Cosmology: The Science of the Universe is a broad introduction to the science of modern cosmology, with emphasis on its historical origins. Great advances in science in the 20th century – in astronomy, physics and biology – have triggered an explosion of powerful new ideas on the nature of the universe. This book explores the realm of receding galaxies, the nature of space and time, black holes, inflation and the expansion of the universe. The subjects range from the subatomic to the extragalactic, from the beginning to the end of time, from terrestrial to extraterrestrial life and encompass the origin of atoms, galaxies, the cosmos and life itself.

Cosmology is more than science. It has no limits. It challenges us with problems (such as the cosmic-edge issue) and with perplexities (such as the containment riddle). Its roots are buried deep in mythology, philosophy and theology. In this unique book, Professor Harrison shows that societies in every age construct universes, or world systems, that seek to make sense of human experience. He also shows how the Babylonian, Pythagorean, Aristotelian, Stoic, Epicurean, Medieval, Cartesian and Newtonian ancestral world systems laid the foundation of the modern physical universe.

The first edition of this best-selling book received world-wide acclaim for its lucid style and wide-ranging exploration of the universe. This eagerly awaited second edition updates and greatly extends the first edition. Seven new chapters explore Early Scientific Cosmology, Cartesian and Newtonian World Systems, Cosmology After Newton and Before Einstein, Special Relativity, Observational Cosmology, Inflation, and Creation of the Universe. All chapters conclude with a section entitled "Reflections" containing topics provocative of thought and debate. The new "Projects" section, also at the end of each chapter, raises questions and issues to challenge the reader.

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2ND EDITION

HARRISON

COSMOLOGY

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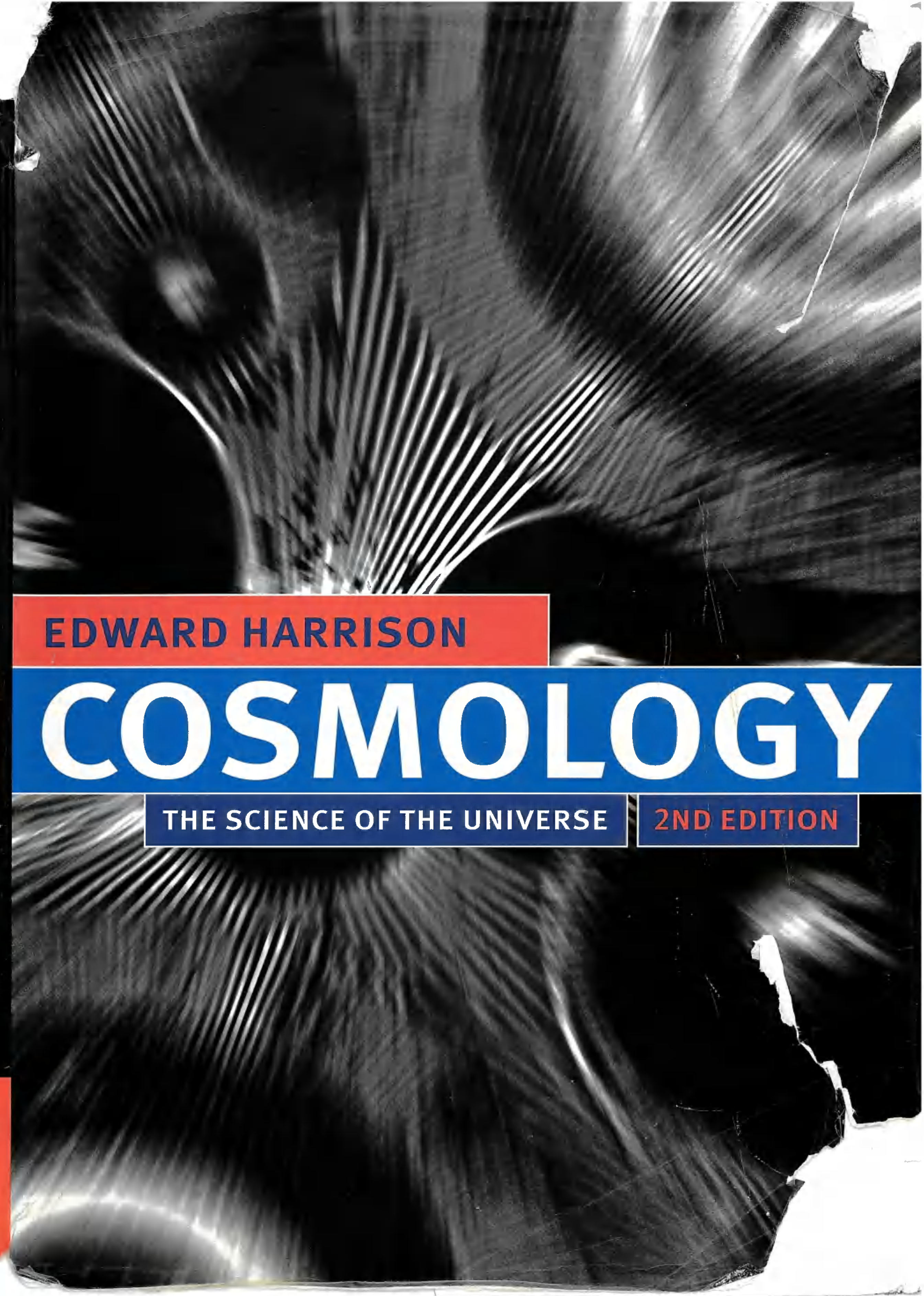

EDWARD HARRISON

COSMOLOGY

THE SCIENCE OF THE UNIVERSE

2ND EDITION

21 HORIZONS IN THE UNIVERSE

I am a part of all that I have met;
Yet all experience is an arch wherethro'
Gleams that untravelled world, whose margin fades
For ever and for ever when I move.
Tennyson (1809–92), Ulysses

WHAT ARE COSMOLOGICAL HORIZONS?

Horizons

We look out in space and back in time and do not see the galaxies stretching away endlessly to an infinite distance in an infinite past. Instead, we look out a finite distance and see only things within the "observable universe." Like the sea-watching folk in Robert Frost's poem, we "cannot look out far" and "cannot look in deep."

The observable universe is normally only a portion of the whole universe. We are at the center of our observable universe; its distant boundary acts as a cosmic horizon beyond which lie things that cannot be observed. Observers in other galaxies are located at the centers of their observable universes that are also bounded by horizons. A person on a ship far from land, who sees the sea stretching away to a horizon, is at the center of an "observable sea." People on other ships are at the centers of their own observable seas that are bounded by horizons. Despite this analogy the horizons of the universe are not as simple as the horizons of the sea.

Particle and event horizons

The subject of cosmic horizons was confusing until Wolfgang Rindler cleared up the muddle in 1956. He showed that in discussing the observable and unobservable we must distinguish between two kinds of observables: things that endure in time and things that have only momentary existence.

World lines in spacetime represent things such as particles and galaxies that endure; they occupy at each instant in time a place in space. Points in spacetime represent events or brief happenings, such as the flash of a firefly or the explosion of a supernova, that occupy a place in space and only a moment in time. World lines are in effect strings of events. In this chapter, the events of main interest emit light, and the world lines of interest, other than the observer's, are of luminous bodies, such as galaxies.

To discover what is observable and what is unobservable we must specify the nature of the things observed. If they are particles or galaxies that endure and have world lines, we discover one kind of answer; if they are events that occur briefly, we discover another kind of answer. For example, if a person is asked, "Have you met Mr. X?" the answer could be quite different to that in response to, "Did you see Mr. X at his wedding?" The first question asks if a world line has been observed at some time or other, and the second asks if an event was observed that occurred at a particular time.

There are thus two types of horizon, a world line horizon and an event horizon, and both are important. Rindler referred to the world line horizon as a "particle" horizon, and because this latter term is now widely adopted we shall continue to use it. It must be understood, however, that the word "particle" in this case means world line and represents anything that endures.

In the following we first define particle and event horizons. With the help of illustrations we try to make clear their significance, first in a static universe, and then in an expanding universe. Our discussion concerns only horizons in universes that are isotropic and homogeneous (i.e., all directions are alike and all places are alike at each instant in cosmic time).

A particle horizon is the surface of a sphere in space that has the observer at the center. This horizon divides the whole of space into two regions: the region inside the horizon contains all galaxies that are visible, and the region outside contains all galaxies that are not visible. Thus the particle horizon is a spherical surface in space that encloses the observable universe. The horizon at sea is of this type; it is a frontier that divides all things into two groups: those inside that are visible and the rest outside that are not visible.

The event horizon divides all events into two groups: those visible at some time or other and those that are never visible. An observer sees events on the backward lightcone. The event horizon is therefore not a surface in space but a null surface in spacetime (in this case the backward lightcone) separating the events that can be observed at some time from the events that can never be observed. The event horizon is not quite so obvious as the particle horizon with its sea-horizon analogy, but this need not cause concern; the next section will help to clarify this obscure subject.

HORIZONS IN STATIC UNIVERSES

The two types of horizon are most easily demonstrated in an infinite and static universe. We forget for a moment that the universe is expanding and suppose that we live in a static universe that contains uniformly distributed galaxies.

