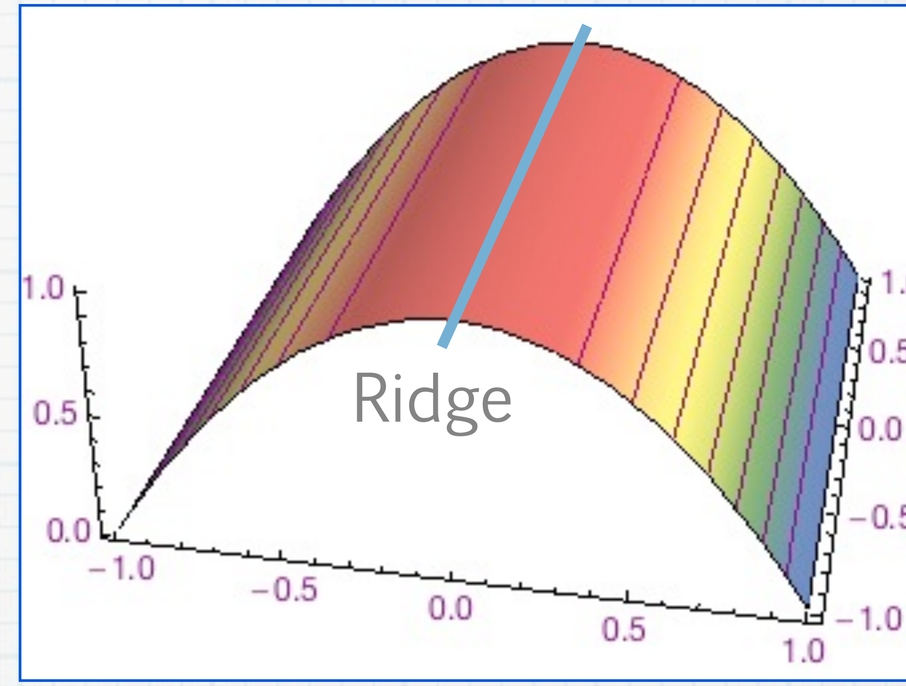


Start with a gaussian distribution

Ridge

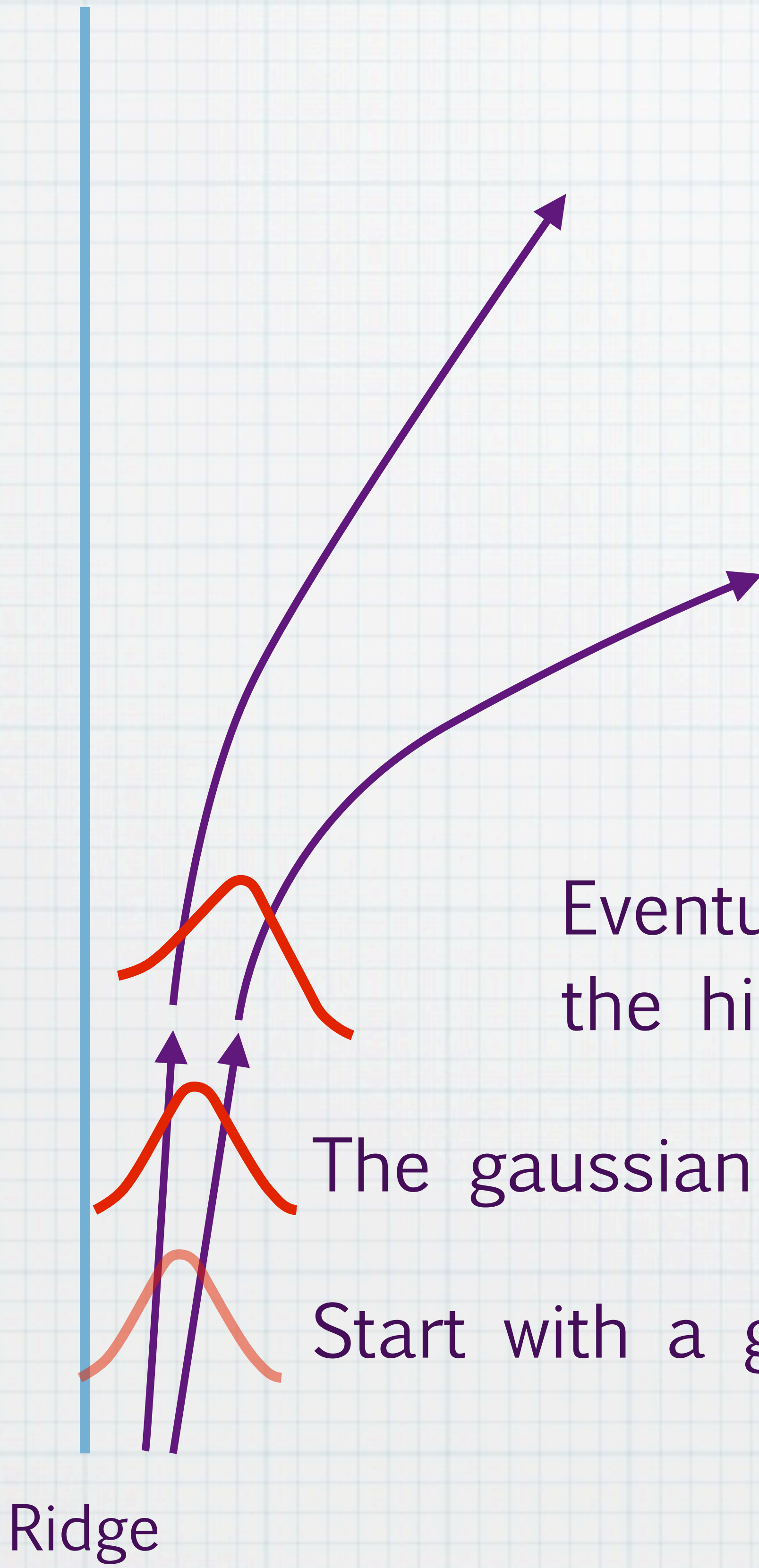


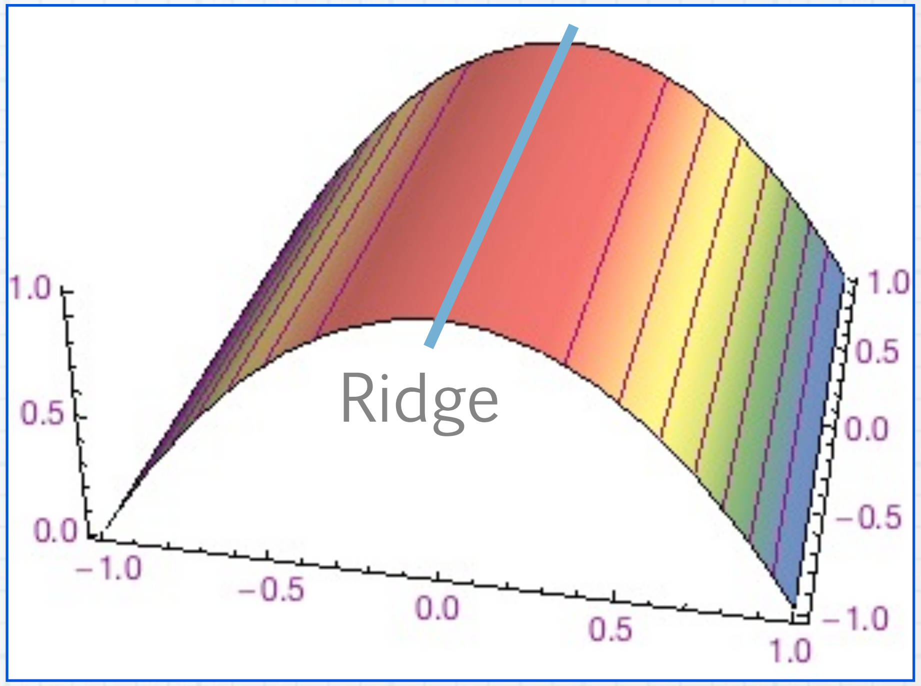


Eventually a few trajectories slide away down the hillside, generating a **heavy tail**

