Second Order CMB Perturbations

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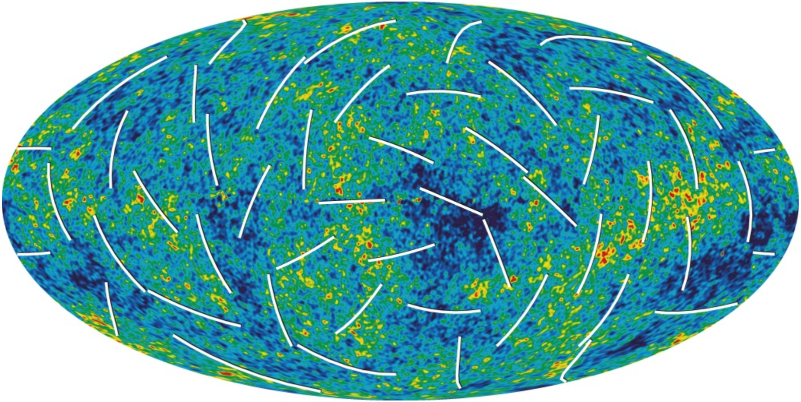
November 2012

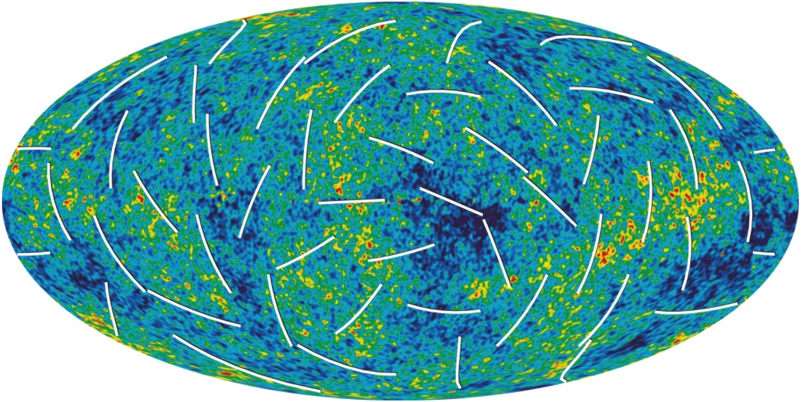
CMB anisotropies



[WMAP]

- Small anisotropies of order 10^{-5} induced from quantum fluctuations
  - ➔ Define cosmological perturbation theory as deviations from homogeneity
- Inflation constrained by power spectra of the CMB fluctuations

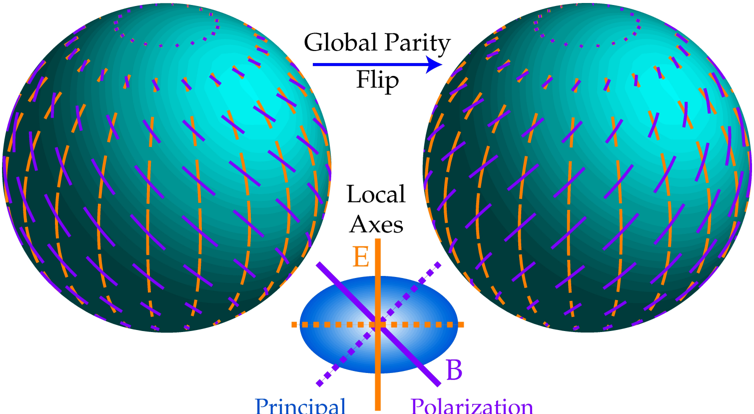
CMB anisotropies



[WMAP]

- Inflation also generates primordial gravity waves
- Tensor to scalar ratio linked to slow roll parameter
  - ➔ Polarisation can be used to measure the primordial tensor perturbations