Particle horizon

We suppose that the galaxies have been luminous for 10 billion years. Either this hypothetical universe was created 10 billion years ago with luminous galaxies or galaxies

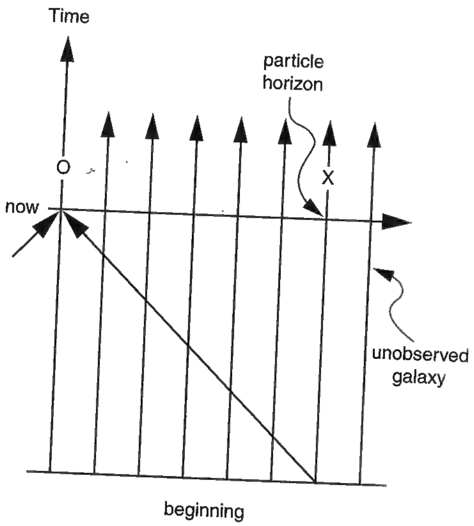

Figure 21.1. This diagram represents a static universe that has a beginning and consists of uniformly distributed galaxies. We are the observer O who looks out now and sees the world lines of luminous galaxies intersecting our backward lightcone. World line X is at the particle horizon. Galaxies having world lines beyond X cannot be seen.

became luminous 10 billion years ago in a preexisting dark universe. The situation is shown in Figure 21.1. The universe consists of world lines of luminous galaxies that commence at a "beginning."

The world line labeled O represents our Galaxy from which we observe the universe. From O, at the instant "now," we look out in space and back in time and see the other galaxies on our backward lightcone. We see galaxies because their world lines intersect our lightcone, and we see each at some instant in its lifetime. All galaxies have been shining for 10 billion years and it is therefore possible to look out and see them stretching away to a distance of 10 billion light years. Galaxies at greater distances cannot be seen because we look back either to a time when the universe was created or to a time before galaxies were born.

A particle horizon divides all luminous sources into those observed and those not observed. Hence, in a static universe the

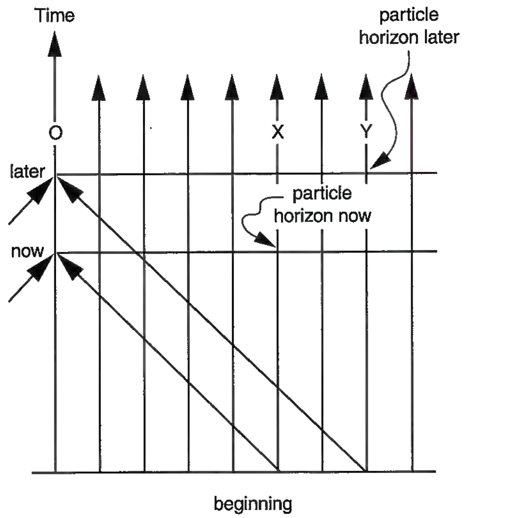

Figure 21.2. At the moment "now" we – the observer O – see no farther than the world line X. Subsequently, at the moment labeled "later," we see beyond X to world line Y. The particle horizon thus recedes in a static universe and the observable universe, bounded by the particle horizon, expands.

particle horizon is at the distance indicated by world line X, and in this example it lies at distance 10 billion light years. Galaxies at distances less than 10 billion light years are visible and lie inside the observable universe, and galaxies at distances greater than 10 billion light years are not visible and lie outside the observable universe.

We wait a period of time – say 1 billion years – and repeat our observations. At the instant "later," shown in Figure 21.2, when galaxies have been shining for 11 billion years, we see them stretching away to a distance of 11 billion light years. The particle horizon has receded to a distance of 11 billion light years. Thus the particle horizon moves outward away from the observer at the speed of light, and although the universe is static, the observable universe actually expands. This is important. In all uniform (i.e., homogeneous and isotropic) universes, static and nonstatic, expanding and contracting, the particle horizon moves outward at the speed of light relative to the galaxies. As time passes we always see more and more of the universe.

Event horizon

We turn now to the event horizon and ask whether in the static universe events exist that can never be seen at any time by an observer. If such events exist, we can divide the universe into two parts: one that contains all the events observable from the observer's world line O; and the other that contains the remaining events unobservable from O. The surface separating the two parts is the event horizon for observer O. ("Observers" in this chapter are immortal; they are born with the universe and die with the universe.)

If the universe is eternal and galaxies shine forever, no event horizon exists. O's lightcone advances up O's world line, and any pointlike event in spacetime will eventually lie on the lightcone and be visible. Hence, in an eternal static universe, in which galaxies are forever luminous, there exists no event horizon and every event in the universe at some time or other is observed by every observer.

An event horizon exists in a universe that has an "end." Either the whole universe terminates, or the galaxies cease to shine and the universe becomes dark. As a result, all world lines of luminous galaxies come to an end, as in Figure 21.3. Figure 21.4 shows clearly that such a universe has an event horizon: it is O's lightcone at the last

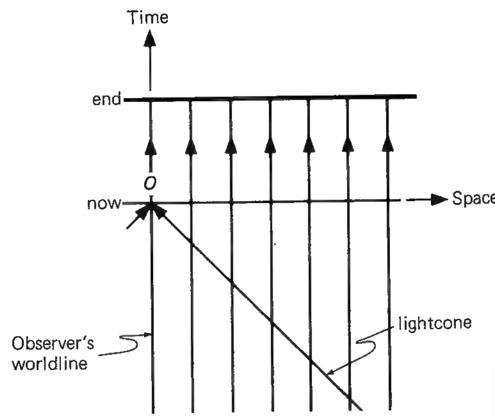

Figure 21.3. A static universe that has an ending. The time labeled "end" is the observer's last moment of observation.

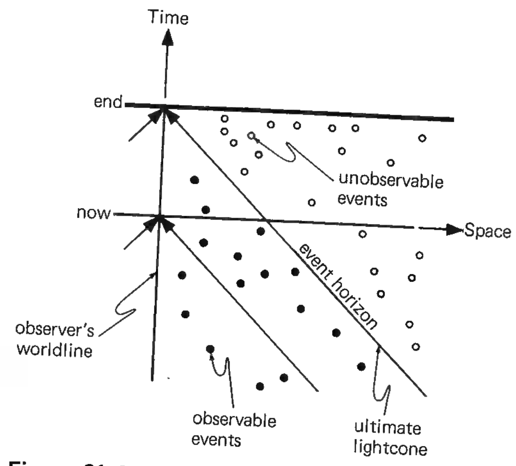

Figure 21.4. The event horizon is the observer's backward lightcone at the moment when the universe ends. Inside this ultimate lightcone are the events that can be observed at some time, and outside are the events that can never be observed.

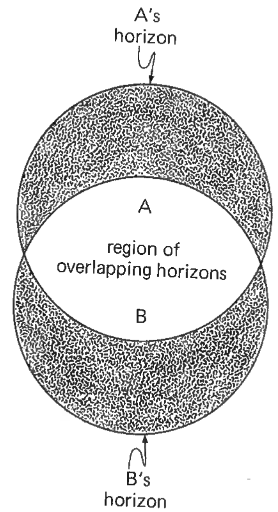

Figure 21.5. Albert (A) and Bertha (B) have overlapping horizons, but each can apparently see things that the other cannot. By communicating with each other can they enlarge their individual horizons into a joint horizon? If they can, then their individual horizons are not true information horizons.

possible moment. Inside the event horizon are the events that have been seen, and outside are the events that can never be seen. The lightcone cannot advance farther into the future and all events outside this ultimate lightcone remain unseen.

The static universe serves to illustrate moderately well the nature of cosmic horizons. From it we learn that beyond the particle horizon are world lines (particles, stars, galaxies) that cannot at the time of observation be seen at any stage in their existence, and beyond the event horizon are events (happenings of short duration) that cannot be seen at any time in the observer's existence.

Before proceeding to nonstatic universes, we must discuss the horizon riddle and the horizon problem.

THE HORIZON RIDDLE

Consider two widely separated observers, A (for Albert) and B (for Bertha). We suppose they can see each other. Each has a horizon such that A cannot see things beyond his horizon and B cannot see things beyond her horizon. Each sees things the other cannot see, as illustrated in Figure 21.5. We ask: Can B communicate to A information that extends A's knowledge of things beyond his horizon? If so, then a third observer C may communicate to B information that extends her horizon, which can then be communicated to A. Hence, an unlimited sequence of observers B, C, D, E,... may extend A's knowledge of the universe to indefinite limits. According to this argument A has no true horizon. This is the horizon riddle.

The riddle arises from our experience with horizons on the surface of the Earth. If A and B are on ships at sea, within sight of each other, they each see the sea stretching away to the horizon. A sees things that B cannot see, and similarly, B sees things that A cannot see. By flag signals or by radio they can keep each other informed of things not directly visible. By communication, A and B share information and succeed in extending their horizons. A pre-twentieth-century admiral had a horizon that embraced his entire fleet.

When we speak of things that are seen or not seen we usually have in mind those that endure and are represented by world lines.