The gaussian is preserved in the early phases

Start with a gaussian distribution





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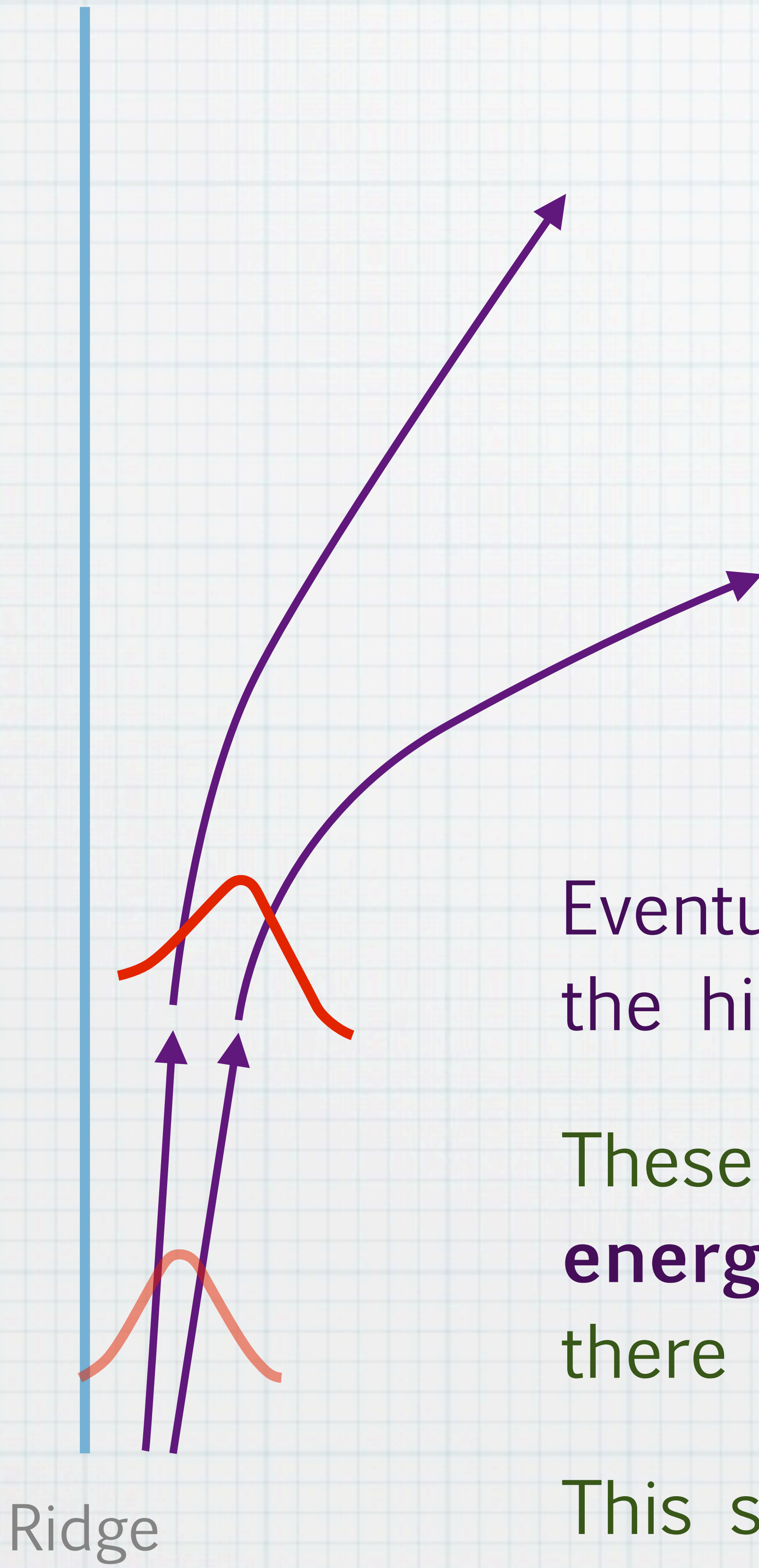
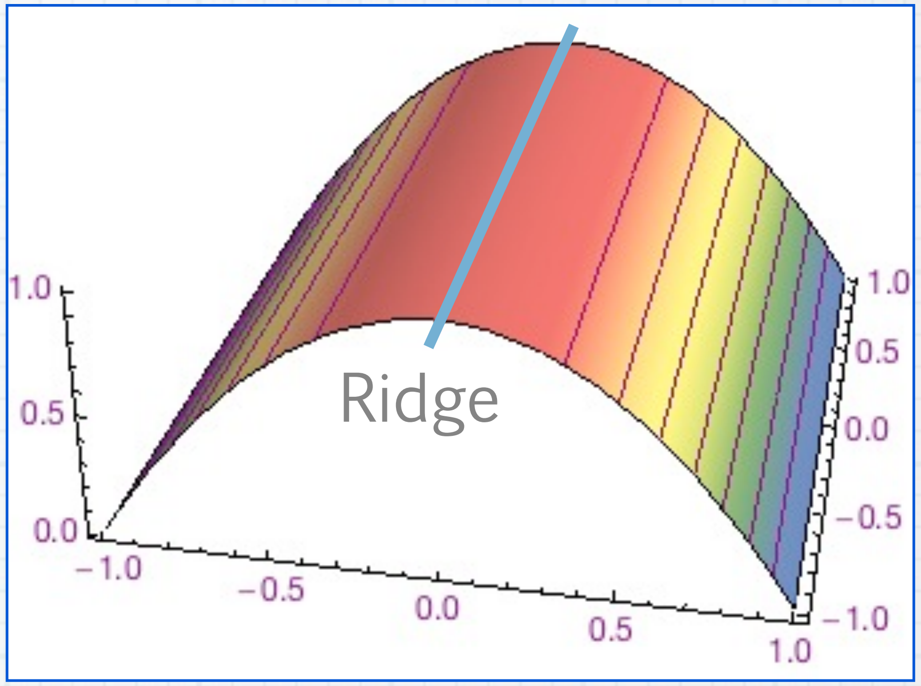
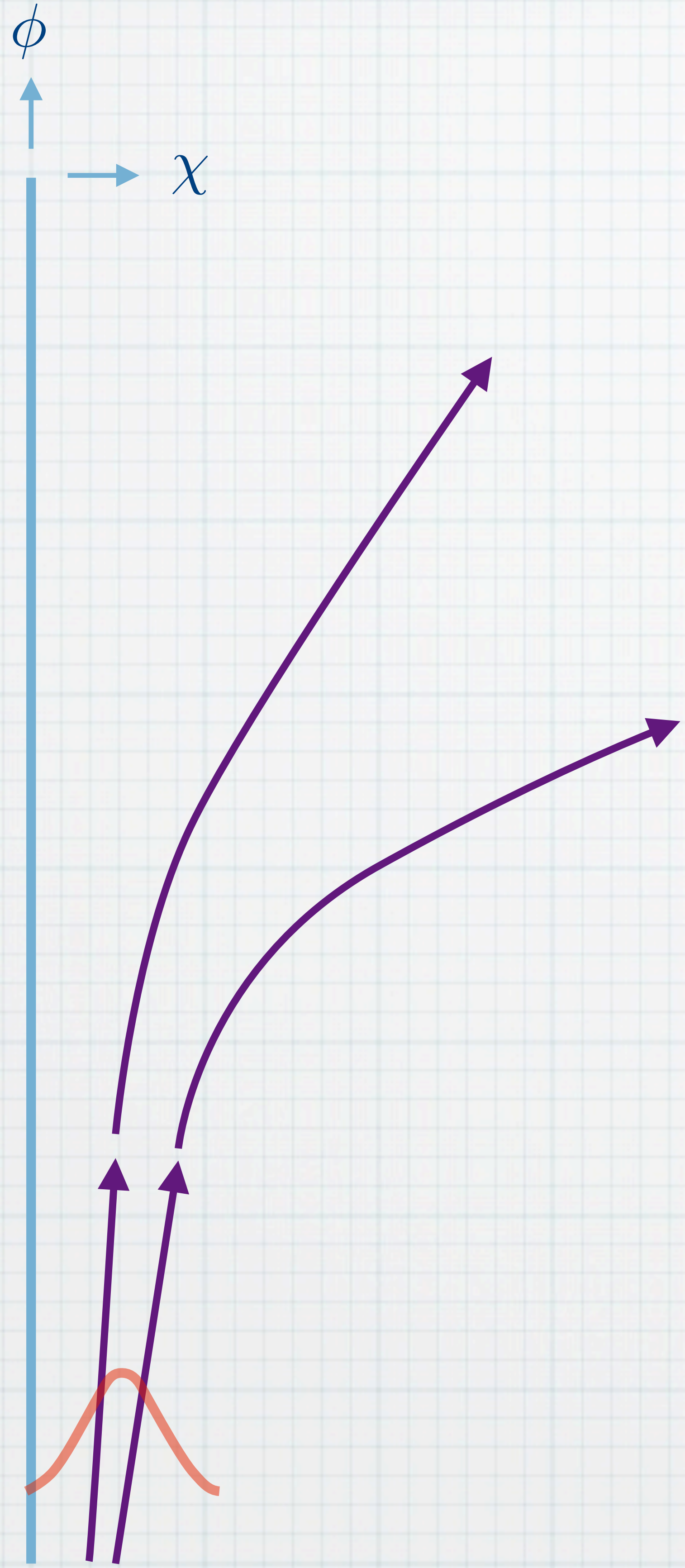


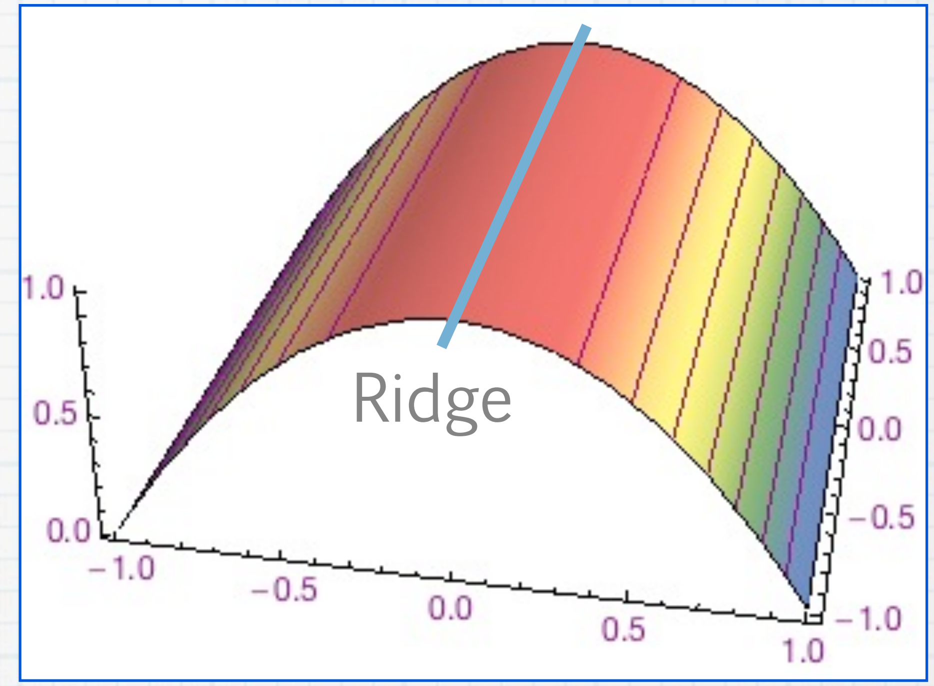


Eventually a few trajectories slide away down the hillside, generating a **heavy tail**

These are all excursions to **larger kinetic energy, smaller potential energy.** Hence, there is **less expansion**, so **negative** δN

