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GRAVITATIONAL LENSING EFFECT ON THE TWO-POINT CORRELATION OF HOT SPOTS IN THE COSMIC MICROWAVE BACKGROUND

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ABSTRACT

We investigate the weak gravitational lensing effect that is due to the large-scale structure of the universe on two-point correlations of local maxima (hot spots) in the two-dimensional sky map of the cosmic microwave background (CMB) anisotropy. According to the Gaussian random statistics, as most inflationary scenarios predict, the hot spots are discretely distributed, with some *characteristic* angular separations on the last scattering surface that are due to oscillations of the CMB angular power spectrum. The weak lensing then causes pairs of hot spots, which are separated with the characteristic scale, to be observed with various separations. We found that the lensing fairly smooths out the oscillatory features of the two-point correlation function of hot spots. This indicates that the hot spot correlations can be a new statistical tool for measuring the shape and normalization of the power spectrum of matter fluctuations from the lensing signatures.

Subject headings: cosmic microwave background — cosmology: theory — gravitational lensing — large-scale structure of universe

1. INTRODUCTION

The temperature anisotropy of the cosmic microwave background (CMB) is the most powerful probe of our universe (e.g., Hu, Sugiyama, & Silk 1997). Therefore, high-precision measurements of the primary CMB anisotropies enable us to determine a lot of fundamental cosmological parameters with unprecedented precision (Jungman et al. 1996; Bond, Efstathiou, & Tegmark 1997; Zaldarriaga, Spergel, & Seljak 1997). On the other hand, gravitational lensing that is due to the large-scale structure between the last scattering surface (LSS) and us deforms the two-dimensional sky map of the CMB anisotropy. In the near future, the weak lensing will be an effective tool for mapping the inhomogeneous distributions of dark matter because it is not affected by uncertainties in the "biasing" problem regarding the extent to which luminous objects trace the mass distribution. Furthermore, a great advantage to studying the lensing effect on the CMB anisotropies is that statistical properties of the unlensed CMB field can be fully specified once we assume that the field obeys the Gaussian random theory (Bardeen et al. 1986, hereafter BBKS; Bond & Efstathiou 1987, hereafter BE). Indeed, most inflationary scenarios support this assumption. However, most previous studies have focused on theoretical predictions of lensed shapes of the two-point correlation function $C(\theta)$ or, equivalently, the angular power spectrum C_l of the temperature fluctuation field (Blanchard & Schneider 1987; Cole & Efstathiou 1989; Sasaki 1989; Tomita & Watanabe 1989; Cayón, Martínez-González, & Sanz 1993a, 1993b; Seljak 1996; Metcalf & Silk 1997; Zaldarriaga & Seljak 1998), while some studies have shown that the lensing induces non-Gaussian signatures (Bernardeau 1997; Zaldarriaga 1999). It is therefore worth exploring another statistical CMB estimator that is sensitive to the lensing signatures by using more directly the advantage of the predictability of the two-dimensional CMB sky map.

