Analytic approaches to large scale structure: perturbation theory and beyond

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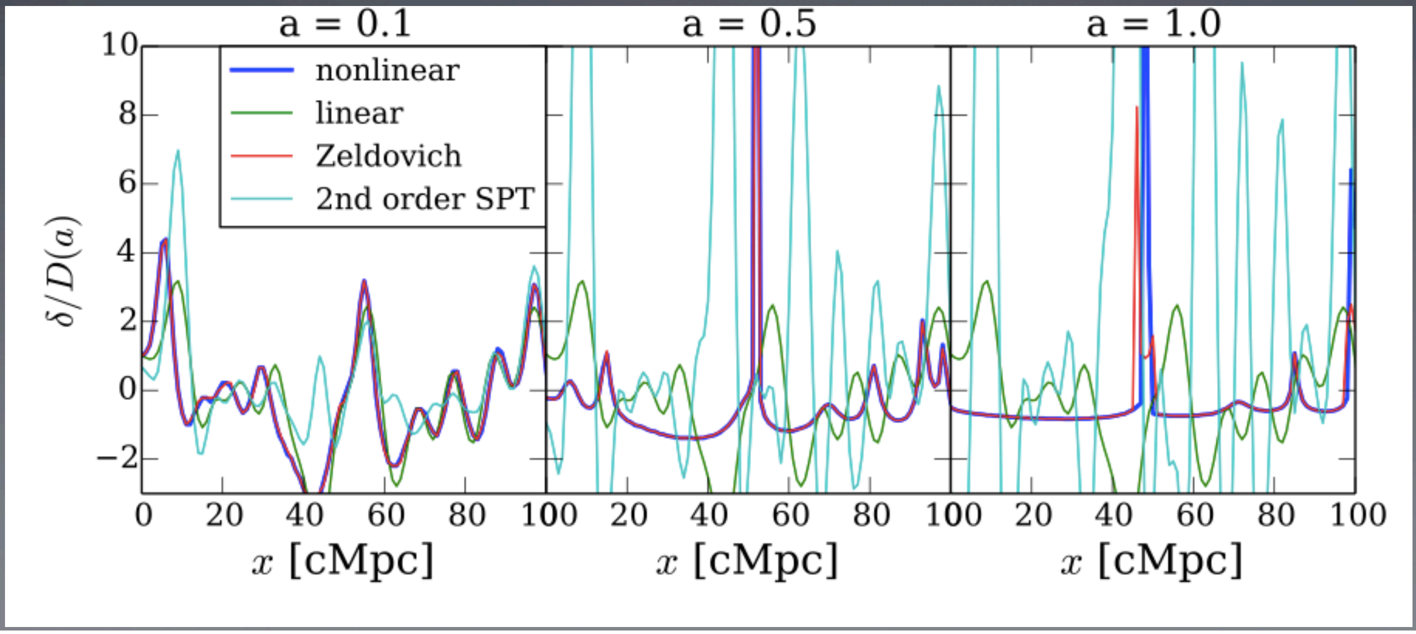
Why do theory

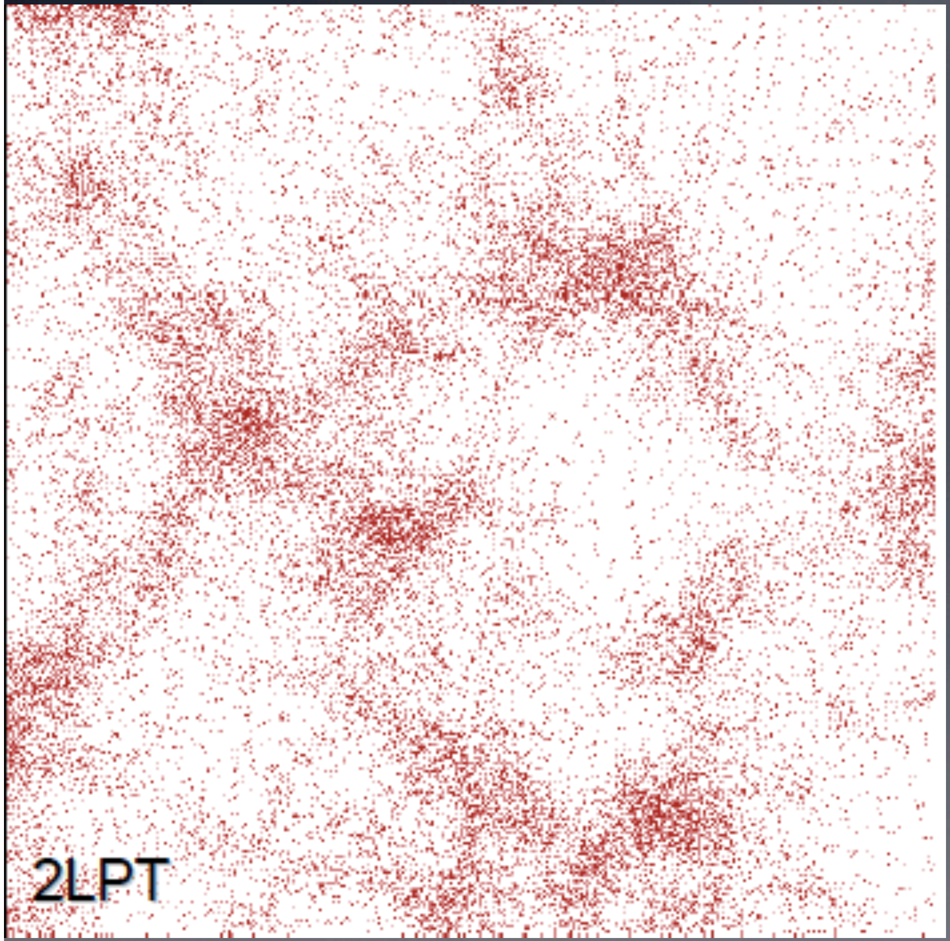
- Because it is fun
- To show off
- Because we are too lazy to run simulations
- Because we like to argue with fellow theorists
- Because it gives useful insights (maybe)

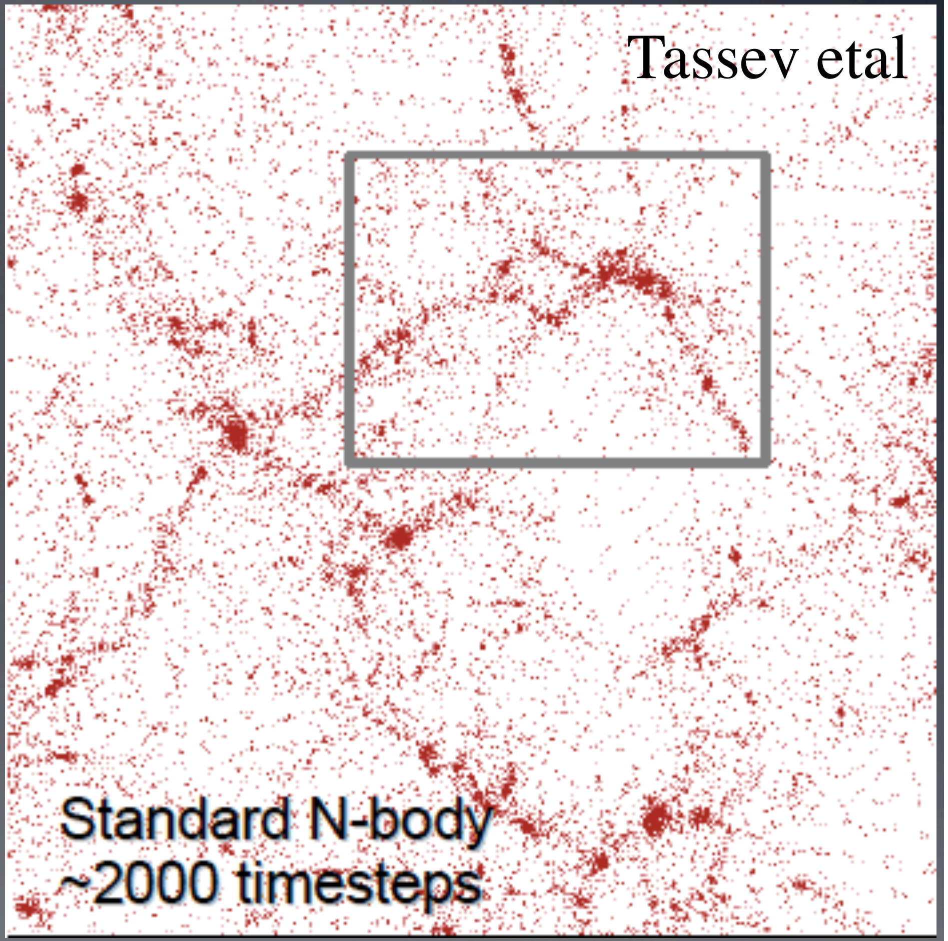
Perturbative approaches

- Perturbation theory: Eulerian (SPT), Lagrangian (LPT)
- Lowest order: 1 loop SPT, Zeldovich
- PT assumes δ<1 and fails at the orbit crossing: halo formation
- 1d example (McQuinn & White): Zeldovich exact up to orbit crossing



