Recent Developments and Applications of Bayesian Data Analysis in Fusion Science: Integrated Data Analysis

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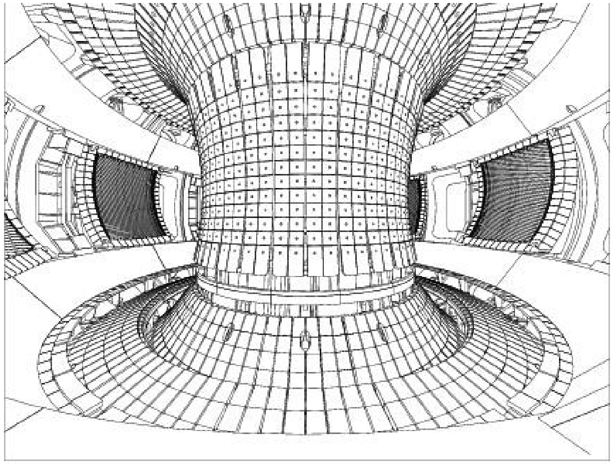
EURATOM Association

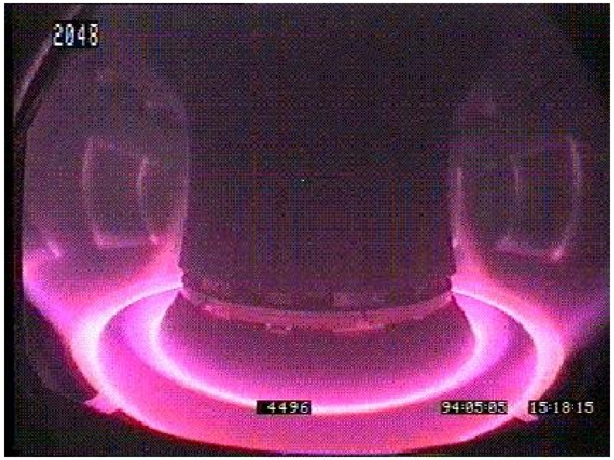
Garching, Oct 12, 2012

Fusion Science

- physical basis of a fusion power plant
- like the sun, such a plant is to generate energy from fusion of atomic nuclei ($E=mc^2$)
- a plasma is a hot (100 Mio °C) ionized gas (hydrogen and deuterium)
- the physical properties are studied experimentally and theoretically
- the link between theory and experiment is given by Data Analysis



ASDEX Upgrade with **divertor I**



ASDEX Upgrade discharge #4496

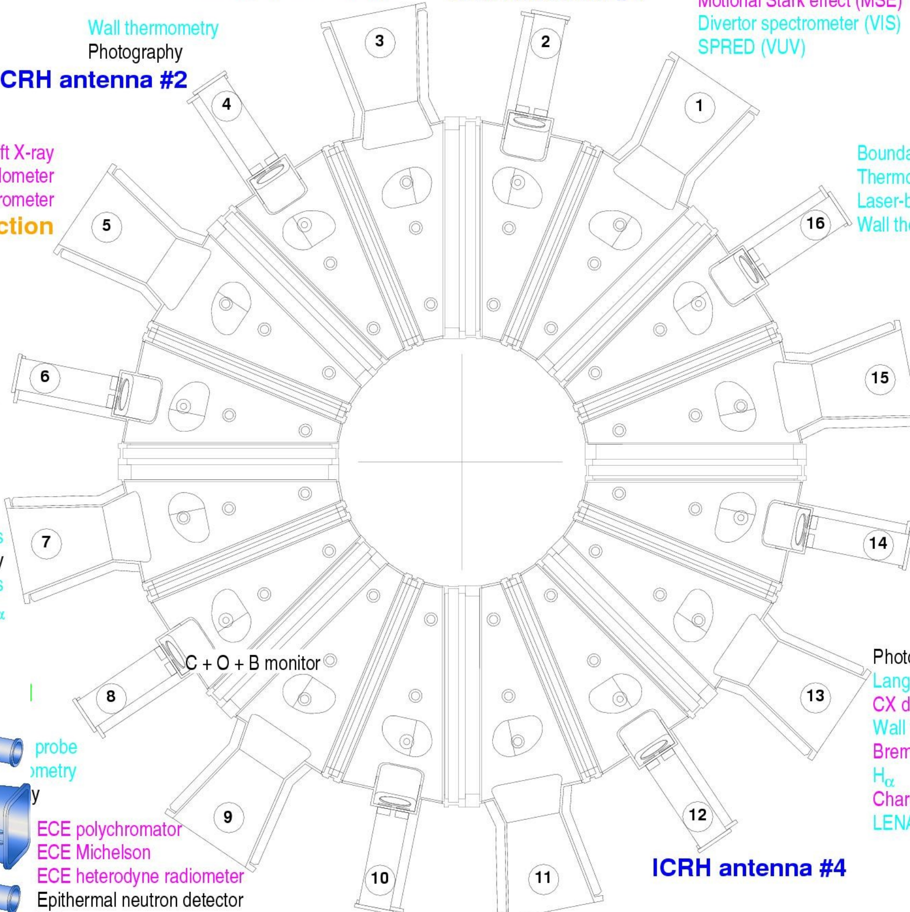
~1 m

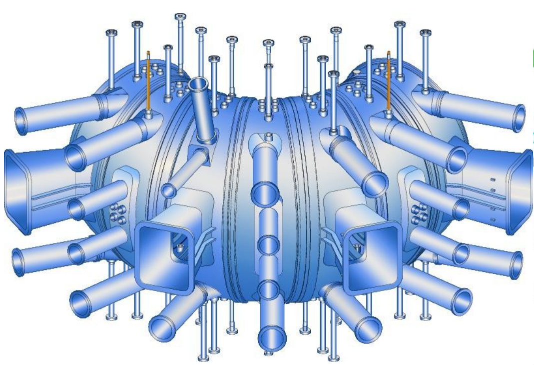
Diagnosing the plasma

Diagnostics
bulk plasma
edge and divertor plasma

Heating
ICRH
NBI
ECRH

Fuelling
pellet injection





each segment:
cooling water calorimetry

HXR: 4 detectors at walls of the experimental hall

IDA for Nuclear Fusion

Different measurement techniques (diagnostics: LIB, DCN, ECE, TS, REF, ...)
for the same quantities (n_e, T_e, …)
and parametric entanglement in data analysis

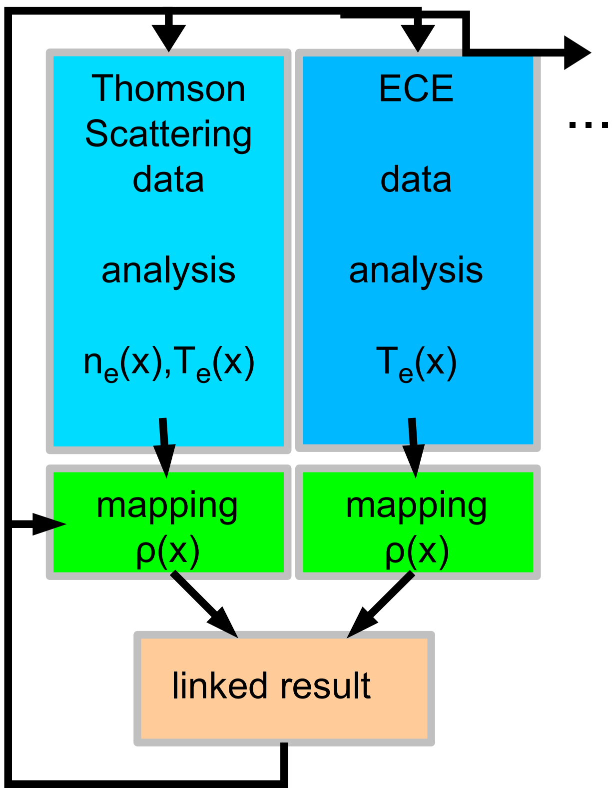
- Redundant data:
  - reduction of estimation uncertainties (combined evaluation, “super fit”)
  - detect and resolve data inconsistencies (reliable/consistent diagnostics)

- Complementary data:
  - resolve parametric entanglement
  - resolve complex error propagation (non-Gaussian)
  - synergistic effects (parametric correlations, multi-tasking tools (TS/IF, CXRS/BES))
  - automatic *in-situ* and *in-vivo* calibration (transient effects, degradation, ...)

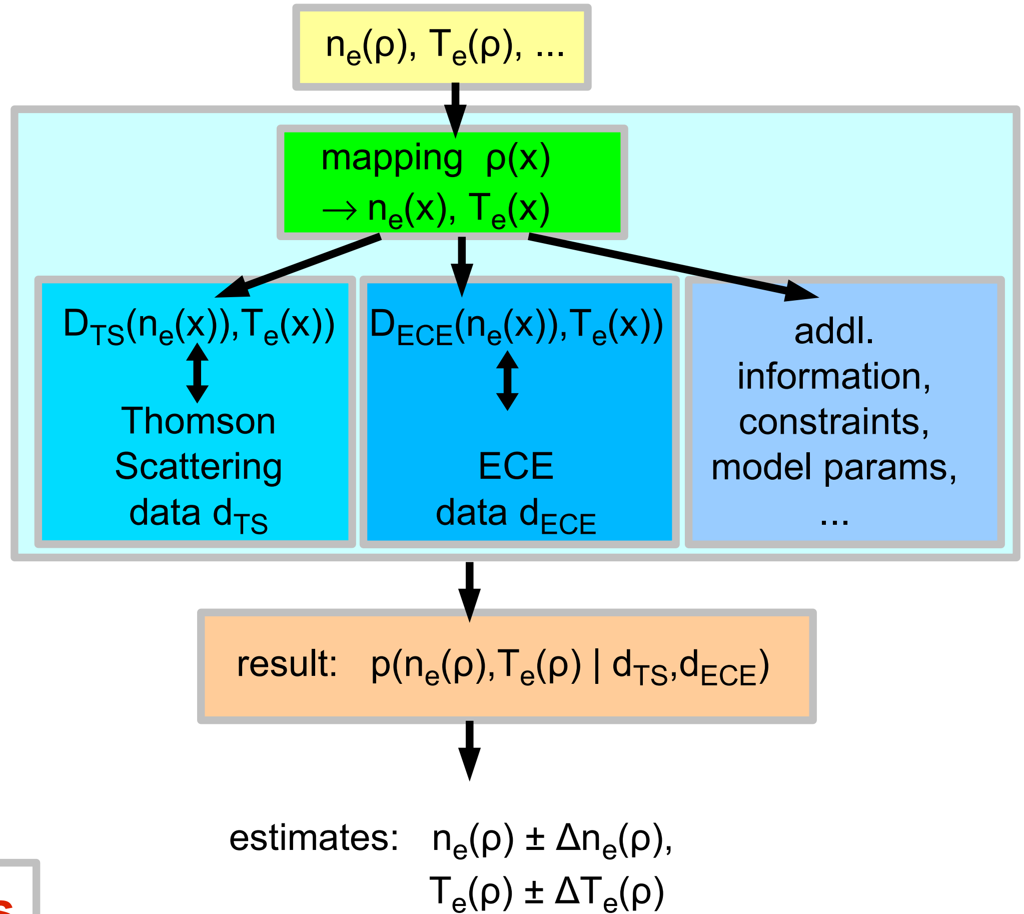
- Goal: Coherent combination of measurements from different diagnostics
  - replace combination of results from individual diagnostics
  - with combination of measured data → one-step analysis of pooled data
  - in a probabilistic framework (unified error analysis!)

