Bayesian mixture models for background-source separation

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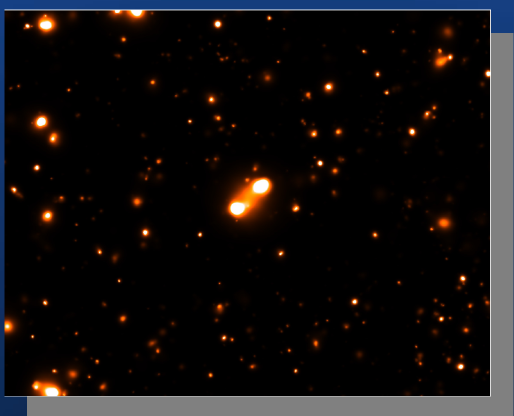
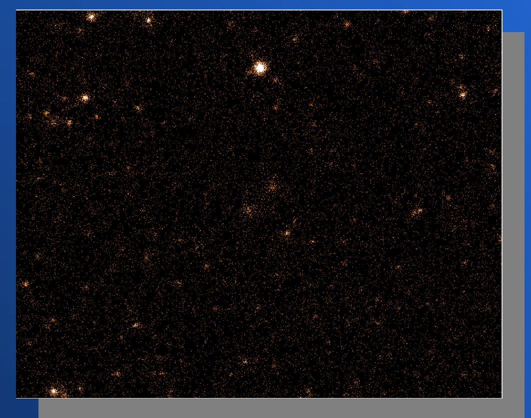
Outline

1. Introduction
2. The Bayesian mixture model technique
   - *Guglielmetti F., Fischer R., Dose V., 2009, MNRAS, 396, 165*
   - Source detection and characterization
3. Applications
4. Summary & Conclusions

Introduction





Simulated eROSITA image for 2 ks observing time. Image includes expected particle and instrumental background, population of AGN, effects due to PSF and Poisson noise.

Simulated sky image

[erg/s/cm²/deg²]

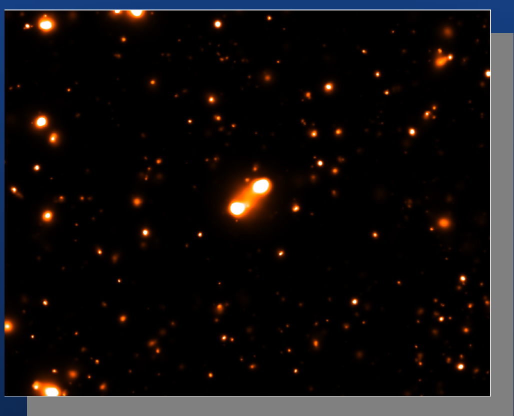
X-ray sky image

[photon count/pixel]

Credits to:
Pace, F. (ICG, UK)
Roncarelli, M. (UniBo, Italy)

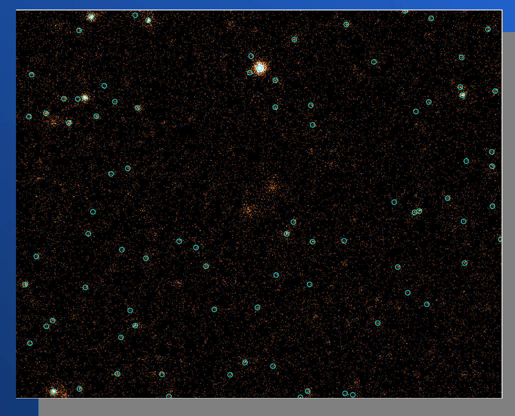
Credits to: Mühlegger, M.

Introduction



Simulated sky image

[erg/s/cm²/deg²]



X-ray sky image

[photon count/pixel]

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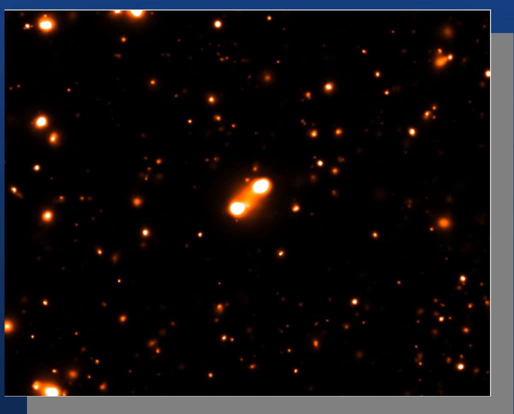
Introduction



Simulated sky image

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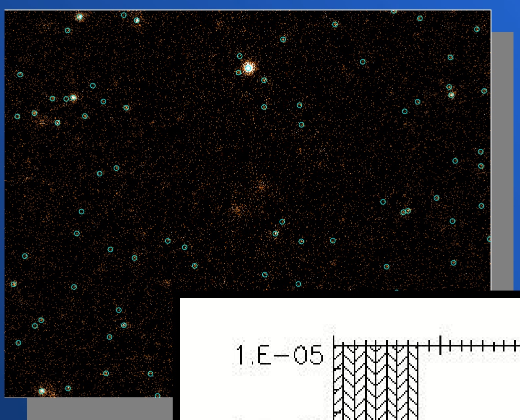
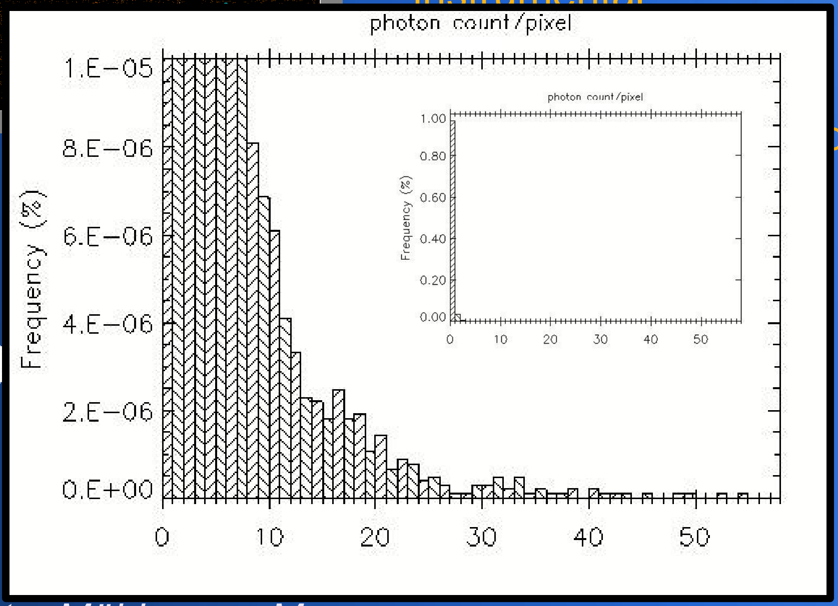


Simulated eROSITA image for 2 ks observing time. Image includes expected particle and instrumental



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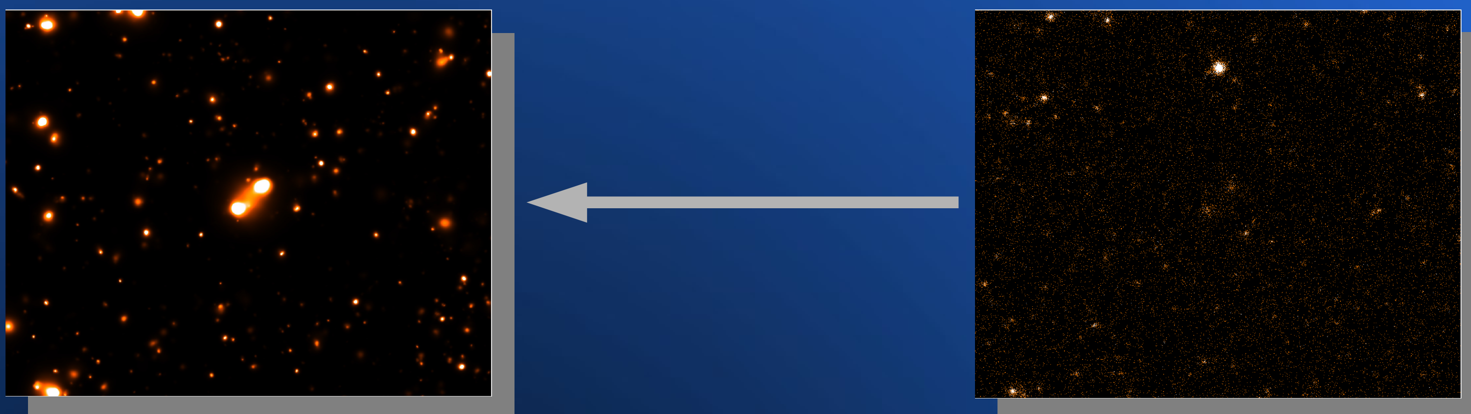
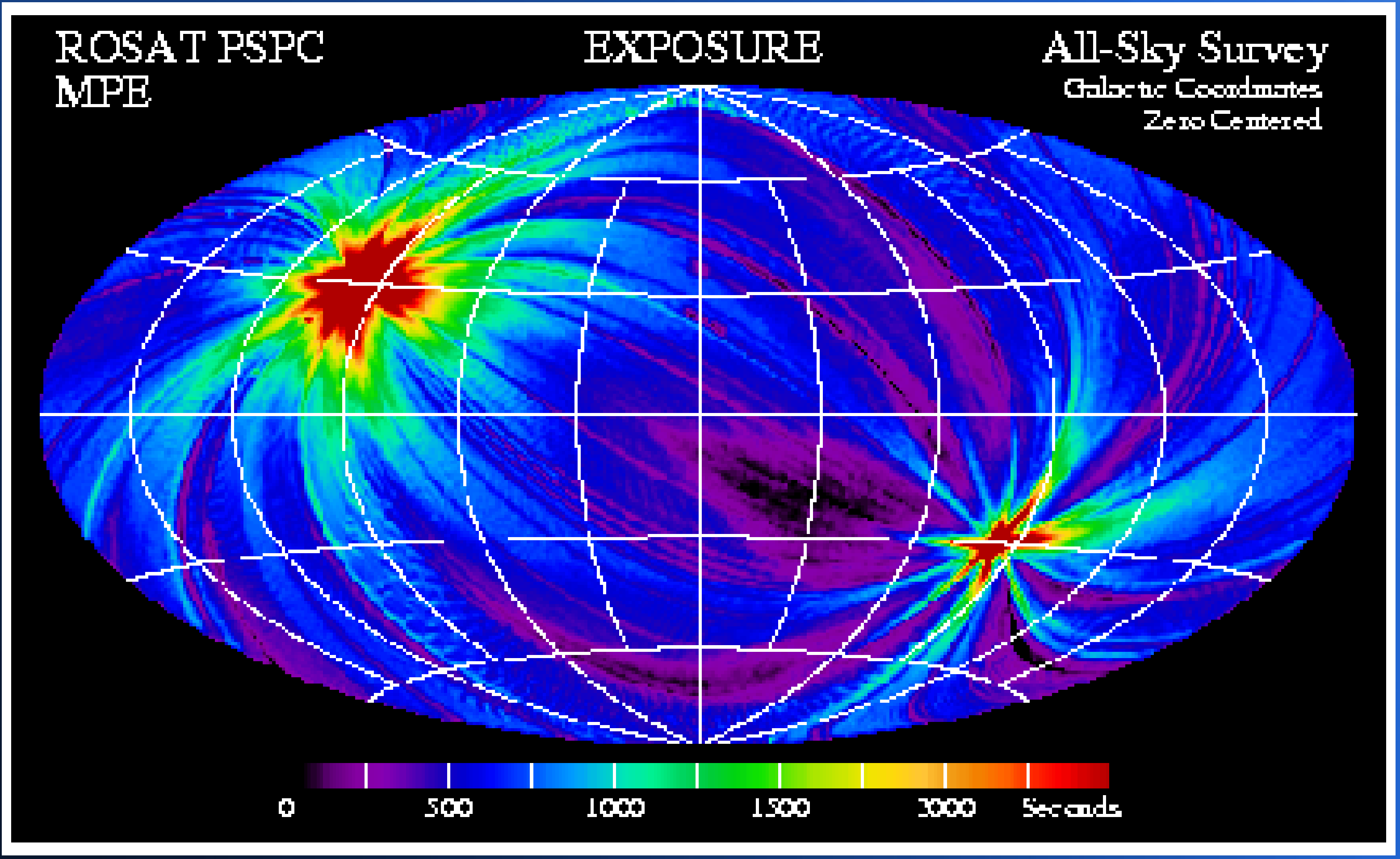
Introduction