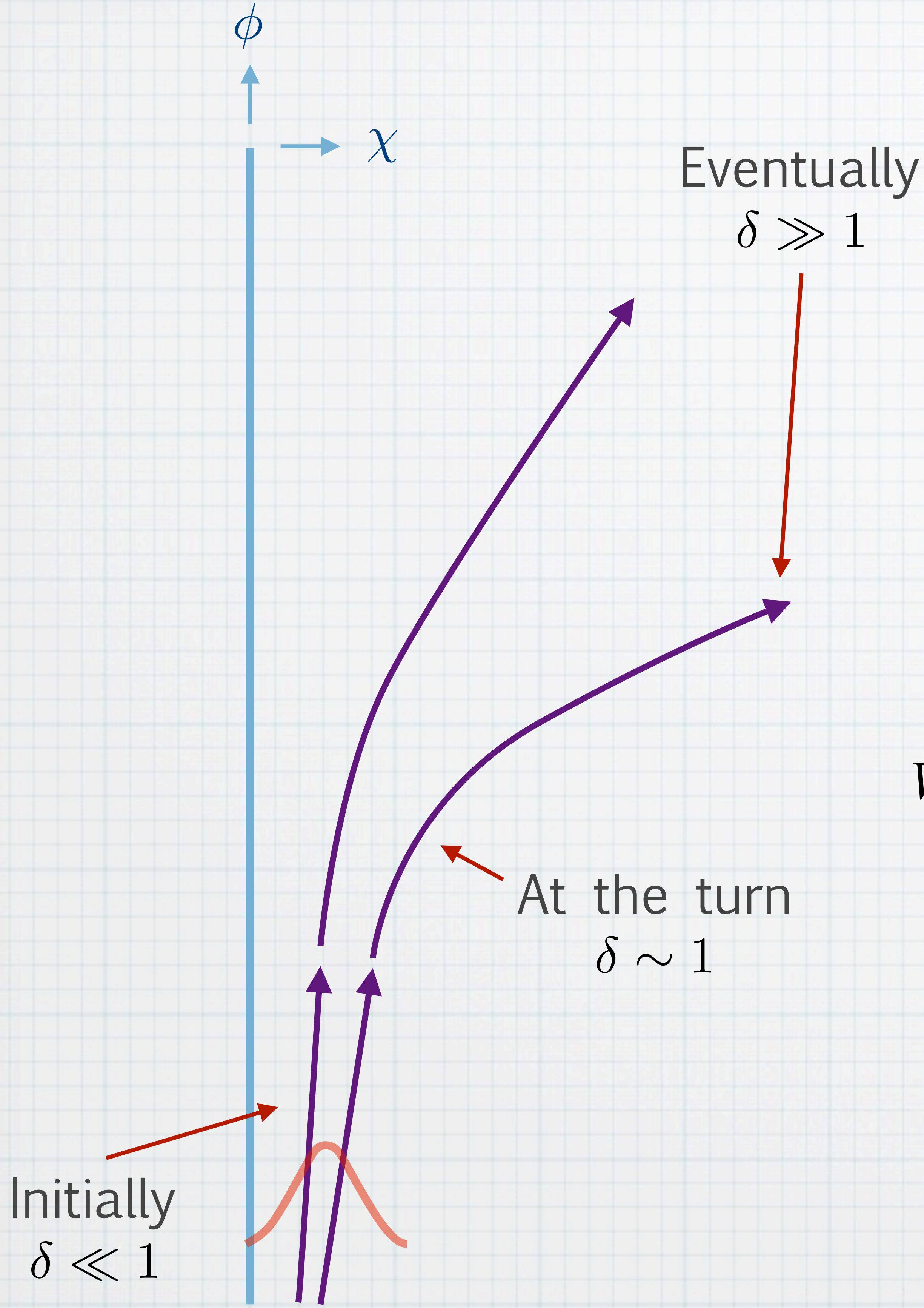
This skews the distribution to negative δN, so gives **negative** f_{NL}

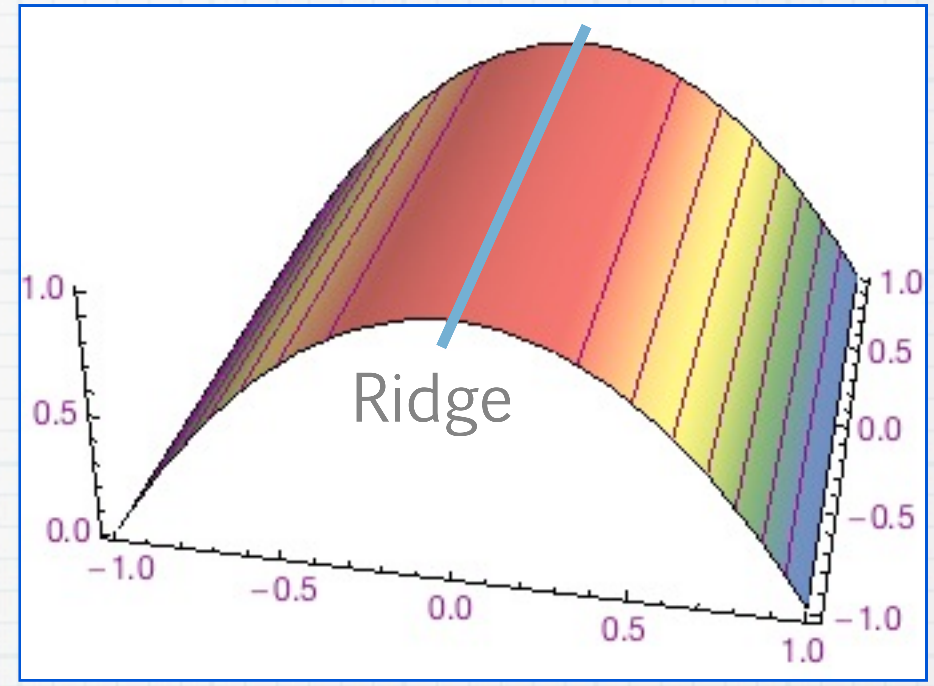




$$W = W_0 + g_0(\phi - \phi_0) - \frac{1}{2}m_\chi^2\chi^2$$

Define $\delta = \dfrac{\dot{\chi}}{\dot{\phi}} = m_\chi^2 \left| \dfrac{\chi}{g_0} \right|$





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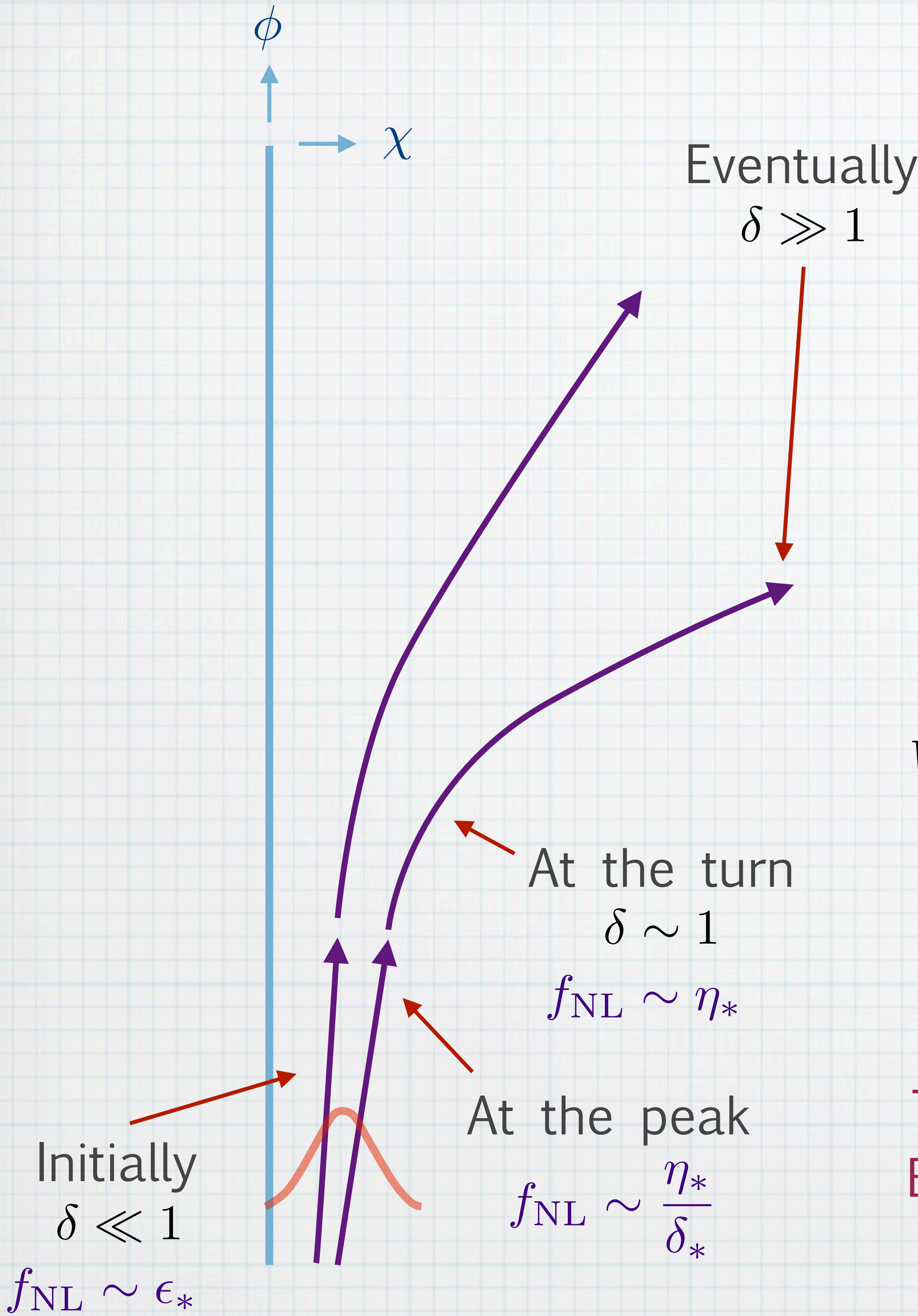