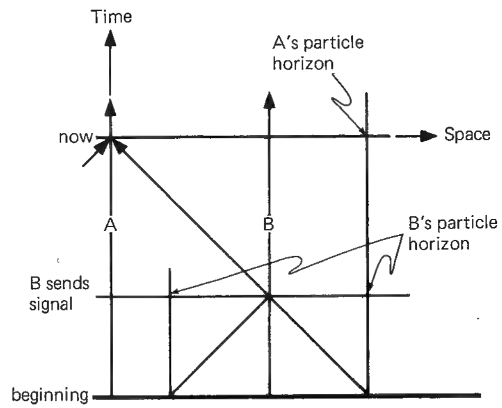

Figure 21.6. Proof that Bertha cannot help Albert to see beyond his horizon (and similarly Albert cannot help Bertha to see beyond her horizon). The horizons in this case are particle horizons: B communicates information to A by sending it at the speed of light on A's backward lightcone; but when B sends the information, her horizon extends no farther than A's horizon, and she cannot see farther than A.

Thus the horizon riddle applies to the particle horizon of the universe. We consider the particle horizon in a static universe (Figure 21.6) and show that the riddle has a simple solution. We have supposed that luminous galaxies originated 10 billion years ago and the particle horizon is therefore at distance 10 billion light years. Observers A and B see each other and have overlapping horizons. Suppose A and B are separated by a distance of 6 billion light years. B sends out information that travels at the speed of light and takes 6 billion years to reach A. Hence A receives from B information that was sent 6 billion years ago when the universe was 4 billion years old. But B's particle horizon in the past at the time when the information was sent was only 4 billion light years distant. Thus B's horizon at that time did not extend beyond A's present horizon. With this argument, and the help of Figure 21.6, we see that neither B nor any other observer can extend A's particle horizon. The particle horizon is a true information horizon and no information can be obtained from other observers concerning what lies beyond. Although we have used the static universe, the argument applies quite generally to particle horizons in all universes.

THE HORIZON PROBLEM

While to deny the existence of an unseen kingdom is bad, to pretend that we know more about it than its bare existence is no better.
Samuel Butler (1835–1902)

For many years cosmologists have debated a subject referred to as the horizon problem. The problem exists in all static and expanding universes that have particle horizons. As an illustration of the problem, consider a static universe of age t_0. An observer cannot see farther than the particle horizon at distance ct_0, where c is the speed of light, or distance t_0 in units of light-travel time. Figure 21.7 shows the observer as a dot at

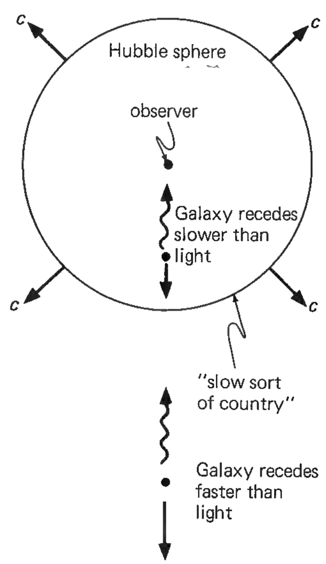

Figure 21.7. A galaxy inside the Hubble sphere recedes from us (the observer) at subluminal velocity, and the light emitted by the galaxy in our direction is able to approach us. A galaxy outside the Hubble sphere recedes from us at superluminal velocity, and the light emitted by the galaxy in our direction is unable to approach us and actually recedes. The edge of the Hubble sphere is the country of the Red Queen – the photon horizon – where the recession velocity of the galaxies is transluminal and light emitted in our direction stands still.

the center of a sphere of radius ct_0. The edge of the sphere is the particle horizon that encloses the observable universe; outside the sphere lies the unobservable universe from which light has not yet reached the observer. The sphere expands as the universe ages and its edge – the particle horizon – recedes from the observer at the speed of light. In the course of time more and more of the universe becomes visible. Thus when the universe was 1 year old, the horizon was at distance 1 light year. When the universe is 10 billion years old, the horizon is at distance 10 billion light years. In the future when the universe is 20 billion years old the horizon will be at distance 20 billion light years. In all universes, static and nonstatic, the particle horizon sweeps past the galaxies at the speed of light and the observer progressively sees more and more of the universe.

Suppose observer O sees A in one direction at distance L and B in the opposite direction also at distance L. How large must L be in order that A and B are unaware of each other's existence at the time when they are seen by O? In a static universe, the answer is $L = \frac{1}{3}ct_0$, as shown in Figure 21.8. More generally, in static and nonstatic universes, A and B cannot see each other when L is greater than $\frac{1}{3}L_P$, where L_P is the distance to the particle horizon. When A and B are each farther away than one-third the particle horizon distance L_P, they see

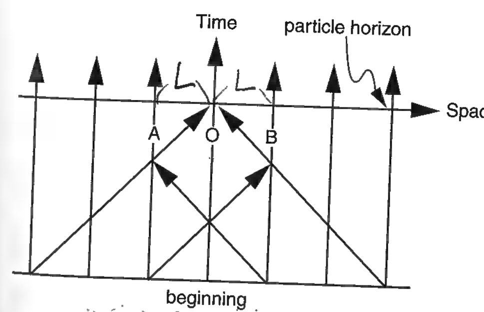

Figure 21.8. The observer sees A and B at equal distances L in opposite directions. When L is greater than one-third the distance to the particle horizon, A and B are unaware of each other's existence.

At the time of observation, A and B have been in causal contact already

the observer but cannot see each other. They do not know that each other exists. To make clear the nature of the problem, imagine that A and B have similar genetic coding. If they do not know that each other exists – are outside each other's horizon – and previously had no history of interaction, how can we explain why they are genetically alike? Stated more generally, why should galaxies, stars, chemical elements, and subatomic particles exist in similar states when their horizons do not overlap?

The particle horizon is important because it determines not only the maximum distance an observer can see, but also the maximum distance between things that are able to communicate and affect one another. It determines the range of causal interactions. A body observed at distance $L = \frac{1}{2}L_P$, and now inside our horizon, was outside at a cosmic age earlier than $\frac{1}{2}t_0$. Normally we look back into the past for causes that explain the way things are now. But how can we explain the way things are now on the scale of 10 billion light years by causes that existed when the universe was less than 10 billion years old? This is the horizon problem.

The horizon problem has no known scientific solution in a static universe. A possible scientific solution in an expanding universe requires a period of accelerated expansion in the early universe. Alan Guth introduced the idea of accelerated expansion as a serious possibility in 1981 and called it inflation, and his inflationary model is discussed in Chapter 22.

HUBBLE SPHERES

Static universes serve to illustrate the fundamental nature of cosmological horizons but are not very realistic. First, in a preambling manner we discuss a few basic properties of expanding universes.

According to the velocity–distance law the recession velocities V of the galaxies increase linearly with distance L:

$$V = HL, \qquad [21.1]$$

where H is the Hubble term and L is the sort of distance one would obtain with a tape measure stretched on a curved surface. The value of H at the present epoch is $H_0 = 100h$ km per second per megaparsec and the coefficient h lies perhaps between 0.5 and 1. When L is doubled the recession velocity V is also doubled. Distances are measured in the world map that covers homogeneous space at a common instant in cosmic time. In a homogeneous universe that stays homogeneous during expansion, the velocity–distance law is necessarily linear in the world map (i.e., velocity must increase in strict proportion to distance). Thus if the universe consists of infinite space, at infinite distance the recession velocity is infinitely large. The velocity–distance law $V = H_0 L$ tells us how fast a galaxy at distance L recedes at the present time.

The recession velocity equals the velocity of light at the Hubble distance

$$L_H = \frac{c}{H_0}, \qquad [21.2]$$

and at the present time

$$L_H = 9.8 \times 10^9 h^{-1} \text{ light years.} \qquad [21.3]$$

The Hubble sphere distance lies somewhere between 10 and 20 billion light years, and for illustration we assume an intermediate value 15 billion light years. From Equations [21.1] and [21.2] we have

$$\frac{V}{c} = \frac{L}{L_H}, \qquad [21.4]$$

and this shows clearly that galaxies at distance L greater than L_H recede faster than the velocity of light. According to the expanding space paradigm, galaxies are stationary in space and recede from one another because of the expansion of intergalactic space. We are at the center of our Hubble sphere, a sphere whose present radius is 15 billion light years. Inside this Hubble sphere are the galaxies that recede slower than light velocity, and outside are those that recede faster than light velocity. The Hubble sphere must not be confused with the observable universe. The observable universe is bounded by the particle horizon, and if the Hubble sphere and observable universe were the same, the observable universe would be infinitely large in a static universe ($H_0 = 0$, $L_H = \infty$). But static universes of finite age have particle horizons at finite distance, and therefore the Hubble sphere cannot be the observable universe.

RECEPTION AND EMISSION DISTANCES

We look out and see galaxies of various redshifts and must be careful about assuming that distance increases always with redshift. Reception distances (measured in the world map) increase always with redshift. The larger the redshift, the greater the distance at the time of reception. But we cannot see galaxies at their present distances; we see them in the past at the time when they emitted the light now seen, and their emission distances do not continually increase with redshift in a big bang universe.

Each receding galaxy has two distances: the distance at the time of reception and the distance at the time of emission. The first is the reception distance measured in the world map and denoted by L, and the second is the emission distance and denoted by L_{emit} (see Figure 15.6). These two distances have the simple redshift relation

$$\frac{L}{L_{\text{emit}}} = 1 + z, \qquad [21.5]$$

Reception distance = Comoving Distance!

where z is the redshift of the galaxy. The reception distance is always greater than the emission distance in an expanding universe.