- <u>Difficulties to overcome in image analysis:</u>
  1. Ill-posed inverse problem
  2. 0-few counts per pixel
  3. Diffuse background plus celestial objects: a) Background is not constant;
     b) Sources show large variety of source morphologies
  4. Instrumental complexities
     - ➔ Increase statistical and systematic errors in the data

Introduction



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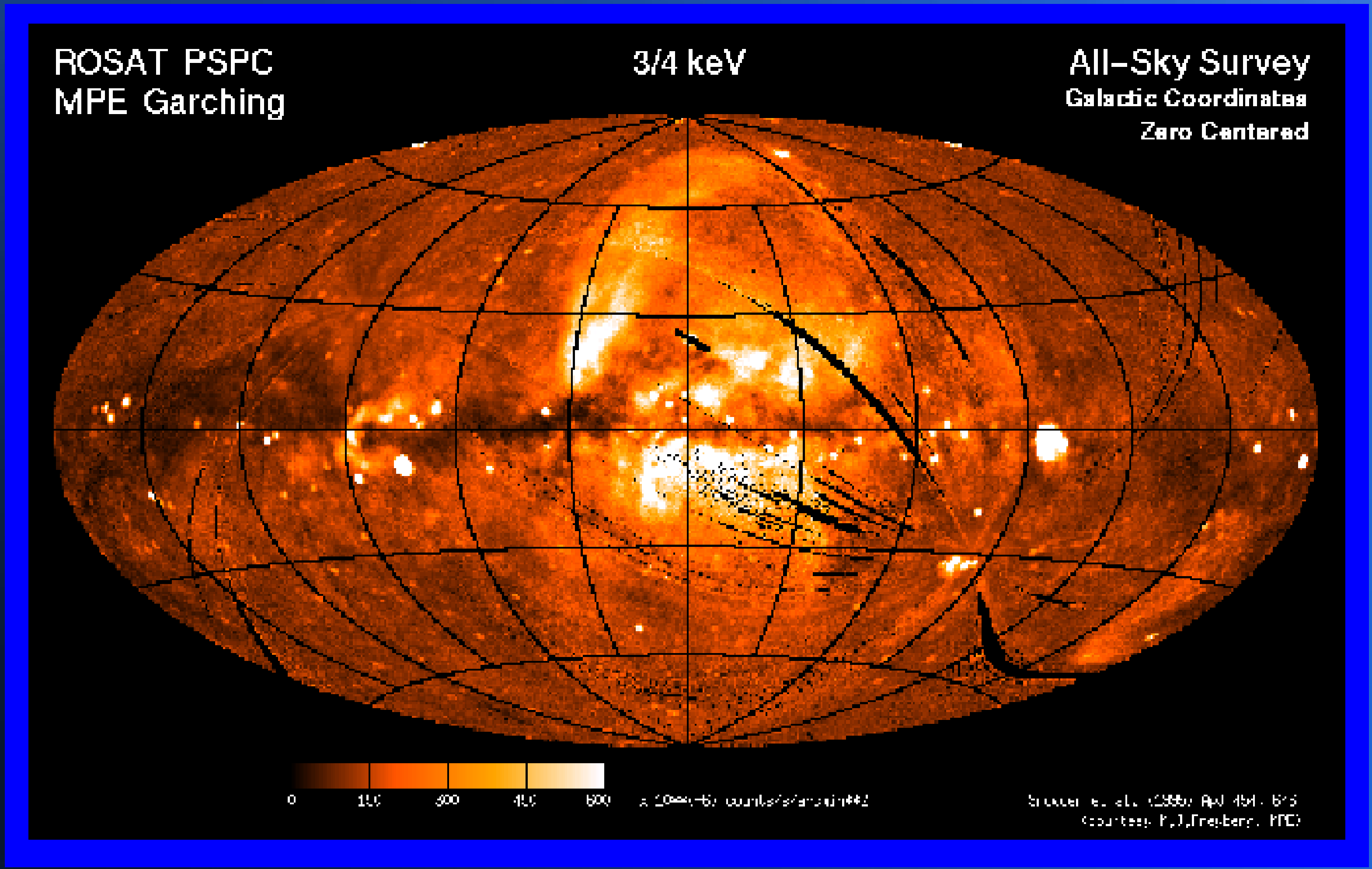


Why is it important?

- Address astrophysical problems, as:
  - Study physical properties of detected objects
  - Test models of structure formation (as for clusters of galaxies)
  - Explore stellar and galaxy evolution
  - Understand the nature of dark energy and dark matter
  - Provide insight for the origin and the ultimate fate of the Universe
  - ....

➡ **We need to detect both point-like and extended sources**

Standard detection approach

- Objects of interest are superposed on a relatively flat signal: *Background signal*
- Background must be accurately estimated, or bias on flux estimation is introduced
- (Common) Background estimation:
  - Cut out of sources (ebox)
  - Histogram after partitioning image into blocks (Median filtering)
- Statistical fluctuations: thresholds are used for tuning the number of false sources
  - False positives and negatives

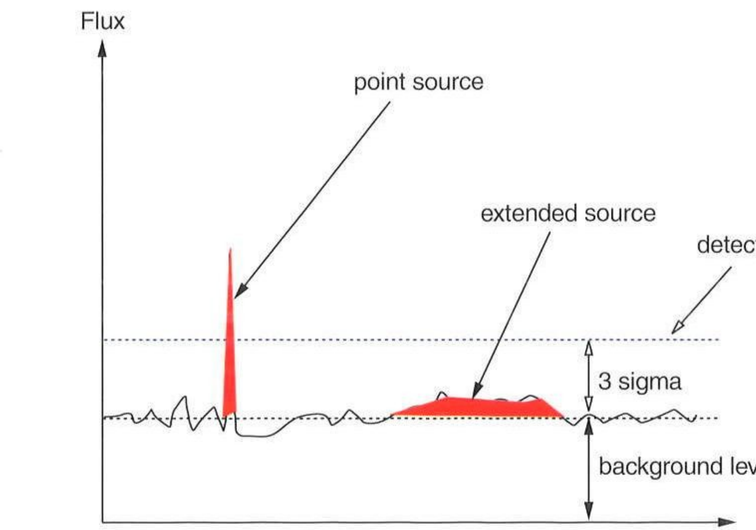


Fig. 4.1. Example of astronomical data: a point source and an extended source are shown, with noise and background. The extended object, which can be detected by eye, is undetected by a standard detection approach.

From: "Astronomical Image and Data Analysis"
Starck, J.-L. and Murtagh, F.
Springer Verlag 2006

Desiderata & Challenges

1. Preserves statistics
2. Detect faint sources
3. Detect point-like and extended sources
4. Reliable background model
5. Properly include exposure
6. Uncertainty of estimates

1. Poisson, background fluctuations
2. Joint background+sources,

   model parameters estimated from the data
3. Large variety of source morphologies
4. Steep gradients
5. Instrumental complexities
6. Quantification

Bayesian mixture models

- Single observed data set:

$$D = \{d_{ij}\} \in \mathbb{N}$$

- Bayesian Probability Theory (BPT)
- Two complementary hypotheses for each pixel:

$$B_{ij} : d_{ij} = b_{ij} + \epsilon_{ij}$$
$$\overline{B}_{ij} : d_{ij} = b_{ij} + s_{ij} + \epsilon_{ij}$$

- Assumptions:

  I. *b* smoother than *s*
  II. $b, s \in \mathbb{R}^{+}$

- 2D spline (Thin-Plate spline)
- BPT with probabilistic mixture model

Thin-Plate spline (TPS)

Number of supporting points

weights

$$t(\bar{x}) = E(\bar{x}) + \sum_{l=1}^{N} \lambda_l \, f(\bar{x} - \bar{x}_l)\ ,\ with\ \bar{x} \in \mathbb{R}^2$$

plane

Radial Basis Function: $f(\bar{x} - \bar{x}_l) = r^2 \ln r^2$

- Requirements:

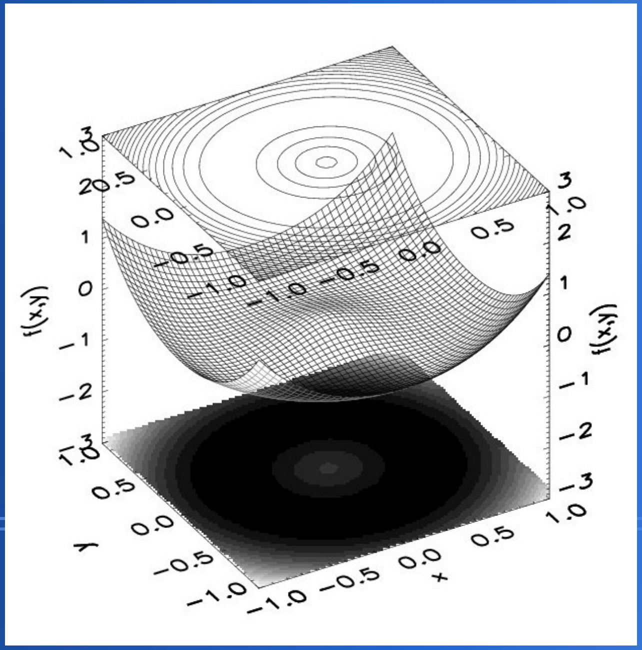
1) $t(\bar{x})$ is 2[nd] order differentiable

2) $t(\bar{x}_i) = z_i$

3) $\|t^2\| = I[f(x,y)] = \iint_{\mathbb{R}^2} (f_{xx}{}^2 + 2 f_{xy}{}^2 + f_{yy}{}^2)$

↓

satisfy the interpolation conditions, evaluate the TPS



Likelihood for mixture models

Poisson Likelihood

$$p(d_{ij}|B_{ij}, b_{ij}) = \frac{b_{ij}^{d_{ij}}}{d_{ij}!} e^{-b_{ij}}, \textit{ when } B_{ij} \textit{ is true}$$

$$p(d_{ij}|\bar{B}_{ij}, b_{ij}, s_{ij}) = \frac{(b_{ij}+s_{ij})^{d_{ij}}}{d_{ij}!} e^{-(b_{ij}+s_{ij})}, \textit{ when } \bar{B}_{ij} \textit{ is true}$$