Conventional vs. Integrated Data Analysis

conventional



IDA (Bayesian probability theory)



Parametric entanglements

Conventional vs. Integrated Data Analysis (2)

Drawbacks of conventional data analysis: iterative

- (self-)consistent results? (cumbersome; do they exist?)
- difficult to be automated (huge amount of data from steady state devices: W7X, ITER, ...)
- information propagation? (Single estimates as input for analysis of other diagnostics?)
- data and result validation? (How to deal with inconsistencies?)
- non-Gaussian error propagation? (frequently neglected: underestimation of the true error?)
- often backward inversion techniques (noise fitting? numerical stability? loss of information?)
- result: estimates and error bars (sufficient? non-linear dependencies?)

Probabilistic combination of different diagnostics (IDA)

✔ uses only forward modeling (complete set of parameters → modeling of measured data)

✔ additional physical information easily to be integrated

✔ systematic effects → nuisance parameters

✔ unified error interpretation → Bayesian Probability Theory

✔ result: probability distribution of parameters of interest

IDA offers a unified way
of combining data (information) from various experiments (sources)
to obtain improved results

Probabilistic (Bayesian) Recipe

Reasoning about parameter θ:

(uncertain) prior information $p(\theta)$ *prior* distribution

\+ (uncertain) measured data $d = D + \epsilon$

\+ physical model $D = f(\theta)$

$$\left.\begin{matrix} d = D + \epsilon \\ D = f(\theta) \end{matrix}\right\} p(d|\theta)$$

likelihood distribution

\+ Bayes theorem

$$p(\theta|d) = \frac{p(d|\theta) \times p(\theta)}{p(d)}$$

posterior distribution

\+ additional (nuisance) parameter β

$$p(\theta|d) = \int d\beta \, p(\theta, \beta|d)$$

$$= \int d\beta \, \frac{p(d|\theta, \beta) \times p(\theta) \times p(\beta)}{p(d)}$$

marginalization (integration)

generalization of Gaussian error propagation laws

\+ parameter averaging (model comparison)

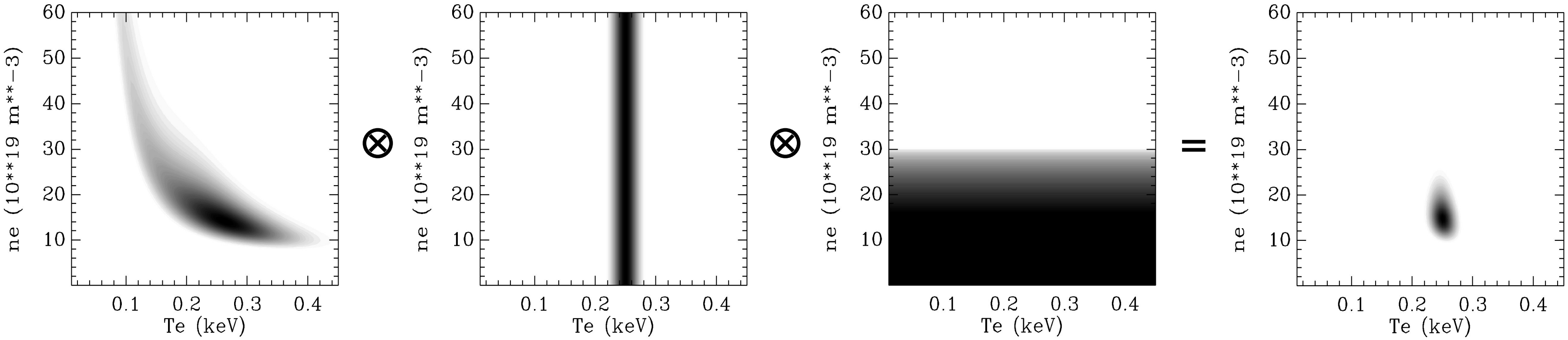
$$p(d|M) = \int d\theta \, p(\theta, d|M) = \int d\theta \, p(d|\theta, M) \, p(\theta)$$

prior predictive value

Integrated Data Analysis

W7-AS: n_e, T_e: Thomson scattering, interferometry, soft X-ray

Using interdependencies:
Combination of results from a *set* of diagnostics (W7-AS)



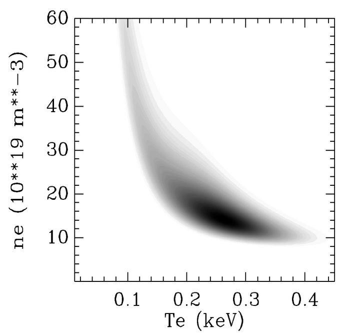
Thomson Scattering | Soft-X-ray | Interferometer Operation | **Integrated Result**

Probabilistic framework

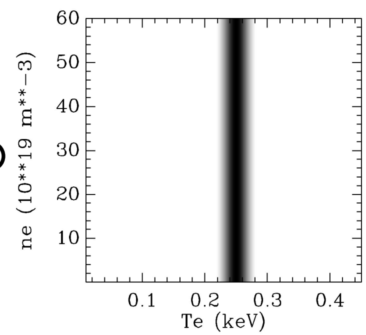
R. Fischer, A. Dinklage, and E. Pasch, Bayesian modelling of fusion diagnostics, PPCF, 45, 1095-1111 (2003)

Application: W7-AS

Using synergism: Combination of results from a *set* of diagnostics

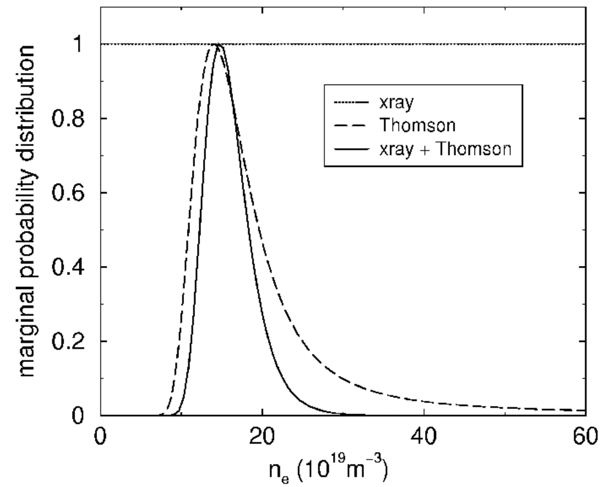
$\int$ 

Thomson Scattering

$\otimes$ 

Soft-X-ray

dT_e =



Electron density

30% reduced error

→ synergism by exploiting
full probabilistic correlation structure

Application: ASDEX Upgrade

(1) profiles of density $n_e(\rho)$, and temperature $T_e(\rho)$:

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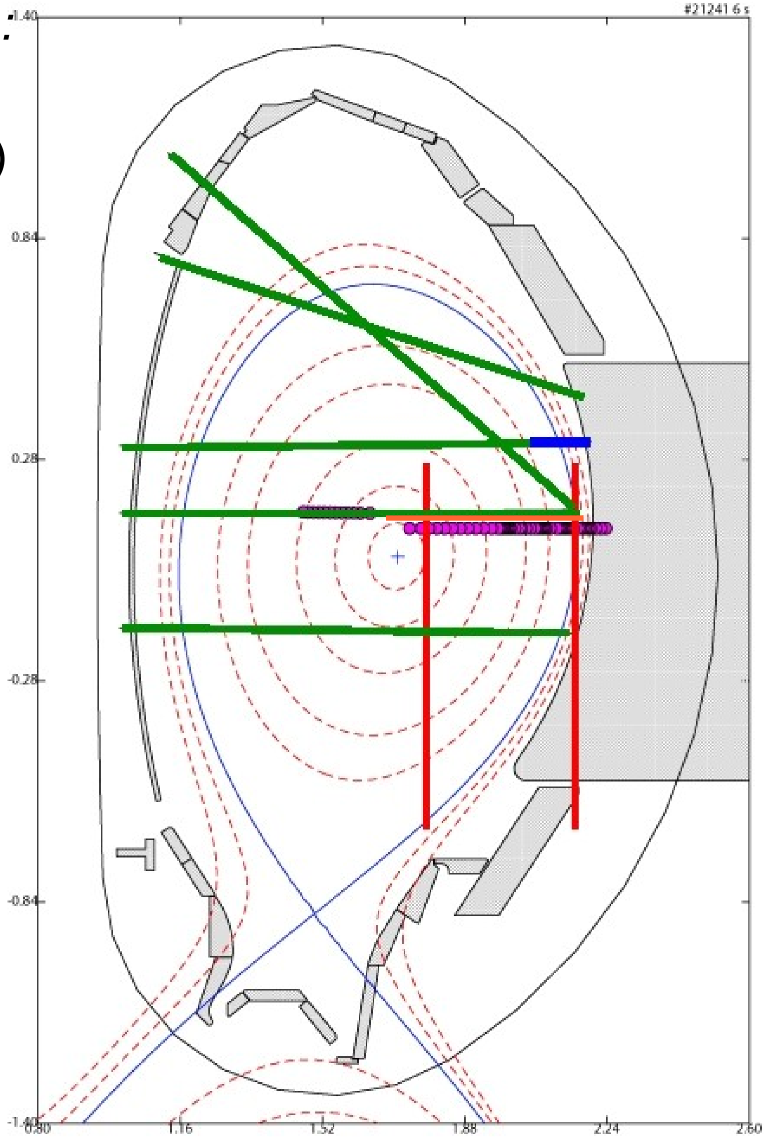
- Electron cyclotron emission (ECE)
- Thomson scattering (TS)
- Reflectometry (REF)
- Equilibrium reconstructions for diagnostics mapping

R. Fischer et al., Integrated data analysis of profile diagnostics at ASDEX Upgrade, Fusion Sci. Technol., 58, 675-684 (2010)



(2) Z_{eff} :

- Bremsstrahlung background from various CXRS spectroscopies
- Impurity concentrations from CXRS

S. Rathgeber et al., Estimation of profiles of the effective ion charge at ASDEX Upgrade with Integrated Data Analysis, PPCF, 52, 095008 (2010)

Bayesian Recipe for IDA: LIB+DCN+ECE+TS+REF

Reasoning about parameter n_e, T_e:

(uncertain) prior information $p(n_e, T_e)$ *prior* distribution

+ experiment 1: $d_{LiB} = D_{LiB}(n_e, T_e) + \epsilon$; $p(d_{LiB} | n_e, T_e)$
+ experiment 2: $d_{DCN} = D_{DCN}(n_e) + \epsilon$; $p(d_{DCN} | n_e)$
+ experiment 3: $d_{ECE} = D_{ECE}(T_e) + \epsilon$; $p(d_{ECE} | T_e)$
+ experiment 4: $d_{TS} = D_{TS}(n_e, T_e) + \epsilon$; $p(d_{TS} | n_e, T_e)$
+ experiment 5: $d_{REF} = D_{REF}(n_e) + \epsilon$; $p(d_{REF} | n_e)$