Recently, Heavens & Sheth (1999, hereafter HS) investigated the two-point correlation function of local maxima (hot spots) above a certain threshold in the two-dimensional CMB map, say, $\xi_{\rm pk-pk}(\theta)$, which can be accurately calculated based on the Gaussian random theory once C_l is given. Their purpose was to propose that $\xi_{\rm pk-pk}(\theta)$ can be a powerful test of the Gaussian hypothesis of the temperature fluctuations, and they showed that the shape of $\xi_{\rm pk-pk}(\theta)$ is largely different from that of $C(\theta)$; $\xi_{\rm pk-pk}(\theta)$ has oscillatory features such as the depression around the angular scale corresponding to the first Doppler peak of C_l and a prominent peak with a damping tail toward the scales that are smaller than the peak scale. The reasons can be explained as follows. In the derivation of $\xi_{\rm pk-pk}(\theta)$, we need statistical properties of the gradient and the second derivative fields of the temperature fluctuation field in order to identify the local maxima in the two-dimensional CMB map. The shape of $\xi_{\rm pk-pk}(\theta)$ thus reflects more strongly the oscillatory features of C_l, such as a series of Doppler peaks, than $C(\theta)$ does because the gradient and the second derivative fields are more sensitive to the oscillations of C_l. The oscillations of $\xi_{\rm pk-pk}(\theta)$ physically mean that the hot spots are discretely distributed with some *characteristic* angular separations on the LSS. Then the weak lensing that is due to the large-scale structure causes pairs of hot spots, which are separated with the characteristic scale on the LSS, to be observed with various separations in the random line of sight. This leads to the expectation that $\xi_{\rm pk-pk}(\theta)$ enables us to see more directly the lensing dispersion of the specific angular separation than when using $C(\theta)$. The purpose of this Letter, therefore, is to develop the formula for calculating the lensing effect on $\xi_{\rm pk-pk}(\theta)$, using the power spectrum approach in order to compute the dispersion of the lensing deflection angle (Seljak 1996), and to present some results in the standard cold dark matter (SCDM) model.

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2. WEAK LENSING EFFECT ON THE HOT SPOT CORRELATION FUNCTION

During the propagation from the LSS to us, a CMB photon is randomly deflected by the inhomogeneous matter distributions in the intervening large-scale structure. Then two CMB photons, observed with a certain angular separation θ, will have a different separation when emitted on the LSS. If we assume that the lensing fluctuations of the relative angular separation, $\delta\boldsymbol{\theta}(= \delta\boldsymbol{\theta}_1 - \delta\boldsymbol{\theta}_2)$, obey the Gaussian random statistics (Seljak 1996), the ensemble average of the following characteristic function can be calculated as

$$\langle \exp\,[i\boldsymbol{l}\cdot(\delta\boldsymbol{\theta}_1 - \delta\boldsymbol{\theta}_2)]\rangle_{|\boldsymbol{\theta}_1-\boldsymbol{\theta}_2|=\theta} \simeq \exp\left[-\frac{l^2}{2}\,\sigma^2_{\rm GL}(\theta)\right], \tag{1}$$

where $\delta\boldsymbol{\theta}_1$ and $\delta\boldsymbol{\theta}_2$ are the angular excursions of the two photons and $\langle\ \rangle_\theta$ denotes the average performed over all pairs with a fixed observed angular separation θ. The $\sigma^2_{\rm GL}(\theta)$ is defined by

$$\sigma^2_{\rm GL}(\theta) = \frac{9}{2\pi}\,H_0^4\Omega^2_{m0}\int_0^\infty \frac{dk}{k}\int_0^{\chi_{\rm rec}} d\chi a^{-2}P_\delta(k,\tau)W^2(\chi,\chi_{\rm rec})[1 - J_0(\chi k\theta)], \tag{2}$$

where τ is a conformal time, $\chi \equiv \tau_0 - \tau$, $J_n(x)$ is the Bessel function of order n, and the subscripts 0 and "rec" denote values at present and a recombination time, respectively. $P_\delta(k,\tau)$ is the power spectrum of the matter fluctuations, and $H_0 = 100\ h$ km s^{-1} Mpc^{-1} and Ω_{m0} denote the present Hubble parameter and the present energy density of matter, respectively. The projection operator $W(\chi,\chi_{\rm rec})$ on the celestial sphere is given by $W(\chi,\chi_{\rm rec}) = 1 - \chi/\chi_{\rm rec}$ in a flat universe. The approximation used in the derivation of equation (1) is valid as long as the $\sigma_{\rm GL}(\theta)/\theta \ll 1$ is satisfied, and we numerically confirmed $\sigma_{\rm GL}/\theta < 0.25$ on the relevant angular scales for all normalizations of the matter power spectrum considered in this Letter.