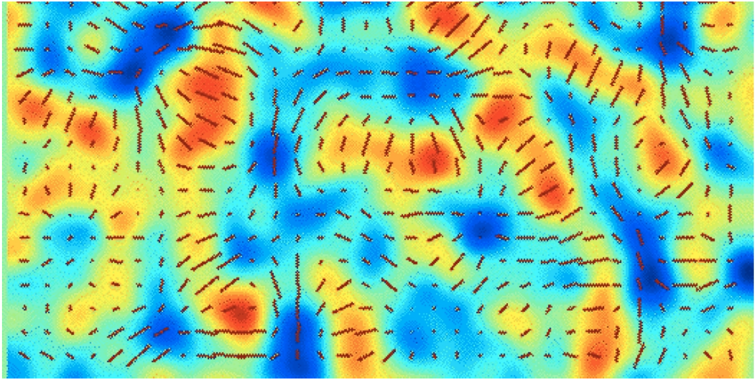
Parity of E and B polarisation



[Hu]

- B polarisation not induced by scalar sources
  - ➔ Can only be generated by gravity waves

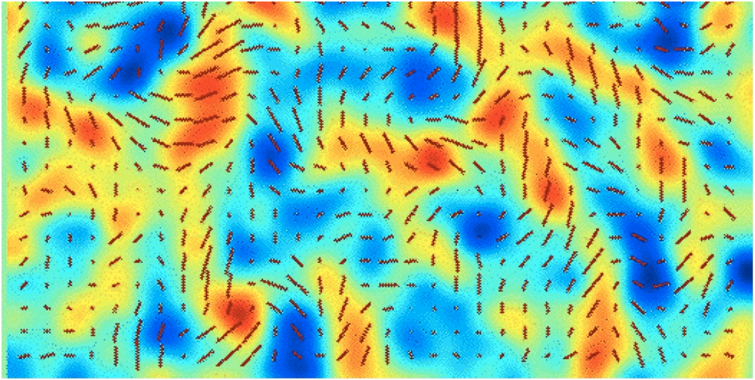
E and B polarisation



[Zaldarriaga]

- E polarisation: pure gradient field in analogy to electric field
  - ➔ Induced by scalar and tensor fluctuations
- B polarisation: pure rotation field in analogy to magnetic field
  - ➔ Only induced by tensor fluctuations

E and B polarisation



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- E polarisation: pure gradient field in analogy to electric field
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- B polarisation: pure rotation field in analogy to magnetic field
  - ➔ Only induced by tensor fluctuations

Higher order effects

Second order non-Gaussianity

Non-Gaussianity is naturally induced at second order:

- At first order Δ_{lm} can be written as:

$$\Delta^{(1)}_{lm} = \Phi(\boldsymbol{k})\, T_l(k)\, Y_{lm}(\boldsymbol{k})$$

$$< \Phi(\boldsymbol{k}_1)\Phi(\boldsymbol{k}_2)\Phi(\boldsymbol{k}_3) >= 0 \quad \Rightarrow \quad < \Delta_{l_1 m_1}\Delta_{l_2 m_2}\Delta_{l_3 m_3} >= 0$$

- Non vanishing bispectrum linked to primordial non-Gaussianity

Higher order effects

Second order non-Gaussianity

Non-Gaussianity is naturally induced at second order:

- At second order we obtain:

$$\Delta^{(2)}_{lm} = \mathcal{K}\left[\Phi(\boldsymbol{k}_1)\Phi(\boldsymbol{k}_2)\,T_{lm}(k,k_1,k_2)\,Y_{lm}(\boldsymbol{k})\right]$$

$$< \Phi(\boldsymbol{k}_1)\Phi(\boldsymbol{k}_2)\Phi(\boldsymbol{k}_3) >= 0 \quad \not\Rightarrow \quad < \Delta^{(2)}_{l_1 m_1}\Delta_{l_2 m_2}\Delta_{l_3 m_3} >= 0$$

- Primordial non-Gaussianity contaminated by second order background

  ➔ Calculation of shape and magnitude of second order non-Gaussianity needed if primordial non-Gaussianity is small

Second order toy equations

$$\dot{\Delta}_n^{(2)} + \boldsymbol{A_n^{(2)}} + \sigma_n + C_{nm}\Delta_m^{(2)} = -|\dot{\kappa}|(\Delta_n^{(2)} + \varsigma_{nm}\Delta_m^{(2)} + \rho_n)$$