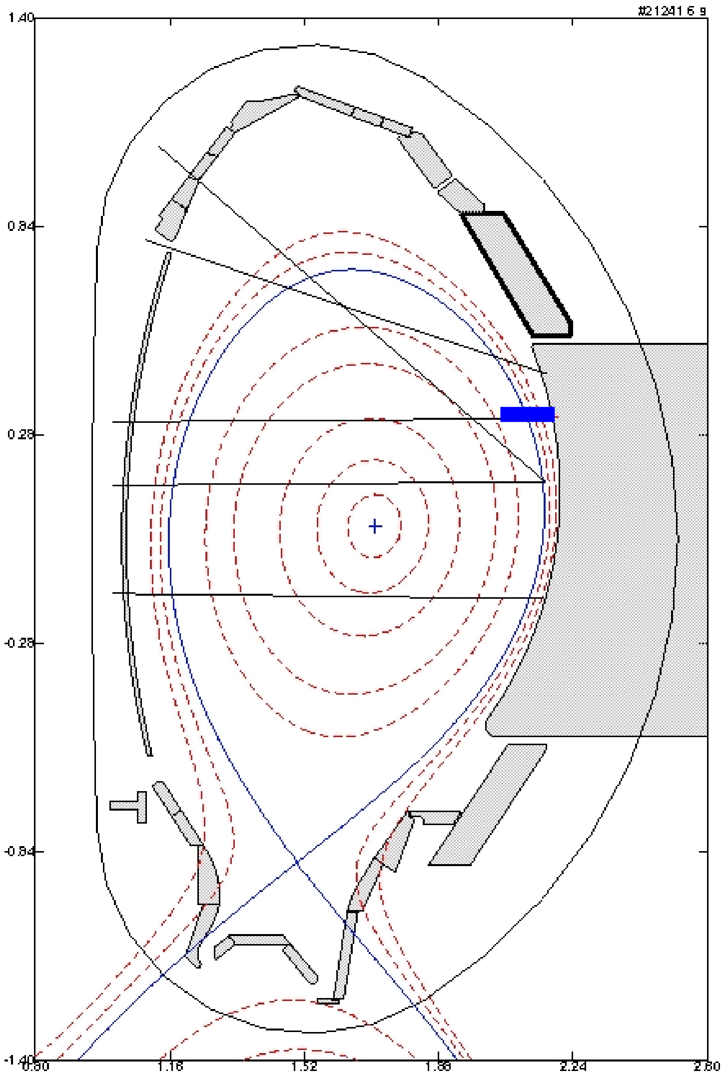
likelihood distributions

+ Bayes theorem

$$p(n_e, T_e | d_{TS}, d_{ECE}, d_{LiB}, d_{DCN}, d_{REF}) \propto p(d_{TS} | n_e, T_e) \times p(d_{ECE} | T_e) \times p(d_{LiB} | n_e, T_e) \times p(d_{DCN} | n_e) \times p(d_{REF} | n_e) \times p(n_e, T_e)$$

posterior distribution

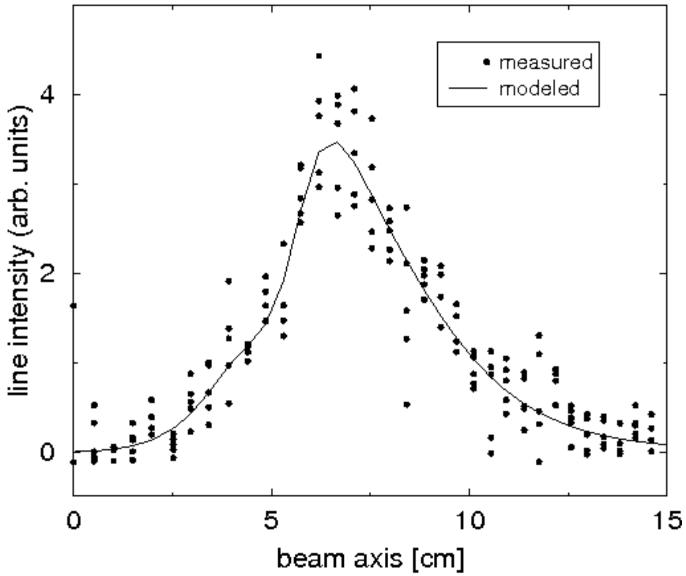
Lithium-beam diagnostic



Lithium beam impact excitation spectroscopy
(LiI: Li(2p) → Li(2s), λ = 670.8 nm)

LiI radiation from neutral Lithium (E=30-80 keV)

$$D_{LiB}[n_e(x_{LiB})]$$



System of coupled linear differential equations:

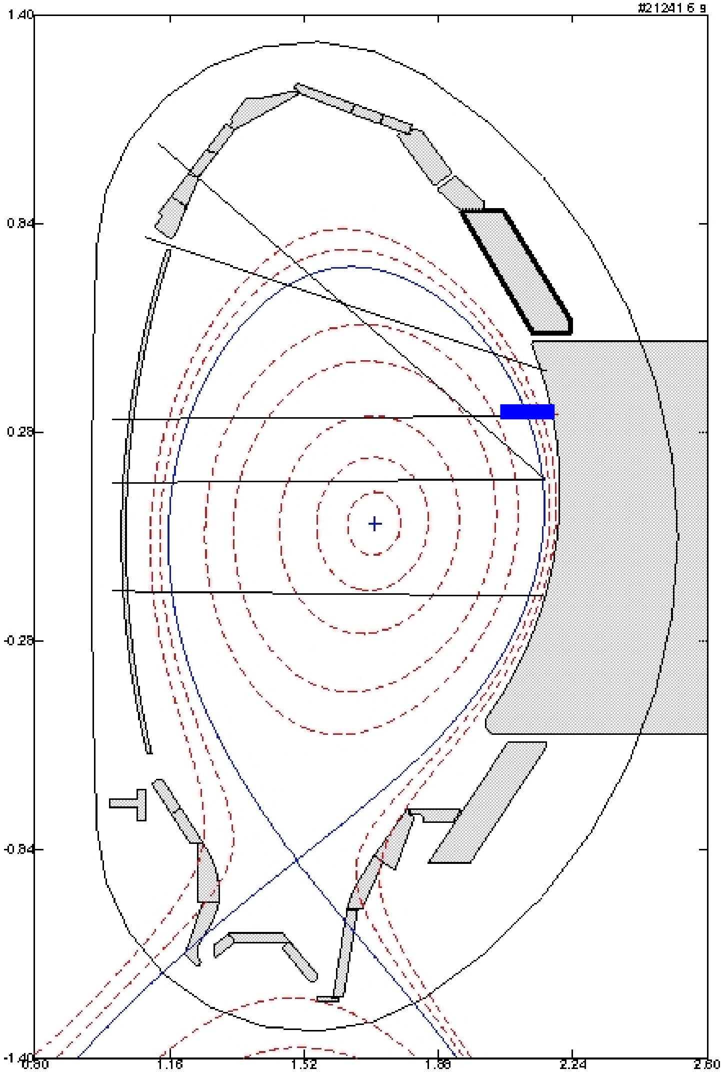
$$\frac{dN_i(x)}{dx} = \sum_{j=1}^{N_{Li}} \{n_e(x) a_{ij}(T_e(x)) + b_{ij}\} N_j(x) \quad ; \quad N_i(x=0) = \delta_{1i}$$

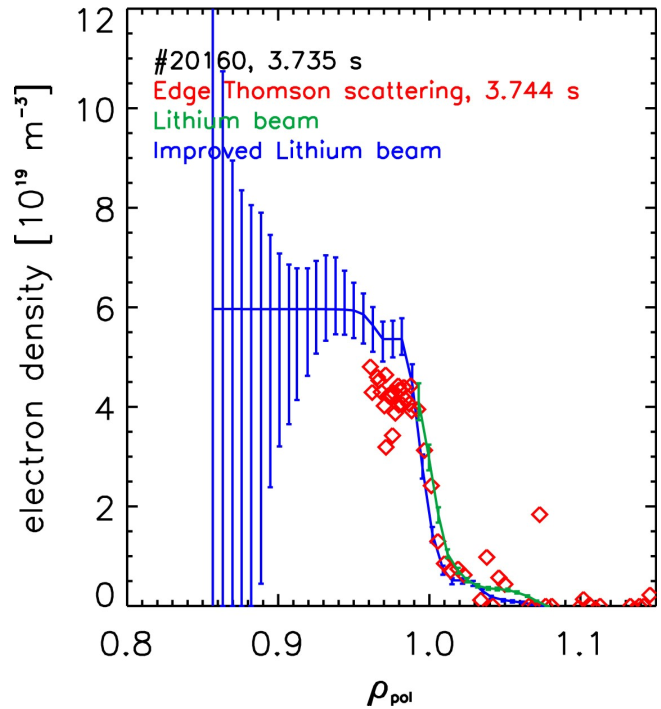
solved for a given profile $n_e(x)$ to obtain occupation density Li(2p): $N_2(x|n_e)$

$$D_{LiB}[n_e(x_i)] = \alpha s_i N_2(x) \quad ; \quad i = 1,..,35$$

Lithium-beam diagnostic: Comparison old-new data analysis

comparison of profiles from old data analysis and new probabilistic method





- Old: density profile stops just before pedestal (temporal binning: 20 ms)
- New: Pedestal well determined
- New: high reliability of profile for $\rho_{pol} > 0.93$
- New: Temporal resolution: 50 µs

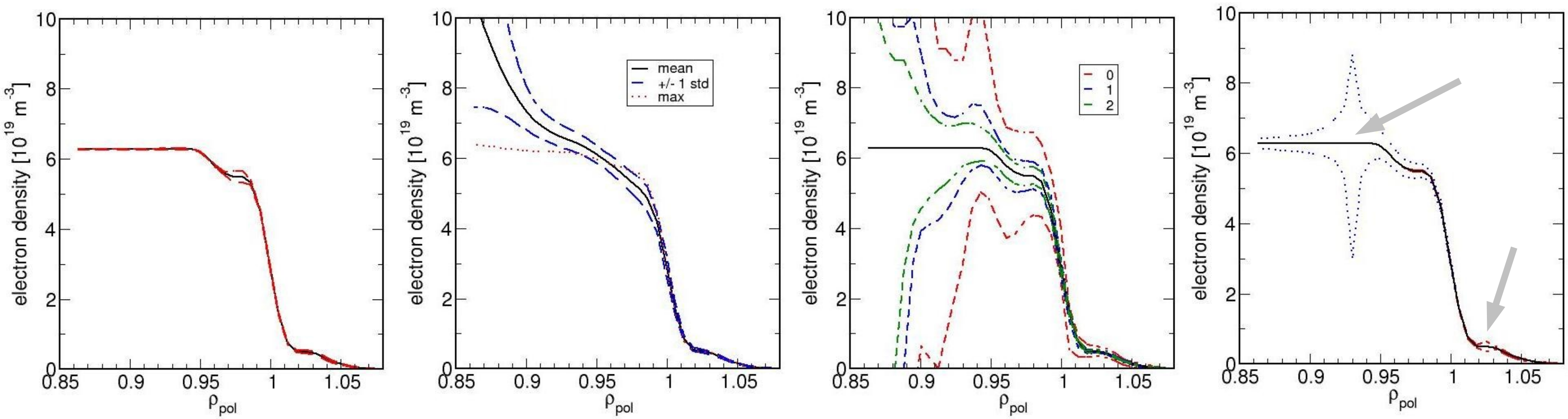
R. Fischer, E. Wolfrum, J. Schweinzer, 34th EPS Conference on Controlled Fusion and Plasma Physics. 2007

LIB: Profile Uncertainties

- Uncertainties are necessary to evaluate the quality of the estimated profile

  error bands, confidence intervals, ...