Marginal Poisson Likelihood

Likelihood for the mixture model

$$p(D|b, \lambda^*, \beta^*) = \prod_{ij} [\beta^* p(d_{ij}|B_{ij}, b_{ij}) + (1-\beta^*) p(d_{ij}|\bar{B}_{ij}, b_{ij}, \lambda^*)]$$

$$p(B_{ij}) = \beta, \; p(\bar{B}_{ij}) = 1-\beta$$

$$p(s_{ij}|\lambda) = \frac{e^{-s_{ij}/\lambda}}{\lambda}$$

mean expected intensity

Likelihood for mixture models

Poisson Likelihood

$$p\,(d_{ij}|B_{ij}\,,\,b_{ij}) = \frac{b_{ij}^{d_{ij}}}{d_{ij}!}\,e^{-b_{ij}}\,,\ when\ B_{ij}\ is\ true$$

$$p\,(d_{ij}|\bar{B}_{ij}\,,\,b_{ij}\,,\,s_{ij}) = \frac{(b_{ij}+s_{ij})^{d_{ij}}}{d_{ij}!}\,\mathrm{e}^{-(b_{ij}+s_{ij})}\,,\ when\ \bar{B}_{ij}\ is\ true$$

Marginal Poisson Likelihood

Likelihood for the mixture model

$$p(D|b,\lambda^*,\beta^*)=\prod_{ij}[\beta^* p(d_{ij}|B_{ij},b_{ij})+(1-\beta^*)\,p(d_{ij}|\bar{B}_{ij},b_{ij},\lambda^*)]$$

$$p\,(B_{ij})=\beta\,,\,p\,(\bar{B}_{ij})=1-\beta$$

$$p(s_{ij}|\lambda,a)=\mathrm{e}^{-a/s_{ij}}\,s_{ij}^{-\lambda}\,\frac{a^{\lambda-1}}{\Gamma(\lambda-1)}$$

Slope Cut-off params

Likelihood for mixture models

Poisson Likelihood

$$p\left(d_{ij}|B_{ij}, b_{ij}\right) = \frac{b_{ij}^{d_{ij}}}{d_{ij}!} e^{-b_{ij}}, \text{ when } B_{ij} \text{ is true}$$

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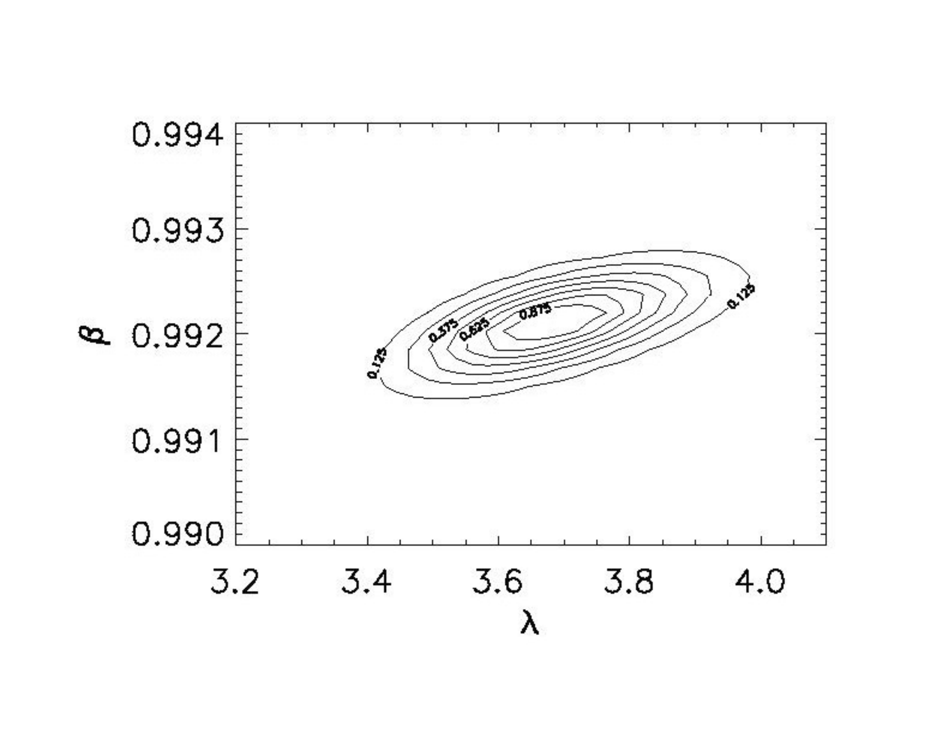
Hyper-parameters: Laplace approximation