The emission distances of galaxies increase at low redshifts and decrease at high redshifts. Faint galaxies of large redshifts seemingly far away were actually nearer to us at the time of emission than bright galaxies of small redshifts. This odd state of affairs occurs in all expanding universes in which the deceleration term q is greater than -1. It does not occur in the de Sitter and steady-state universes in which q is equal to -1.

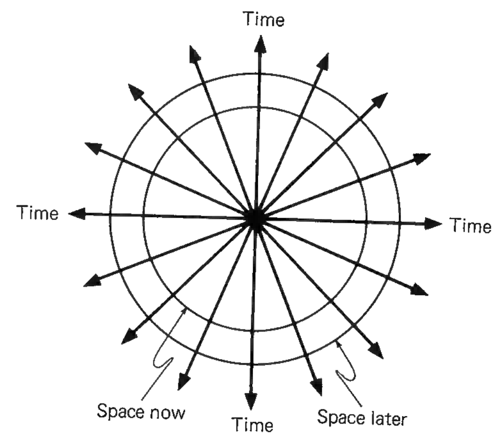

Figure 21.9. A big bang with world lines diverging in all directions. Do not let this diagram mislead you into thinking that the big bang occurs at a point in space. Time is measured along the world lines, and space is represented by any spherically curved surface perpendicular to the world lines.

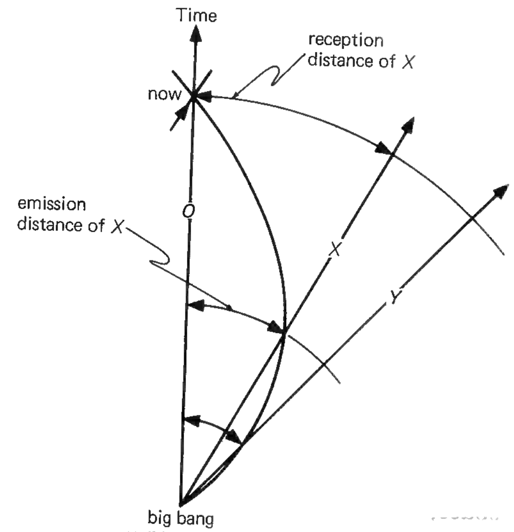

Figure 21.10. O's lightcone curves back into the big bang, and for this reason we are able to observe the cosmic background radiation that has traveled freely since it decoupled in the early universe. This diagram shows the reception and emission distances of galaxy X. Although galaxy Y has a greater reception distance, its emission distance is smaller than that of X. Thus Y, which is now farther away than X, was closer to us than X at the time of emission of the light we now see.

A spacetime diagram convenient for our immediate purpose is shown in Figure 21.9, in which world lines diverge radially in all directions from a big bang. Space is represented by spherical surfaces perpendicular to the world lines, and time is measured along the radial world lines. World lines are galaxies fixed in space, and as time advances, they recede from one another because of the expansion of space. An expanding spherical balloon is a helpful analogy. Galaxies are points marked on the surface of the balloon and as the balloon inflates the "galaxies" recede from one another. The two-dimensional surface of the balloon represents our three-dimensional space. The radial direction in which the balloon expands represents time and should not be confused with the third dimension of space.

Any world line – it does not matter which – is chosen as the observer and labeled O, as in Figure 21.10. At any instant in time – call it "now" – the observer's lightcone stretches out and back and intersects other world lines such as X and Y. Because of the expansion of space, the lightcone does not stretch out straight as in a static universe, but contracts back into the big bang. All world lines and all backward lightcones converge into the big bang. The observer, by looking in any direction, looks back into the big bang, and the light the observer receives from the big bang is the cosmic background radiation. The luminous events on the backward lightcone are redshifted, and the closer they are to the big bang, the larger is their redshift. Thus redshift increases steadily as we proceed along the lightcone toward the big bang.

Figure 21.11 shows the emission and reception distances of two galaxies X and Y. X's emission distance is measured in space at the time X emitted the light that O now sees, and X's reception distance is measured in space at the time its light reaches O. Similarly with Y. As shown, X's reception distance is smaller than Y's and we may say X is now nearer than Y. Also X's redshift is smaller than Y's redshift.

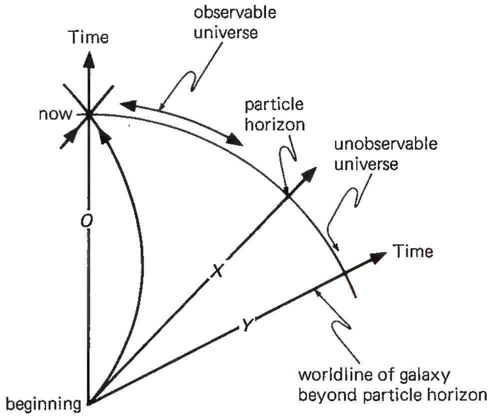

Figure 21.11. At the instant labeled "now" the particle horizon is at worldline X. In a big bang universe, all galaxies at the particle horizon have infinite redshift.

Redshift always increases with reception distance. As we move back along the lightcone, the emission distance at first increases, then reaches a maximum, and thereafter decreases. In Figure 21.10, the emission distance of Y is less than that of X, even though Y is now farther away and has the greater redshift. The maximum emission distance L_{max} in the Einstein–de Sitter universe is 8/27 of the Hubble distance L_H, or almost 5 billion light years, and has redshift of 1.25. Galaxies at distance L_{max} at the time of emission had a recession velocity equal to the velocity of light; they were at the edge of the observer's Hubble sphere at the time of emission. Notice in Figure 21.10 that when Y emitted light toward O, the backward lightcone was diverging away from O's world line. The light rays leaving Y and moving toward O were at first dragged away from O by the expansion of space, and then, on reaching the maximum distance, began to approach O.

THE PHOTON HORIZON IN COSMOLOGY (= Hubble sphere)

Country of the Red Queen

A galaxy outside the Hubble sphere emits a ray of light in our direction, as shown in Figure 21.7. Although the ray hurries toward us, it actually recedes; it travels through space at the speed of light, but the space through which it travels recedes from us faster than light. As Eddington in 1933 wrote: "Light is like a runner on an expanding track with the winning-post receding faster than he can run." All light rays emitted in our direction within the Hubble sphere approach us, whereas all light rays emitted outside the Hubble sphere recede from us. At the edge of the Hubble sphere, the light rays traveling toward us stand still; they hurry toward us at the same velocity that expanding space carries them away. "Now, here, you see, it takes all the running you can do, to keep in the same place," said the Red Queen to Alice.

All galaxies inside the Hubble sphere recede subluminally (slower than light) and all galaxies outside recede superluminally (faster than light). The edge of the Hubble sphere is what might be called the photon horizon, a curious sort of horizon, not of particles but of photons. Light rays moving toward us inside the photon horizon approach us, all light rays outside must recede. The photon horizon, where recession is transluminal, is the country of the Red Queen.

Galaxies outside the Hubble sphere recede superluminally, and their light rays recede, but this does not mean that galaxies and their events outside the photon horizon are permanently hidden from the observer's view. If that were so, the photon horizon would also be an event horizon. In most universes the Hubble term H is not constant. In a decelerating universe, in which the Hubble term decreases with time, the Hubble distance $L_H = c/H$ increases, and the Hubble sphere expands in the comoving frame. Hence, it expands faster than the universe and the edge of the Hubble sphere – the photon horizon – overtakes the receding galaxies. Light rays outside the Hubble sphere moving toward us may therefore eventually be overtaken by the photon horizon; they will then be inside the Hubble sphere and will at last start approaching us. Eddington's runner sees the winning-post receding, but he must keep running and not give up; the expanding track is slowing down and eventually the winning-post will be reached.

wrong answer
But the particle horizon increases faster than that
correct answer
$L_H = \frac{c}{H}$ increases as H decreases!

How fast does the Hubble sphere expand? Its radius is $L_H = c/H$, and it expands at velocity $U_H = dL_H/dt$, and this can be shown to be (Equation 14.29)

$$U_H = c(1+q), \qquad [21.6]$$

where q is the deceleration term. This is the recession velocity of the photon horizon. Galaxies at the photon horizon recede at the velocity of light c, whereas the horizon itself recedes at the velocity $c(1+q)$. In a decelerating universe, such as a Friedmann model, the deceleration term q is positive and the Hubble sphere expands faster than the universe. Thus the photon horizon overtakes the galaxies and sweeps past them at relative velocity cq. The deceleration term in the Einstein–de Sitter universe, for example, has a value 0.5, and hence $U_H = 1.5c$, and the photon horizon sweeps past the galaxies at $0.5c$.

THE PARTICLE HORIZON

The receding particle horizon

We recall that beyond the particle horizon are galaxies that at the present time cannot be observed at any stage in their evolution. Their world lines do not intersect the observer's backward lightcone. Inside the particle horizon are the galaxies whose world lines do intersect the backward lightcone, and they comprise the observable universe.