  - how to be estimated? → different techniques
  - for what purpose? → use the error bars for further analysis
  - interpretation? → what do error bars tell us at all?
  - correlations ($p_e = n_e T_e$)?



Laplace approximation

Markov Chain Monte Carlo (MCMC)

χ^2 -binning method

error star

R. Fischer, E. Wolfrum, J. Schweinzer, PPCF **50** (2008) 085009

Application: ASDEX Upgrade

(1) profiles of density $n_e(\rho)$, and temperature $T_e(\rho)$:

- Lithium beam impact excitation spectroscopy (LIB)
  $\rightarrow n_e(\rho)$ at plasma edge
- Interferometry measurements (DCN)
  $\rightarrow n_e(\rho)$ line integrated
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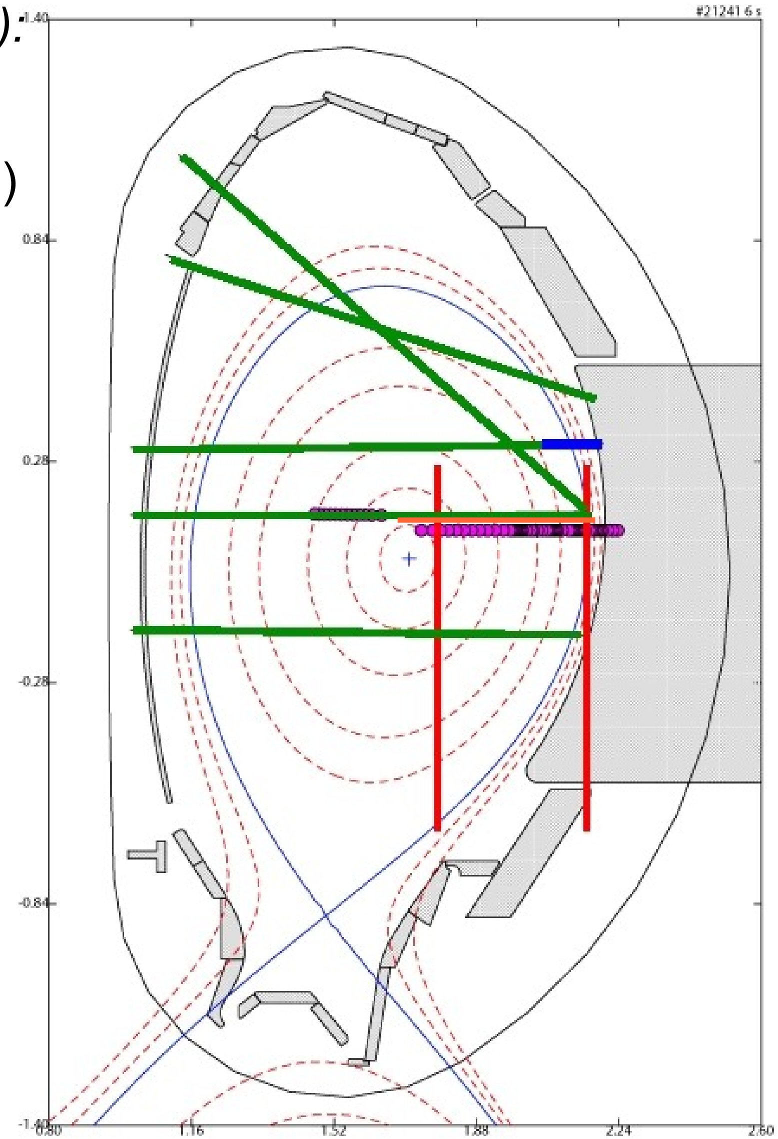
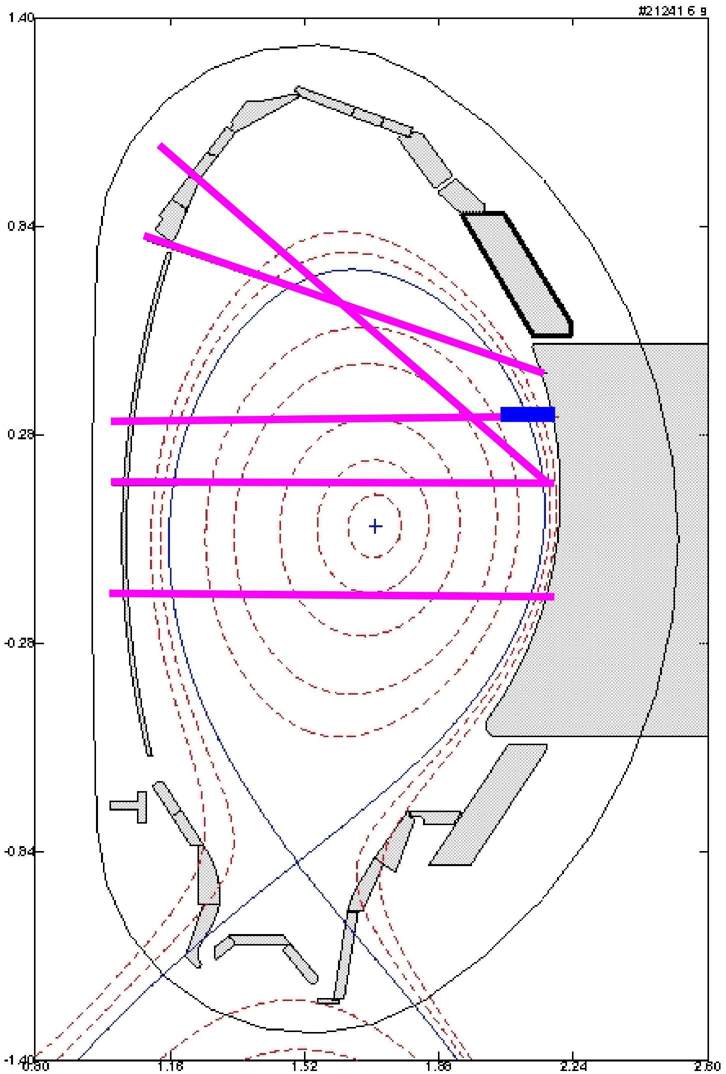
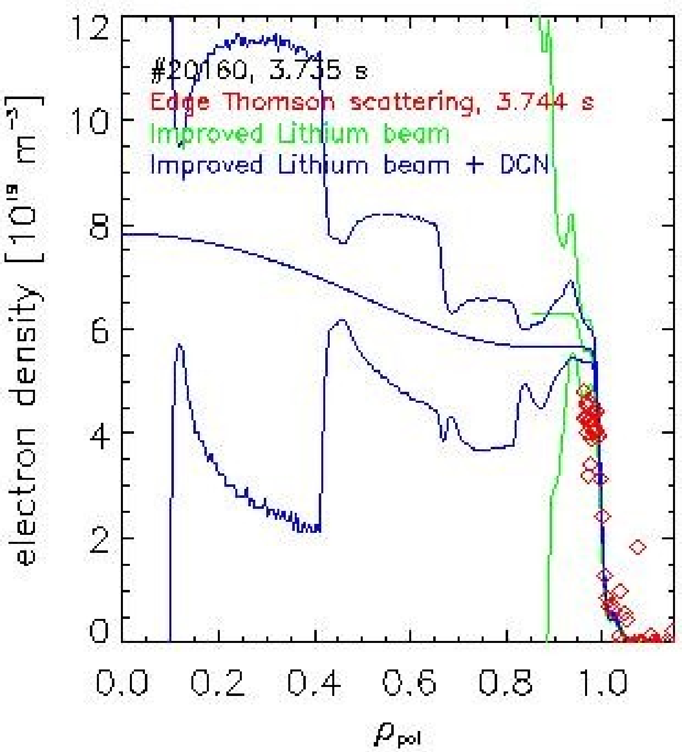
- Thomson scattering (TS)
  $\rightarrow n_e(\rho), T_e(\rho)$
- Reflectometry (REF)
  $\rightarrow n_e(\rho)$
- Equilibrium reconstructions for diagnostics mapping: $(x,y) \rightarrow \rho$



R. Fischer et al., Integrated data analysis of profile diagnostics at ASDEX Upgrade, Fusion Sci. Technol., 58, 675-684 (2010)

LIB + DCN Interferometry

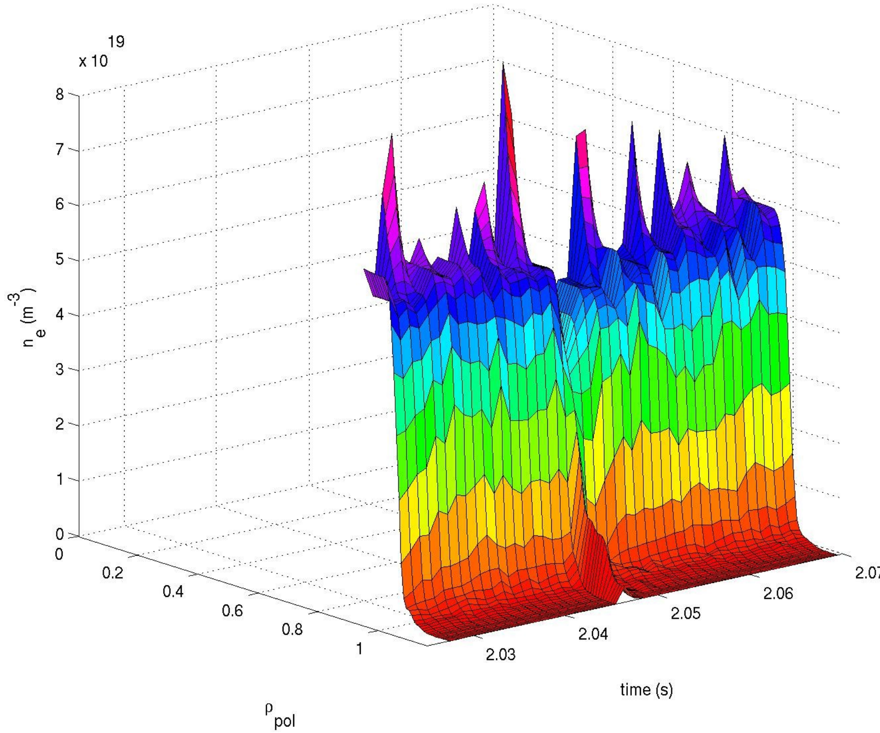




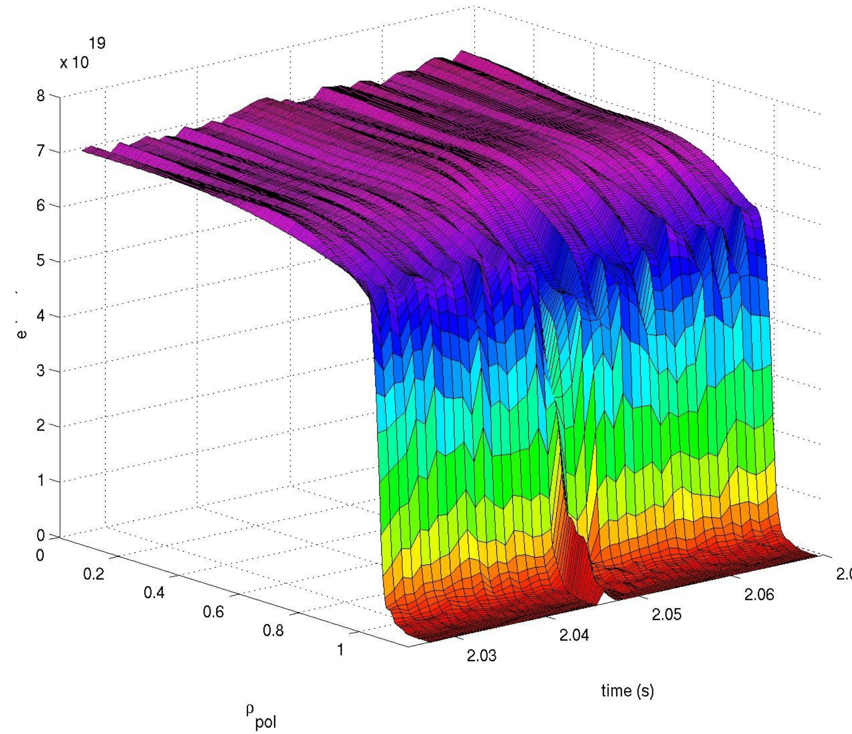
- full density profiles
- smooth profile with steep edge
- uncertainties:
  - → ρ_{pol} > 0.95: Li-Beam
  - → ρ_{pol} < 0.95: DCN (5 LOS)