LPT vs simulations: 3d





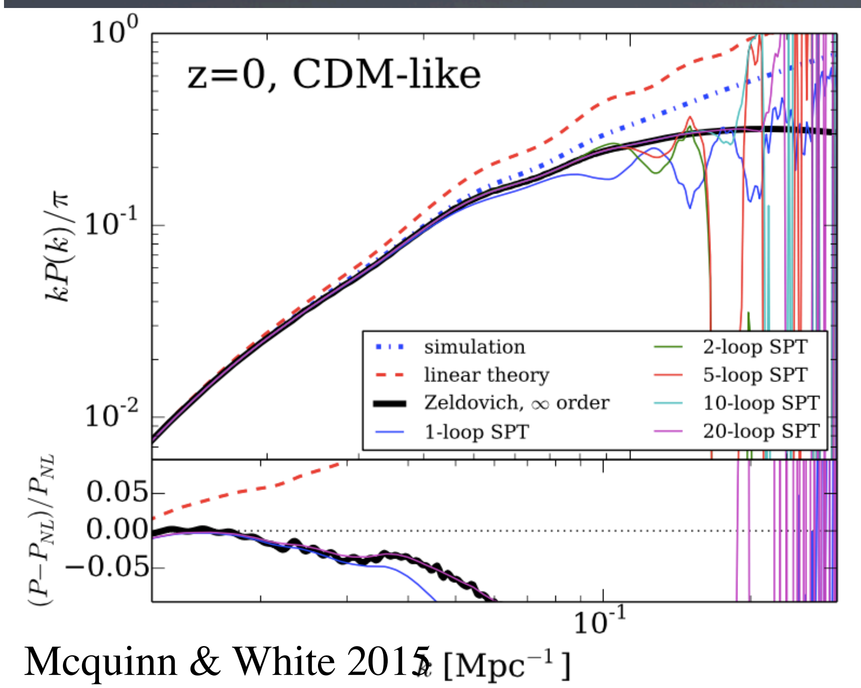
LPT gets LSS right, but does not form high density halos
In Zeldovich approximation particles just stream along straight line

Beyond PT: parametrize PT ignorance

take the ratio. One can write the ratio of true P(k) to 1LPT P(k) as a function $T^2(k)$ describing the PT ignorance, which starts as k^2: effective field theory (EFT) approach

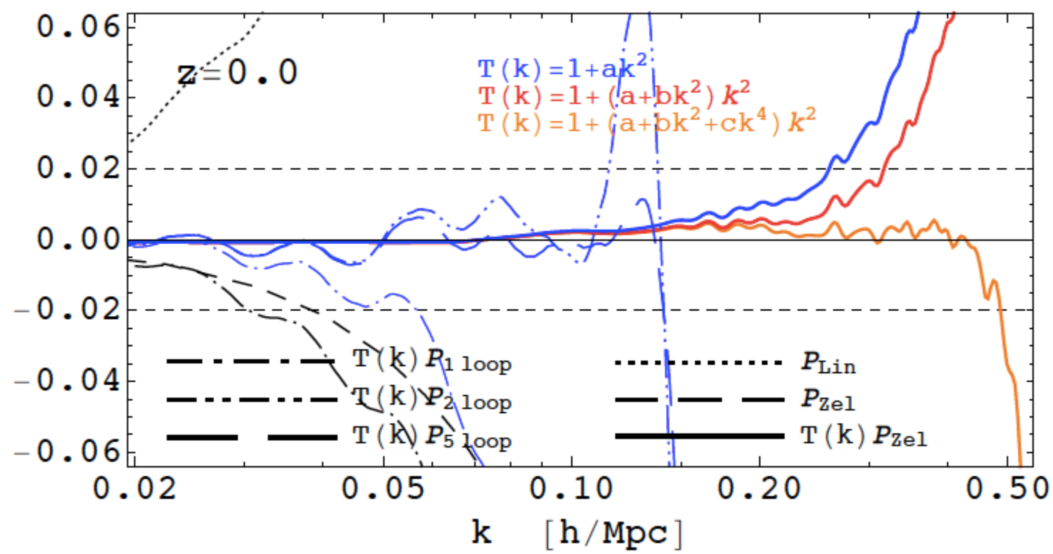
$$\frac{P_{\rm dm}(k)}{P_{1LPT}(k)} \equiv T^2_{1LPT}(k) = 1 + \alpha_{1LPT}(k)k^2 \equiv \left(1 + \sum_{i=1}^{\infty} \alpha_{i,1LPT} k^{2i}\right)$$

EFT α is a parameter describing 1LPT error, of order (few Mpc)^2



Mcquinn & White 2015

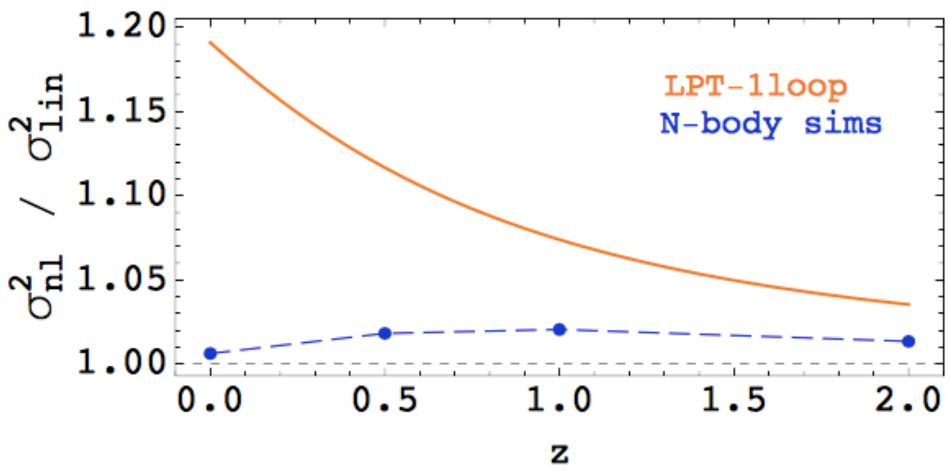
Vlah & US 2015

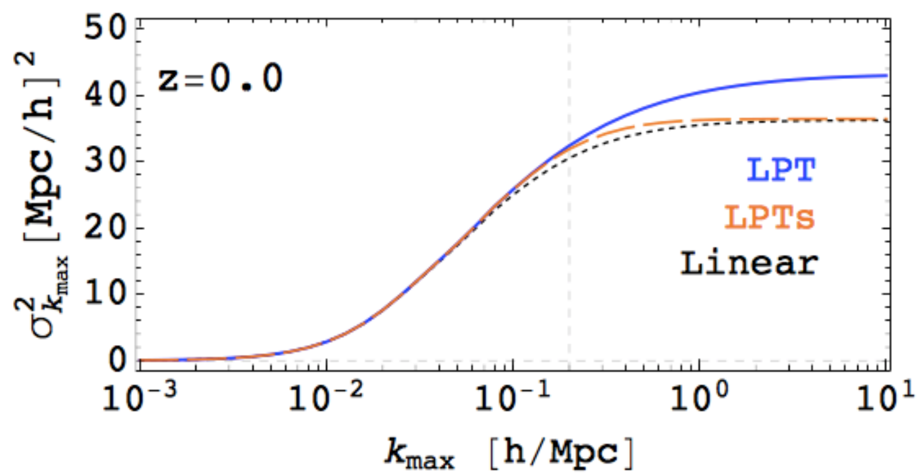


PT challenges in 3-d

- In 1-d SPT expansion is convergent: e.g. 2 loop starts as $k^4 P_L$, hence smaller than 1 loop $k^2 P_L$ at low k
- In 3-d no longer so: higher order loops all start at $k^2 P_L$ and receive large contributions from small scales which are spurious: in reality DM is trapped inside dark matter halos (shell crossings) on small scales. PT in 3-d is harder
- As a result higher loop terms can be completely wrong in their dominant terms