The spacetime diagrams shown in Figures 21.11 and 21.12 illustrate the nature of the particle horizon in an expanding universe. In Figure 21.11 we see how space at the moment "now" divides into two regions: the nearer contains all world lines intersecting the observer's backward lightcone; the farther contains all world lines not intersecting the backward lightcone. The first region is the observable universe, the second region is the unobservable universe, and the particle horizon separates the two. The distance of the particle horizon is measured in the observer's world map (the space at

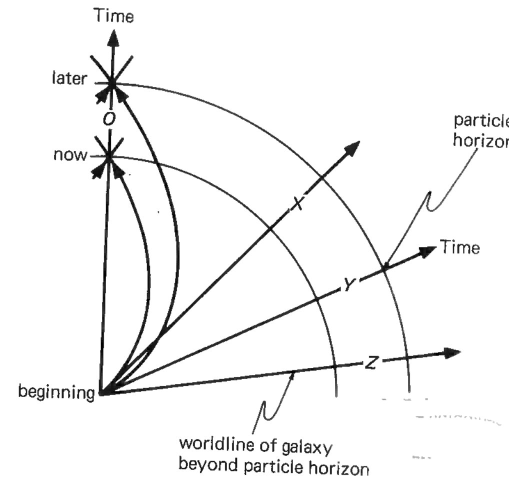

Figure 21.12. At the instant labeled "later" the particle horizon has receded to world line Y. Notice the distance of the particle horizon is always a reception distance, and the particle horizon always overtakes the galaxies and always the fraction of the universe observed increases.

time "now") and is the reception distance of galaxies of infinite redshift. The redshift is infinite because L_{emit} in Equation [21.5] is zero.

We consider a later instant, labeled "later" in Figure 21.12, when the universe is more expanded. Clearly, the observer's lightcone intersects more world lines and the particle horizon is at a greater distance. Thus the particle horizon recedes and the observable part of the universe expands faster than the universe itself. We have seen that in a static universe the particle horizon sweeps out past the galaxies at the speed of light, and it can be shown quite generally in all universes, static and nonstatic, that the particle horizon sweeps past the galaxies at the speed of light. The observed fraction of the universe always increases. The particle horizon at distance L_P recedes at velocity $U_P = dL_P/dt$, and it can be shown

$$U_P = c + H_0 L_P. \qquad [21.7]$$

At the particle horizon the galaxies recede at velocity $V_P = H_0 L_P$, and the particle horizon itself recedes at $U_P = c + V_P$; hence the

$H^{-1} = \frac{3}{2}t$
$L_P = 3ct = 2cH^{-1}$

horizon overtakes the galaxies at the speed of light c.

The Einstein–de Sitter universe illustrates what happens. We find that the particle horizon is at twice the Hubble distance. Thus the observable universe has a radius twice that of the Hubble sphere. The recession velocity of the galaxies at the particle horizon, according to the velocity–distance law, is twice the speed of light. Because the particle horizon overtakes the galaxies at the speed of light, the particle horizon in the Einstein–de Sitter model recedes at three times the speed of light. The redshift of the galaxies at the photon horizon is $z = 3$, and at the particle horizon is infinite.

$z \to \infty$ at L_P.

Notice that galaxies at the photon horizon recede at velocity c and yet have finite cosmological redshift. (The redshift for the special relativity Doppler effect would be infinite, demonstrating once again that cosmological redshifts are not a Doppler effect.) In universes of constant positive deceleration q, the distances of the particle and photon horizons have the ratio $L_P/L_H = 1/q$. In the radiation-dominated early universe, $q = 1$, and the Hubble sphere and observable universe have the same size; the photon and the particle horizons are coincident and both are often referred to as the "horizon," although they have distinctly differently properties. Generally, when q is not constant, comoving bodies can be inside and outside the Hubble sphere at different times. But not so for the observable universe; once inside, always inside. Horizons are like membranes; the photon horizon acts as a two-way membrane (comoving bodies can cross in both directions depending on the value of q), and the particle horizon acts like a one-way membrane (comoving bodies always move in and never out).

Universes without particle horizons

Some universes, such as the Milne, de Sitter, and steady-state universes, lack particle horizons. In these universes, all world lines intersect an observer's backward lightcone and all galaxies in the universe are visible at some stage in their evolution. To show why such universes are possible we use a different type of spacetime diagram. This diagram, shown in Figure 21.13, depicts comoving rather than ordinary space coordinates. All comoving bodies are separated from one another by constant comoving distances, and in this new diagram all world lines are parallel. But light rays are not straight lines. The backward lightcone does not diverge as in the static universe but flares out. In universes with particle horizons, such as the Friedmann versions, the lightcone extends back to the beginning of the universe at finite comoving distance and a particle horizon exists at world line X. In other universes, however, such as the Milne model (in which the scale factor $R = t$ and $H = 1/t$, $q = 0$), the lightcone reaches the beginning of time $t = 0$ at an infinite comoving distance and there is no particle horizon. The observable universe fills the entire actual universe and all galaxies are in principle visible. The de Sitter and steady-state universes are of this kind, but are more complicated and will be considered when we discuss event horizons.

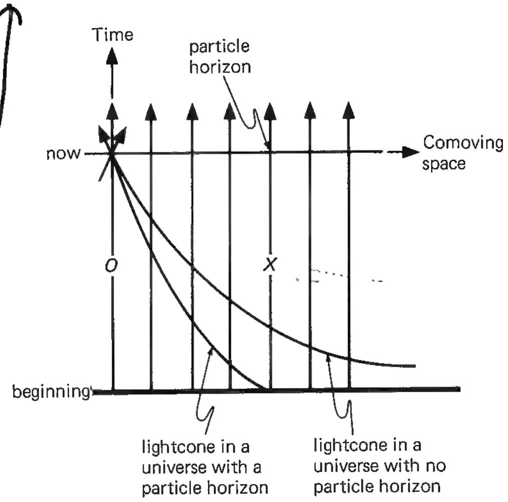

Figure 21.13. A spacetime diagram of comoving space (in which all world lines are parallel) and cosmic time. Some universes have particle horizons and in their case the lightcone stretches out and back to the beginning at a finite comoving distance indicated by the world line X. In universes without particle horizons, the lightcone stretches out to an unlimited distance and intersects all world lines.

CONFORMAL DIAGRAMS

The time has come to introduce the reader to a powerful tool. Quite simply, we transform the units of space and time so that the spacetime diagram looks the same as for a static and flat universe. Everything that we have learned about horizons in the simple static Euclidean universe then applies to all universes, static and nonstatic, flat and curved. This mathematical tool is known as a conformal transformation. In our case it is conformal because the spacetime diagram resembles (conforms to) the Minkowski diagram of special relativity and leaves unchanged spatial angles.

In Figure 21.13, comoving space takes the place of ordinary space. This kind of diagram has the advantage that all world lines of comoving bodies are parallel to one another. They look like the world lines of stationary bodies in a static universe. But light rays are not straight and the lightcone spreads out awkwardly, as in Figure 21.13, and the horizons are not obvious. We have already altered the intervals of space from ordinary to comoving space; we now in addition alter the intervals of time so that light rays are straight and the lightcone is conical, as in the static universe. We then have a spacetime diagram of conformal space and conformal time (Figure 21.14) that looks like the ordinary spacetime diagram of a static universe of Euclidean geometry. Because we know how to handle horizons in a static universe, we now know how to handle horizons in nonstatic universes.

The spacetime interval between two events close together is given by the line element

$$(\text{spacetime interval})^2 = (\text{time interval})^2 - (\text{space interval})^2, \qquad [21.8]$$

in which space intervals are measured in light-travel time. We have already changed space intervals into $R \times$ intervals of comoving (conformal) distance, and this suggests we change time intervals to $R \times$ intervals of conformal time. In the new conformal line element the spacetime interval becomes

$$(\text{spacetime int.})^2 = R^2[(\text{conf. time int.})^2 - (\text{conf. space int.})^2], \qquad [21.9]$$

with obvious abbreviations. All light rays follow paths – technically called null geodesics – on which spacetime intervals are zero. Thus if a light ray travels 1 light second in 1 second, the spacetime interval is zero, and from Equation [21.9] the equation for the lightcone is

$$(\text{conf. space int.})^2 = (\text{conf. time int.})^2,$$

and hence

$$\text{conf. space int.} = \pm\text{conf. time int.}, \qquad [21.10]$$

where the plus sign is for the forward lightcone into the future and the minus sign is for the backward lightcone into the past. This last relation (Equation 21.10) between intervals of conformal space and time is for a spacetime, shown in Figure 21.14, in

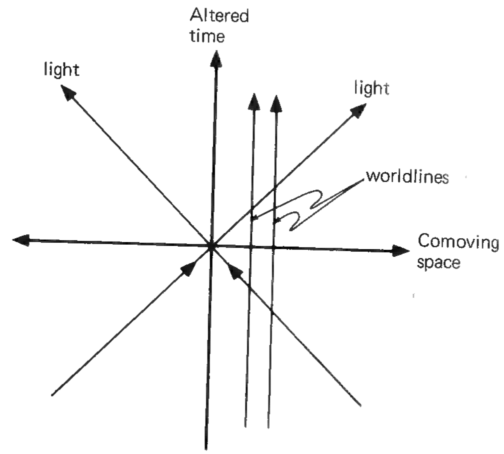

Figure 21.14. A spacetime diagram of conformal coordinates consisting of comoving space and altered time. When we straighten out the lightcone by altering the intervals of time, the spacetime diagram of a nonstatic universe looks like that of a static universe. This coordinate transformation allows us to study horizons in nonstatic universes just as easily as in static universes.

which world lines are parallel and light rays are straight as in a static universe. The advantage of this kind of diagram is that it allows us to treat horizons in the same way as for a static universe. Four possibilities must be considered:

(a) The universe has a beginning and an ending in conformal time, as in Figure 21.15. The closed Friedmann universe that begins and ends with big bangs belongs to this class.