LIB + DCN: Temporal resolution

#22561, 2.045-2.048 s, H-mode, type I ELM



LIN: Lithium beam only



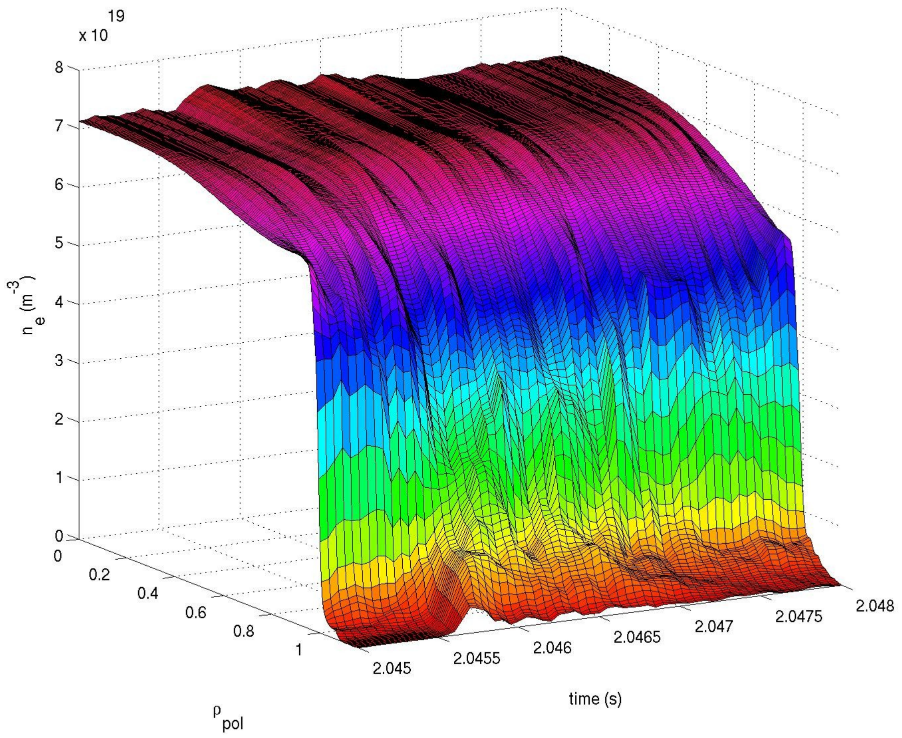
IDA: Lithium beam + DCN Interferometry

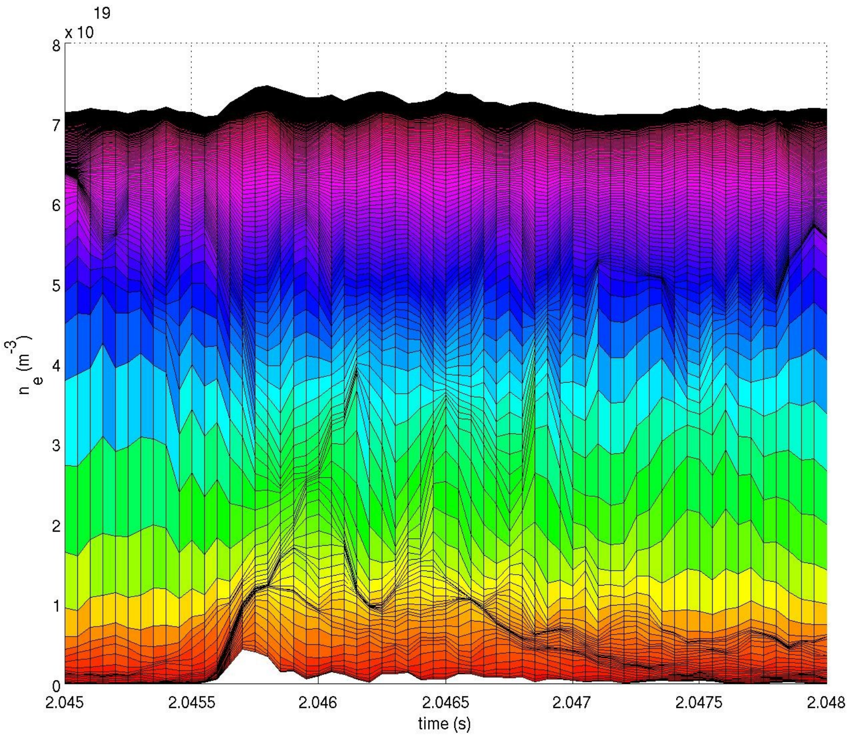
density profiles with temporal resolution of 1 ms (routinely)

R. Fischer, E. Wolfrum, J. Schweinzer, PPCF **50** (2008) 085009

IDA: ELM resolved profiles

#22561, 2.045-2.048 s, H-mode, type I ELM





- temporal resolution 50 µs (new in 2012: 5 µs)
- no ELM averaging → single ELMs
- no correlation in data analysis of neighboring time frames!

R. Fischer, E. Wolfrum, Ch. Fuchs, 35th EPS Conference on Controlled Fusion and Plasma Physics. 2008

Application: ASDEX Upgrade

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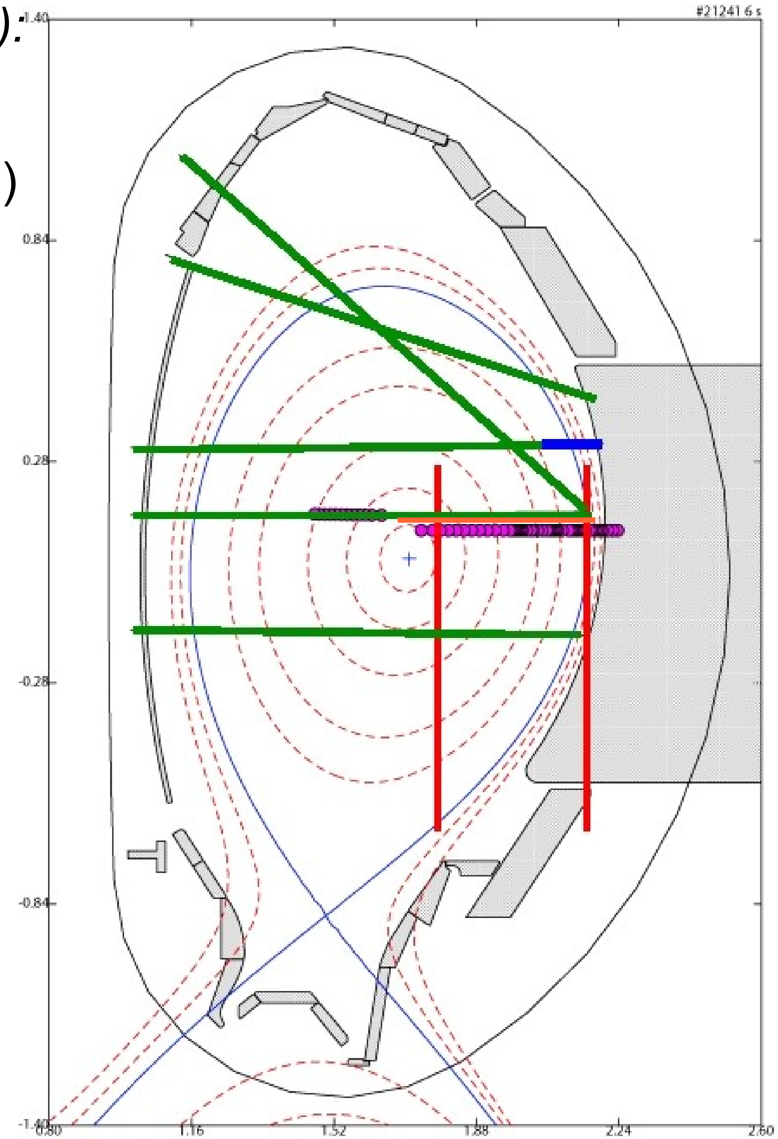
- Electron cyclotron emission (ECE)
  → $T_e(\rho)$
- Thomson scattering (TS)
  → $n_e(\rho)$, $T_e(\rho)$
- Reflectometry (REF)
  → $n_e(\rho)$
- Equilibrium reconstructions for diagnostics mapping: (x,y) → ρ