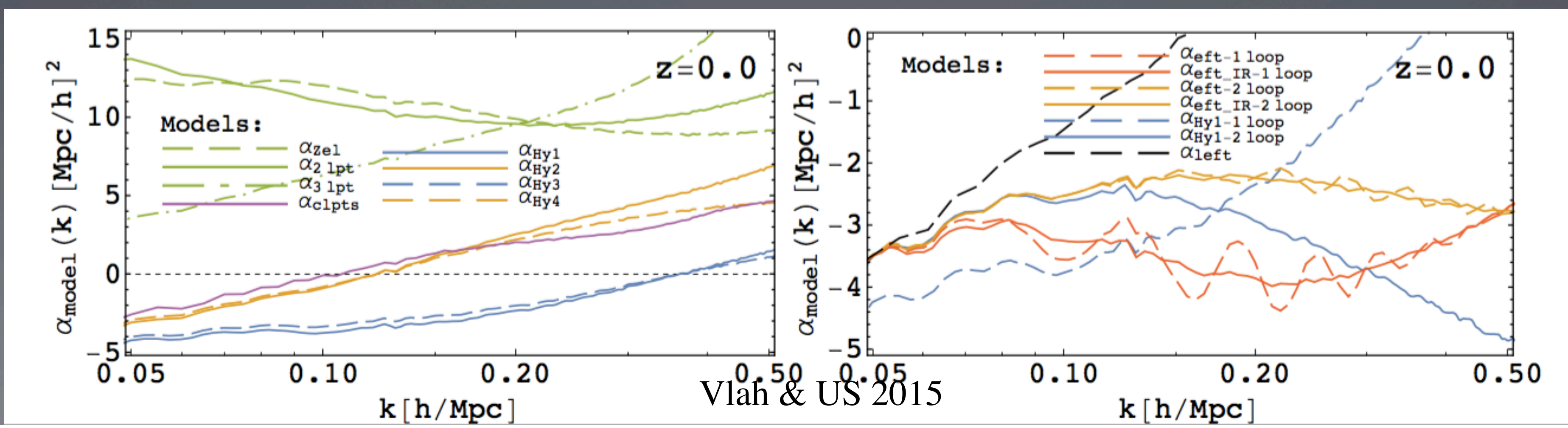
Vlah etal 2014





3-d PT/EFT situation

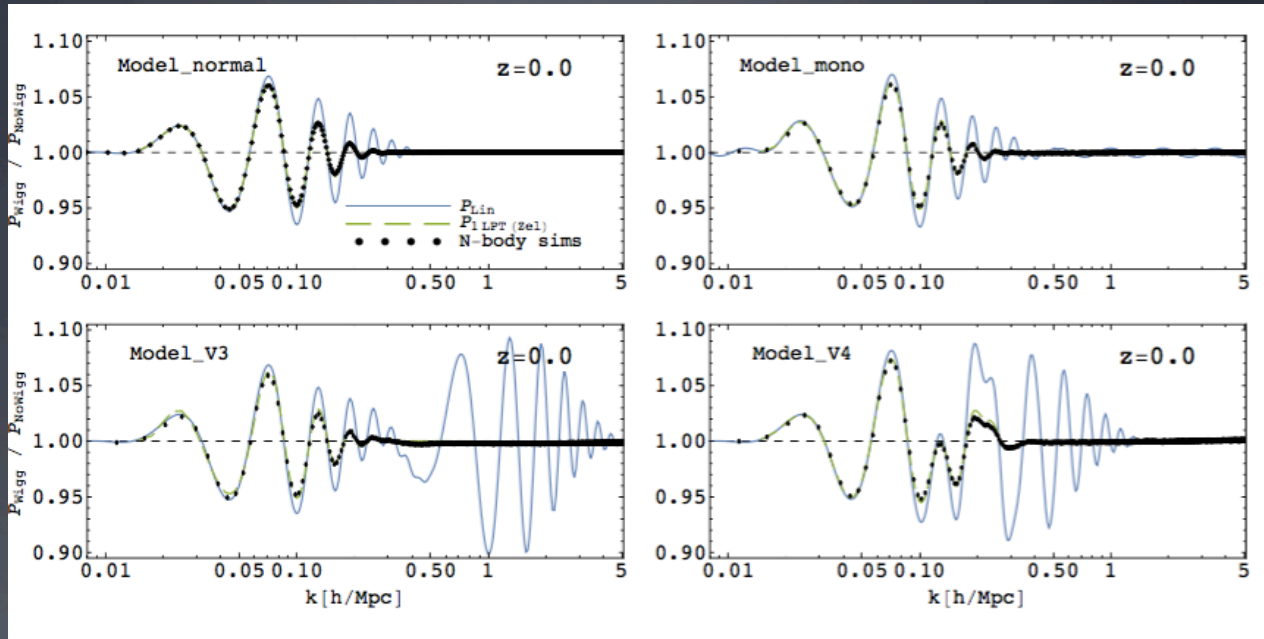
- One can define many different PT approaches and many different ways to parametrize the ignorance: SPT 1 loop, SPT 2 loop, 1LPT, 2LPT, 3LPT… Higher order not obviously better. One can also define many different EFT schemes (EFT in SPT, LPT, IR resummation…)
- EFT needs to absorb a lot of PT problems: many EFT parameters, typical value of α of order (1-3Mpc/h)^2
- One can quantify the success by asking whether EFT parameter in front of k^2P_L is a constant (but beware of numerical issues at low k)
- EFT 1 loop SPT works to k=0.1h/Mpc, 2 loop to 0.2h/Mpc with 1 parameter (Baldauf etal, Foreman etal)



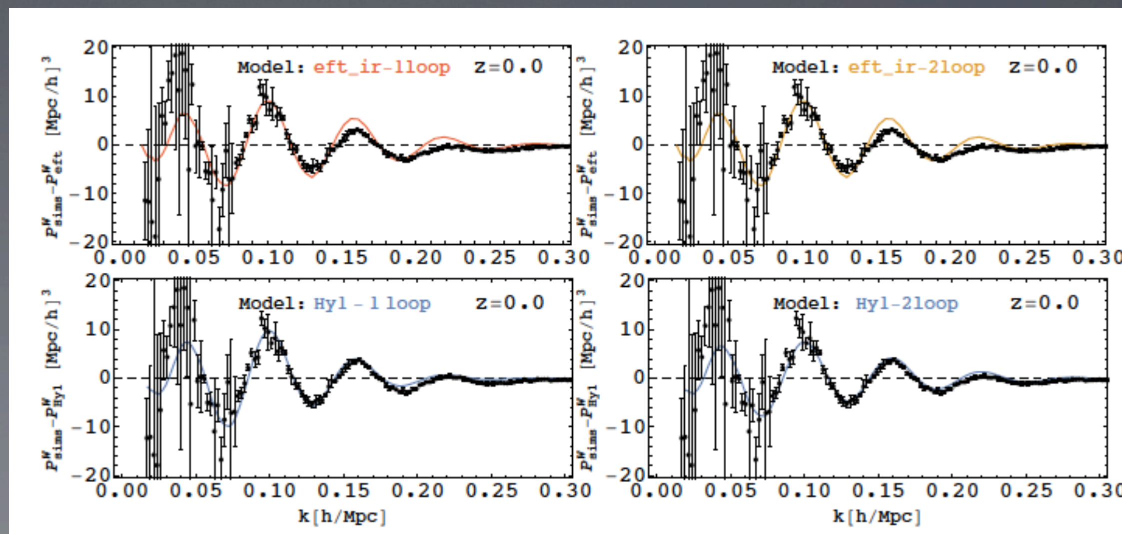
Vlah & US 2015

Primordial+BAO wiggles

- Zeldovich predicts wiggles well



PT+EFT predicts even better: tiny <1% effects, require special simulations to see them



EFT challenges for broadband

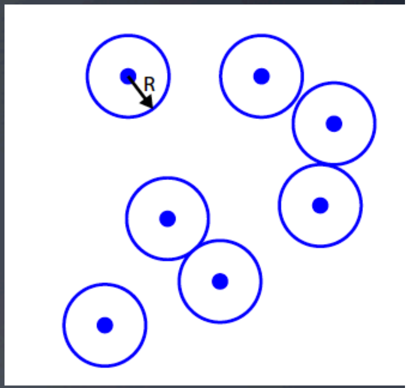
- one parameter EFT is only an approximation, one needs to add more free parameters at higher k
- EFT has free parameters which are fit from simulations, so one can always choose a k where the fit is perfect: danger of overfitting (e.g Baldauf etal 2015)
- For k>0.2h/Mpc stochastic terms become important, and EFT needs additional terms
- Parametrizing ignorance with a single parameter works best at low k, but this is the regime where sampling variance errors dominate and linear theory works well
- There is a limited dynamic range where EFT with a single parameter is applicable and relevant: 0.05h/Mpc<k<0.2h/Mpc (z=0)
- For higher k additive 1-halo term model is more suitable