(b) The universe has a beginning but no ending in conformal time, as in Figure 21.16. The Einstein–de Sitter universe and the Friedmann universe of negative curvature, which begin with a big bang and expand forever, belong to this class.

(c) The universe has an ending but no beginning in conformal time, as in Figure 21.17. The de Sitter and steady-state universes belong to this class.

(d) The universe has no beginning and no ending in conformal time, as in Figure 21.18. The Einstein static, the Eddington–Lemaître, and the Milne universes are members of this class.

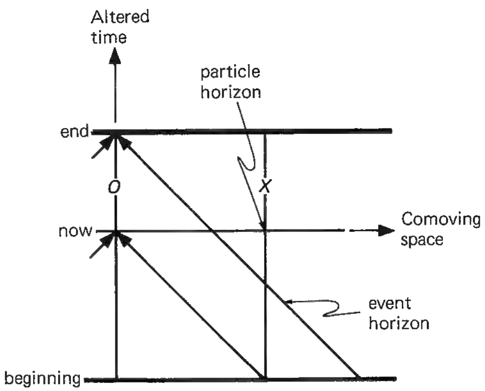

Figure 21.15. A conformal spacetime diagram in which altered time has a beginning and an ending. The world line X is at the particle horizon. Notice the existence of an event horizon.

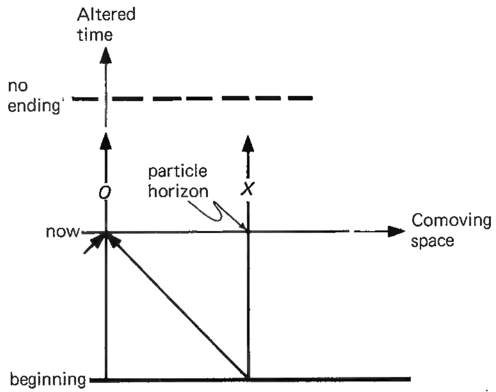

Figure 21.16. A conformal spacetime diagram in which altered time has a beginning but no ending. The particle horizon is at world line X, and no event horizon exists.

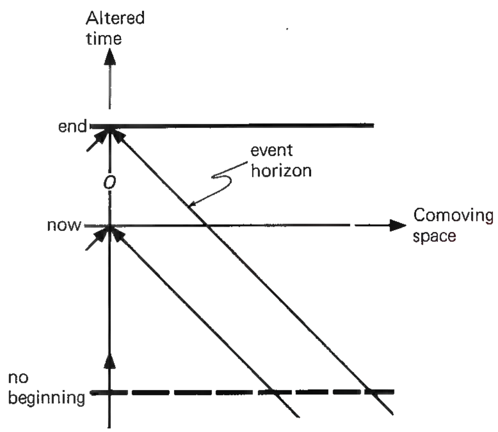

Figure 21.17. A conformal spacetime diagram in which altered time has an ending but no beginning. Only an event horizon exists.

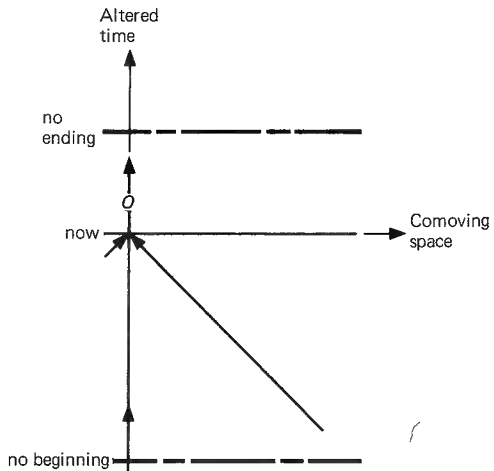

Figure 21.18. A conformal spacetime diagram in which altered time has no beginning and no ending. There are no particle and event horizons.

We can now see which universes have particle horizons. The necessary condition for a particle horizon is that conformal time has a beginning, as in class (a) shown in Figure 21.15, and in class (b) shown in Figure 21.16. The observer's lightcone stretches back and terminates at the lower boundary where the universe begins. When conformal time has no beginning, hence no lower boundary, as in class (c) shown in Figure 21.17, and in class (d) shown in Figure 21.18, the lightcone stretches back without limit and intersects all world lines in the universe. In these universes there are no particle horizons. Note that a beginning in conformal time does not necessarily mean a beginning in ordinary time.

By constructing spacetime diagrams with coordinates that are conformal with those of a static universe, we find that particle horizons exist when conformal time has a beginning. As in ordinary time in the static universe, the observer's lightcone advances in conformal time into the future and the particle horizon recedes. Once a galaxy is inside the particle horizon, and part of the observable universe, it remains always inside the particle horizon, as seen in Figure 21.19.

EVENT HORIZONS

The ultimate lightcone

Inside the observer's event horizon exist events that can be observed at some time or other, and outside exist events that can never be observed. With our new spacetime diagrams of conformal time and conformal (or comoving) space, event horizons are easy to understand. Let us consider an event located somewhere in spacetime (Figure 21.20). The observer's lightcone advances up O's world line, sweeping through spacetime, and the event eventually lies on the lightcone and the observer sees it. In this way, in the course of time, all events are disclosed to the observer.

But this is not so when conformal time has an ending, as in class (a) shown in Figure 21.15, and in class (c) shown in Figure 21.17. In these classes there exist events that can never be seen, as in Figure 21.20: O's lightcone cannot advance beyond the upper limit of conformal time, and the events that never lie on O's lightcone will never be observed.

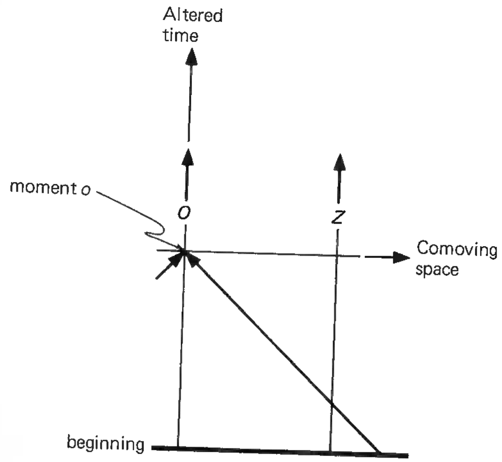

Figure 21.19. As the moment of observation *o* advances up the observer's world line O, the particle horizon recedes. Once a world line, such as Z, lies within the particle horizon, it remains inside forever. This means that a galaxy inside the observable universe will remain inside and always observable as long as it exists.

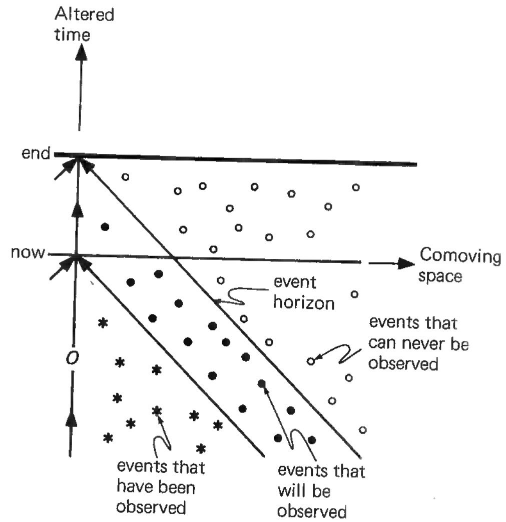

Figure 21.20. O's lightcone cannot advance beyond the end of altered time. Hence there are events that can never be observed by O, and the ultimate lightcone is the event horizon.

The necessary condition for an event horizon is that conformal time has an ending. The event horizon in cosmology is thus nothing more than the observer's ultimate lightcone at the end of conformal time. All the events inside the event horizon (the ultimate lightcone) are at some time observed, and all events outside are never observed. Note that an end in conformal time does not necessarily mean an end in ordinary time.

Blueshifts and redshifts at event horizons

Universes that end in big bangs, such as the closed Friedmann universe, have event horizons. An observer (world line O) in a collapsing universe receives signals from event *a* (Figure 21.21) at redshift

$$z + 1 = R_0/R, \qquad [21.11]$$

where R_0 is the value of the scaling factor at the time of reception and R the value at the time of emission. Because the universe is collapsing, R_0 is less than R, and the redshift is negative. A negative redshift means that light is blueshifted toward the blue end of the visible spectrum. At the last moment of observation R_0 is zero. Hence the redshift is -1 and the blueshift is maximum. Everything seen close to the event horizon happens rapidly, and at the last possible moment, at the event horizon, happens infinitely rapidly.

There are universes that expand forever and yet have endings in conformal time. The de Sitter and steady-state universes are of this kind and therefore have event horizons. But R_0 is now not zero but infinity. At the event horizon all events seen have infinite redshift and happen infinitely slowly.