The unlensed $C(\theta)$ in the context of the small-angle approximation developed by BE is given by $C(\theta) = \langle\Delta(\boldsymbol{\theta}_1)\Delta(\boldsymbol{\theta}_2)\rangle = \int (l\,dl/2\pi)S(l)J_0(l\theta)$, where $|\boldsymbol{\theta}_1 - \boldsymbol{\theta}_2| = \theta$ and $\Delta(\boldsymbol{\theta}) \equiv [T(\boldsymbol{\theta}) - T_{\rm CMB}]/T_{\rm CMB}$. The two-dimensional power spectrum of the temperature fluctuation field $S(l)$ is identical to C_l in the limit of $l \gg 1$.

First, we briefly present the derivation of the unlensed two-point correlation function of hot spots in the two-dimensional Gaussian random CMB field developed by HS. The moments of the power spectrum are defined by $\sigma_i^2 \equiv \int_0^\infty (l\,dl/2\pi)S(l)l^{2i}$. At a hot spot, the gradients of the temperature fluctuation field $\Delta_i \equiv \partial\Delta/\partial\theta^i$ vanish, and the eigenvalues of the second-derivative matrix $\Delta_{ij} \equiv \partial^2\Delta/\partial\theta^i\partial\theta^j$ are negative. We thus need six independent variables $\boldsymbol{v} = (\Delta, \Delta_x, \Delta_y, \Delta_{xx}, \Delta_{xy}, \Delta_{yy})$ to specify one local hot spot. The probability density function of the 12 variables $\boldsymbol{v} \equiv (\boldsymbol{v}_{(1)}, \boldsymbol{v}_{(2)}) = [\boldsymbol{v}(\boldsymbol{\theta}_1), \boldsymbol{v}(\boldsymbol{\theta}_2)]$ for the two hot spots separated with the angle $\theta(= |\boldsymbol{\theta}_1 - \boldsymbol{\theta}_2|)$ is defined by

$$p_2(\boldsymbol{v}_{(1)}, \boldsymbol{v}_{(2)}) = \frac{1}{(2\pi)^6\,|M|^{1/2}}\exp\left(-\frac{1}{2}\,v_iM^{-1}_{ij}v_j\right), \tag{3}$$

where M_{ij} is the covariance matrix: $M_{ij} \equiv \langle(v_i - \langle v_i\rangle)(v_j - \langle v_j\rangle)\rangle$, $M = \det(M_{ij})$, and M^{-1}_{ij} is the inverse of M_{ij}. Note that $\langle v_i\rangle = 0$ in the present case. It is then convenient to introduce the variables defined by $\nu \equiv \Delta/\sigma_0$, $\eta_i \equiv \Delta_i/\sigma_1$, $X \equiv -(\Delta_{xx} + \Delta_{yy})/\sigma_2$, $Y \equiv (\Delta_{xx} - \Delta_{yy})/\sigma_2$, and $Z \equiv 2\Delta_{xy}/\sigma_2$. All nonzero components of M_{ij} for the two hot spots characterized by $\nu_{(1)} \equiv \nu(\boldsymbol{\theta}_1)$ and $\nu_{(2)} \equiv \nu(\boldsymbol{\theta}_2)$ and so on were explicitly calculated by HS. Employing the BBKS formalism, we can obtain the following unlensed correlation function of two hot spots above the certain thresholds ν_1 and ν_2, respectively:

$$\begin{aligned}1 + \xi_{\rm pk-pk}(\theta|>\nu_1,\ >\nu_2) \equiv{}& \frac{1}{\bar n_{\rm pk}(>\nu_1)\bar n_{\rm pk}(>\nu_2)}\,\langle n_{\rm pk}(\boldsymbol{\theta}_1)n_{\rm pk}(\boldsymbol{\theta}_2)\rangle = \frac{1}{2^2\theta_*^4\bar n_{\rm pk}(>\nu_1)\bar n_{\rm pk}(>\nu_2)}\int_{\nu_1}^\infty d\nu'_{(1)}\int_{\nu_2}^\infty d\nu'_{(2)}\int_0^\infty dX_{(1)}\int_0^\infty dX_{(2)}\\
&\times\int_{-X_{(1)}}^{X_{(1)}} dY_{(1)}\int_{-X_{(2)}}^{X_{(2)}} dY_{(2)}\int_{-X^2_{(1)}-Y^2_{(1)}}^{X^2_{(1)}-Y^2_{(1)}} dZ_{(1)}\int_{-X^2_{(2)}-Y^2_{(2)}}^{X^2_{(2)}-Y^2_{(2)}} dZ_{(2)}\,(X^2_{(1)} - Y^2_{(1)} - Z^2_{(1)})\,(X^2_{(2)} - Y^2_{(2)} - Z^2_{(2)})\\
&\times p_2\big(\nu'_{(1)}, X_{(1)}, Y_{(1)}, Z_{(1)}, \eta_{(1)i} = 0, \nu'_{(2)}, X_{(2)}, Y_{(2)}, Z_{(2)}, \eta_{(2)i} = 0\big),\end{aligned} \tag{4}$$

where $\theta_* = \sqrt{2}\sigma_1/\sigma_2$, and $n_{\rm pk}(\boldsymbol{\theta})$ and $\bar n_{\rm pk}(>\nu)$ are the filed number density and the mean number density of hot spots above height ν, respectively. To obtain $\xi_{\rm pk-pk}(\theta)$, we performed a six-dimensional numerical integration of equation (4) because the two integrals can be done analytically.

Next we derive the lensed correlation function of the hot spots. In this Letter, we focus on the lensing contributions to $\xi_{\rm pk-pk}(\theta)$ as a secondary effect that comes from a mapping of the intrinsic discrete distributions of hot spots on the LSS. The procedure is appropriate because the power of anisotropies generated by the lensing is small (Seljak & Zaldarriaga 2000; Zaldarriaga 1999). For this purpose, we define the fluctuation field of $n_{\rm pk}(\boldsymbol{\theta})$ as $\delta n(\boldsymbol{\theta}) \equiv [n_{\rm pk}(\boldsymbol{\theta}) - \bar n_{\rm pk}]/\bar n_{\rm pk}$. Likewise, since the gravitational lensing makes us observe, at a certain angular position $\boldsymbol{\theta}$, a hot spot at a different position $\boldsymbol{\theta} + \delta\boldsymbol{\theta}$ on the LSS, the lensed (observed) fluctuation field $\delta\tilde n(\boldsymbol{\theta})$ can be expressed as $\delta\tilde n(\boldsymbol{\theta}) \equiv \delta n(\boldsymbol{\theta} + \delta\boldsymbol{\theta}) = \int d^2\boldsymbol{l}/(2\pi)^2\delta n(\boldsymbol{l})\exp[i\boldsymbol{l}\cdot(\boldsymbol{\theta} + \delta\boldsymbol{\theta})]$, where $\delta n(\boldsymbol{l})$ is the Fourier component of $\delta n(\boldsymbol{\theta})$. If we assume that the lensing fluctuations and the intrinsic CMB anisotropy field are statistically independent,