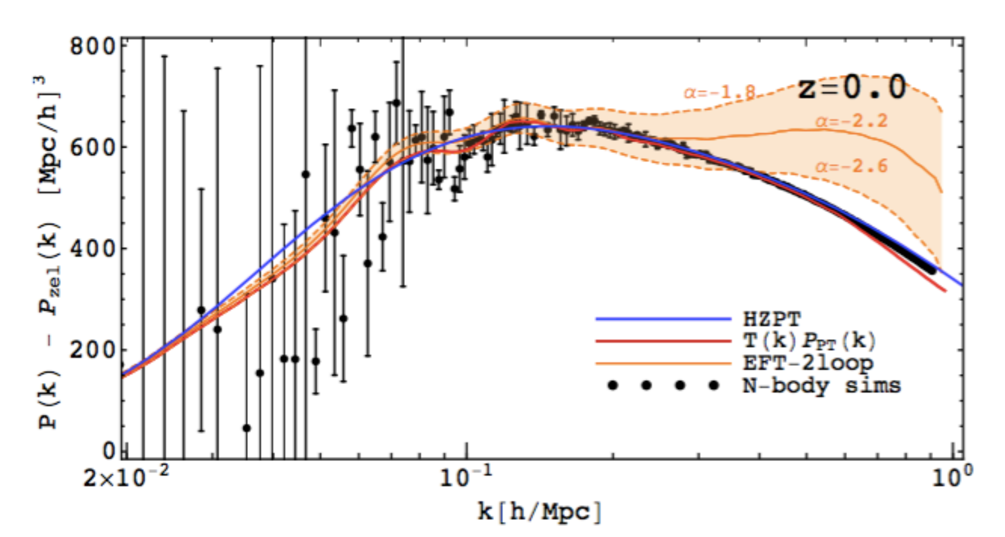
Halo model (with I. Mohammed, Z. Vlah)

- Approach 2: use PT (e.g. Zeldovich) approximation to describe large scales
- Use halo model to describe small scales and add
- Halo description: mass, profile: take moments
- Halo mass function dn/dlnM=f(ν)dν
- $P_{1halo}=A_0k^0+A_2k^2+A_4k^4+\ldots$

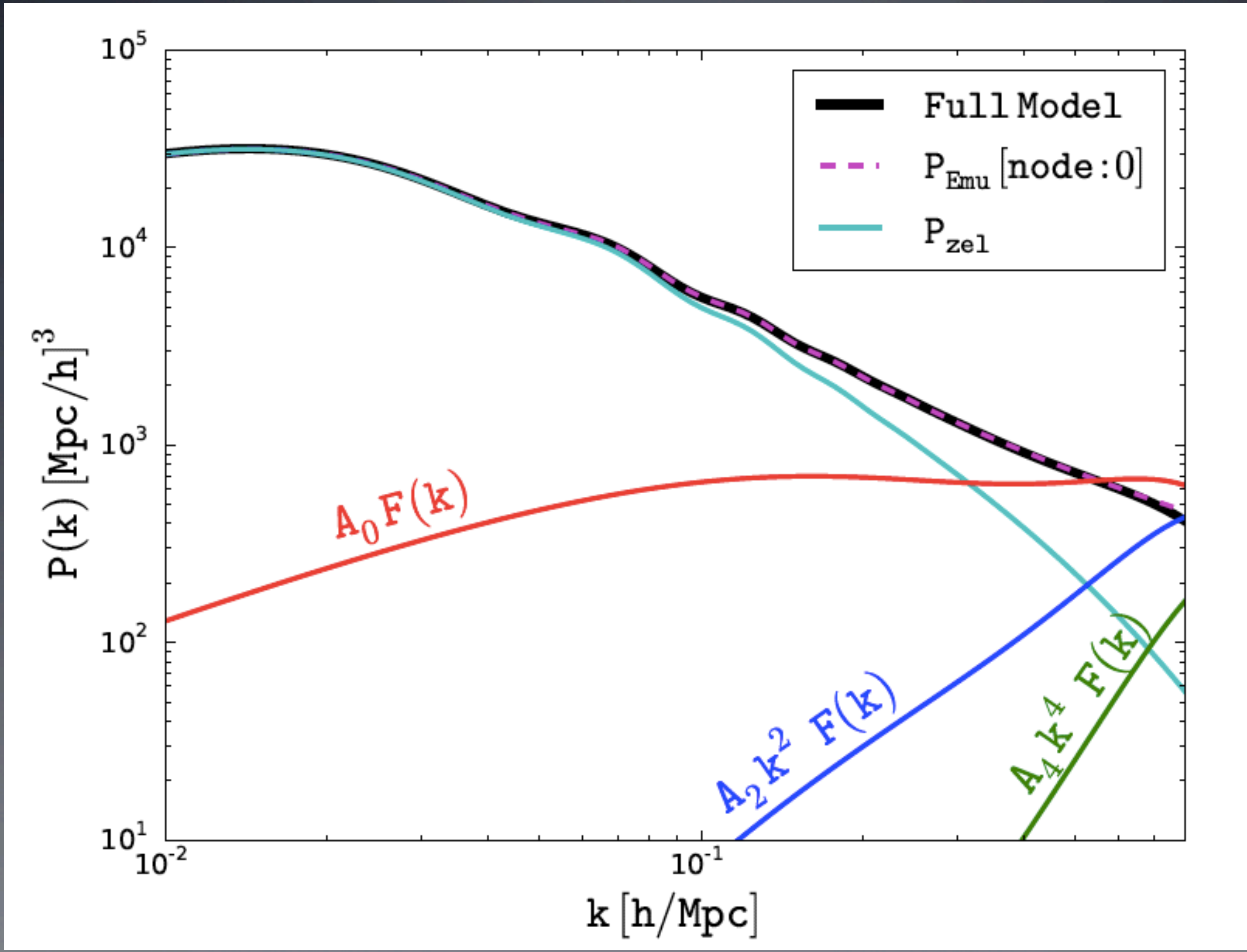
  Halo model not accurate enough, use simulations to determine A_0, A_2, etc

At low k we need to impose mass and momentum conservation, forcing 1 halo term to vanish