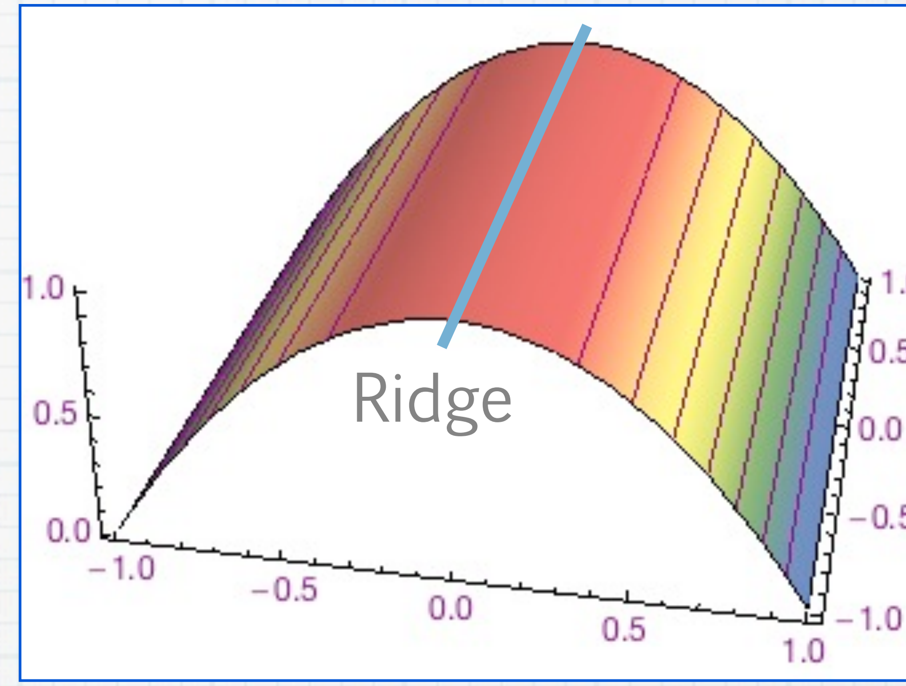

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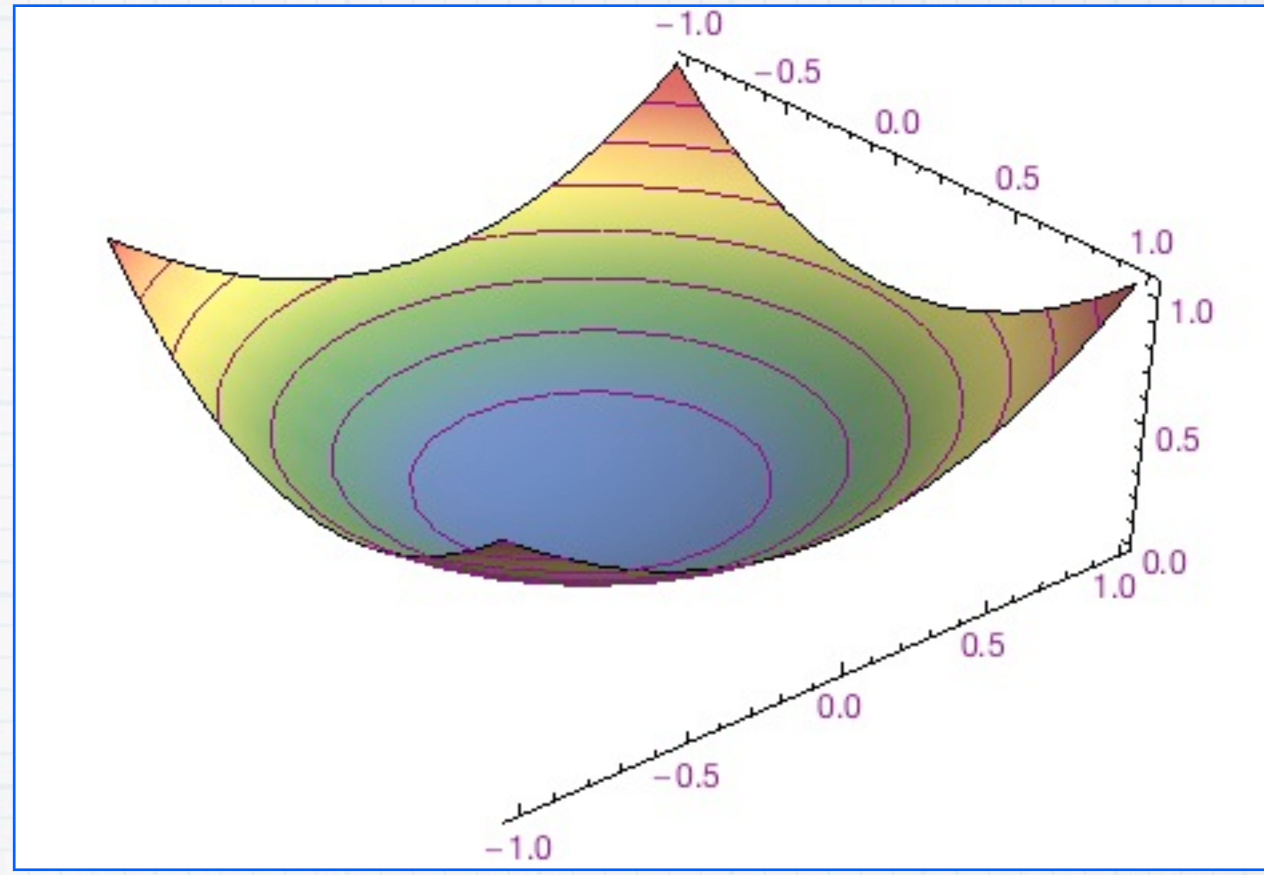
The peak $f_{\rm NL}$ occurs *before* the turn. By the turn $f_{\rm NL}$ has decayed back to a small value

Something similar happens when converging into a valley



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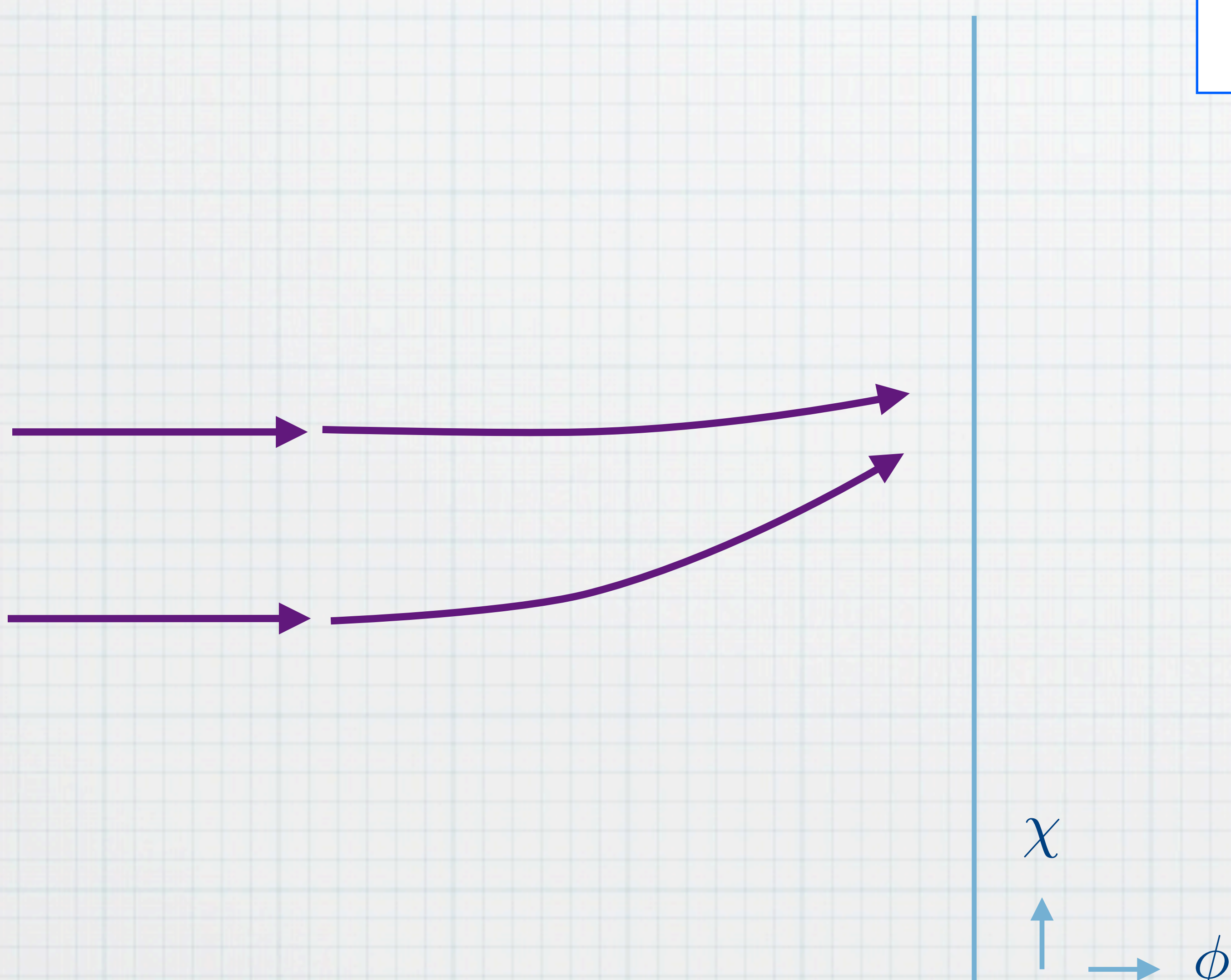