$$\max_{\beta,\lambda} p(\beta, \lambda|D) \rightarrow \beta^*, \lambda^*$$

Likelihood for mixture models



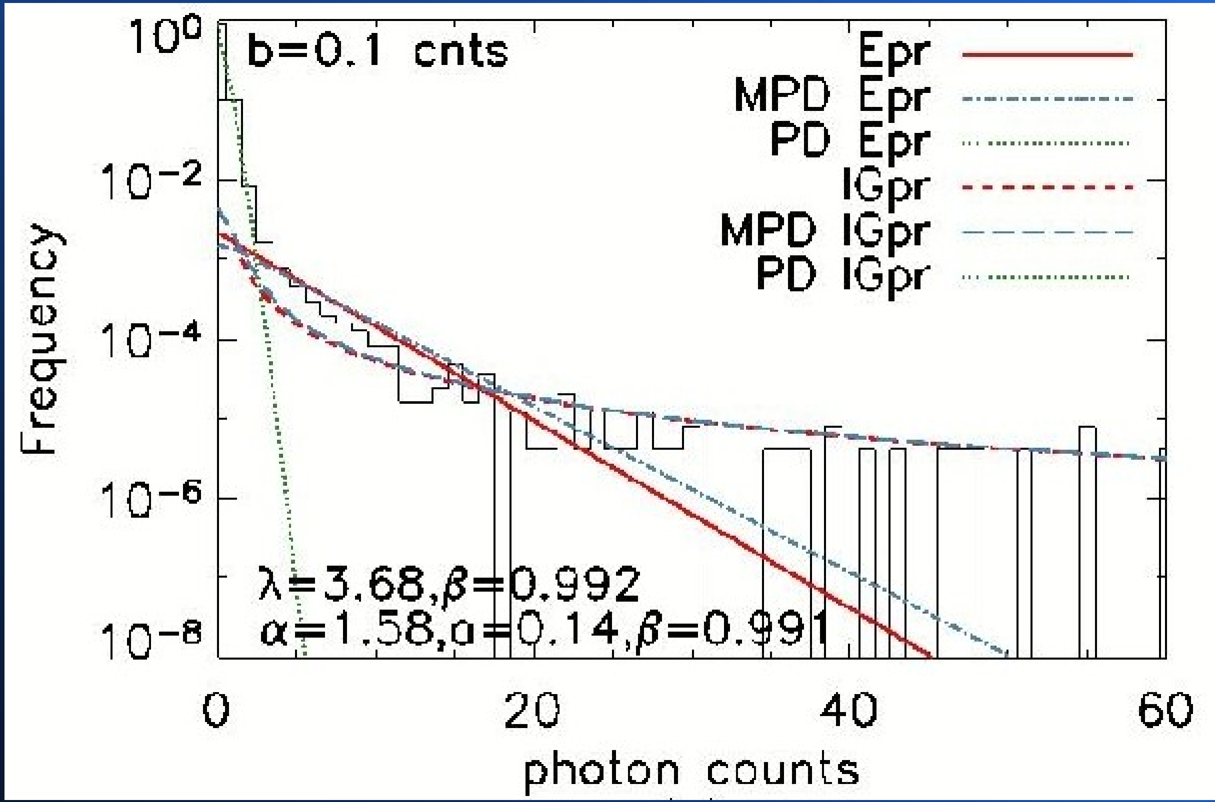
Hyperparameters: Laplace approximation

$$p(\beta, \lambda | D) = \frac{p(D|\lambda, \beta) p(\lambda) p(\beta)}{p(D)}$$

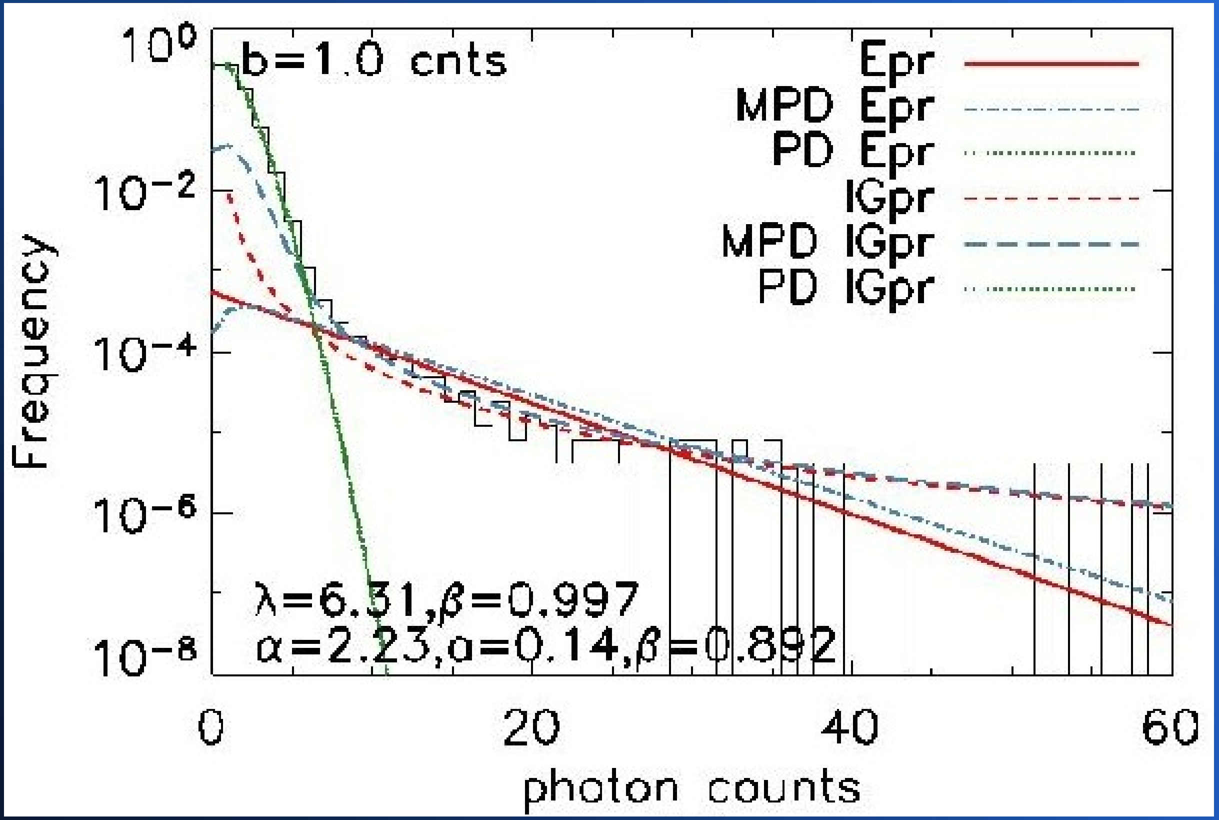
↑

$$\max_{\beta, \lambda} p(\beta, \lambda | D) \rightarrow \beta^*, \lambda^*$$

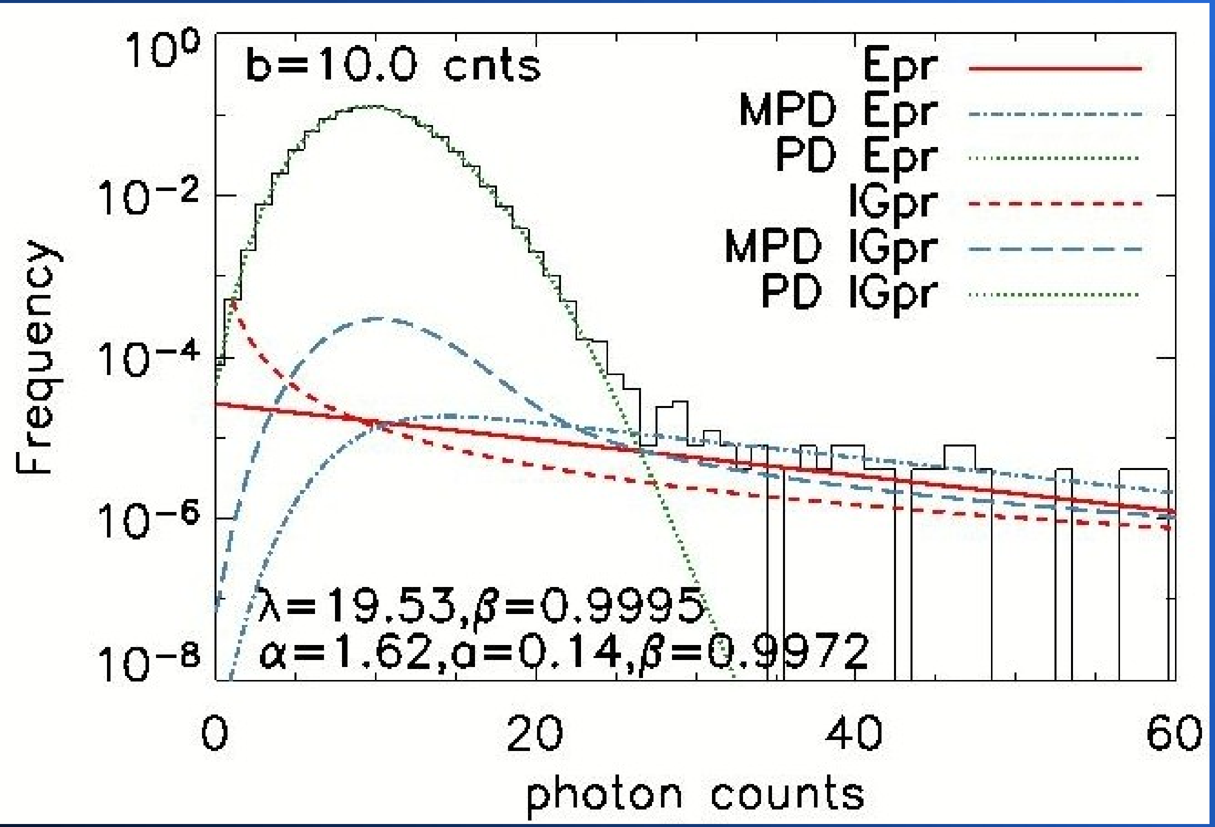
Likelihood for mixture models



Likelihood for mixture models



Likelihood for mixture models



Background model

TPS → $t(\bar{x}) \times \tau(\bar{x})$ ← Observatory exposure time

↓

Posterior PDF of background:

$$p(b|D) \propto p(D|b, \lambda^*, \beta^*) \times p(b)$$

↓

$$\max_b p(b|D) \rightarrow b^*$$

← Estimate of Background Map

Posterior pdf for source detection

$$p(\bar{B}_{ij}|d_{ij}) \approx \frac{1}{1+\frac{\beta^*}{1-\beta^*}\cdot\frac{p(d_{ij}|B_{ij},b_{ij}^*)}{p(d_{ij}|\bar{B}_{ij},b_{ij}^*,\lambda^*)}}$$

Posterior pdf for source detection

$$b^* = \{b_{ij}^*\}$$

$$p(\bar{B}_{ij}|d_{ij}) \approx \frac{1}{1 + \frac{\beta^*}{1-\beta^*} \cdot \frac{p(d_{ij}|B_{ij}, b_{ij}^*)}{p(d_{ij}|\bar{B}_{ij}, b_{ij}^*, \lambda^*)}}$$

Posterior pdf for source detection

$$p(\bar{B}_{ij}|d_{ij}) \approx \frac{1}{1+\frac{\beta^*}{1-\beta^*}\frac{p(d_{ij}|B_{ij},b^*_{ij})}{p(d_{ij}|\bar{B}_{ij},b^*_{ij},\lambda^*)}}$$

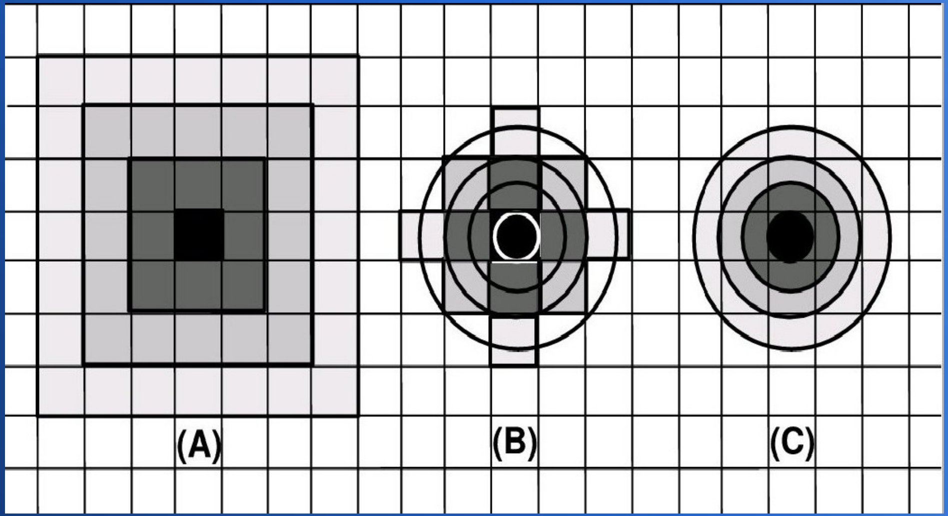
Bayes factors

Detection of faint sources and complex morphologies

1. **Multi resolution analysis:**

   pdfs assigned correlating the information of neighbouring pixels:

   Source Probability Maps (SPM)