Total power spectrum



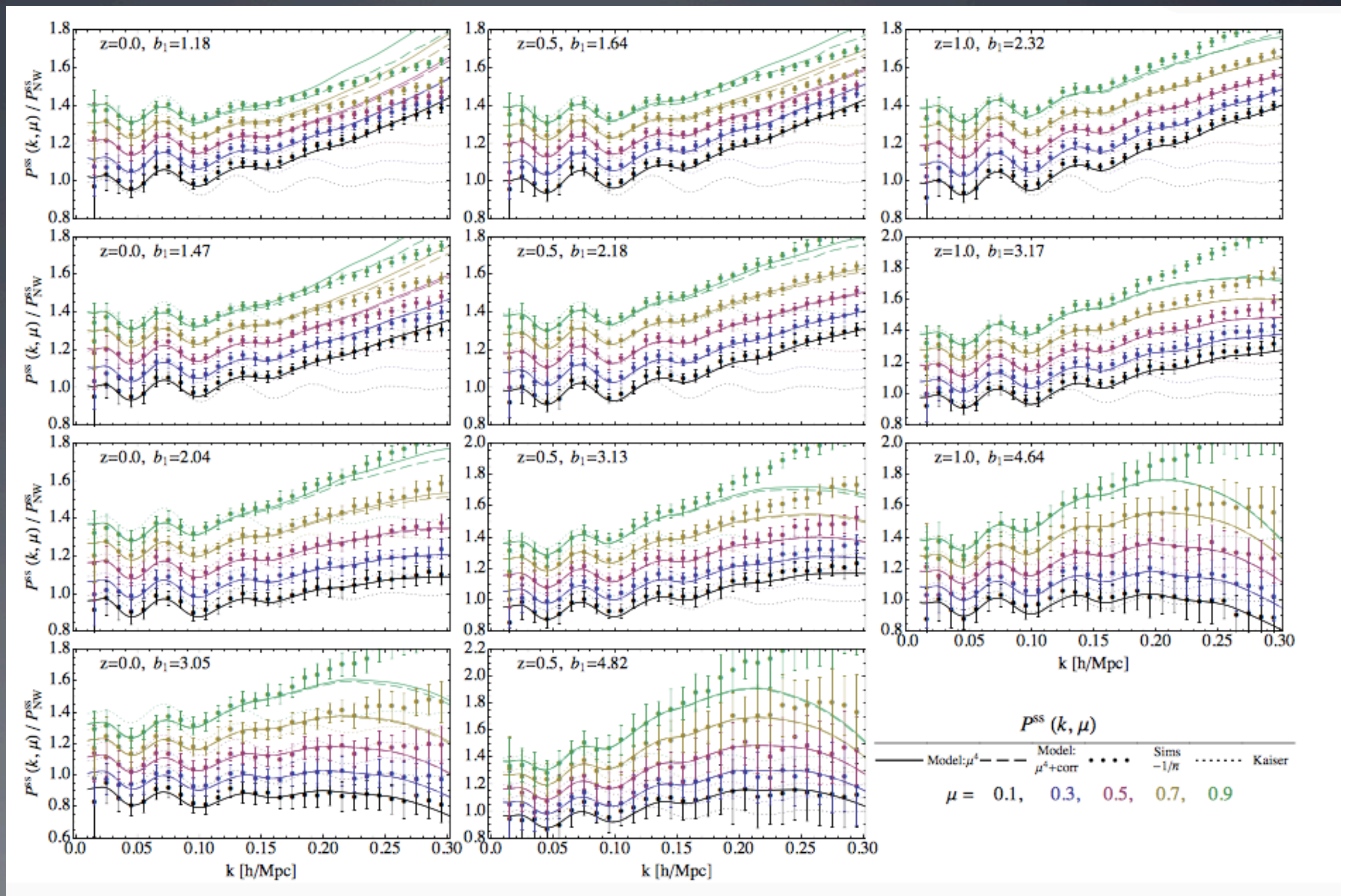
PT model for galaxies

- Need to develop nonlinear models that are sufficiently general to allow for any reasonable nonlinear effects present in the data, while preserving as much of cosmological information as possible
- Some can be modeled by perturbation theory (PT)+biasing
- There is always more information on small scales, but most of it is hopelessly corrupted by nonlinear effects that cannot be modeled in PT
- One needs to model our ignorance obeys all symmetries (e.g. $k^2P_L(k)$ at low k) and all physics (the biasing parameters are physical, e.g. FoG is determined by halo mass…)
- In recent years a workhorse has been the halo model+biasing+PT

PT+halo model for halos

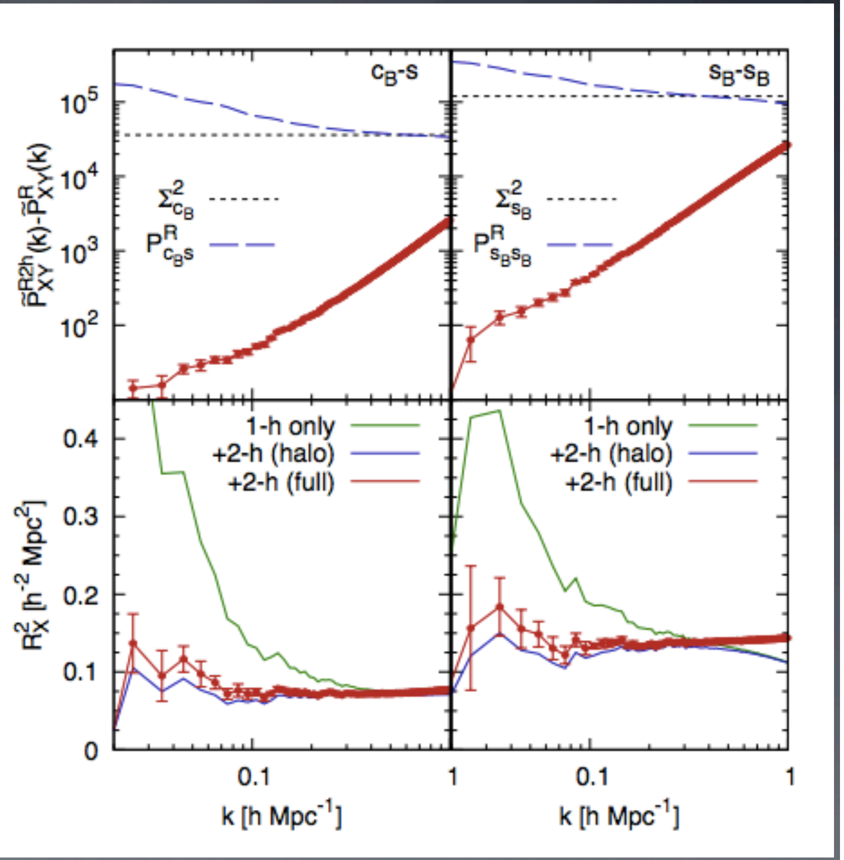
- Use PT to model halos, account for all bias terms
- Biasing: local, non-local, k^2, stochastic…
- General principle: everything that is allowed by symmetry is also present in reality (many biasing terms)
- Example: 1 loop SPT modeling of RSD for halos (Vlah etal 2013)

RSD is never linear for k>0.1h/Mpc



Beyond PT: effect of satellites in real space

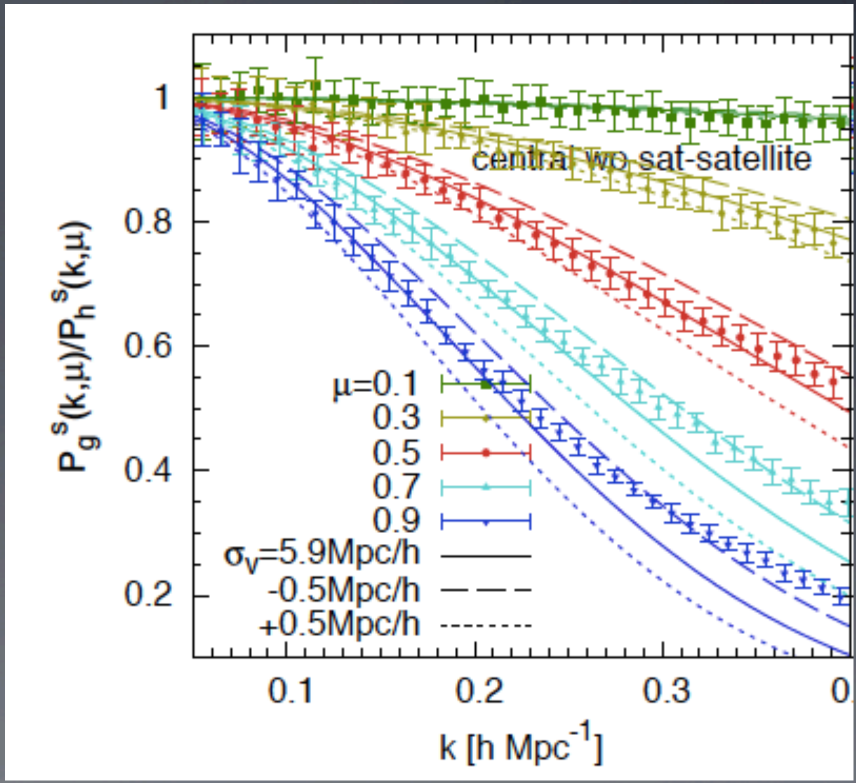
- Satellite-central pairs and satellite-satellite pairs inside halos create additional 1 halo term
- Leading term at low k: additional Poisson shot noise amplitude $\Sigma^2 = V/N_{cs}$ given by number of central-satellite pairs
- Leading correction due to radial distribution $\Sigma^2(1-k^2R^2_{vir})$, same also for 2-halo term
- Must vanish at high k to give 0 or 1/n



Okumura et al 2015

Beyond PT: redshift space distortions

- Supplemented by satellite velocities inside the halos (Fingers of God), inducing 2 halo term and 1 halo term
- FoG term is large: virial theorem $\sigma_{vir}^2 = 50R_{vir}^2$, must use resummed version, e.g. $\exp(-k^2\mu^2\sigma_{vir}^2)$
- Leading term: 2 halo correlation between central galaxies and satellites in different halos
- FoG also applies to 1 halo term