- $\boldsymbol{A_n^{(2)}}$: Second order metric sources
  - → Second order SW and ISW
- σ_n: Weak lensing and time delay
- $C_{nm}\Delta_m^{(2)}$: Free streaming term
- $-|\dot{\kappa}|\Delta_n^{(2)}$: Suppression term
- $-|\dot{\kappa}|\varsigma_{nm}\Delta_m^{(2)}$: Counter term
- $-|\dot{\kappa}|\rho_n$: Scattering source term

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- $\boldsymbol{A_n^{(2)}}$: Second order metric sources
- $\boldsymbol{\sigma_n}$: Weak lensing and time delay
  - ➔ Convolutions over first order photon and metric perturbations
  - ➔ Sources exist at any multipole moment
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- $A_n^{(2)}$: Second order metric sources
- σ_n: Weak lensing and time delay
- $C_{nm}\Delta_m^{(2)}$: Free streaming term
  - ➔ Couples neighbouring moments, generating higher moments over time
- $-|\dot{\kappa}|\Delta_n^{(2)}$: Suppression term
- $-|\dot{\kappa}|\varsigma_{nm}\Delta_m^{(2)}$: Counter term
- $-|\dot{\kappa}|\rho_n$: Scattering source term

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- $A_n^{(2)}$: Second order metric sources
- σ_n: Weak lensing and time delay
- $C_{nm}\Delta_m^{(2)}$: Free streaming term
- $-|\dot{\kappa}|\Delta_n^{(2)}$: Suppression term
  - ➔ Induces gradient suppressing every moment
  - ➔ Only relevant before recombination when $|\dot{\kappa}|$ is large
- $-|\dot{\kappa}|\varsigma_{nm}\Delta_m^{(2)}$: Counter term
- $-|\dot{\kappa}|\rho_n$: Scattering source term

Second order toy equations

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- $-|\dot{\kappa}|\Delta_n^{(2)}$: Suppression term
- $-|\dot{\kappa}|\varsigma_{nm}\Delta_m^{(2)}$: Counter term
  - ➔ Counters suppression term for monopole
  - ➔ Couples dipole and electron velocity
- $-|\dot{\kappa}|\rho_n$: Scattering source term

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- $-|\dot{\kappa}|\rho_n$: Scattering source term
  - ➔ Convolutions over first order photon and electron perturbations
  - ➔ Sources exist at any multipole moment, but high multipoles are suppressed

Transfer functions

Problem: First oder solutions needed, but only statistical properties can be calculated

Transfer functions

$$\begin{aligned}\Delta^{(1)}_{lm}(\boldsymbol{k}) &= \Phi(\boldsymbol{k})\, T^{(1)}_{lm}(k)\, Y_{lm}(\boldsymbol{k}) \\ \Delta^{(2)}_{lm}(\boldsymbol{k}) &= \mathcal{K}\left[\Phi(\boldsymbol{k}_1)\Phi(\boldsymbol{k}_2)\, T^{(2)}_{lm}(k, k_1, k_2)\, Y_{lm}(\boldsymbol{k})\right]\end{aligned}$$

- We obtain a system of ordinary differential equations for $T^{(2)}_{lm}$
- Statistical properties can be related to properties of primordial perturbations
- Introduces additional k_1 and k_2 dependance in the transfer function

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Line-of-sight integration

Problem: We have to deal with a infinite system of coupled equations

- The line-of-sight integration is an analytical solution of the differential equation
$$\dot{\Delta}_n^{(2)} + C_{nm}\Delta_m^{(2)} = -|\dot{\kappa}|\Delta_n^{(2)} + \xi_n$$
- We identify
$$\xi_n = -A_n^{(2)} - \sigma_n - |\dot{\kappa}|(\varsigma_{nm}\Delta_m^{(2)} + \rho_n)$$
- Which leads to the integral equation

$$\begin{aligned}\Delta_n^{(2)}(\eta_0) &= \int_0^{\eta_0} d\eta\, |\dot{\kappa}| e^{-\kappa(\eta)} j_{nm}(k(\eta_0-\eta))(\varsigma_{mp}\Delta_p^{(2)}(\eta) + \rho_m(\eta)) \\ &\quad + \int_0^{\eta_0} d\eta\, e^{-\kappa(\eta)} j_{nm}(k(\eta_0-\eta))(A_m^{(2)}(\eta) + \sigma_m(\eta))\end{aligned}$$