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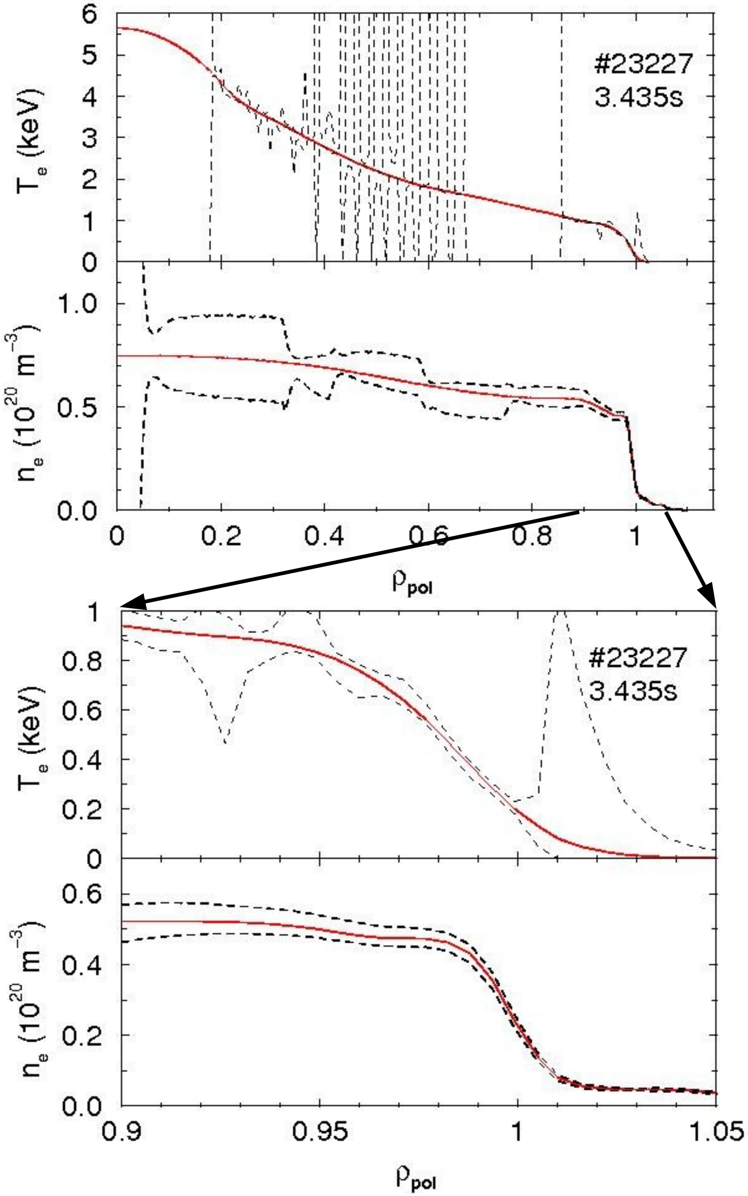
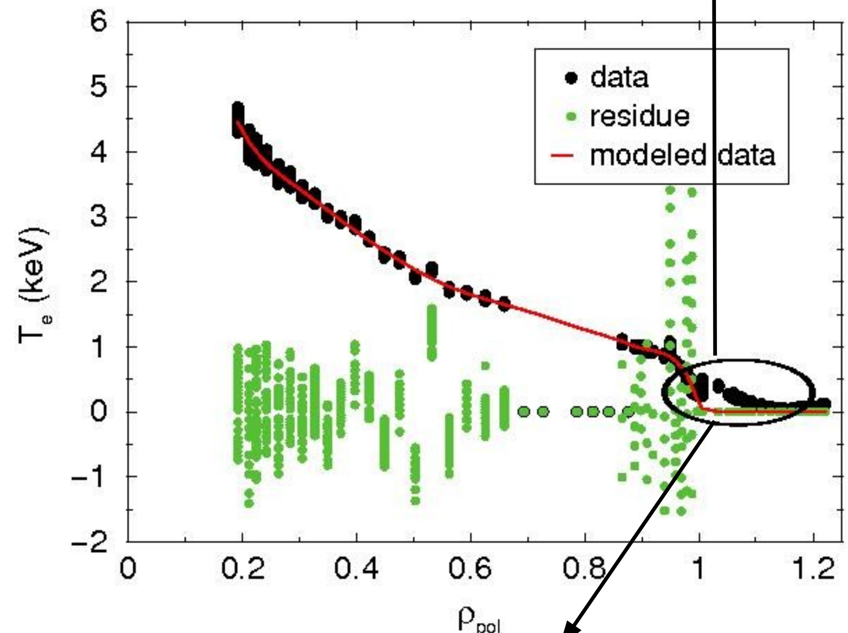
IDA: LIB + DCN + ECE



- ➔ simultaneous:
  - ✔ full density profiles
  - ✔ (partly) temperature profiles
  - → pressure profile
- ➔ $n_e > 0.95 * n_{e,cut\text{-}off}$ → masking of ECE channels
- ➔ opt. depth ~ $n_e T_e$ → masking of ECE channels



New: Electron Cyclotron Forward Model (ECFM)

S. K. Rathgeber, et al., to be published
S. K. Rathgeber, PhD thesis

Application: ASDEX Upgrade

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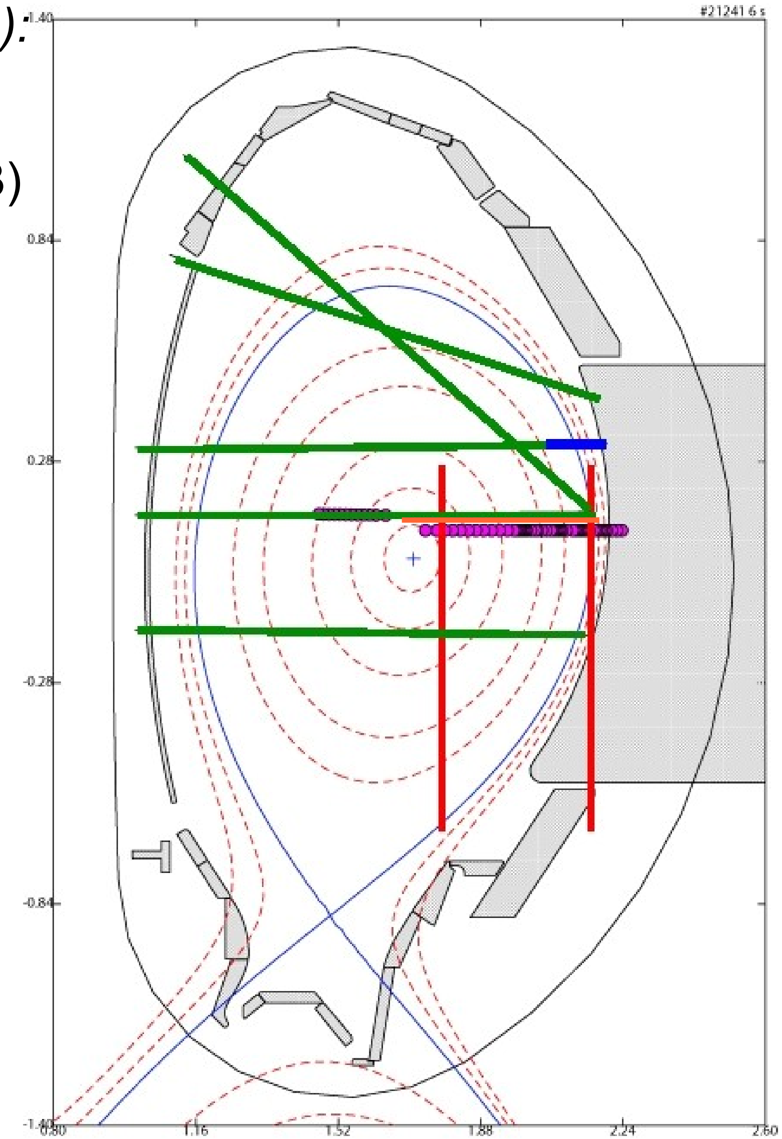
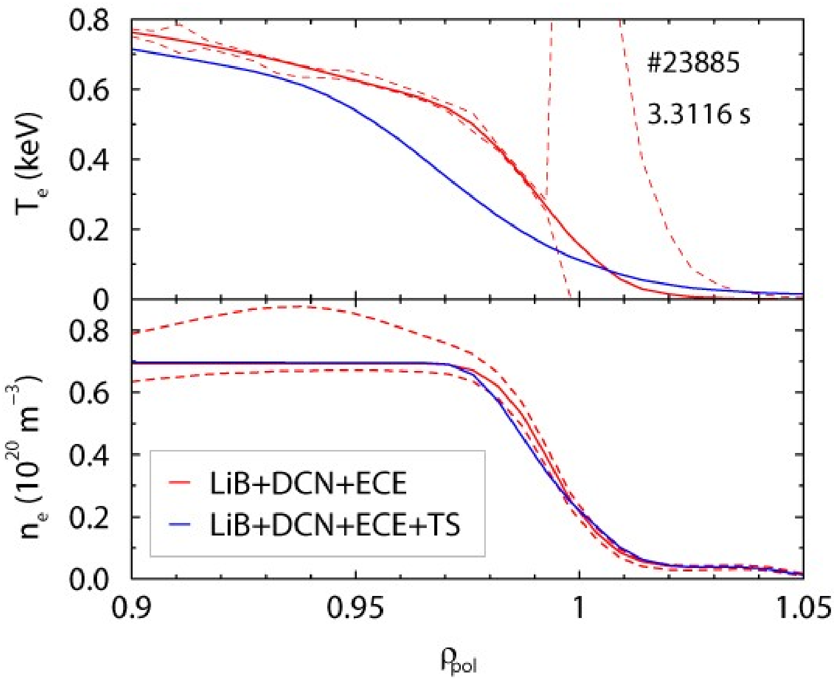


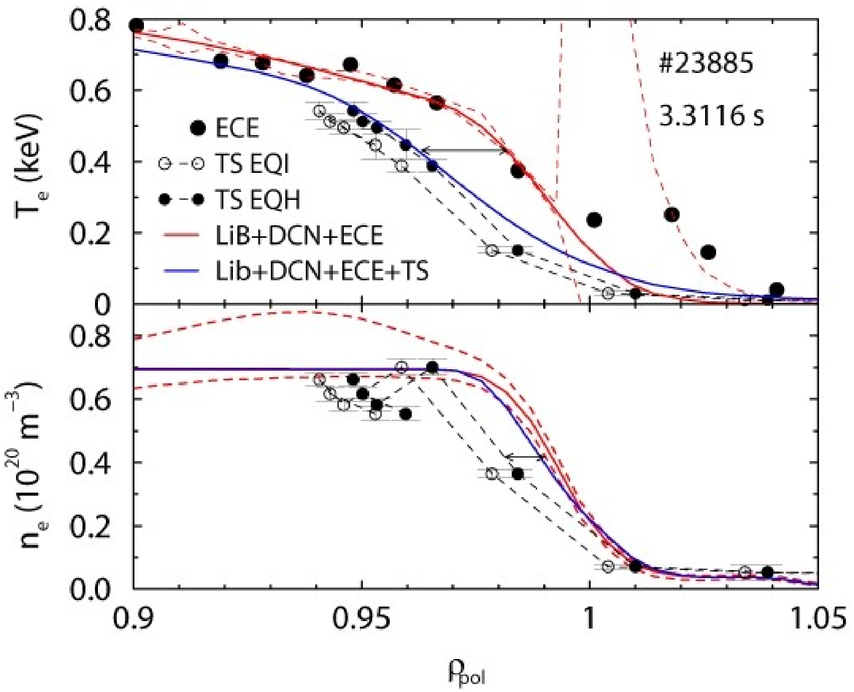
IDA: LIB + DCN + ECE + TS

➔ profiles from different diagnostics (TS, LIB, ECE) to not perfectly match at plasma edge

➔ 2 additional parameters: scale of the LIB "ρ_{pol}"