Okumura etal 2015

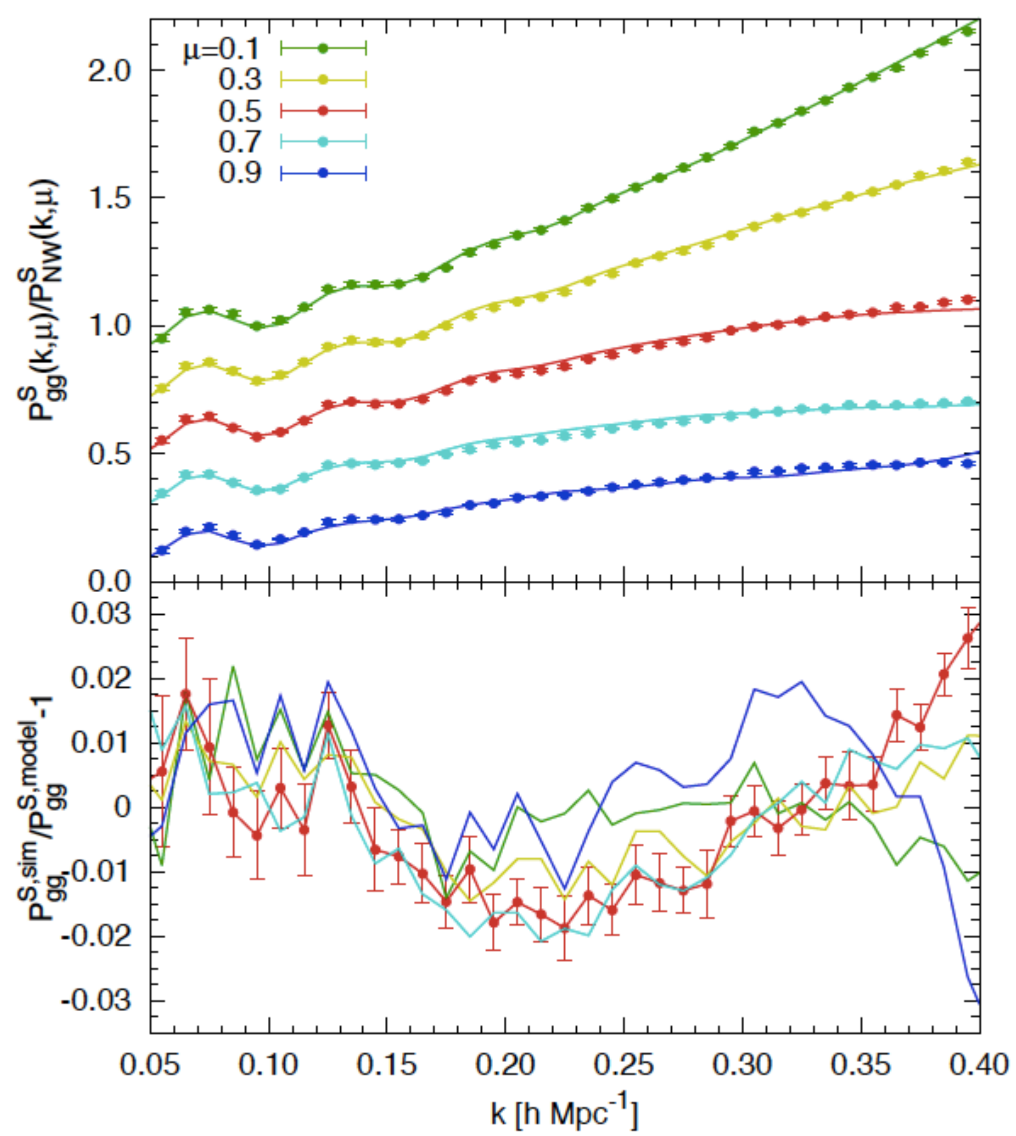
Example: modeling of CMASS in simulations

Okumura etal 2015

Linear Kaiser never a good model

With SPT, 1 shot noise term and 1 FoG term one can go to k=0.2h/Mpc (current state of the art: e.g. Beutler et al, based on models by Saito, Taruya, Scoccimarro)

To go beyond one needs more PT biasing parameters. New NL model achieves 1% to k=0.4h/Mpc by introducing many physical parameters in the PT model: central and satellite galaxies, each with a bias and FoG, 1-halo contribution from central-satellite pairs, halo exclusion…



Covariance matrix

- simulations have a hard time converging on covariance matrix, its inverse is "hard": e.g. 12,000 simulations in Blot et al. 2014
- Disconnected part: "gaussian" is easy: we should compute it analytically using window functions (note: this is not done currently)
- Connected part: smooth response to long wavelength modes

$$\mathrm{Cov}(P(k_i), P(k_j)) = P(k_i)P(k_j)V^{-1}\left(\frac{4\pi^2}{k_i^2\Delta k}\delta_{ij} + \delta_{A_0}^2\right)$$

PT approach to Covariance

- Modes from outside the survey (do not average to zero): tree level effects from survey window function very important (supersample variance), easy to calculate, depend on whether the mean density is computed from within the survey or not (Li, Takada, Hu 2014)

$$\delta \ln P(k) = \left(\frac{47}{21} - \frac{1}{3} \frac{d \ln P}{d \ln k} \right) \delta_b = \left(\frac{68}{21} - \frac{1}{3} \frac{d \ln (k^3 P)}{d \ln k} \right) \delta_b$$

- Use 26/21 instead of 68/21 for local mean density
- Modes inside the survey (average to zero): use PT trispectrum

PT trispectrum

- Tree-level calculation (Scoccimarro etal 1999)

$$C_{ij} \equiv \langle \hat{P}(k_i)\hat{P}(k_j)\rangle - \langle \hat{P}(k_i)\rangle\langle \hat{P}(k_j)\rangle = V_f\ [\frac{2P_i^2}{V_s(k_i)}\delta_{ij} + \bar{T}(k_i,k_j)]$$

$$\bar{T}(k_i,k_j) = \int_{k_i}\frac{d^3\boldsymbol{k}_1}{V_s(k_i)}\int_{k_j}\frac{d^3\boldsymbol{k}_2}{V_s(k_j)}\,T(\boldsymbol{k}_1,-\boldsymbol{k}_1,\boldsymbol{k}_2,-\boldsymbol{k}_2)$$

$$\bar{T}(k_i,k_j) = \int_{k_i}\frac{d^3\boldsymbol{k}_1}{V_s(k_i)}\int_{k_j}\frac{d^3\boldsymbol{k}_2}{V_s(k_j)}\Big[12F_3(\boldsymbol{k}_1,-\boldsymbol{k}_1,\boldsymbol{k}_2)P_1^2P_2 + 8F_2^2(\boldsymbol{k}_1-\boldsymbol{k}_2,\boldsymbol{k}_2)P(|\boldsymbol{k}_1-\boldsymbol{k}_2|)P_2^2$$
$$+\ 16F_2(\boldsymbol{k}_1-\boldsymbol{k}_2,\boldsymbol{k}_2)F_2(\boldsymbol{k}_2-\boldsymbol{k}_1,\boldsymbol{k}_1)P_1P_2P(|\boldsymbol{k}_1-\boldsymbol{k}_2|) + (\boldsymbol{k}_1\leftrightarrow\boldsymbol{k}_2)\Big]$$

- 1-loop terms: sample variance of low k modes (Mohammed & US)