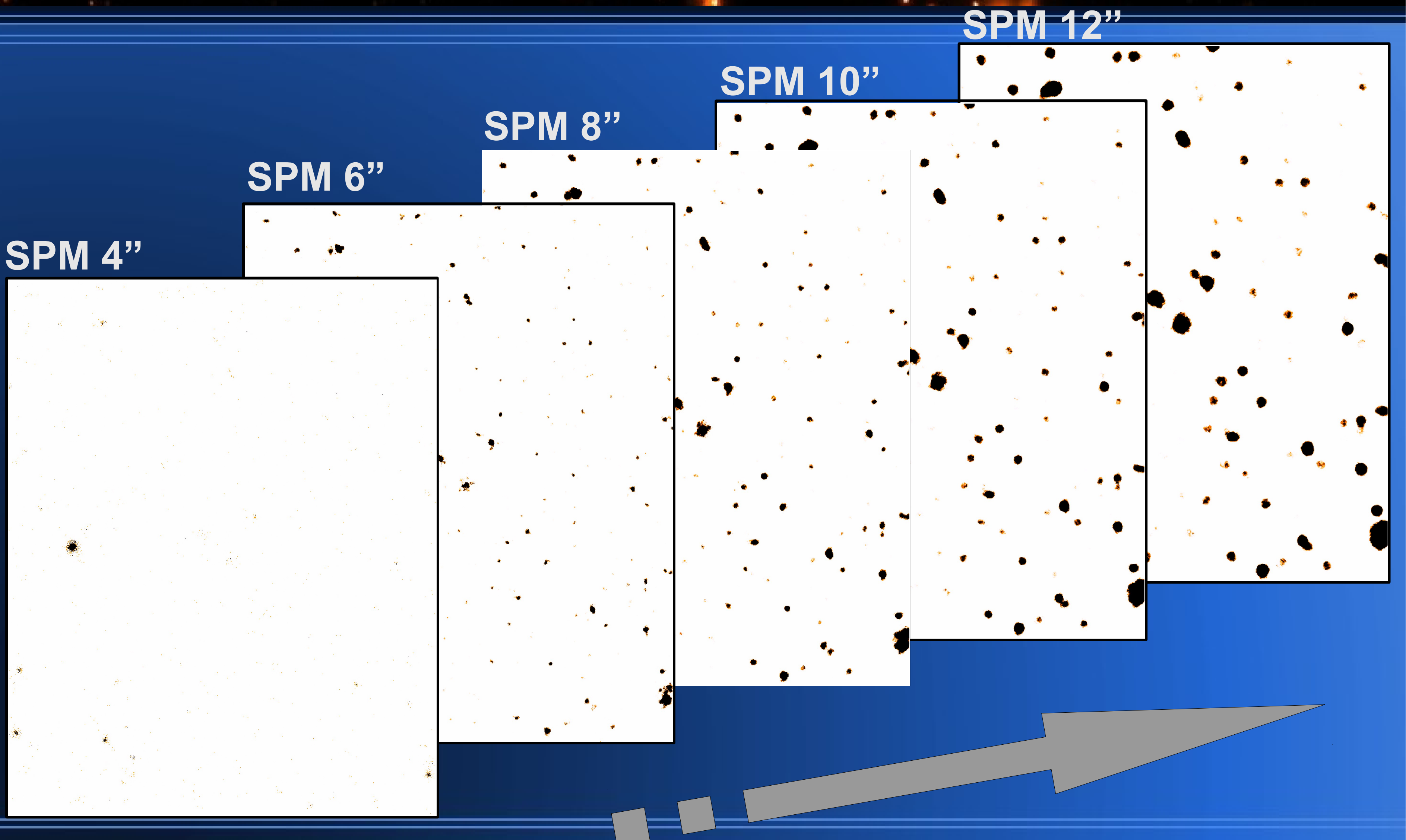


2. **Multi band analysis:**

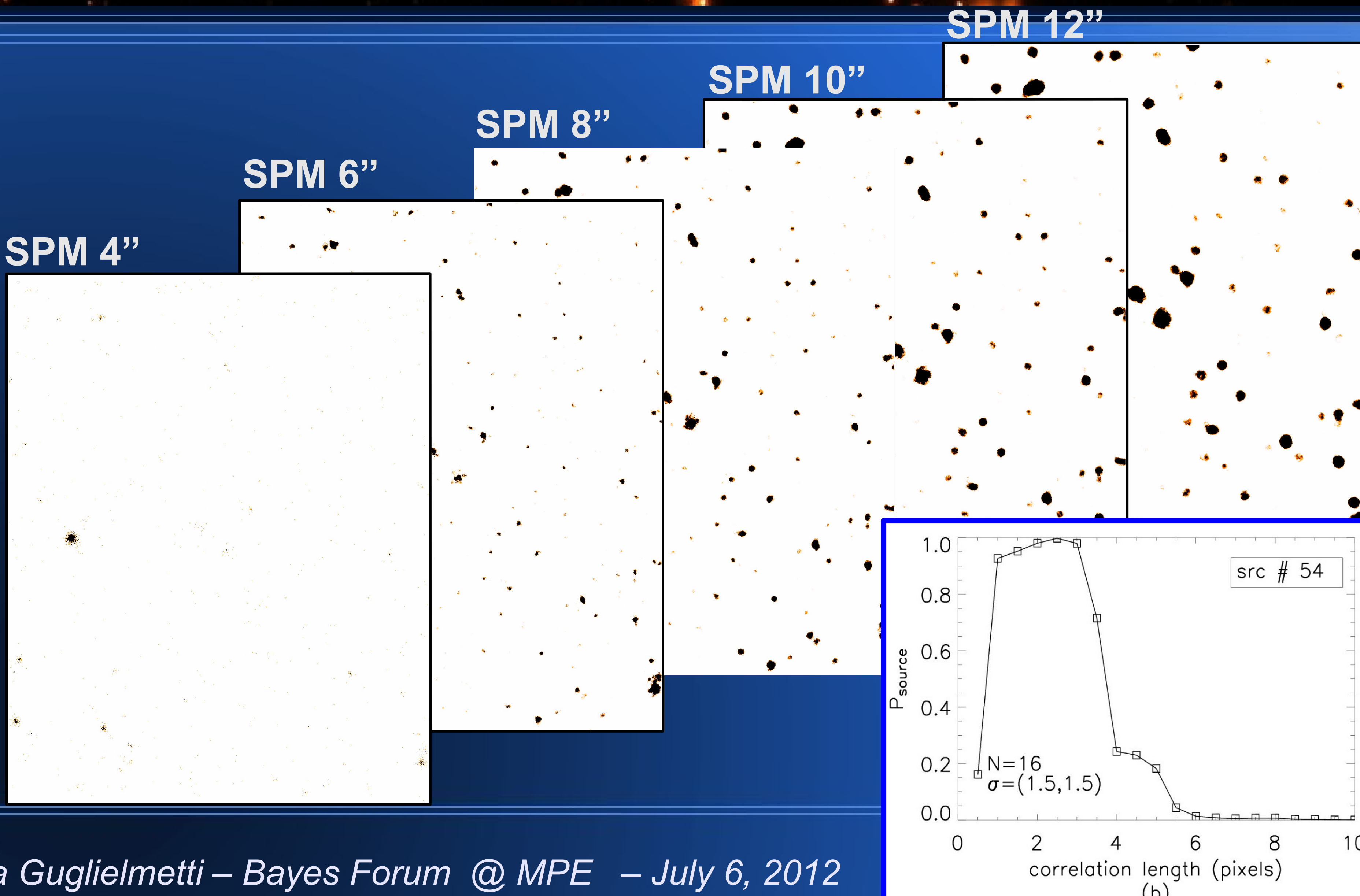
   Statistical combination of data from different energy bands

$$p(\overline{B_{ij}}|d_{ij})_{comb} = 1 - \prod_{k=1}^{n} \left[1 - p(\overline{B_{ij}}|d_{ij})_k\right]$$

Detection of faint sources and complex morphologies

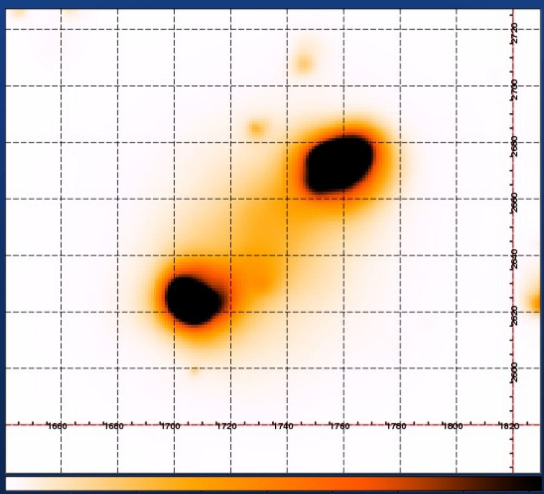


Detection of faint sources and complex morphologies

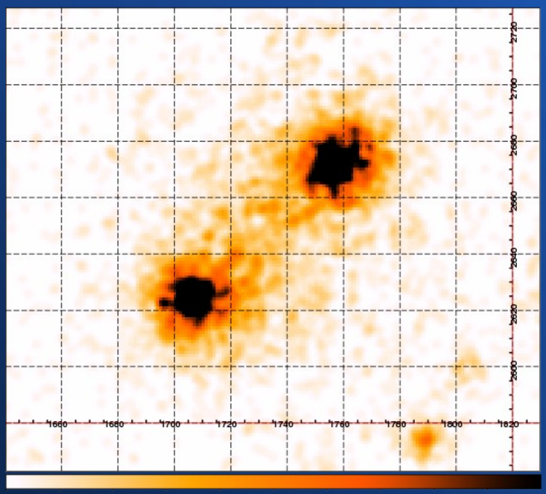


Detection of faint sources and complex morphologies

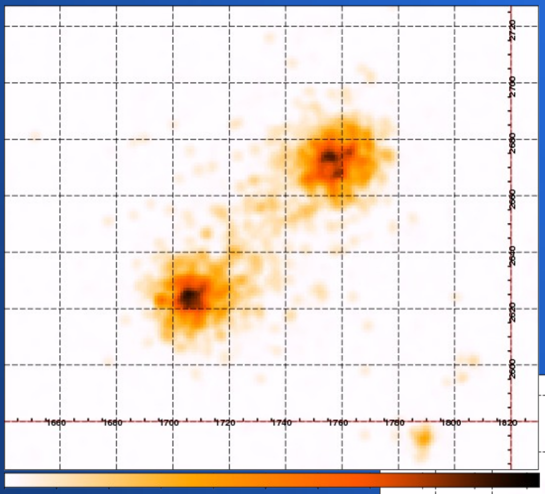
LSS image



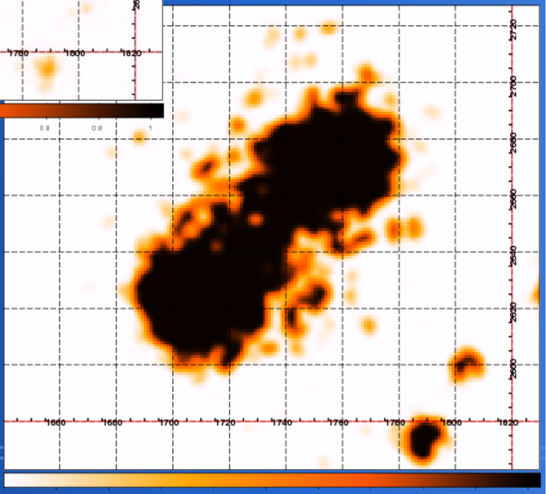
Photon count smoothed



SPM 4''



SPM 6''



Detection of two clusters
(merging)

Photometry

$$D_{ij} = b_{ij} + G_{ij} \qquad \forall \{ij\} \in \{k\}$$

Data of a source in detection area 'k'

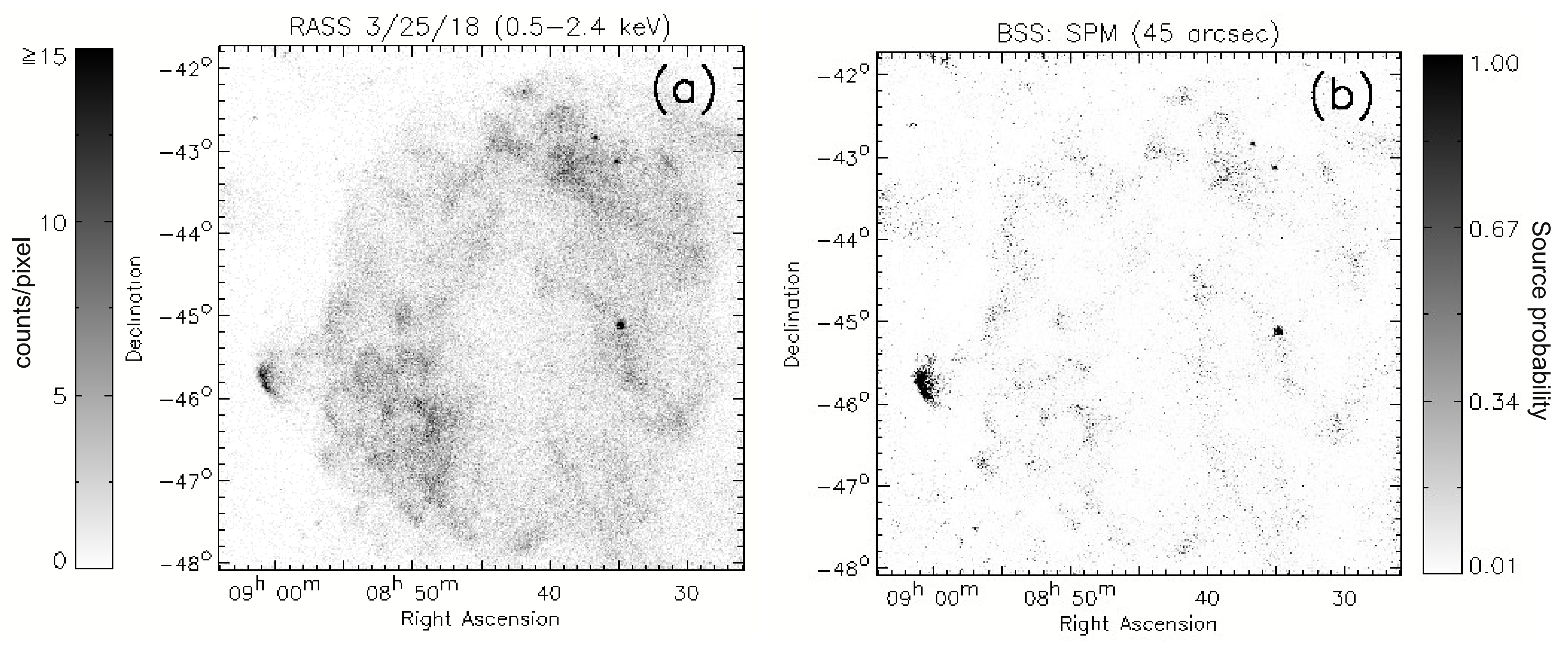
Function describing the photon counts distribution of detected sources