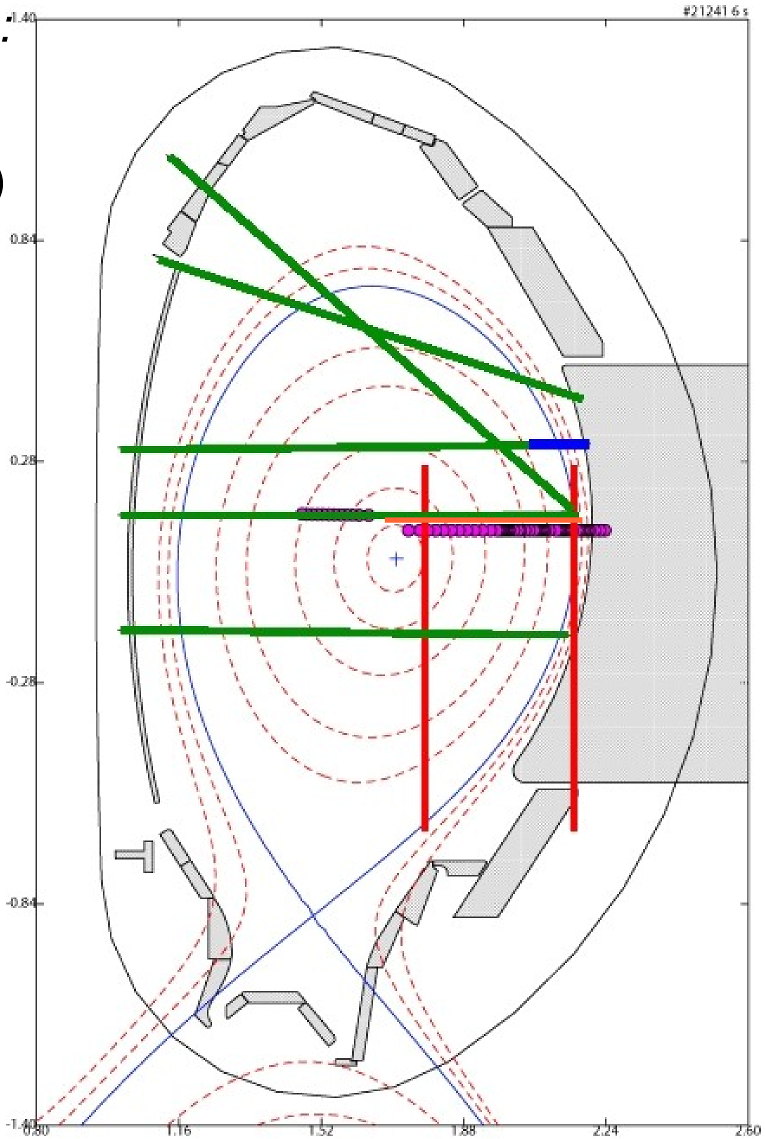
- $\rho_{pol,ECE} = s_{ECE} * \rho_{pol,LiB}$ → $\Delta\rho_{pol,ECE}$ rel. shift of ECE channels at plasma edge
- $\rho_{pol,TS} = s_{TS} * \rho_{pol,LiB}$ → $\Delta\rho_{pol,TS}$ rel. shift of TS channels at plasma edge





Application: ASDEX Upgrade

(1) profiles of density $n_e(\rho)$, and temperature $T_e(\rho)$:



- Lithium beam impact excitation spectroscopy (LIB)
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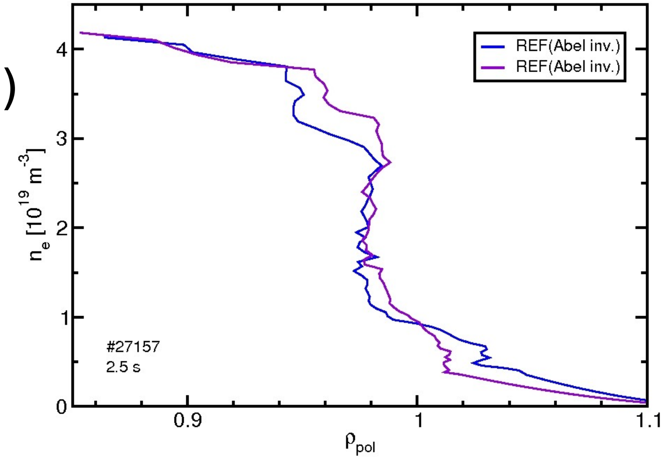
Reflectometry

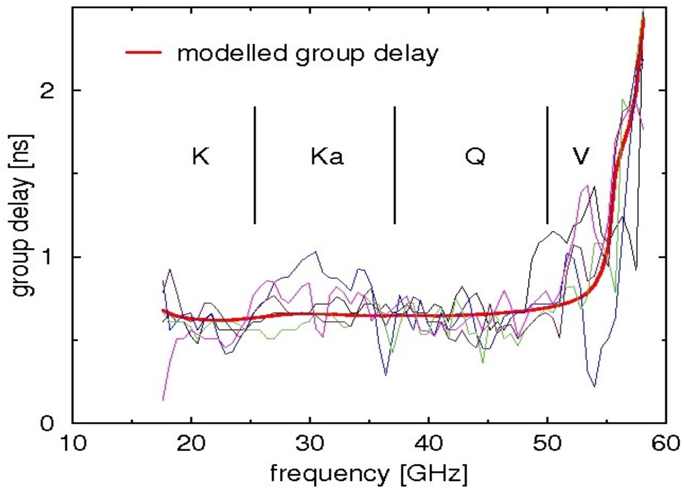


- Goal: n_e profiles, plasma position control (ITER)
- Classical analysis: Abel inversion (O-mode)
  → location of cutoff layer
- Problems:
  - *unphysical* profiles
  - multiple analysis steps (phase of reflected wave → group delay → density)
  - error treatment/propagation; profile uncertainties
  - density initialization outside first cutoff layer
- IDA
  - Forward modeling of measured data for given density profile
  - Benefit:
    - Additional data available → density initialization
    - Alignment

Reflectometry forward modelling

Time delay of the reflected beam (group delay):

$$\tau(f) = \frac{1}{2\pi}\frac{\partial \phi}{\partial f}$$



Phase of reflected beam:

$$\phi = \frac{4\pi f}{c}\int_{r_c(f)}^{r_1} \mu(r)\,dr - \frac{\pi}{2}$$

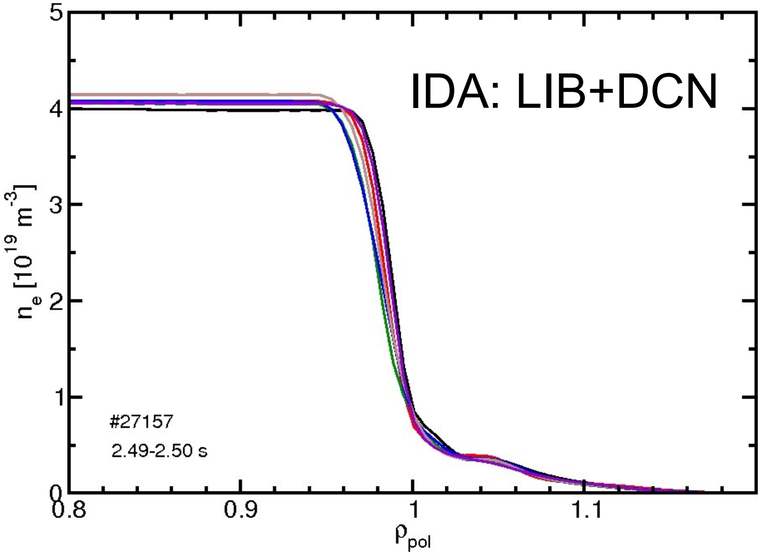
Refractive index:

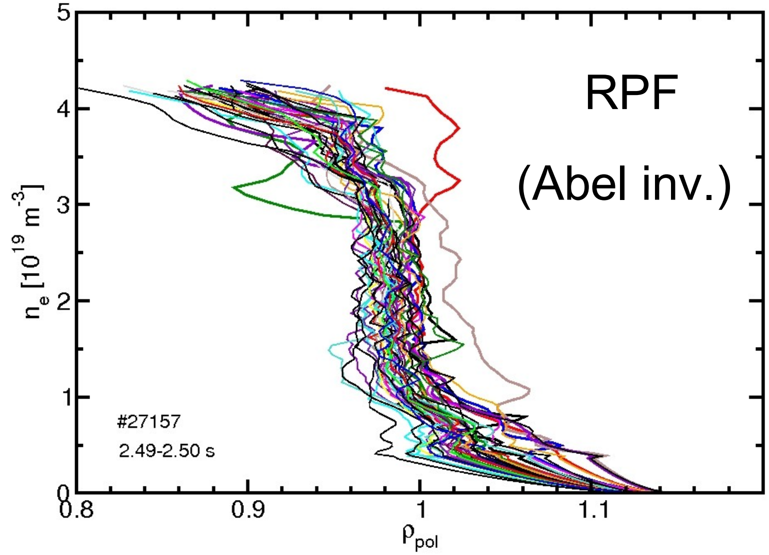
$$\mu(r) = \sqrt{1 - \frac{n(r)}{n_c(f)}}\ ;\ n_c(f) = \frac{4\pi^2 \epsilon_0 m_e f^2}{e^2}$$

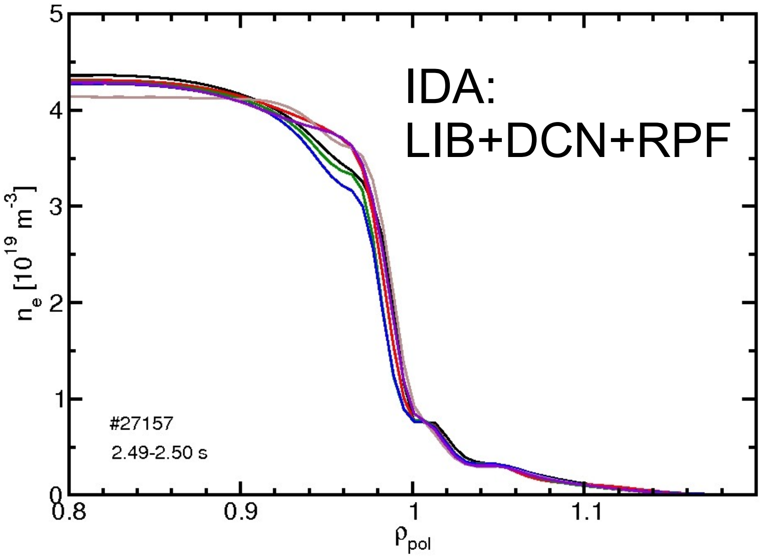
Forward model for group delay for a given density profile:

$$\tau(f, n(r)) = \frac{2}{c}\int_0^{\sqrt{r_c - r_1}} \frac{2x}{\sqrt{1 - \frac{n(r_c - x^2)}{n_c}}}\,dx$$

IDA: LIB + DCN + Reflectometry







- Only physically reasonable profiles possible (spline)
- Alignment is ok (< 5 mm)
- Modification of n_e at pedestal top
- Systematic deviances in residues