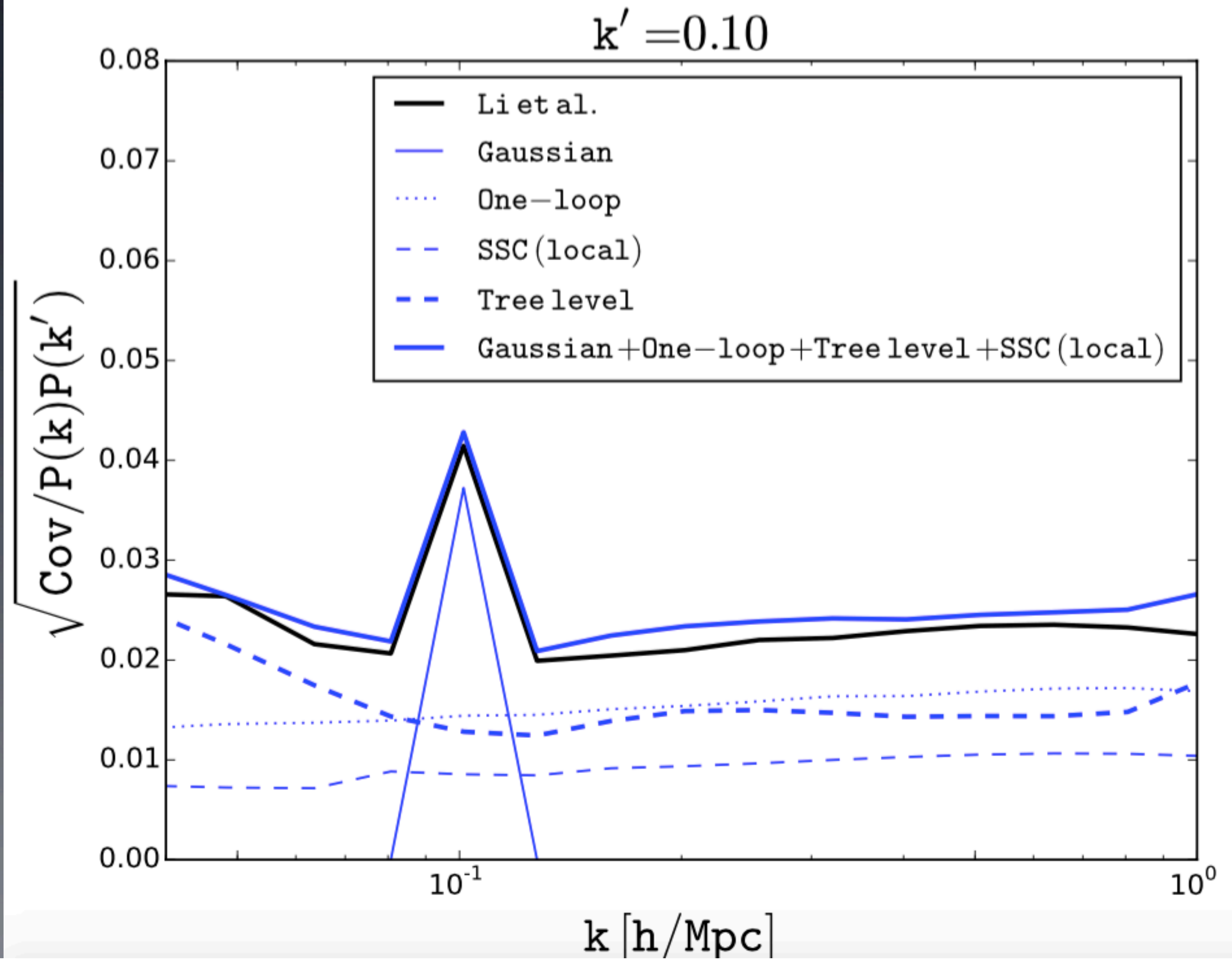
$$P = P_{22}+P_{13} = \{\frac{2519}{2205}P_{S0}(k) - \frac{47}{105}kP'_{S0}(k) + \frac{1}{10}k^2P''_{S0}(k)\}\langle\delta_L^2\rangle$$

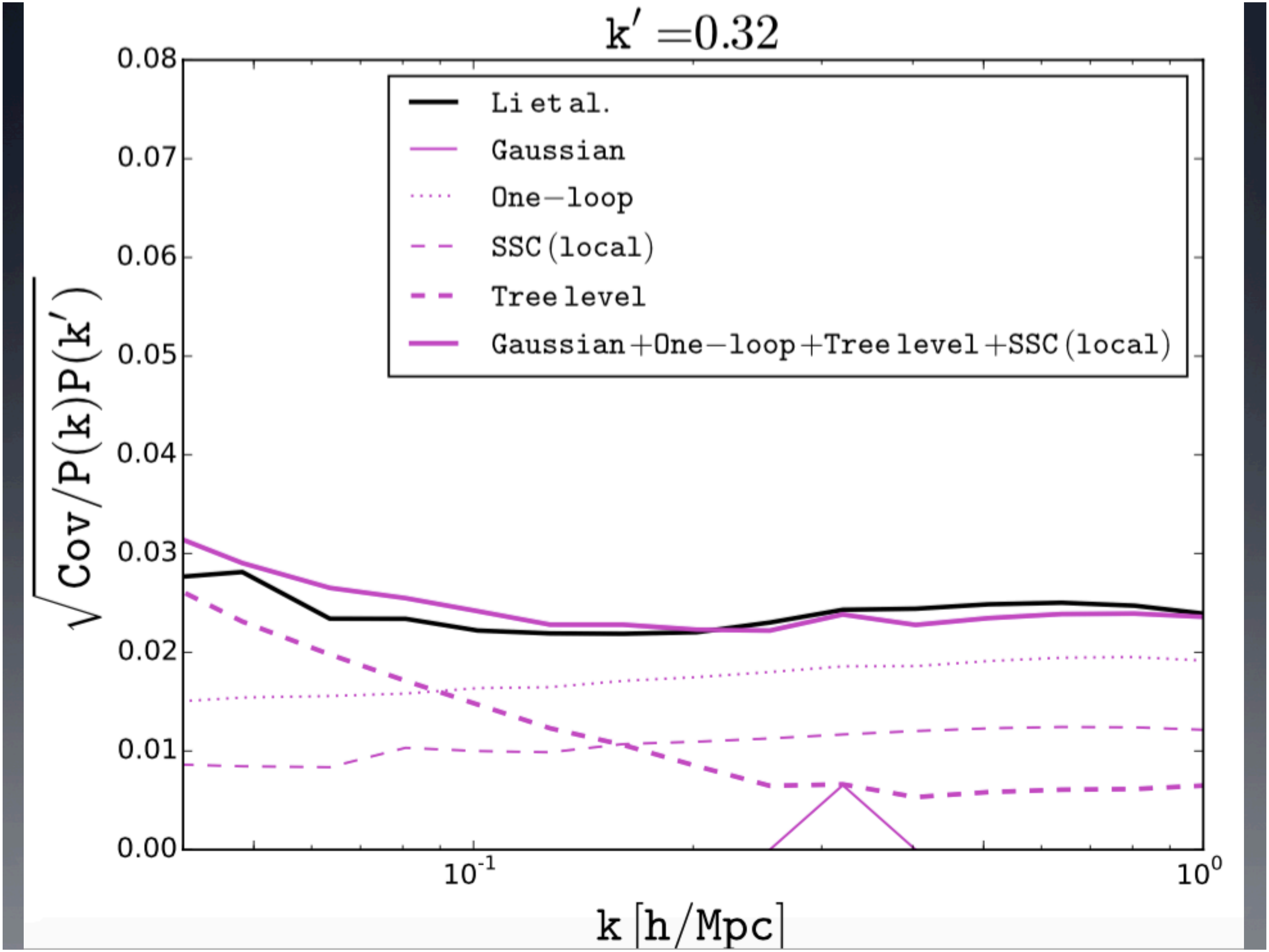
$$\frac{\mathbf{Cov}_{ij}}{P(k_i)P(k_j)} = \left(\frac{1}{\pi^2}\int P_{\rm Lin}^2(k)k^3 d\ln k\right)\mathbf{W}_i\mathbf{W}_j$$