An alternative way of looking at the de Sitter and steady-state universes considers a spacetime diagram of conformal (comoving) space and ordinary cosmic time, as in Figure 21.22. The observer O at moment *o* sees event *a*. As the moment of observation advances into the unlimited future, the lightcone moves upward and approaches more and more slowly but

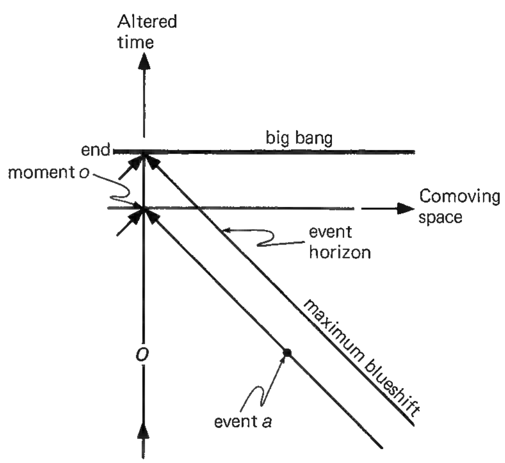

Figure 21.21. Collapsing universes that terminate in big bangs have an end in altered time (and in cosmic time), and therefore possess event horizons. At moment *o*, observer O looks back into the past and sees all events, such as *a* on A's world line, blueshifted. As the moment *o* of observation advances and approaches the end, events are seen with increasing blueshift, and at the last possible moment, all events on the observer's backward lightcone are seen to happen infinitely rapidly. In this case, the event horizon has maximum blueshift.

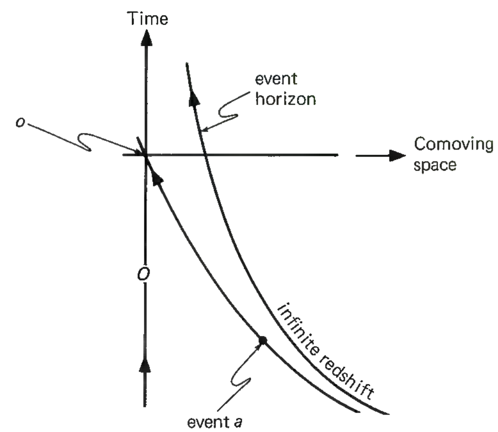

Figure 21.22. This shows the de Sitter universe in a spacetime diagram of comoving space and cosmic time. The event horizon is as shown. As the moment *o* now advances into the infinite future, the observer's backward lightcone approaches the event horizon more and more closely. Events close to this horizon have large redshifts because of expansion, and at the horizon all events have infinite redshift.

never reaches the event horizon. The event horizon is the observer's lightcone in the infinite future. Events outside this horizon can never be observed.

Exponentially expanding universes

Admittedly, our comments so far have not greatly clarified what happens in the de Sitter, steady-state, and other exponentially expanding universes, such as the inflationary universe. We first note that the Hubble term is fixed and the Hubble sphere has constant radius. At the edge of the Hubble sphere – the photon horizon – the recession velocity equals the velocity of light, and calculation shows that in these universes the redshift at the photon horizon is infinite. The Hubble sphere is the observable universe. Why is the boundary of the Hubble sphere an event horizon and not a particle horizon? The observed galaxies are carried farther and farther away from the observer and become progressively more redshifted. What happens to these galaxies – do they eventually cross the edge of the Hubble sphere and disappear from view? These questions, which perplexed many cosmologists in the past, can be answered with the help of Figure 21.23.

The boundary of the Hubble sphere has become a true horizon. It is not a particle horizon because at any instant all galaxies in the universe are visible to an observer. Any galaxy now beyond the Hubble sphere, no matter how far away, had in the past a part of its world line inside the Hubble sphere and is therefore observable at some stage in its history. All galaxies recede and move out of the Hubble sphere, yet the observer never sees them crossing the Hubble boundary.

Exponentially expanding universes have spacetime diagrams in comoving space and conformal time of the kind shown in Figure 21.17. We see that these universes have event horizons but no particle horizons. In the case of these universes, however, it is more convenient to use a diagram of ordinary space and ordinary time, as in Figure 21.23. The Hubble sphere is of constant radius, as shown, and all world lines diverge away from the observer's world line O. The lightcone approaches asymptotically the edge of the Hubble sphere, intersecting all world lines in the universe, and never extends beyond the Hubble sphere. No particle horizon exists because all world lines intersect the observer's lightcone. The edge of the Hubble sphere is an event horizon because it is the observer's ultimate lightcone, and all events outside the Hubble sphere can never be observed.

Figure 21.23 makes clear that the farther the observer looks out in space and back in

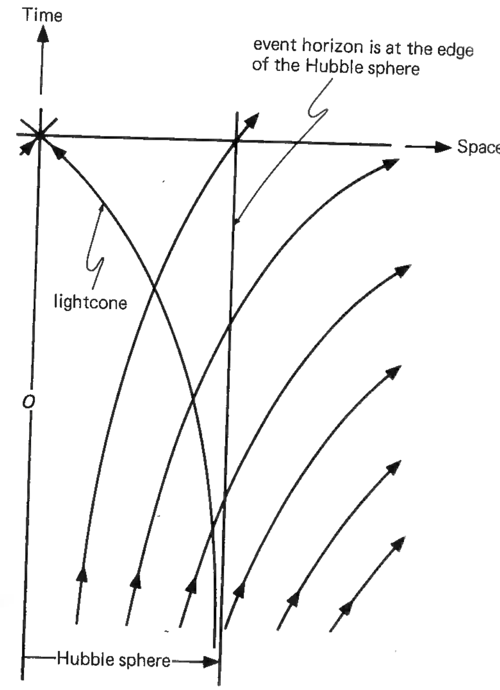

Figure 21.23. The de Sitter universe displayed in ordinary space and ordinary time. The Hubble sphere has constant radius about the observer's world line O. All world lines diverge away from O, and every world line intersects the edge of the Hubble sphere and at some time is inside the Hubble sphere. The observer's lightcone curves back, as shown, and approaches but never crosses the edge of the Hubble sphere. The observer sees all world lines and there is no particle horizon. Events outside the Hubble sphere are never observed and the edge of the Hubble sphere is the event horizon, and is the observer's ultimate backward lightcone.

time the closer the galaxies approach the edge of the Hubble sphere and the greater becomes their redshift. But the observer never sees the galaxies disappearing across the edge. At the edge of the Hubble sphere are crowded an infinite number of infinitely redshifted galaxies.

Galaxies, of course, do not shine forever, and as luminous sources their world lines are therefore of finite length. This is something for the reader to think about and make suitable amendments where necessary in what has previously been said. World lines of finite length, as in Figure 21.24,

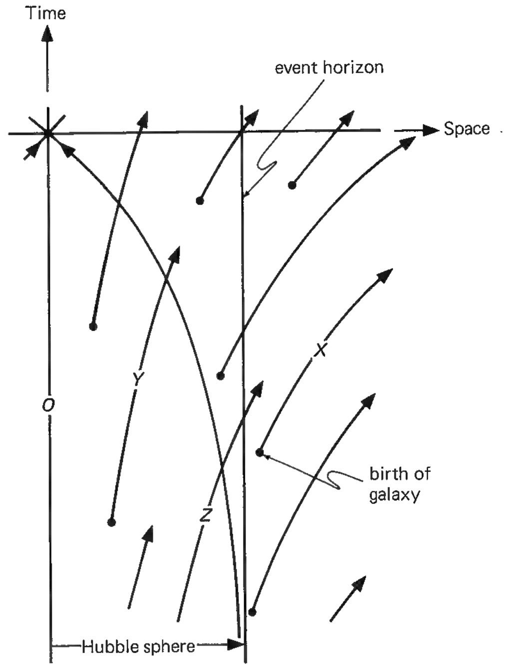

Figure 21.24. In the steady-state expanding universe, galaxies are continually created so as to maintain a constant average density. Galaxies are born, and in this diagram die when they cease to be luminous. Most galaxies, such as X, are born outside the Hubble sphere and are never seen by O; some galaxies, such as Y, are born inside the Hubble sphere and may die before reaching the Hubble edge; and other galaxies, such as Z, cross the Hubble edge while still luminous. As in the de Sitter universe, the number of galaxies of infinite redshift at the event horizon is infinitely great. Thus there are an infinite number of galaxies crowded at the edge of the Hubble sphere.

were of particular interest in the case of the steady-state universe. This universe expands in the same way as the de Sitter universe and has an event horizon at the edge of the Hubble sphere. New galaxies continually form everywhere, maintaining a constant average density of matter, and it is therefore not true to say that all galaxies are observable in a continuous creation steady-state universe. Galaxies do not originate in the infinite past inside the observer's Hubble sphere, but in the finite past and mostly outside the Hubble sphere. World line X in Figure 21.24 is an example of a galaxy never seen by the observer. A galaxy formed inside the Hubble sphere may die and become nonluminous before it reaches the Hubble boundary, as indicated by world line Y. The number of luminous galaxies crossing the Hubble edge, however, having world lines such as Z, is still infinite. At the event horizon, where the redshift is infinite and light rays emitted in our direction stand still, there exists an infinite number of galaxies.