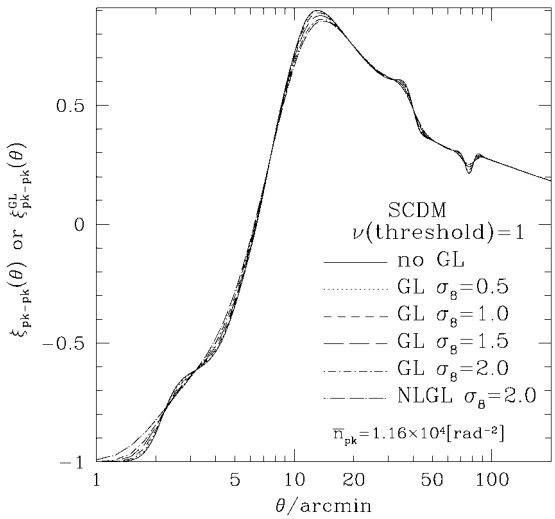

FIG. 1.—Correlation function of hot spots vs. θ with and without the gravitational lensing effect in the SCDM model. The solid line is the unlensed correlation function, while the dotted, short-dashed, long-dashed, and dot-dashed lines represent the corresponding lensing cases with $\sigma_8 = 0.5$, 1.0, 1.5, and 2.0, respectively. The long dot-dashed line denotes the result that includes the nonlinear correction with $\sigma_8 = 2.0$ (see text); $h = 0.6$ and $\Omega_{b0}h^2 = 0.015$ are assumed. The threshold of the hot spots is $\nu = 1$. The mean density of the hot spots is then $\bar{n}_{\rm pk} = 1.16 \times 10^4$ rad^{-2}.

then the lensed two-point correlation function of hot spots can be calculated with help of equation (1) as

$$\xi^{\rm GL}_{\rm pk-pk}(\theta) \equiv \left\langle \delta\tilde{n}(\boldsymbol{\theta}_1)\delta\tilde{n}(\boldsymbol{\theta}_2)\right\rangle \simeq \frac{1}{\sigma^2_{\rm GL}(\theta)} \int d\theta'\theta' \xi_{\rm pk-pk}(\theta') \exp\left[-\frac{\theta^2+\theta'^2}{2\sigma^2_{\rm GL}(\theta)}\right] I_0\left[\frac{\theta\theta'}{\sigma^2_{\rm GL}(\theta)}\right], \tag{5}$$

where $I_0(x)$ is the modified zeroth-order Bessel function. This equation indicates that the lensing contribution at a certain scale θ arises only from the lensing dispersion $\sigma_{\rm GL}(\theta)$ at the same scale all by itself. Hence, detecting the scale dependence of the lensing contributions to $\xi_{\rm pk-pk}(\theta)$ enables us to reconstruct the lensing dispersion; more importantly, it enables us to reconstruct the projected matter power spectrum by using equation (2).

3. RESULTS

In the following, we show some of the results from the SCDM model with $\Omega_{m0} = 1$. The Hubble parameter and the present baryon density are $h = 0.6$ and $\Omega_{b0}h^2 = 0.015$, respectively. To compute the intrinsic CMB angular power spectrum $S(l)$, we used the CMBFAST code developed by Seljak & Zaldarriaga (1996). As for the matter power spectrum in the lensing dispersion (eq. [2]), we employed the Harrison-Zeldovich spectrum and the BBKS transfer function with the shape parameter from Sugiyama (1995). The free parameter in our model is only the normalization of the present-day matter power spectrum, which is conventionally expressed in terms of the rms mass fluctuations within a sphere of 8 h^{-1} Mpc, i.e., σ_8.