Direction of valley floor

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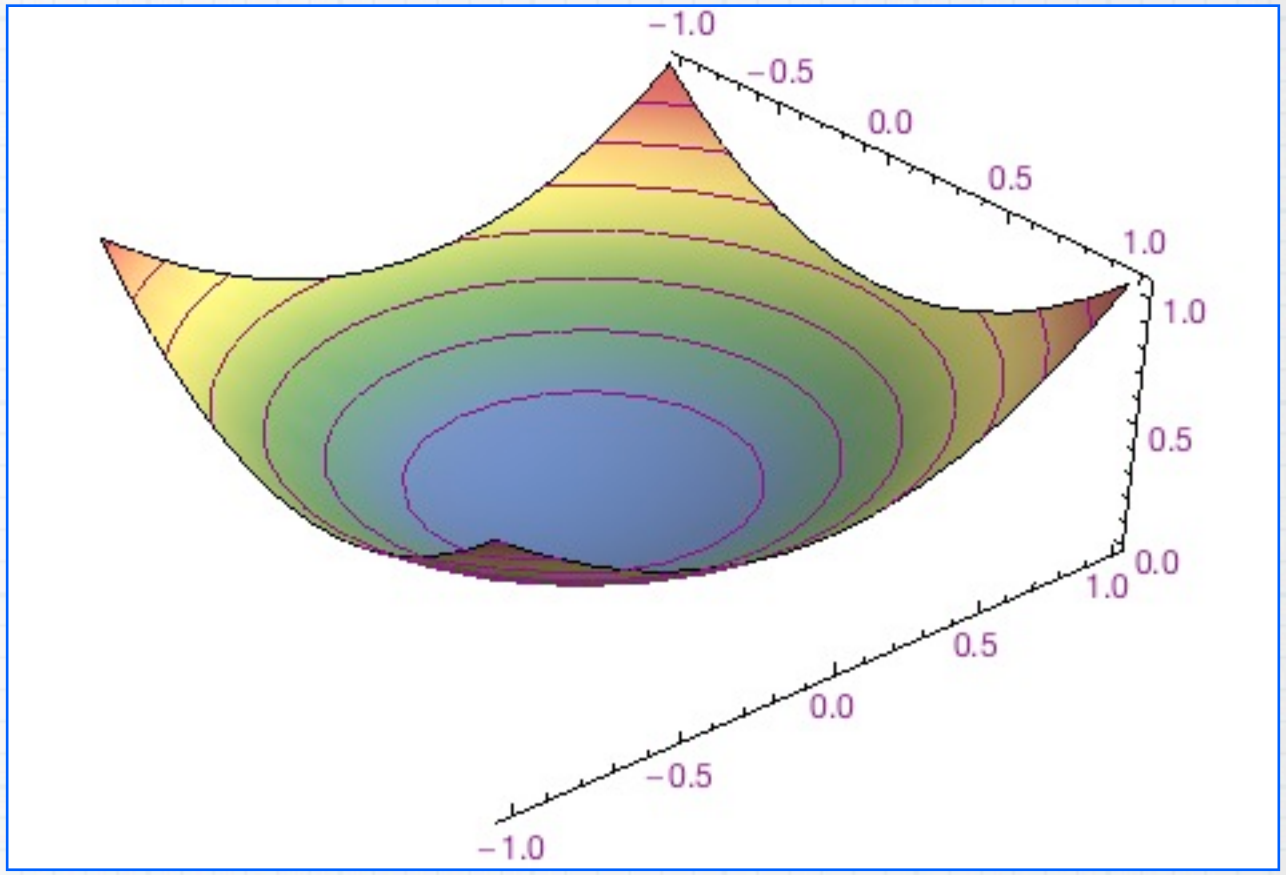

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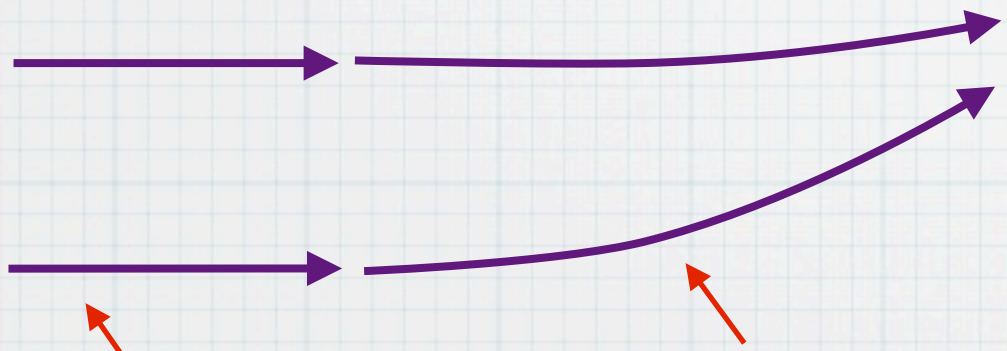

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At the turn $\delta \sim 1$

χ

ϕ

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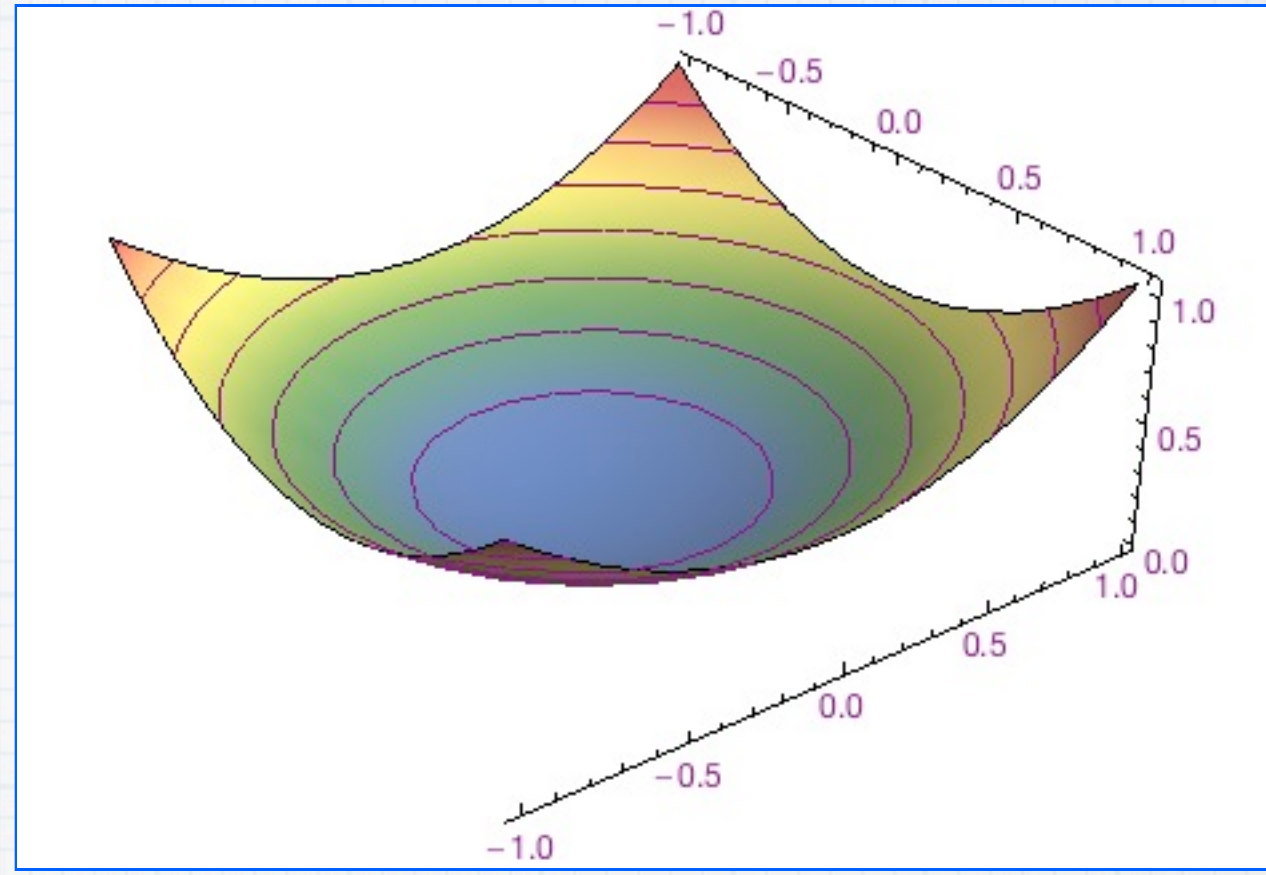

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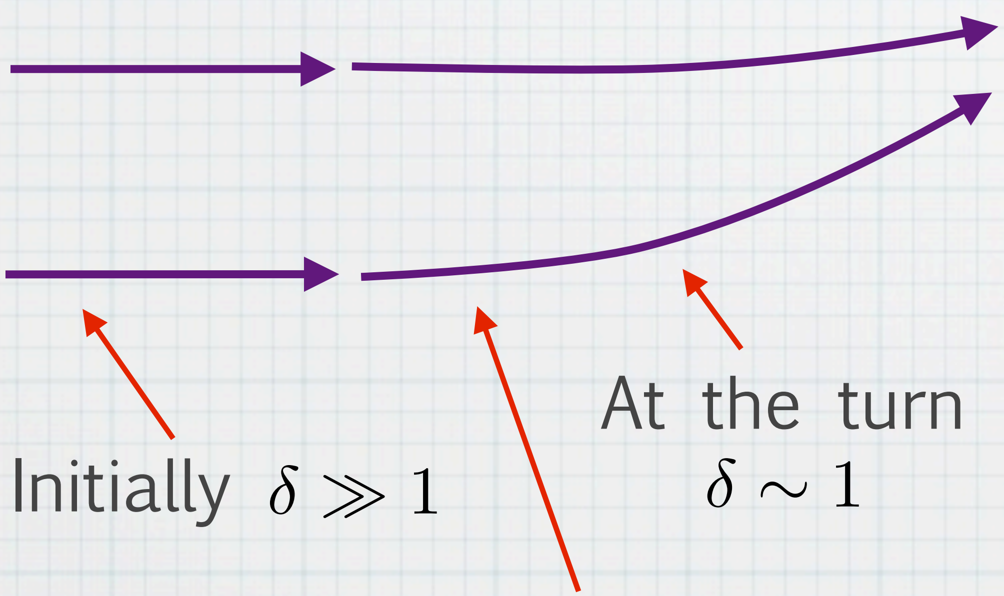

Initially $\delta \gg 1$

At the turn $\delta \sim 1$

At the peak $f_{\rm NL} \sim \eta_* \delta_*$

The peak $f_{\rm NL}$ is achieved before the turn, as for the ridge.
What happens afterwards is model dependent. Either $f_{\rm NL}$ can decay, making a spike as before, or it can plateau.