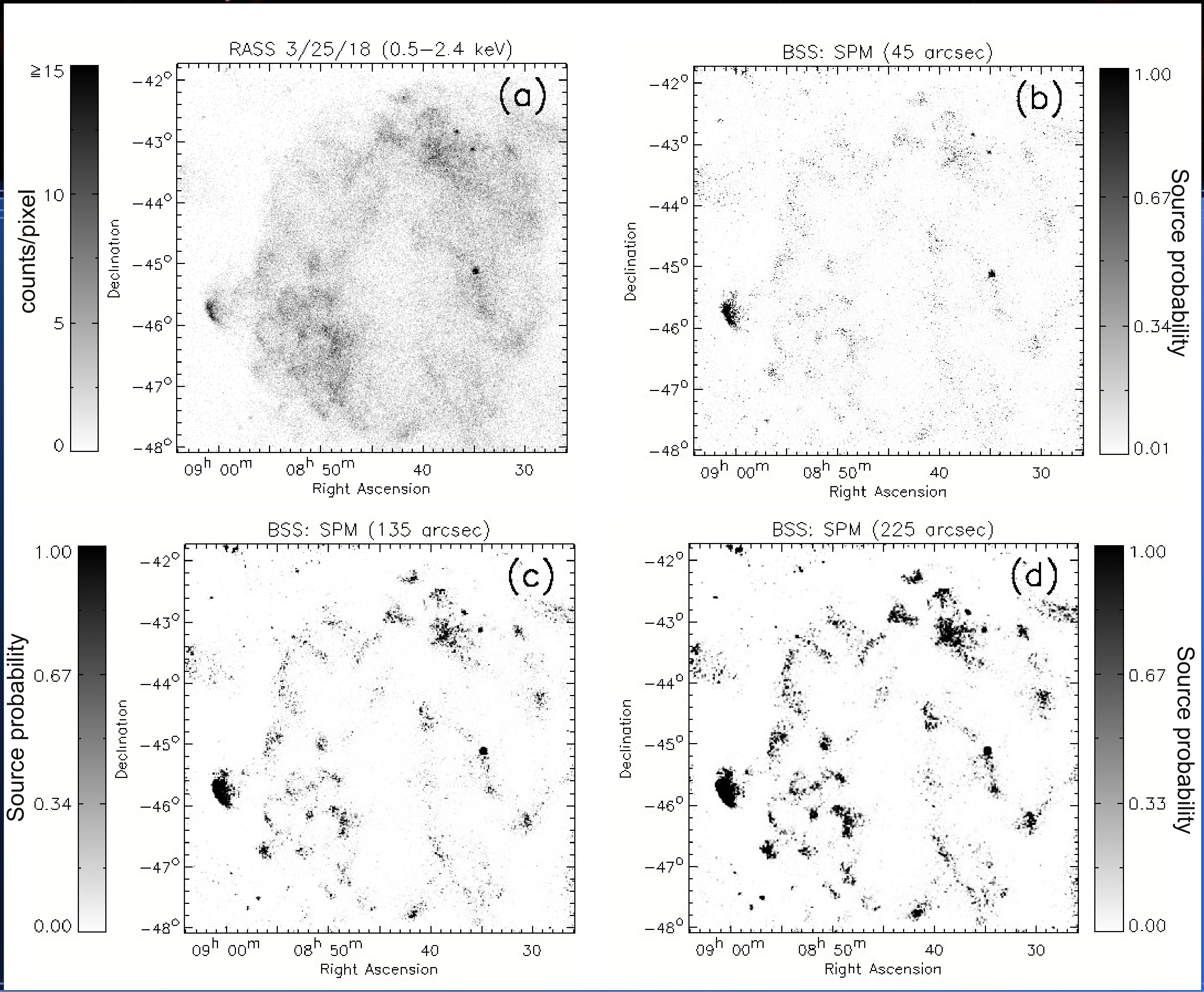
Max of posterior pdf:

$$p(x, y, \sigma_x, \sigma_y, \rho, I | b, d) \propto \prod_{ij} D_{ij}^{d_{ij}} \frac{e^{-D_{ij}}}{d_{ij}!} \qquad \forall \{ij\} \in \{k\}$$

Application to RASS

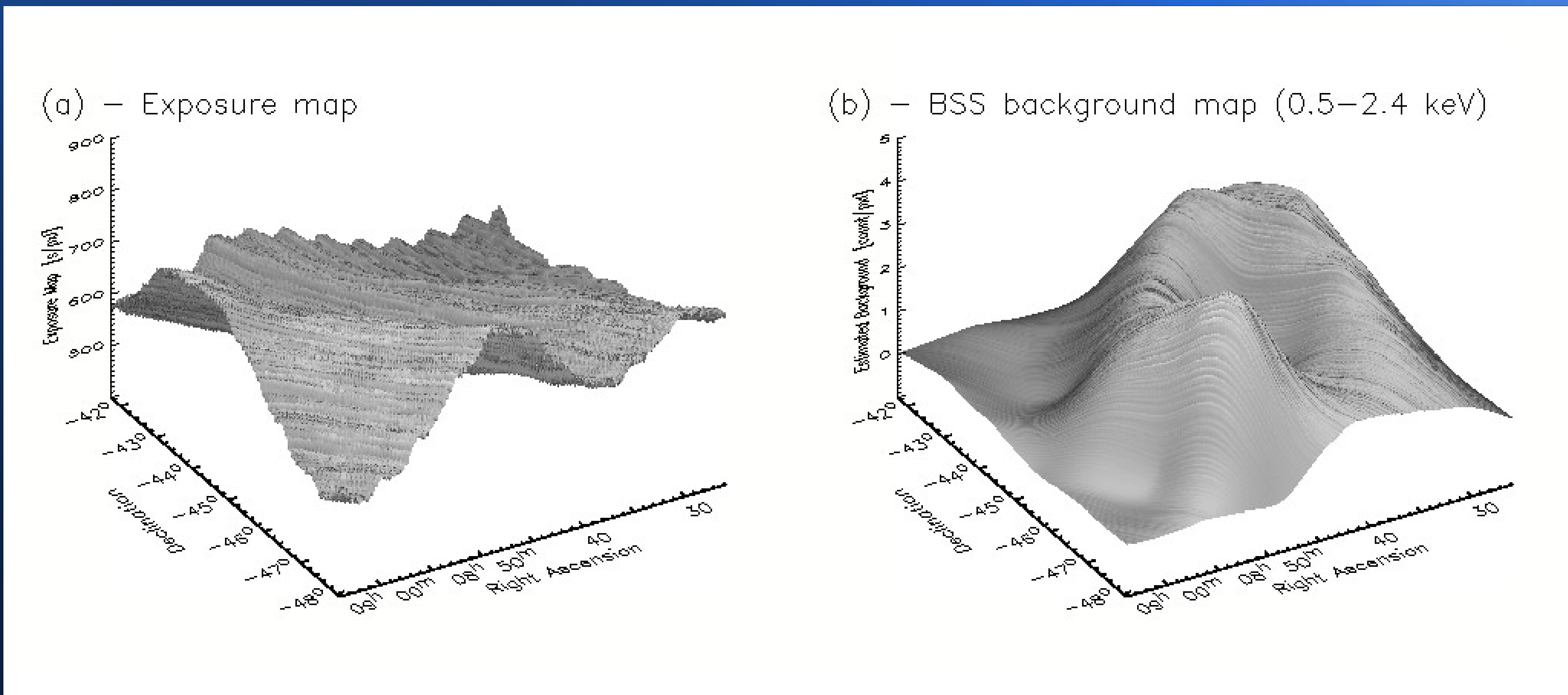
The Vela SNR d=(250 ± 30) pc *(Cha et al.1999)*
Age=(18000 ± 9000) yr *(Aschenbach et al. 1995)*



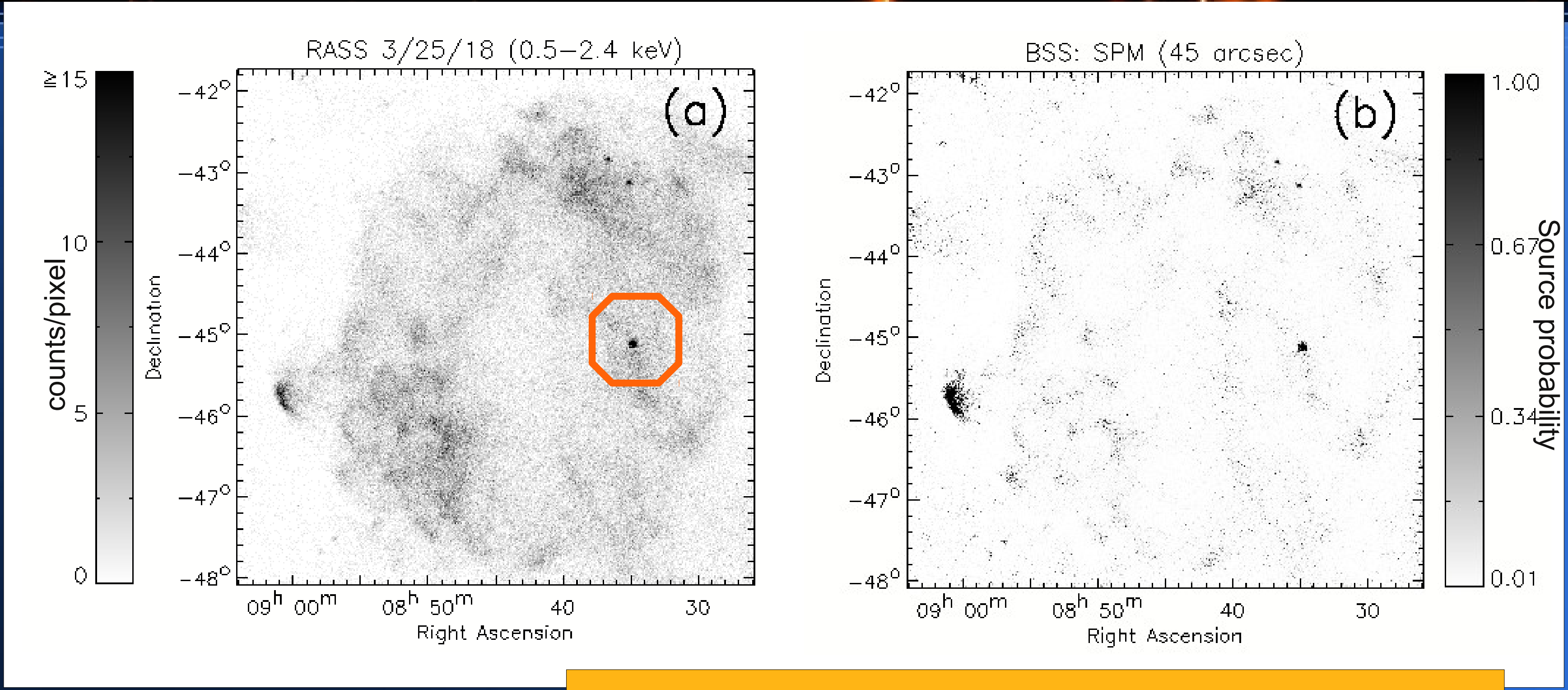


RASS 3/25/18 (0.5–2.4 keV)
(a)
≥15
10
5
0
counts/pixel
Declination
−42°
−43°
−44°
−45°
−46°
−47°
−48°
09h 00m
08h 50m
40
30
Right Ascension
BSS: SPM (45 arcsec)
(b)
1.00
0.67
0.34
0.01
Source probability
BSS: SPM (135 arcsec)
(c)
1.00
0.67
0.34
0.00
Source probability
BSS: SPM (225 arcsec)
(d)
1.00
0.67
0.33
0.00
Source probability