Application: ASDEX Upgrade

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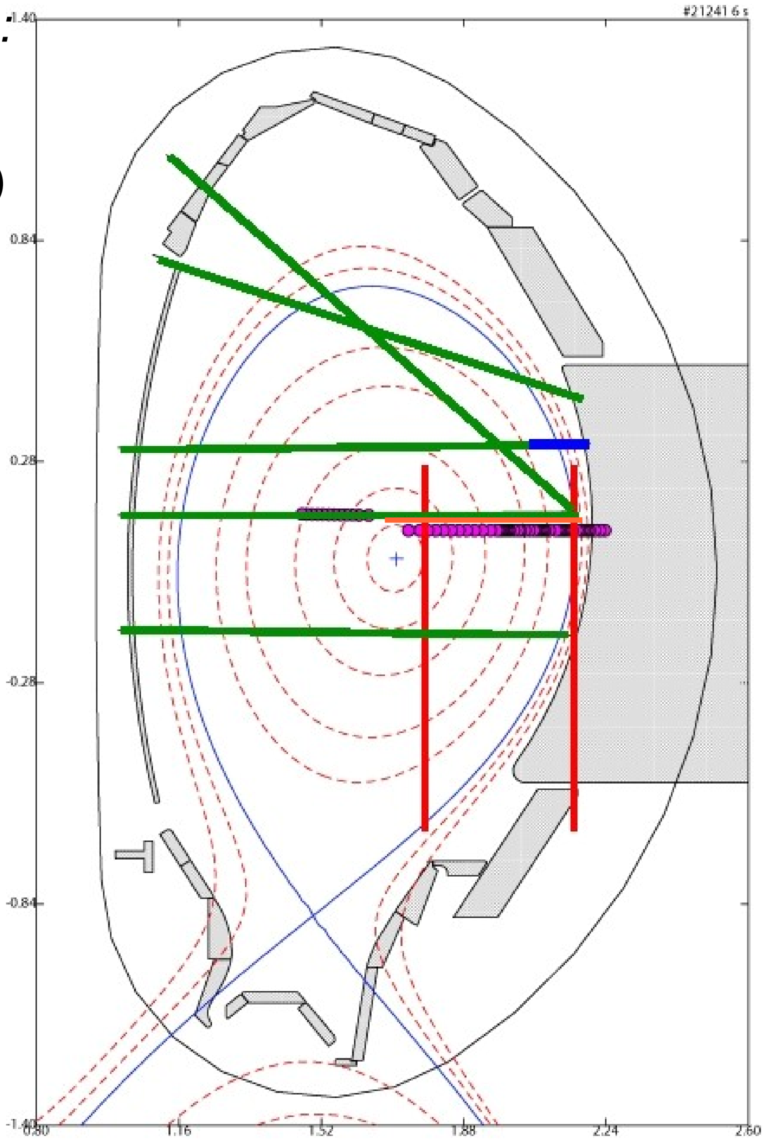


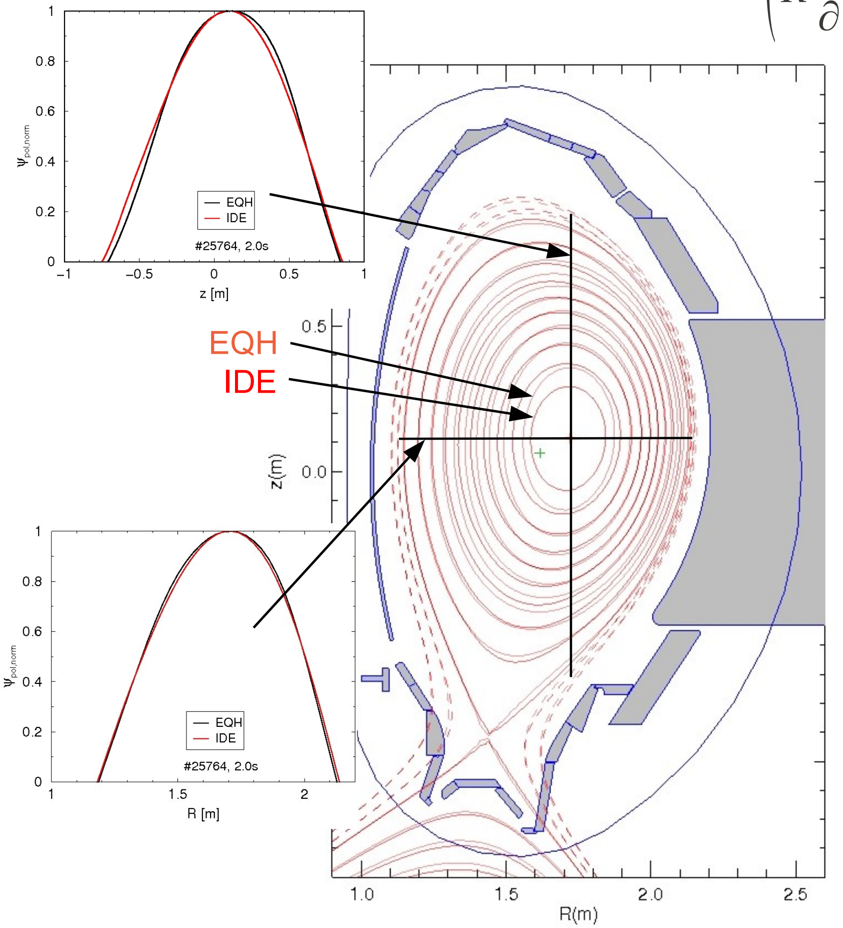
IDA and the Magnetic Equilibrium

- Combine profile diagnostics LIB, DCN, ECE, TS, REF
  → n_e and T_e profile fits to all data at once
- Mapping on a common coordinate grid using an existing equilibrium (EQH/EQI/FPP)
- Inconsistency: Equilibrium is not evaluated with kinetic profiles from IDA
  - ✗ Position of magnetic axis, separatrix, inner flux surfaces?
  - ✗ DCN: H2-H3 vertical plasma position often seems to be wrong up to ~1cm.
  - ✗ ECE: (r,z) depends on equilibrium
  - ✗ TS: vertical system relies very much on equilibrium
  - ✗ Alignment of TS, ECE, LIB (with separatrix T_e) → uncertainties in the equilibrium ???
- Goal: combine data from profile diagnostics with magnetic data
  for a joint estimation of profiles and the magnetic equilibrium
- Needs equilibrium code:
  - ✗ CLISTE very successful, but code too sophisticated to be adapted to the IDA code
  - ✗ New code based on the ideas (success) of CLISTE
    (P. McCarthy, L. Giannone, P. Martin, K. Lackner, S. Gori)
  - ✗ Extra: Parallel Grad-Shafranov solver → real-time equilibrium (~100 μs)
    (R. Preuss, M. Rampp, K. Hallatschek, L. Giannone)

Grad-Shafranov solver

Grad-Shafranov equation: Ideal magnetohydrodynamic equilibrium for poloidal flux function Ψ for axisymmetric geometry

$$\left(R\frac{\partial}{\partial R}\frac{1}{R}\frac{\partial}{\partial R}+\frac{\partial^2}{\partial z^2}\right)\Psi=-(2\pi)^2\mu_0\left(R^2P'+\mu_0FF'\right)$$

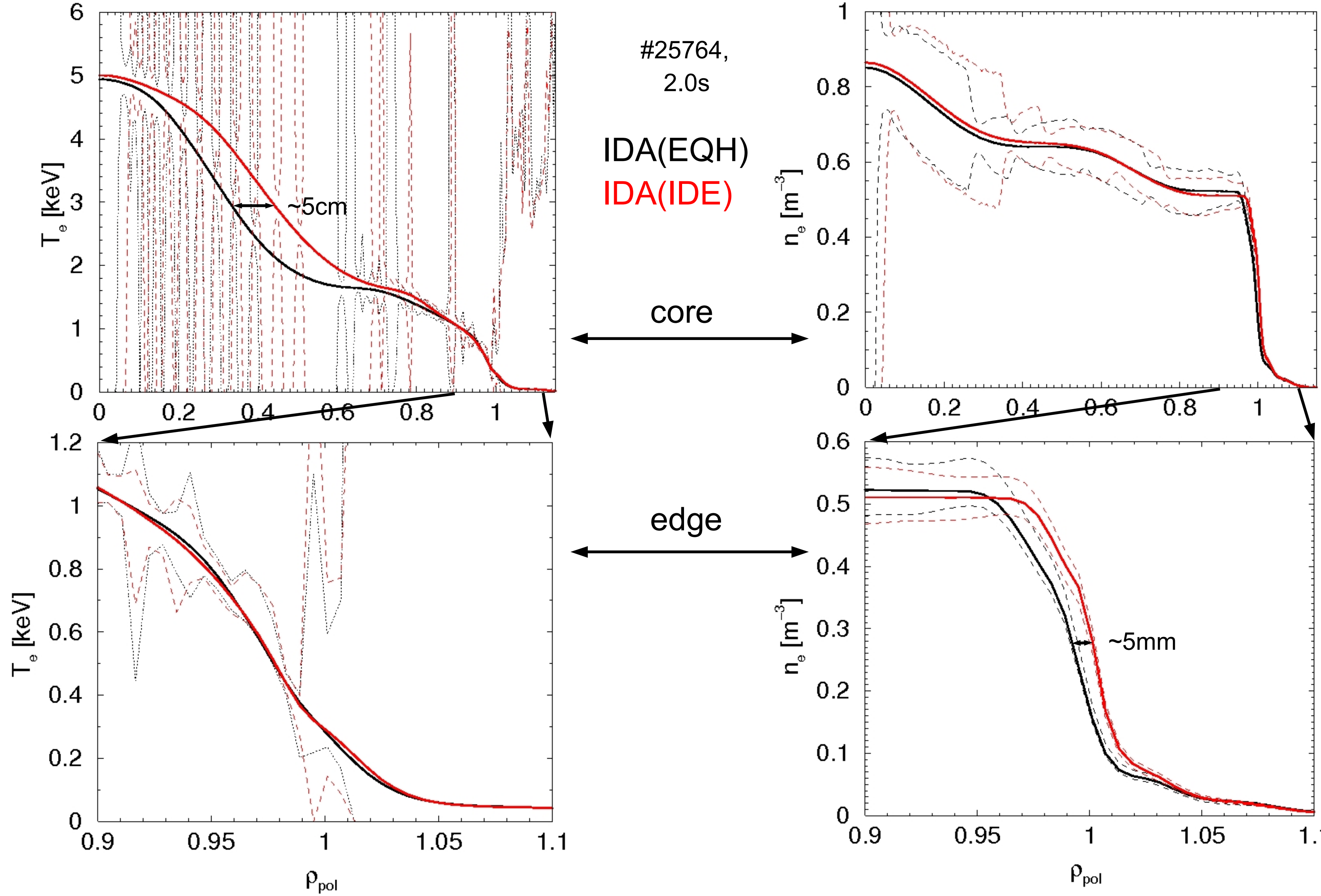


- ✔ New code developed capable to be used in the IDA concept:
- ✔ Magnetic measurements
- ✔ Profiles from other diagnostics
- ✔ Real-time solver (~100 µs)

R. Preuss et al., IPP-Report R/47 (2012)

M. Rampp et al., Fusion Science and Technol., accepted

Comparison EQH/IDE: Temperature and density



Summary: IDA

- Probabilistic modeling of individual diagnostics
  - ✔ forward modeling only (synthetic diagnostic)
  - ✔ probability distributions: describes all kind of uncertainties
  - ✔ multiply probability distributions, marginalization of nuisance parameters
  - ✔ parameter estimates and uncertainties

- Probabilistic combination of different diagnostics
  - ✔ systematic and unified error analysis is a must for comparison of diagnostics
  - ✔ error propagation beyond single diagnostics
  - ✔ more reliable results by larger (meta-) data set (interdependencies, synergism)
  - ✔ redundant information → resolve data inconsistencies
  - ✔ advanced data analysis technique → software/hardware upgrades

- Applications at W7-AS (Garching, Germany), JET (Culham, UK), TJ-II (Madrid, Spain), and ASDEX Upgrade (Garching, Germany), W7-X (Greifswald, Germany), Tore Supra (Cadarache, France)