χ

ϕ

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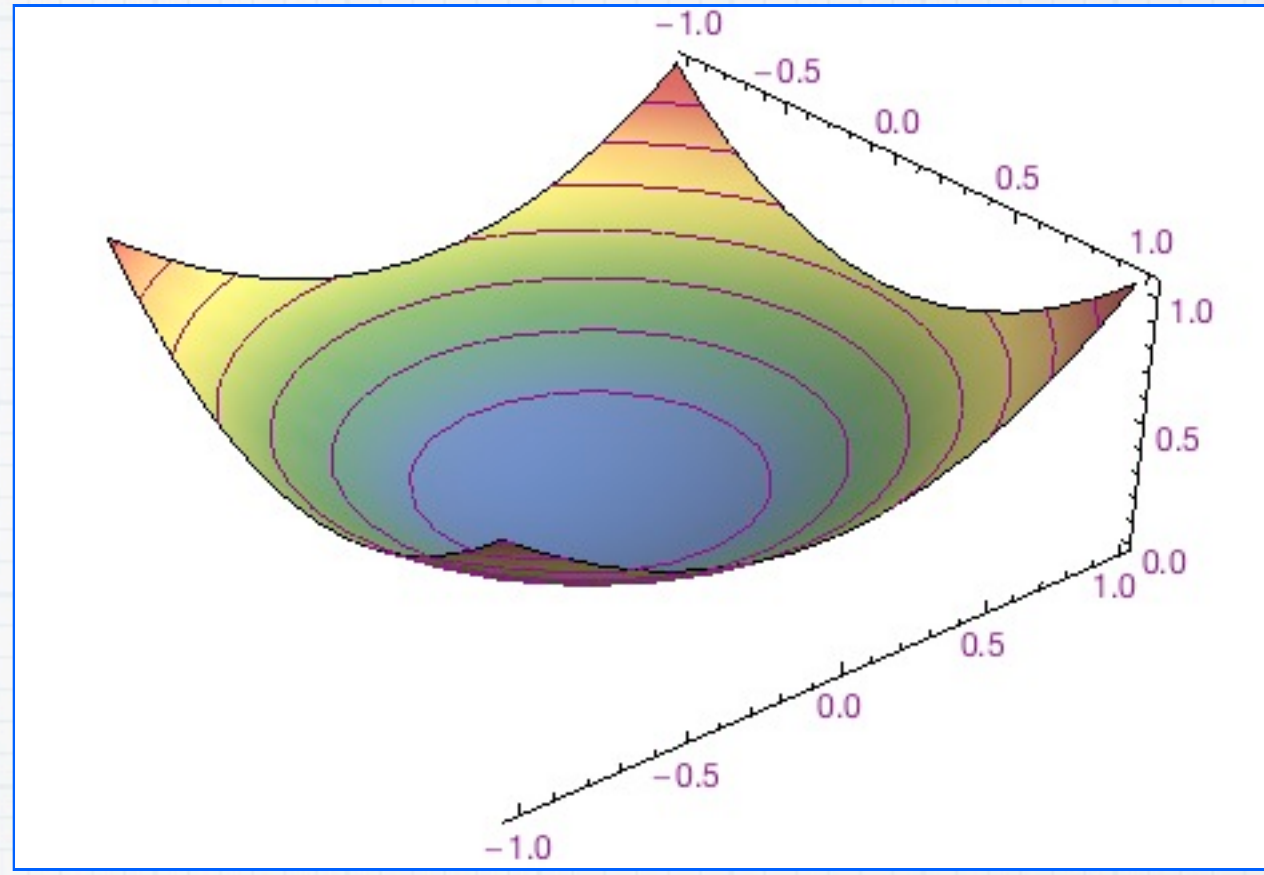
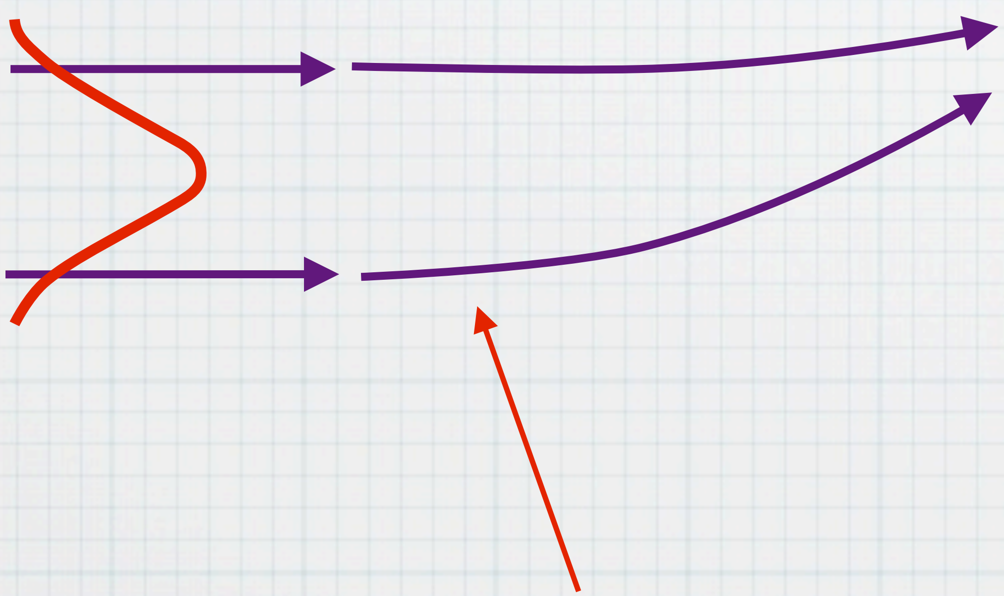




At the peak

$f_{\rm NL} \sim \eta_* \delta_*$

This time, the "uphill" edge of the bundle is compressed towards the centre, which again generates a heavy tail on the "downhill" side.

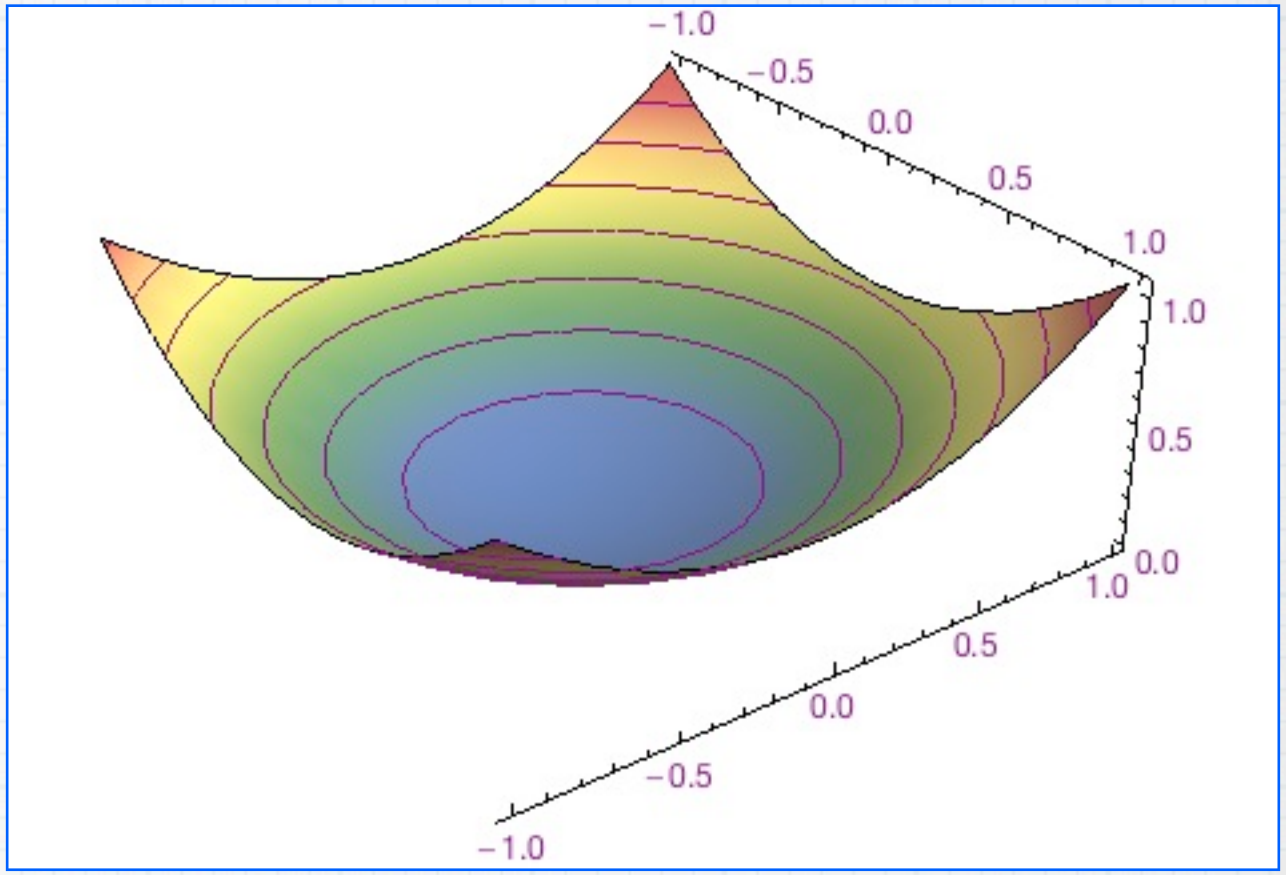
This enhances excursions to *positive* δN, giving positive $f_{\rm NL}$.