Vela SNR background model



The Vela Pulsar

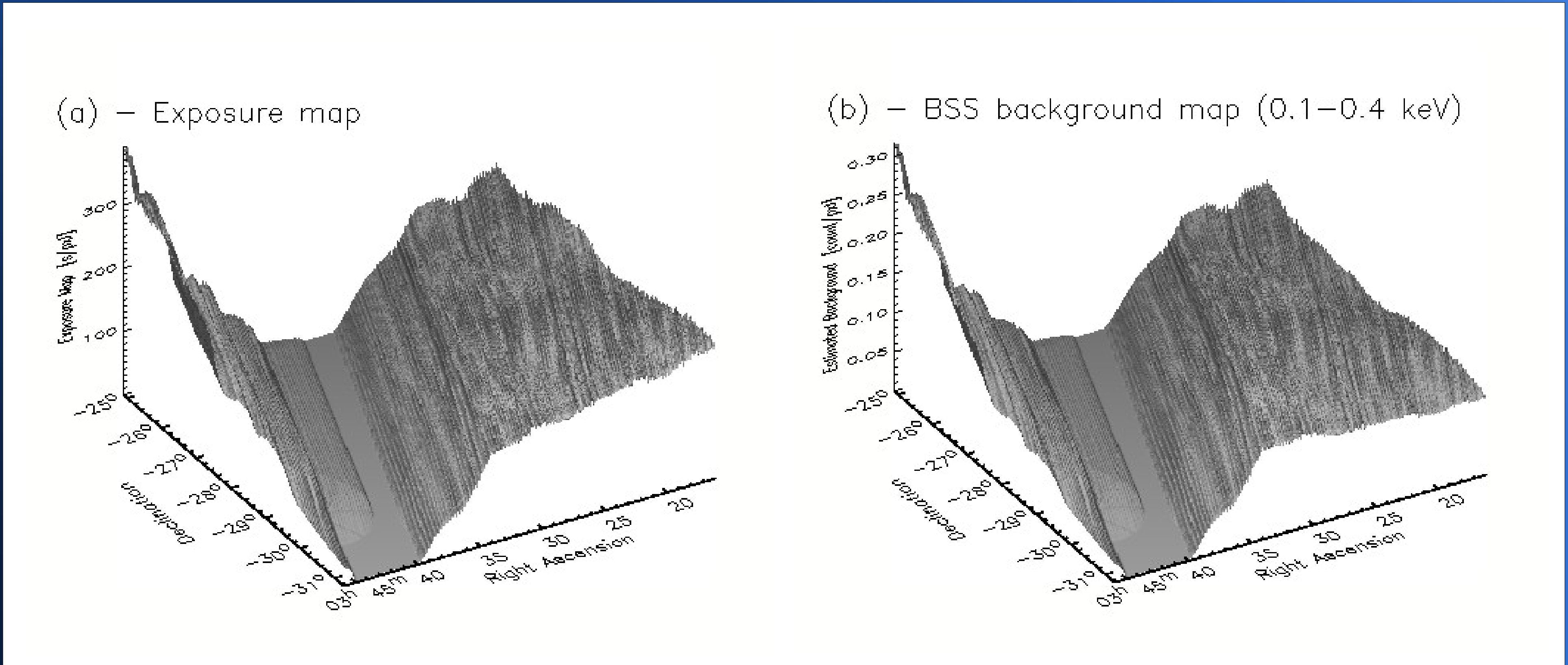


BSS technique: $F_X([0.5-2.0]keV)=(3.176\pm0.009)\times10^{-11}\,erg/s/cm^2$

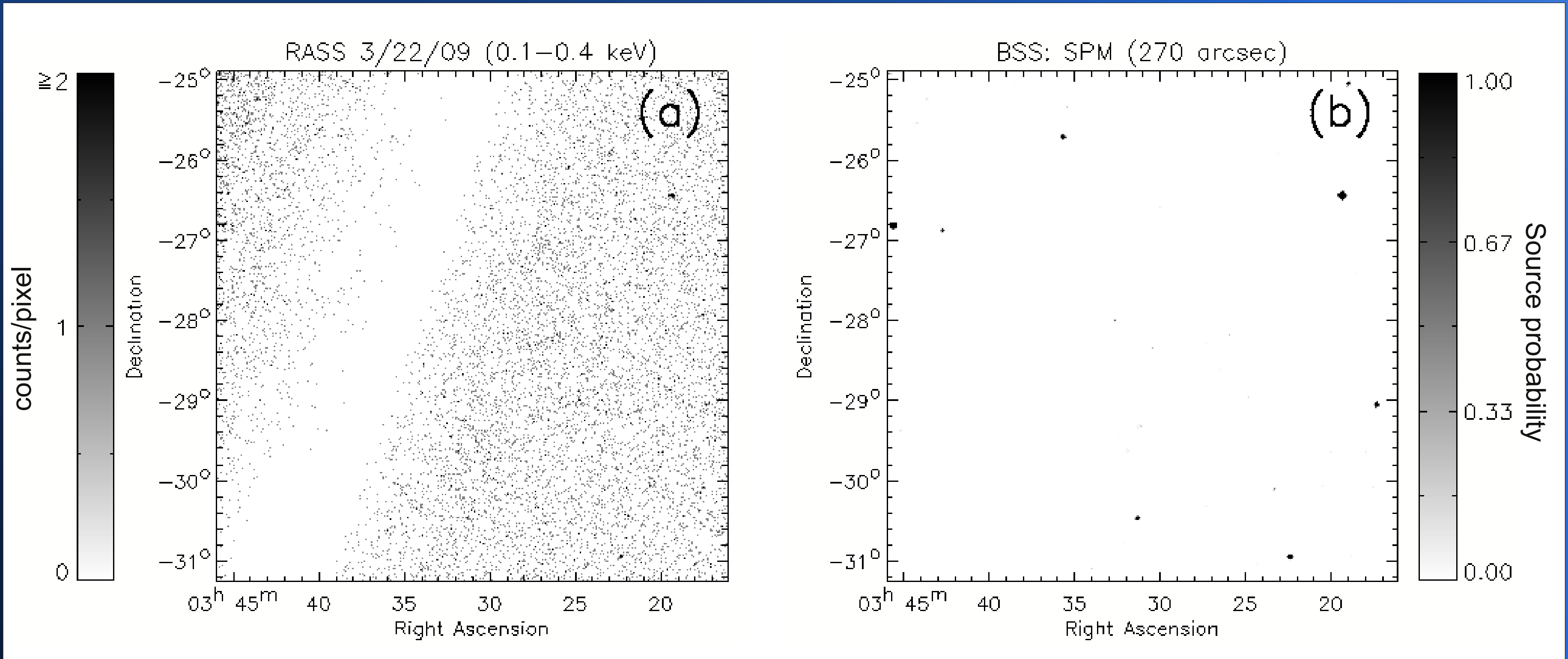
XMM-Newton (2008): $F_X([0.5-2.0]keV)=(3.285\pm0.004)\times10^{-11}\,erg/s/cm^2$