REFLECTIONS

1 *Cosmological horizons were investigated in 1956 by Wolfgang Rindler in a classic paper ("Visual horizons in world-models") in which he wrote, "A horizon is here defined as a frontier between things observable and things unobservable," and he distinguished between two kinds of "things," events and world lines (particles), thus leading to two kinds of horizon: the event horizon and the particle horizon.*

2 *The rational method explains present conditions as the result of past conditions: "things are as they are because things were as they were." This method becomes embarrassing when initial conditions must be arranged in special and even improbable ways to explain present conditions. We are left wondering what explains the special initial conditions. Perhaps this is true of all rational universes?*

3 *"We are unable to obtain a model of the universe without some specifically cosmological assumptions that are completely unverifiable" (George Ellis, "Cosmology and verifiability," 1975). The problem is that we observe isotropy, which we cannot explain, and we assume homogeneity, which we cannot verify. We observe only those events that lie on our backward lightcone, as in Figure 21.25, and the rest of spacetime – except for a small region about our world line – is unobserved. All our knowledge of the universe is limited to a small region surrounding the Earth's world line and around the backward lightcone. The theory of inflation (Chapter 22) changes this picture, but so far inflation is itself an unverified theory.*

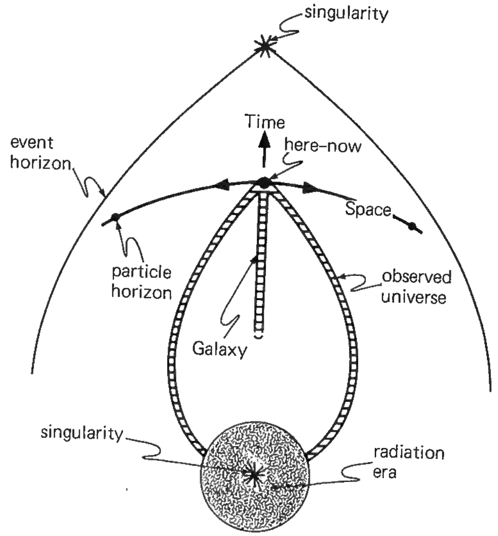

Figure 21.25. The observed universe consists of only those events that lie on the observer's backward lightcone. A small region about the observer's world line contains events not on the lightcone whose existence can be inferred from the immediate environment. The history of the Galaxy, the Solar System, the Earth, and the human race is confined to this region. All the rest of spacetime contains events that at present are unobserved. If there is an event horizon, then beyond this horizon lie all the events that can never be observed.

4 *How can the properties of the universe be explained by causes in the past when interactions over large distances could not exist? The horizon problem became more urgent with the discovery of the smoothness and isotropy of the cosmic background radiation that decoupled at the end of the early universe (Chapter 20) when the age of the universe was a few hundred thousand years. The difference in the radiation from opposite sides of the sky is less than 1 part in 100 000. Yet the emitting regions at the time of decoupling were far apart and outside each other's particle horizons. If these emitting regions do not "know" that each other exists, how can they be in identical states? The beauty of inflation is that it solves this problem.*

5 *Consider two visible bodies at equal distances in opposite directions from us, as shown by world lines A and B in Figure 21.8. We see these bodies, but can they see each other? Let T be the time (in units of conformal time) that light takes to travel to us from A and B. The time that light takes to travel from A to B, or from B to A, is obviously $2T$. Hence when the universe is older than $3T$, we not only see A and B, but they also see each other at the time they emit the light we now see. If the universe is younger than $3T$, and older than T, we see A and B, but they cannot yet see each other. There is thus a maximum distance beyond which the observed bodies A and B do not know that each other exists. By examining Figure 21.8, we see that this maximum distance is one-third the distance to the particle horizon. The answer to our question is that bodies equidistant in opposite directions, farther away from us than one-third the distance to the particle horizon, cannot see each other. In a matter-dominated Einstein–de Sitter universe, this distance is $\frac{1}{3}L_P = \frac{2}{3}L_H$ at redshift $z = 1.25$.*

This highlights the apparently insoluble problem of understanding why the universe is homogeneous (all places are alike). Regions visible to us in opposite directions at large redshifts have not had time to influence each other and are unaware of each other's existence. Yet they exist in identical states. How can things be exactly similar when they lie outside one another's horizons? This is the horizon problem.

6 *Let $\mathrm{d}r = \mathrm{d}L/R$ be an interval of comoving distance and $\mathrm{d}\tau = \mathrm{d}t/R$ an interval of conformal time, as in Chapter 14. Thus $\mathrm{d}L = R\,\mathrm{d}r$ is an interval of proper (or tape-measure) distance and $\mathrm{d}t = R\,\mathrm{d}\tau$ is an interval of cosmic*

time. Equation [21.10] states $dr = -d\tau$ *on the backward lightcone, and on integrating we find*

$$r = \tau_0 - \tau, \qquad [21.12]$$

where

$$\tau = \int_0^t \frac{dt}{R}, \qquad [21.13]$$

is the conformal time of an event on O's backward lightcone measured from the beginning to time t, and

$$\tau_0 = \int_0^{t_0} \frac{dt}{R}, \qquad [21.14]$$

is the conformal time at the moment of observation, also measured from the beginning of t. Alternatively,

$$\tau_0 - \tau = \frac{1}{R_0}\int_0^z \frac{dz}{H}, \qquad [21.15]$$

in terms of redshift (Equations 14.36 and 19.21), where H is a function of z.

The particle horizon corresponds to $\tau = 0$, *and hence* $r_P = \tau_0$, *and the proper distance to the particle horizon is* $R_0\tau_0$, *or*

$$L_P = R_0\tau_0, \qquad [21.16]$$

and τ_0 *is found from Equation [21.14], or*

$$\tau_0 = \frac{1}{R_0}\int_0^\infty \frac{dz}{H}. \qquad [21.17]$$

7 *Assume that the scale factor R varies as* t^n, *where t is the age of the universe and n a constant number. In these power-law big bang universes:* $H = n/t$, *hence n is a positive number in an expanding universe;* $q = (1-n)/n$, *hence n is less than 1 in a decelerating universe. The Hubble distance, where galaxies recede at the velocity of light c, is*

$$L_H = ct_0/n = ct_0(1+q). \qquad [21.18]$$

The Hubble sphere itself expands at velocity $U_H = dL_H/dt$, *or*

$$U_H = c/n = c(1+q), \qquad [21.19]$$

and the edge of the Hubble sphere, or photon horizon, overtakes and sweeps past the galaxies when n is less than unity. The particle horizon is at the distance

$$L_P = \frac{nL_H}{1-n} = \frac{L_H}{q} \qquad [21.20]$$

and this distance – the radius of the observable universe – is greater than, equal to, or less than the Hubble distance when n is greater than, equal to, or less than 0.5. The observable universe expands at velocity $U_P = dL_P/dt$, *or*

$$U_P = c + V_P = \frac{c}{1-n} = c\left(1+\frac{1}{q}\right), \qquad [21.21]$$

and equals the recession velocity of the galaxies $V_P = HL_P$ *at the particle horizon plus the velocity of light c; the particle horizon always overtakes the galaxies at the velocity of light. In the matter-dominated Einstein–de Sitter universe of* $n = 2/3$, *we have* $L_H = 1.5ct_0$, $U_H = 1.5c$, $L_P = 3ct_0$, *and* $U_P = 3c$; *and in the radiation-dominated version of this universe of* $n = 0.5$, *we have* $L_H = 2ct_0$, $L_P = 2ct_0$, *and* $U_P = 2c$. *When n is equal to or greater than unity, there is no particle horizon, and the observer sees all luminous objects in the universe. Thus Milne's universe of* $n = 1$ *lacks particle and event horizons, and he regarded the absence of horizons in his universe as a distinct advantage.*

The maximum distance of the lightcone from an observer's world line is found to be

$$L_{max} = n^{1/(1-n)}L_H, \qquad [21.22]$$

and the redshift of sources at this maximum distance is

$$z\,(\text{at } L_{max}) = n^{-n/(1-n)} - 1, \qquad [21.23]$$

and this gives $L_{max} = 8L_H/27$ *and* $z = 1.25$ *for* $n = 2/3$, *and* $L_{max} = L_H/4$ *and* $z = 1$ *for* $n = 0.5$. *The recession velocity of sources at maximum emission distance, at the time of emission, is always equal to the velocity of light; these sources at maximum distance are therefore at the edge of the observer's Hubble sphere at the time they emitted the light that is now seen.*

PROJECTS

1 If light traveled with infinite speed, the world map and the world picture would be identical. What would happen to the horizons?

2 Find when the observable universe and the Hubble sphere are the same in size. Discuss the behavior of the Hubble sphere and the observable universe in the Dirac universe of $n = 1/3$.

3 As time passes, the observable universe contains more and more galaxies; is this true also in a collapsing universe?

4 Discuss the maximum proper (tape-measure) distance of an observed world line in an expanding big bang universe. How is it possible at the time of emission that a source of redshift $z = 2$ is nearer than a source of $z = 1$?

5 Can you think of cosmic horizons that might exist because of the observer's forward lightcone? Such horizons determine the observer's ability to influence future events and particles elsewhere in the universe.

6 Most physical scientists like to use formulas rather than words. As distinct from most humanists they also like to use lots of diagrams. Is there a reason for this fondness of diagrams?

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