Line-of-sight integration

$$\int_0^{\eta_0} d\eta \, |\dot{\kappa}| e^{-\kappa(\eta)} j_{nm}(k(\eta_0 - \eta))(\varsigma_{mp} \Delta_p^{(2)}(\eta) + \rho_m(\eta))$$

- Scattering contributions
  - ➔ Visibility function $|\dot{\kappa}| e^{-\kappa(\eta)}$ is non-vanishing only around recombination
  - ➔ ς_{mp} vanishes for $l_p > 2$
  - ➔ ρ_m vanishes for large l_m at early times
- Second order metric contributions
- Lensing and time-delay

Line-of-sight integration

$$\int_0^{\eta_0} d\eta \, e^{-\kappa(\eta)} j_{nm}(k(\eta_0 - \eta)) A_m^{(2)}(\eta)$$

- Scattering contributions ✓
- Second order metric contributions
  - ➔ Important also at late times
  - ➔ During matter domination and later $A_m^{(2)}$ evolves almost independent of photon perturbations
- Lensing and time-delay

Line-of-sight integration

$$\int_0^{\eta_0} d\eta \, e^{-\kappa(\eta)} j_{nm}(k(\eta_0 - \eta))\sigma_m(\eta)$$

- Scattering contributions ✓
- Second order metric contributions ✓
- Lensing and time-delay
  - ➔ Only depends on first order perturbations
  - ➔ Sources at any l_m

The code

Structure of the code

1) Solves the Boltzmann-Einstein equations, calculating the second order transfer functions
2) Computes line-of-sight integration generating the temperature transfer functions today
3) Computes full-sky bispectrum by integration over transfer functions (highly oscillatory 4d integration)

Implementation

- Based on CLASS
  - ➔ Accessible and modular
- Parallelised code using methods fit to the problem
  - ➔ Computes in order of hours on normal machines
- Developing two independent codes, employing different methods

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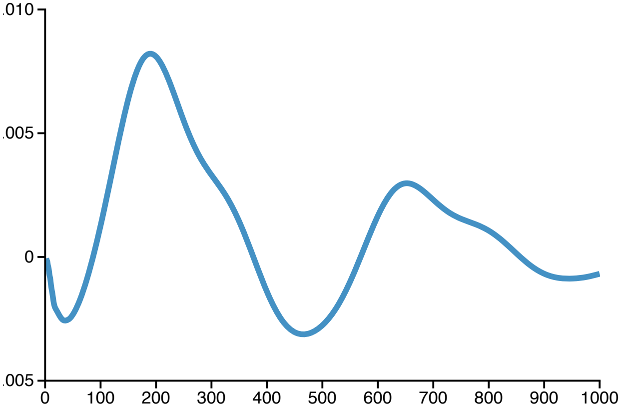
Numerical checks

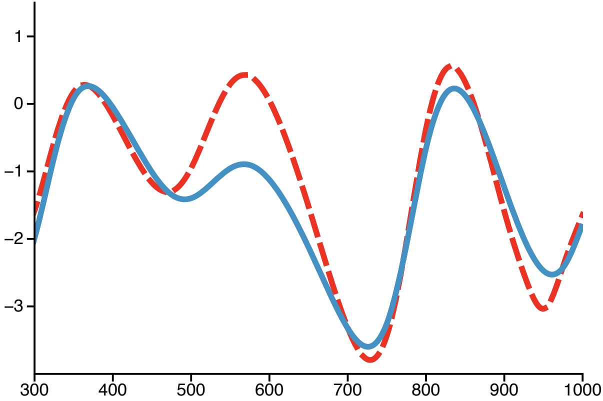
Large f_{NL} run



Check 1: Bispectrum in equilateral configuration

Numerical checks

Analytic squeezed limit



Check 2: Normalised bispectrum in squeezed configuration $l1 = 20$, $l2 = l3$

Outlook

- We have written a second order CMB code which computes
  - ➔ Second order non-Gaussianity
  - ➔ Second order B-polarisation
- Induced from
  - ➔ Scattering sources
  - ➔ Metric sources
- Work in progress
  - ➔ Check numerical stability and convergence
  - ➔ Lensing and time delay

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`Thank you for your attention!`