χ

ϕ

Direction of valley floor

Something similar happens when converging into a valley



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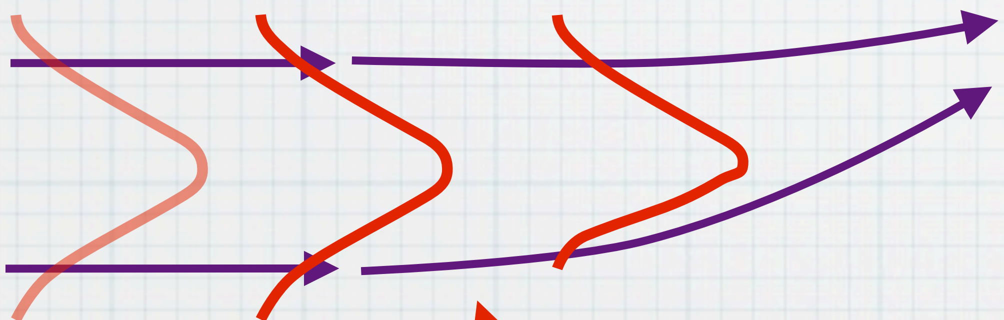
In both cases, $f_{\rm NL}$ inherits its sign from a local η parameter, enhanced by a large dimensionless factor



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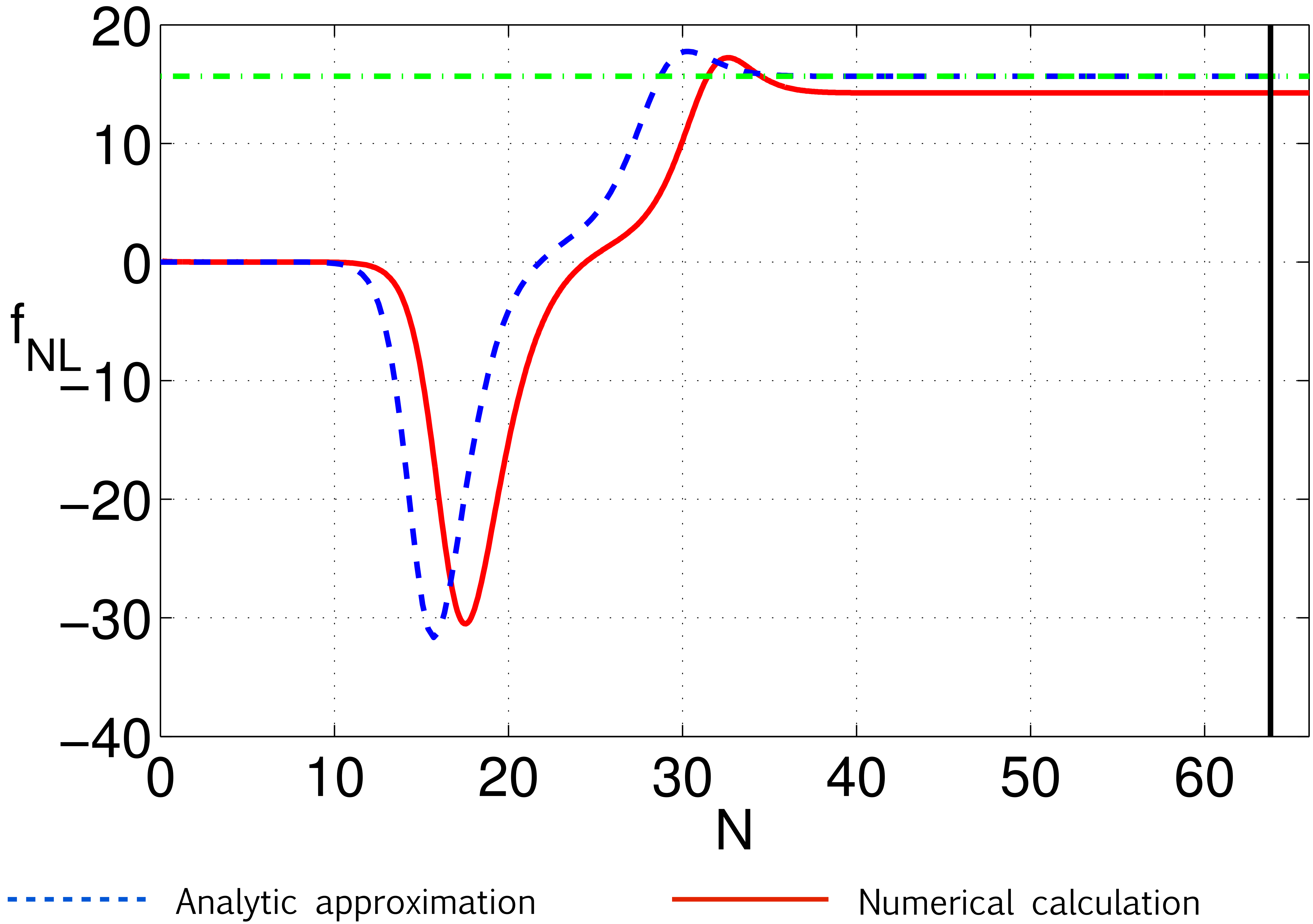
$$V = \frac{1}{2}m^2\phi^2 + \Lambda^4\left(1 - \cos\frac{2\pi\chi}{f}\right)$$



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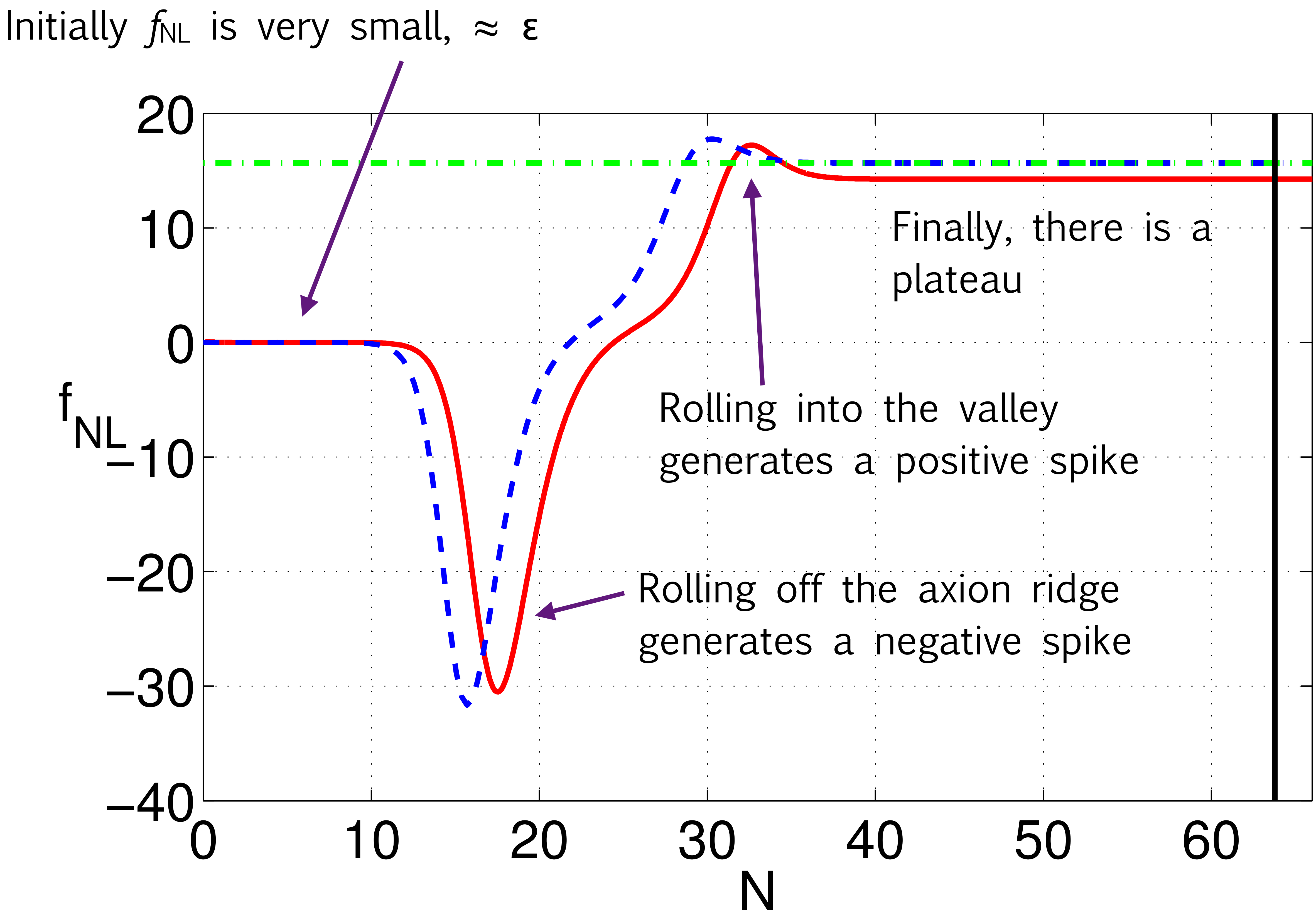

Analytic approximation   Numerical calculation

Begin with an axion potential $V = \Lambda^4 \left(1 - \cos\frac{2\pi\phi}{f}\right)$