In Figure 1, we show the two-point correlation function of hot spots with a height above the threshold $\nu = 1$ with and without the gravitational lensing effect, $\xi^{\rm GL}_{\rm pk-pk}(\theta)$ and $\xi_{\rm pk-pk}(\theta)$, respectively. Note that the intrinsic $\xi_{\rm pk-pk}(\theta)$ can be calculated by using only C_l. The mean number density of hot spots is then $\bar{n}_{\rm pk}(>\nu) = 1.16 \times 10^4$ rad^{-2}. The dependence of the lensing effect on the normalization of mass fluctuations is demonstrated by the four choices of $\sigma_8 = 0.5$, 1, 1.5, and 2, where $\sigma_8 = 0.5$ and 1 roughly correspond to normalizations of the cluster abundance (Eke, Cole, & Frenk 1996; Kitayama & Suto 1997) and the *COBE* measurements (Bunn & White 1997), respectively. The figure clearly shows that $\xi_{\rm pk-pk}(\theta)$ has more oscillatory features than $C(\theta)$. This is because the gradient and the second-derivative fields of $\Delta(\boldsymbol{\theta})$ used in the derivation of $\xi_{\rm pk-pk}(\theta)$ have power spectra of $C_l l^4/(2\pi)$ and $C_l l^6/(2\pi)$ per logarithmic interval in l, respectively, and thus they strongly enhance the oscillations of $l^2C_l/(2\pi)$, while $C(\theta)$ is fully determined by $l^2C_l/(2\pi)$. By using the relation $l \approx \pi/\theta$, there are indeed correspondences between both oscillations of $\xi_{\rm pk-pk}(\theta)$ and l^2C_l, such as the depression at $\theta \approx 75'$ corresponding to the scales around the first Doppler peak of C_l, a prominent peak at $\theta \approx 13'$, and a damping tail at $\theta < 10'$ associated with Silk damping. Moreover, Figure 1 shows that the gravitational lensing appears as a larger smoothing at scales in which $\xi_{\rm pk-pk}(\theta)$ has more oscillatory features since pairs of hot spots separated with the characteristic angular scales corresponding to the oscillations of $\xi_{\rm pk-pk}$ are redistributed by the weak lensing, and thus the power of $\xi_{\rm pk-pk}(\theta)$ is transferred from (or to) the scale to (or from) nearby scales. It should be stressed especially that even the lensing effect on the depression at $\theta \approx 75'$ associated with the first Doppler peak of C_l is distinguishable where the deviation $\delta\xi$ [$\equiv (\xi^{\rm GL}_{\rm pk-pk} - \xi_{\rm pk-pk})/\xi_{\rm pk-pk}$] reaches up to $\approx$20% for $\sigma_8 = 2.0$, while $\delta C < 1.0\%$ in all cases (see Fig. 2). Since both $\xi^{\rm GL}_{\rm pk-pk}$ and $\xi_{\rm pk-pk}$ give the same results as the shape becomes flat on larger scales ($\theta > 80'$), the large-scale amplitude of $\xi_{\rm pk-pk}$ gives the normalization, independent of the lensing contributions. We also demonstrate the nonlinear corrections to the matter power spectrum in order to clarify how the nonlinear evolution of matter fluctuations affects our analysis. Using the semianalytic formulae developed

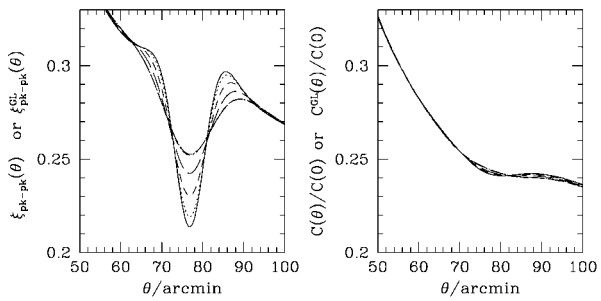

FIG. 2.—Two-point correlation functions of the hot spots and the temperature aniostropy field around the first Doppler peak with and without the gravitational lensing effect.

by Peacock & Dodds (1996) for the extreme case of $\sigma_8 = 2$, where the universe has the power (which is too large) of the present-day small-scale matter fluctuations, we found that the nonlinear correction is small at $\theta > 5'$. Most importantly, Figure 1 indicates that the magnitude of the lensing contributions to ξ_{pk-pk} is fairly sensitive to the amplitude of σ_8. Furthermore, measuring the scale dependence of the lensing contributions such as $\delta\xi(\theta)$ at $\theta = 2'$, $16'$, $36'$, $44'$, and $75'$ will allow us to reconstruct the shape of the projected power spectrum of matter fluctuations in the range of $3\ h^{-1}$ Mpc $\lesssim \lambda \lesssim 50\ h^{-1}$ Mpc.