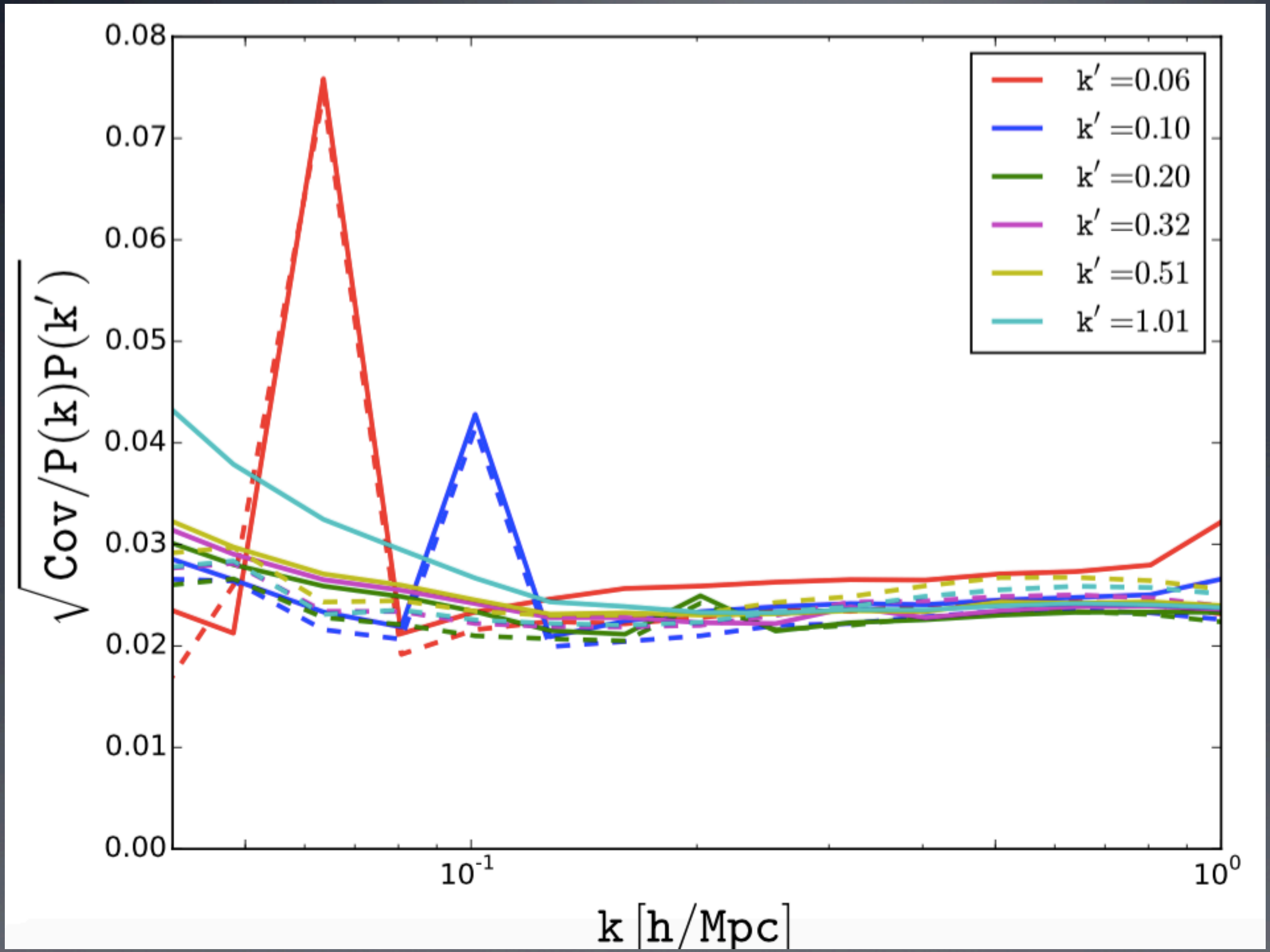
Large contribution from low k, hence large volume needed

$$\mathbf{W}_i = \frac{2519}{2205}E_2(k_i) - \frac{47}{105}\frac{d\ln P(k_i)}{d\ln k_i} + \frac{1}{10}\frac{d^2\ln P(k_i)}{d\ln k_i^2}$$

PT vs simulations




k′ =0.32
0.08
0.07
0.06
0.05
0.04
0.03
0.02
0.01
0.00
Li et al.
Gaussian
One−loop
SSC (local)
Tree level
Gaussian +One−loop +Tree level +SSC (local)
√Cov/P(k)P(k′)
10^-1
10^0
k [h/Mpc]



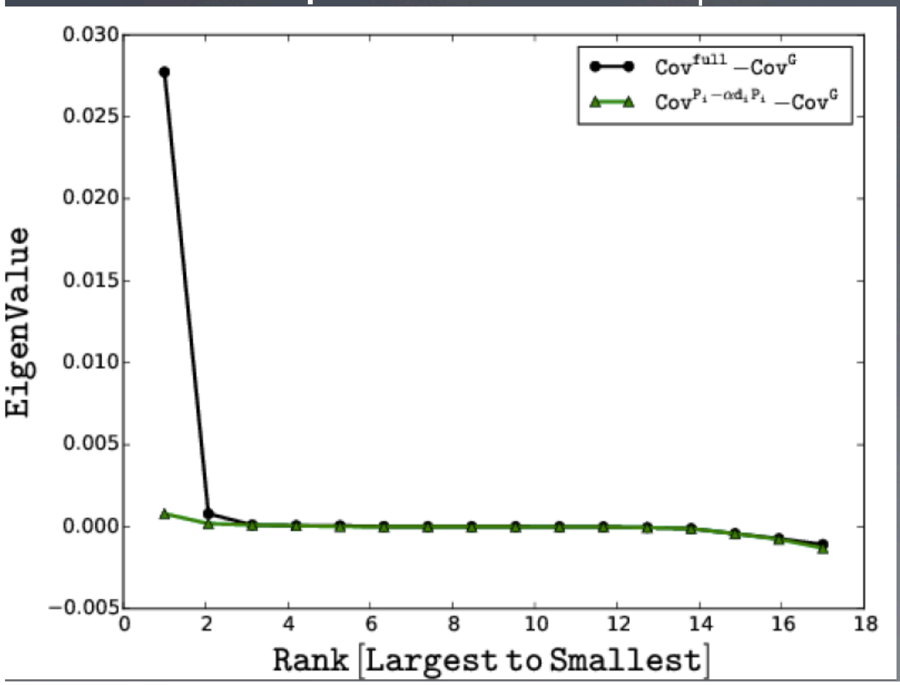
Lessons: connected part of covariance is very correlated
For k>0.2h/Mpc all the modes are strongly correlated
Correlations generated by long wavelength modes:
beware of jackknife/bootstrap methods

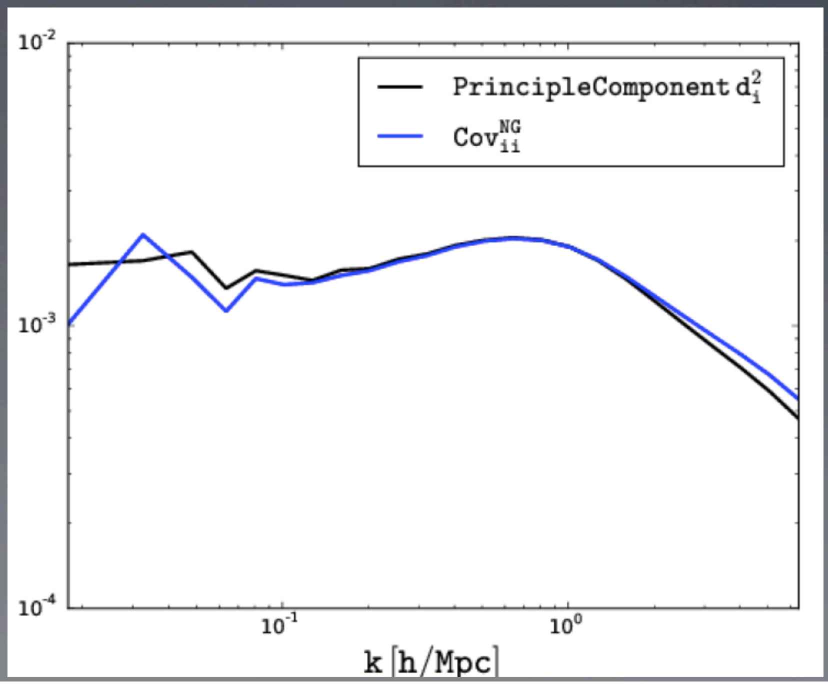
Covariance matrix as an external parameter

- Most of the connected covariance comes from a small scale response to long wavelength modes

- The connected part can be written as a single eigenmode

  $C_{ij}=Ad_id_j$, where i represents k_i amplitude and d_i is a response at that k_i





Covariance as a fit for additional external nuisance parameter

Treat d_i as an external nuisance parameter:

$P(k_i)=P_{fid}(k_i)+\alpha d_i$

Marginalize over α, possibly with a theoretical prior

Non-gaussian covariance matrix reduced by a factor of 100 ($k>0.1$)

Degeneracy with amplitude at low k (prior useful), lifted at higher k (prior not needed)

Preliminary: same concept also works on galaxies

Conclusions

- PT is alive and well: it may be able to describe clustering up to orbit crossing (1 halo term formation), but not beyond: in 3-d PT fails even before shell crossing
- PT alone cannot give accurate results, needs to be supplemented by non-PT modeling of halos: EFT, halo model…
- PT+EFT useful at low k (k<0.2h/Mpc), halo model also at higher k
- Galaxy biasing: all terms that can exist by symmetry exist, and at the halo level may be predictable (i.e. function of halo mass)
- 1-halo terms and RSD FoG terms non-perturbative and needed
- PT+halo model successful in RSD models up to k=0.4h/Mpc
- PT 1 loop SPT successful for modeling DM covariance matrix, and suggests covariance can be modeled as external nuisance parameter
- Full theory (beyond PT) still not available (except simulations)