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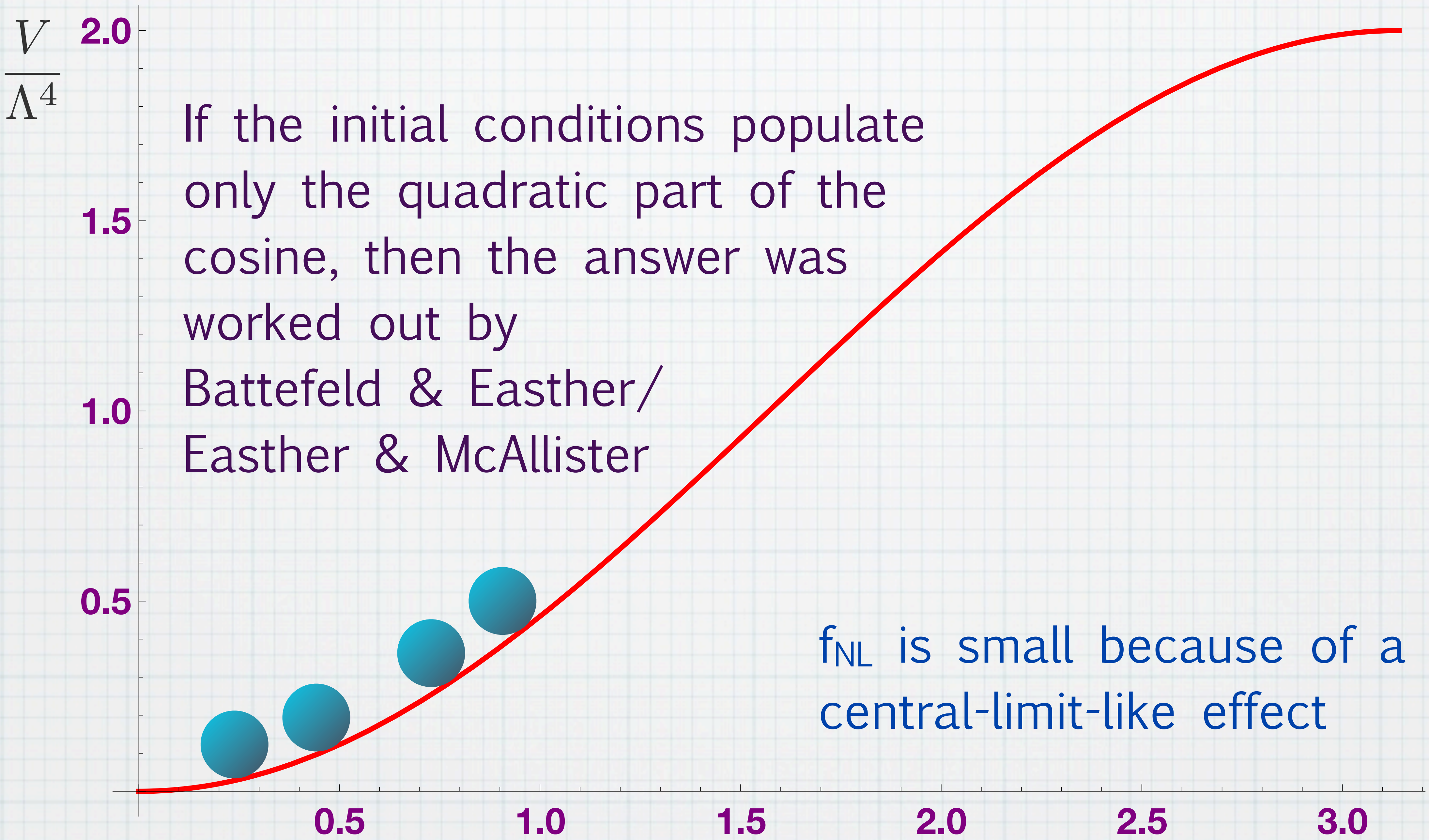

If the initial conditions populate only the quadratic part of the cosine, then the answer was worked out by Battefeld & Easther/ Easther & McAllister

f_{NL} is small because of a central-limit-like effect

Alabidi & Lyth (2006), Kim & Liddle (2006), Battefeld & Easther (2006), Battefeld & Battefeld (2007)

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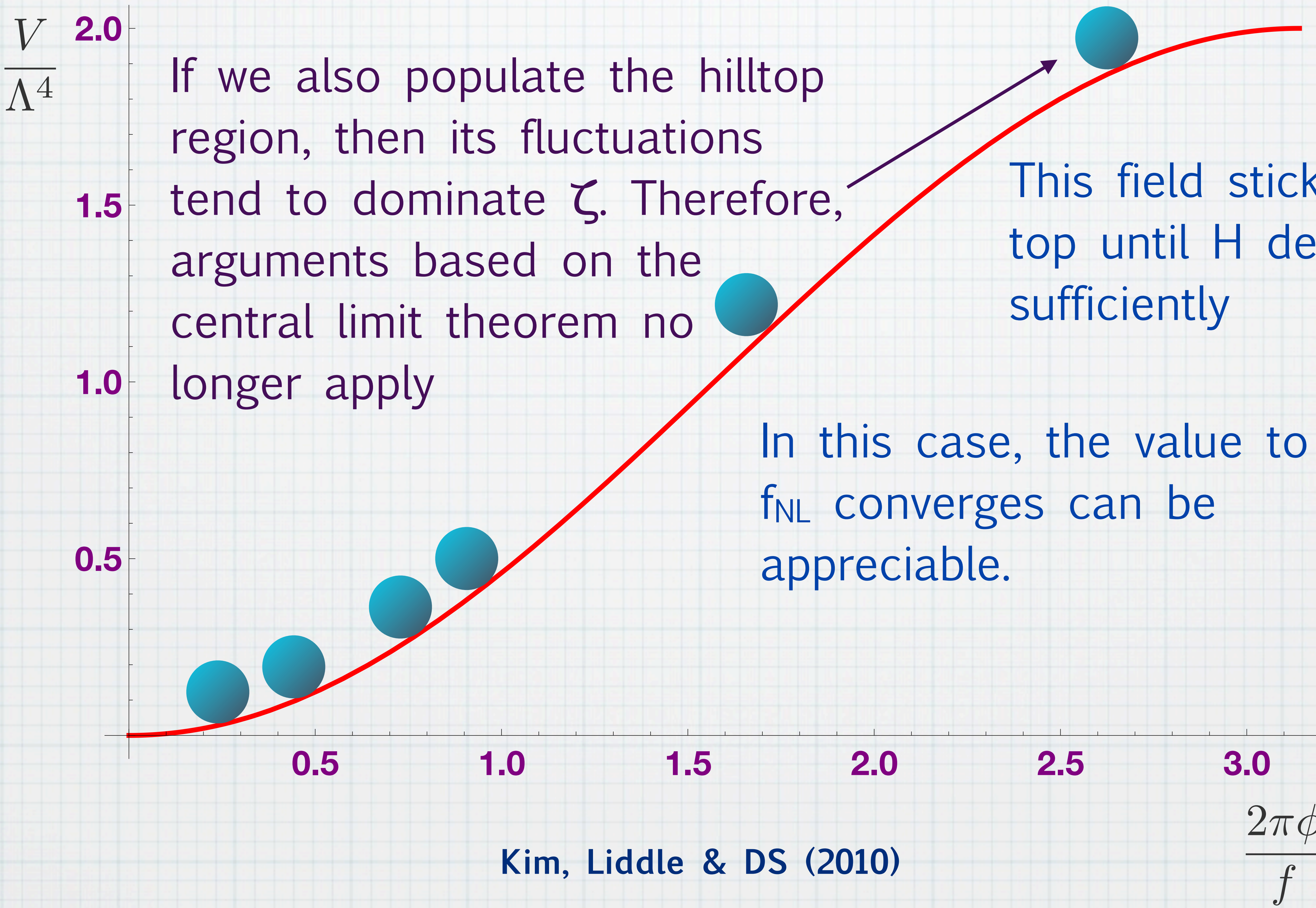

If we also populate the hilltop region, then its fluctuations tend to dominate ζ. Therefore, arguments based on the central limit theorem no longer apply

This field sticks at the top until H decays sufficiently

In this case, the value to which f_{NL} converges can be appreciable.

Kim, Liddle & DS (2010)

Now,

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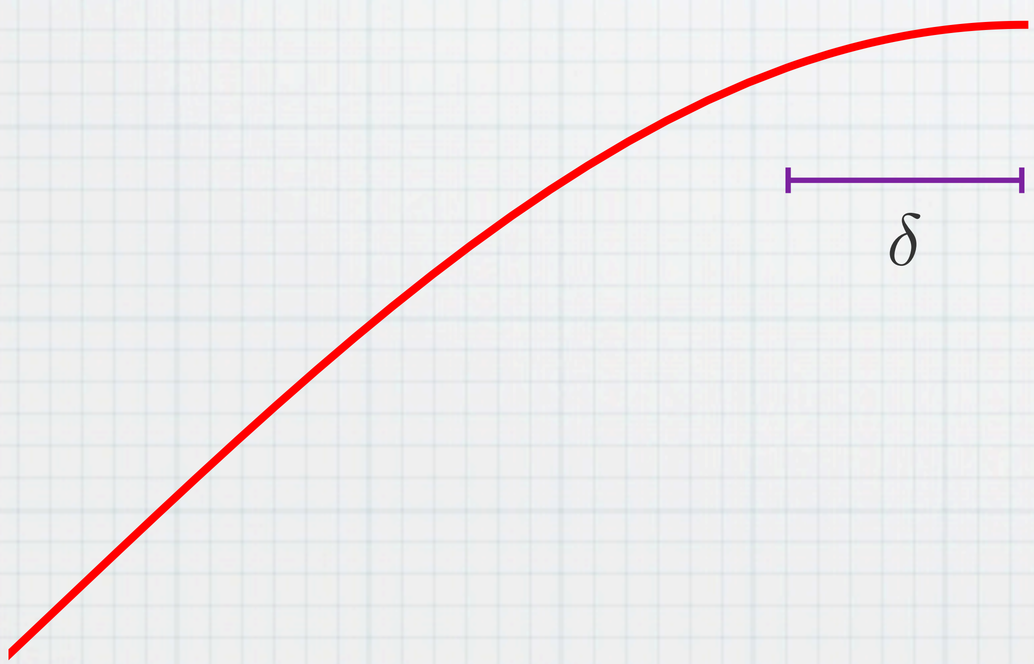
f(k_i) is a complicated function of the k_i with well-defined limits, finite everywhere

near the hilltop

$$V \approx 2\Lambda^4 \left(1 + \frac{\eta\delta}{2M_{\mathrm{P}}^2}\right)$$



It turns out that

$$\epsilon_* \approx 0$$

$$\eta* \approx -2\pi^2 \frac{M_{\mathrm{P}}^2}{f^2}$$

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≈ 20

So, when the attractor is reached and any f_{NL} generated by shear, divergence, focusing, etc., has decayed, f_{NL} asymptotes to a rather large number

Conclusions

- Can recover δN formula directly from the underlying quantum field theory.
  [Caveats: leading logarithm approximation; perturbative in mass]

- Naturally leads to an interpretation in terms of flows
  à la Callan-Symanzik equation

- Typical multiple field models generate nongaussianity through
  dispersion from a ridge
  focusing into a valley
  inheritance from a subdominant field [similar to curvaton]

- For inflation, these all seem to require some form of hierarchy in their initial conditions.