4. DISCUSSIONS AND CONCLUSIONS

In this Letter, we have investigated the weak lensing effect on the two-point correlation of hot spots in the CMB. One of the great advantages to studying the lensing effect on the CMB anisotropies is that statistical properties of the unlensed CMB field are fully predictable by the Gaussian random theory. For example, Bernardeau (1998) has investigated how the lensing changes the probability distributions of the ellipticity defined from the local temperature curvature matrix, and he has shown that the method can be a statistical indicator that is sensitive to the lensing signatures.

It is now expected that the weak lensing can be a powerful probe for measuring the power spectrum of matter fluctuations. In case of normalization based on measurements of the large angular scale CMB anisotropies, such as the *COBE* normalization (e.g., Bunn & White 1997), we need to employ the very accurate matter transfer function that depends very highly on a lot of cosmological parameters in order to determine the amplitude of small-scale matter fluctuations. In fact, even the most sensitive future experiments, such as the *Planck Surveyor*,[3] will constrain σ_8 up to only 20% accuracy (see, e.g., Bond et al. 1997). Thus, we wish to propose that $\xi_{pk-pk}(\theta)$ can be a new effective statistical tool for measuring the amplitude and shape of the matter power spectrum.

For this purpose, we have to perform quantitative predictions in order to find out whether or not the future satellite missions, the *Microwave Anistropy Probe* (*MAP*)[4] and *Planck*, can measure the lensing contributions to $\xi_{pk-pk}(\theta)$ and constrain σ_8 with high accuracy. In particular, we have to estimate not only the uncertainties of the cosmic variance but also the observational errors in identifying the angular positions of hot spots in a realistic two-dimensional sky map of the CMB anisotropies. It is then important to take into account the following experimental effects. (1) The finite beam size of the detectors may incorporate some hot spots within the beam size that are the result of the smoothing of the lensed CMB anisotropies that were filed. Hence, the effect decreases the mean number density of hot spots and suppresses the two-point correlations of hot spots on scales below the beam size. (2) The detector noise may make spurious hot spots in the observed CMB map. HS estimated the errors for the measurements of the unlensed hot spot correlation functions that were expected from *MAP* and *Planck* by using numerical sky realizations, and indeed they show that the theoretical predictions, including the above experimental effects, are in good agreement with the numerical results. Following their method, we are now investigating the detectability of the lensing signatures to $\xi_{pk-pk}(\theta)$, and we have obtained the following preliminary result. By supposing that the data are produced by the *Planck* specification with 65% sky coverage, we find that the expected signal-to-noise ratios for the lensing signatures are 1.3, 1.8, 1.5, and 3.1 at $\theta = 16'$, $36'$, $44'$, and $75'$, respectively, for the $\sigma_8 = 1.5$ model.

Furthermore, we expect that the statistical significance of these signal-to-noise ratios could be increased by combining the measurements of the two-point correlations of *cold spots*. The observed correlations of cold spots should be identical to that of hot spots since they are subject to the Gaussian random statistics. These works are now in progress and will be presented in detail elsewhere.

We finally comment on the possibility that the lensing effect on ξ_{pk-pk} may break the so-called *geometrical degeneracy* (Bond et al. 1997; Metcalf & Silk 1998; Stompor & Efstathiou 1999) inherent in cosmological parameter determinations from measurements of the CMB anisotropies. We have confirmed that the global shape of $\xi_{pk-pk}(\theta)$ is sensitive to fundamental cosmological parameters such as h, Ω_b, Ω_{m0}, and so on. We thus expect that detecting the θ-dependence of the lensing contributions to $\xi^{GL}_{pk-pk}(\theta)$ can yield an additional constraint on the σ_8-Ω_{m0} plane if we assume that this constraint takes on the shape of the matter power spectrum. Hence, the degeneracy could be broken by observations of the CMB anisotropies alone, without the help of other astronomical measurements.

[3] See the *Planck* home page located at http://astro.estec.esa.nl/SA-general/Projects/Planck/.
[4] See the *MAP* home page located at http://map.gsfc.nasa.gov, which is maintained by C. L. Bennett, D. N. Spergel, and G. Hinshaw.

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