~3% difference < 5% expected divergence

RASS: varying exposure

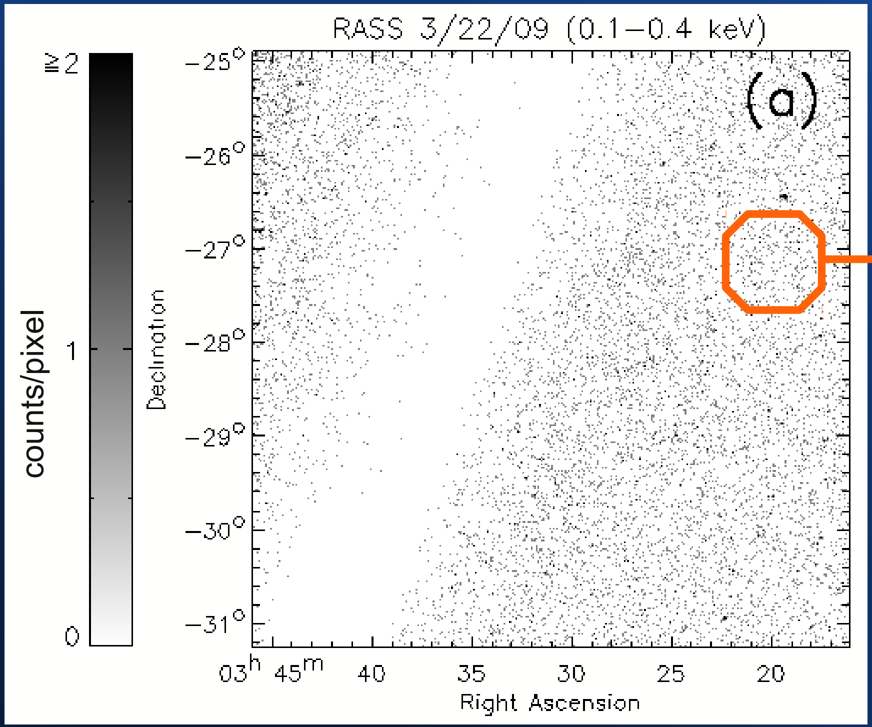


RASS: varying exposure

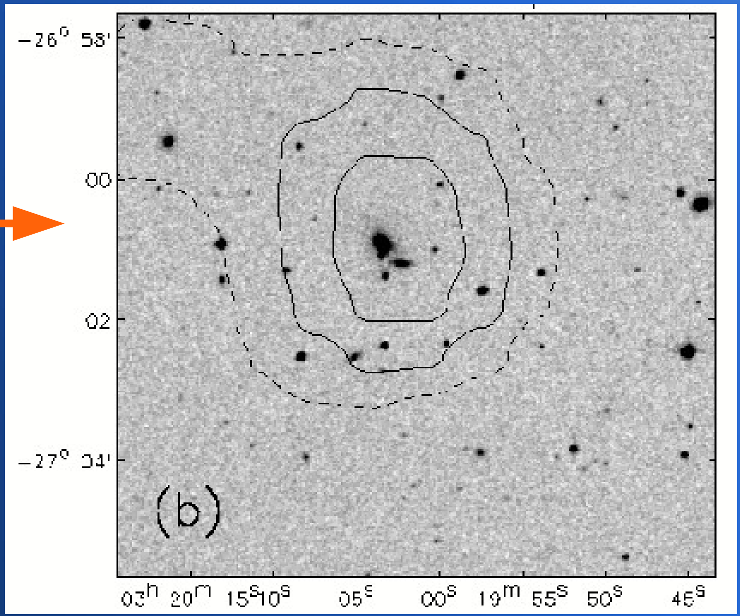


(0-9) counts/pixel

RASS: varying exposure

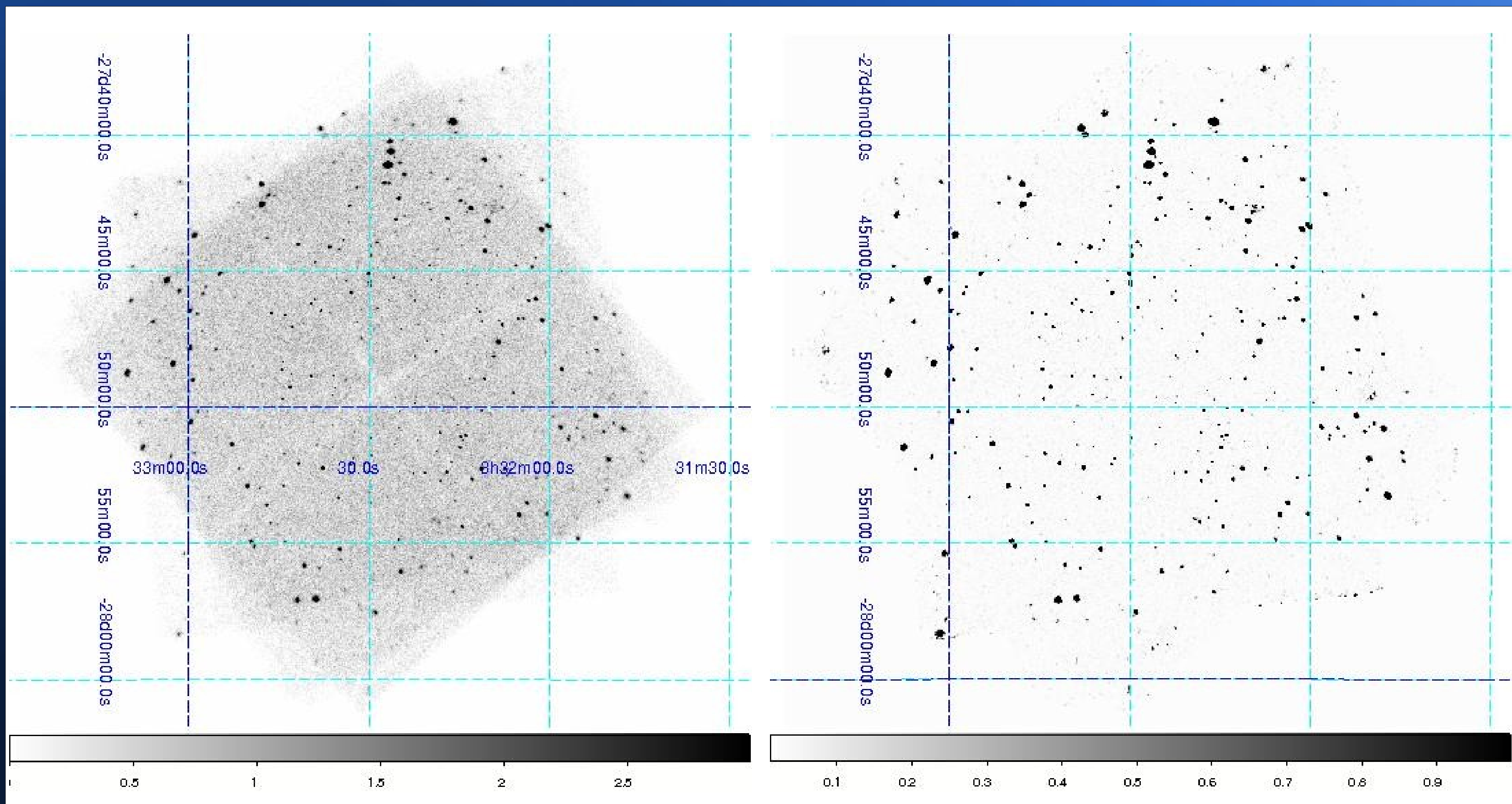


(0-9) counts/pixel



ACO S 340 *(Abell et al, 1989)*
z=0.068 (De Propris et al., 2002)

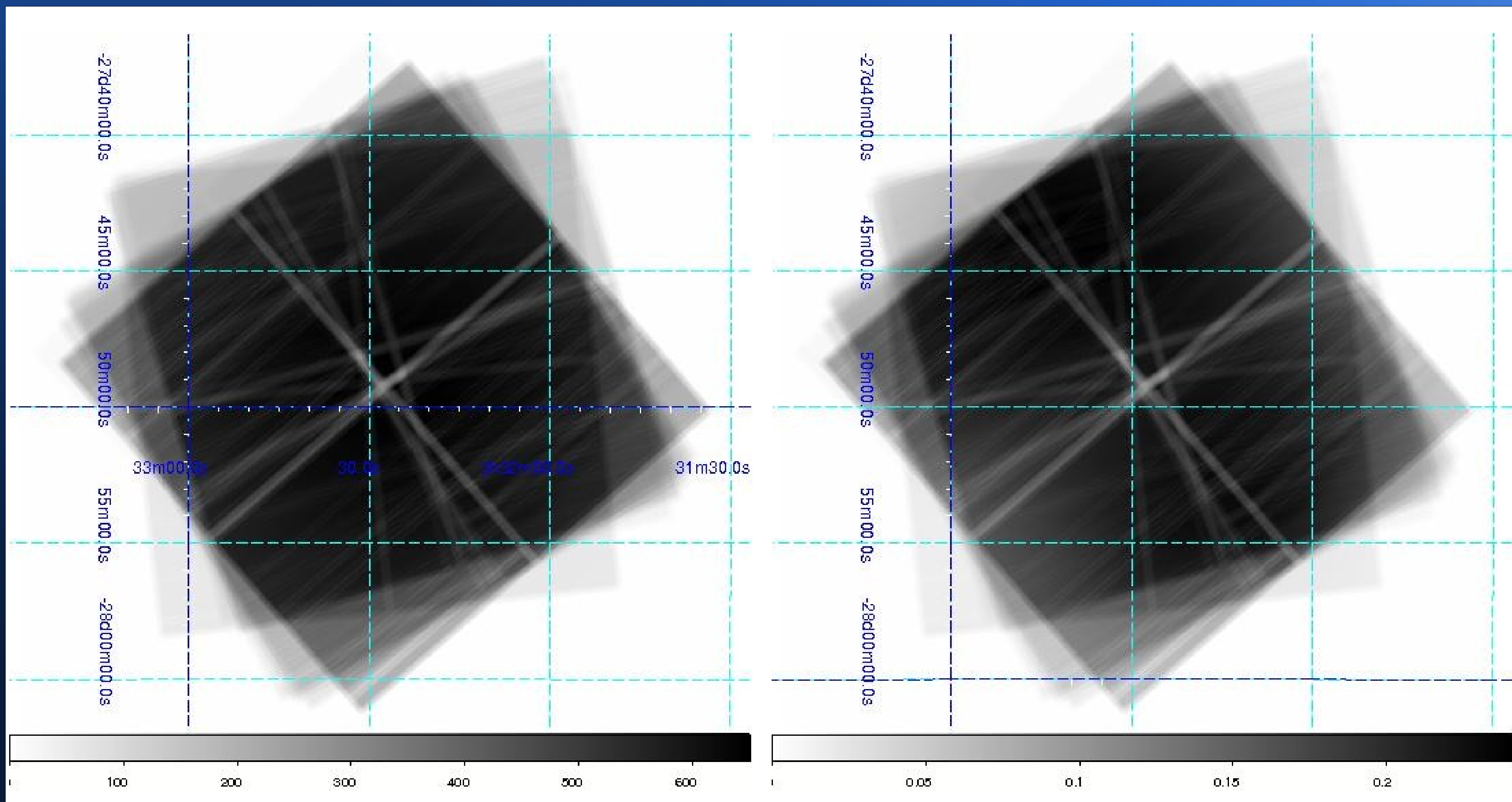
CDF-S



Photon counts/pixel (smoothed)

SPM (3 arcsec)

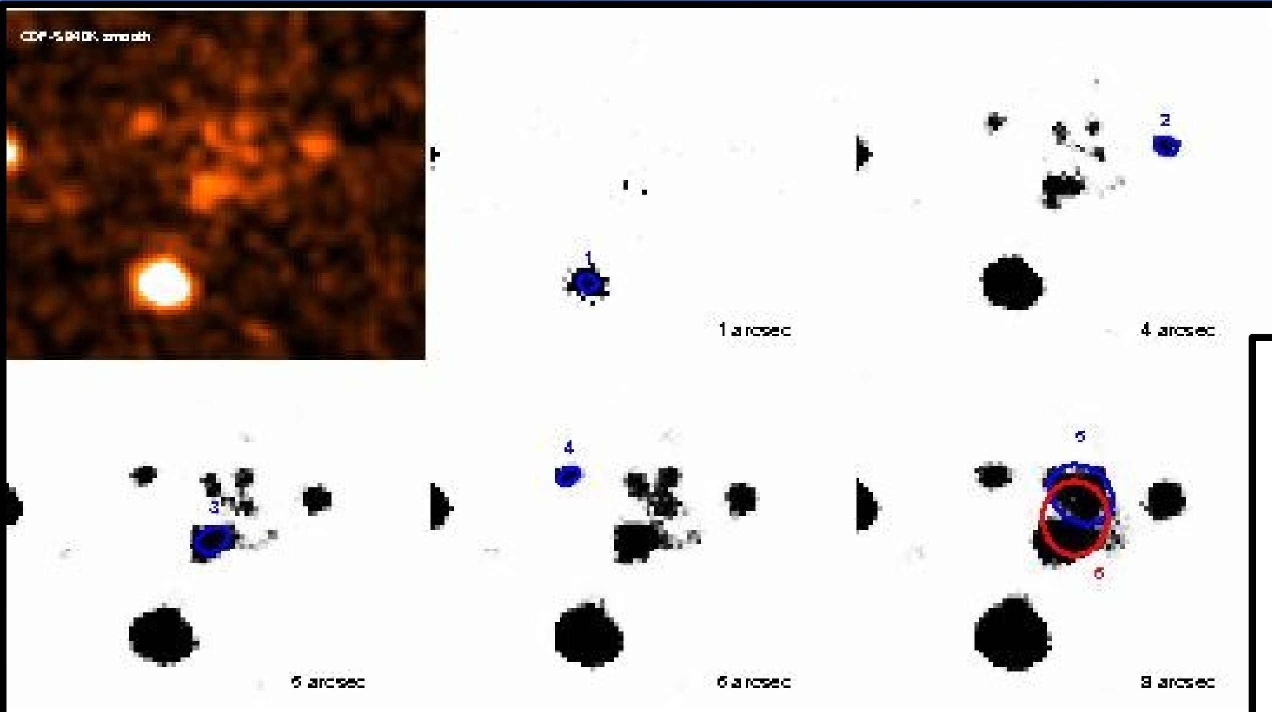
CDF-S



Ms/pixel

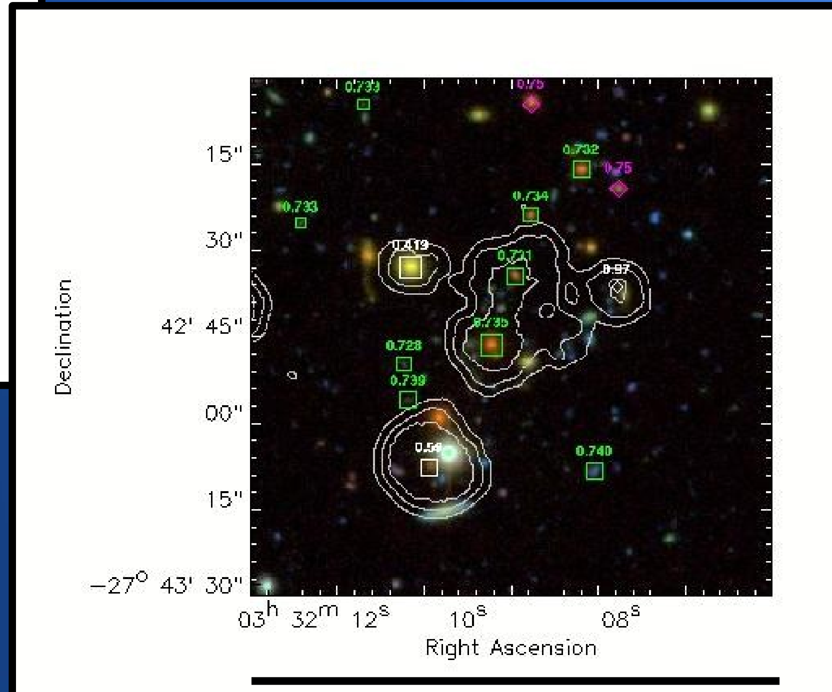
Background counts/pixel

CDF-S: XID 594, z~0.735



Color composite image (B,R,I) - WFI telescope
Superposed: X-ray contours (SPM, 20")

Cluster of galaxy with cD galaxy (#3) not located at the centre of the cluster and contaminated by surrounding galaxies



1.5 arcmin [638 kpc]

Summary & Conclusions

- Analysis of Poisson images is awkward because of:
  - Few counts per pixel, Poisson noise, instrumental complexities, large variety of source morphologies
- BPT supplies a general and consistent frame for logical inference
- BPT combined with a probabilistic mixture model allows one to gain insight into the coexistance of background and sources
- The BSS technique:
  - Provide detection of both point-like and extended soucres
  - Is capable to automatically separate point-like from diffuse emission
  - Is capable to detect sources independently to their morphology also features as filaments
- The BSS method is currently under a feasibility study for being applied to eROSITA mission, with the goals to provide important insight for the quests of: Dark matter, Dark Energy and distribution of matter in the Universe